



The Impact of the Angle-average Approximation in the Partial Frequency Redistribution Modeling of the Mg II h–k Doublet Stokes Profiles

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Abstract

In recent years, the Mg II h and k lines have become increasingly important for the diagnostics of magnetic fields in the solar chromosphere, motivated both by the advances in the modeling capabilities of their polarization and by the unprecedented spectropolarimetric observations provided by the CLASP missions. In this paper, we investigate the impact of the angular and frequency coupling in coherent scattering processes in the radiative transfer modeling of the Mg II doublet polarization profiles. In particular, we aim to assess how the widely used angle-average approximation affects the predicted polarization. By solving the radiative transfer problem in 1D semiempirical atmospheric models, we find that this approximation is suitable in the absence of magnetic fields, but there can be measurable differences in magnetized models, mainly in the linear polarization profiles close to the core of the Mg II k line. We then test the suitability of the approximation in more realistic scenarios, solving the radiative transfer problem pixel-by-pixel in a 3D atmospheric model resulting from a magnetohydrodynamic simulation and mimicking the degradation of the CLASP2 instrument, as well as in several 1D atmospheric models resulting from the Stokes inversion of the CLASP2 data. We find that the impact of the angle-average approximation is greatly diminished at the resolution of the CLASP2 observations. We also find that to suitably include angle-dependent effects in 1D radiative transfer modeling, it is sufficient to initialize the problem with the angle-average solution and then perform a few (angle-dependent) iterations, dramatically reducing the computational cost.

Unified Astronomy Thesaurus concepts: Spectropolarimetry (1973); Radiative transfer (1335); Solar chromosphere (1479)

1. Introduction

A number of the currently open questions in solar physics, such as the sudden energy release in solar flares and the heating of both the solar chromosphere and corona, are closely related to the structure and evolution of the magnetic field in the solar chromosphere. The spectral line polarization carries information about the magnetic field in the region of the atmosphere where the photons were last scattered or emitted. Most of the spectral lines forming in the upper chromospheric layers are found in the ultraviolet (UV) region of the spectrum. For this reason, some of the strongest UV resonance lines, such as the Mg II h and k lines, have been receiving growing attention for magnetic field diagnostics during the last years, from both the theoretical and observational points of view (see the review by J. Trujillo Bueno & T. del Pino Alemán 2022).

Acquiring spectropolarimetric observations in UV lines is challenging. These lines are mostly inaccessible from ground-based facilities due to the absorption of the Earth's atmosphere.⁷

⁷ The Ca II H–K doublet in the near-UV is an exception to this “rule,” being observable from ground-based facilities. However, most telescopes and instruments are not optimized for such blue wavelengths, resulting in very poor throughput and signal-to-noise in polarization.

The first spectropolarimetric observations of the Mg II h and k lines came from the Ultraviolet Spectrometer and Polarimeter (J. Calvert et al. 1979; B. E. Woodgate et al. 1980) aboard the Solar Maximum Mission (J. D. Bohlin et al. 1980). This mission provided the first evidence of significant scattering polarization in the far wings of these lines (W. Henze & J. O. Stenflo 1987; R. Manso Sainz et al. 2019). Later, in 2013, the Interface Region Imaging Spectrograph (see B. De Pontieu et al. 2014) was launched by NASA and still continues to provide excellent observations in the UV solar spectrum, including the Mg II h and k lines, albeit without polarization. More recently, the Chromospheric LAYer SpectroPolarimeter (CLASP2; D. Song et al. 2022) and its reflight (CLASP2.1) provided unprecedented spectropolarimetric observations of the spectral region around the Mg II h and k lines. Launched on 2019 April 11, CLASP2 observed two targets in the solar atmosphere, a quiet-Sun region with the slit crossing the solar limb, and a position closer to the disk center with the slit crossing a plage region. The analysis of these observations confirmed some theoretical predictions on the polarization in these spectral lines (L. A. Rachmeler et al. 2022) and allowed for the first mapping of the longitudinal component of the magnetic field from the photosphere to the upper chromosphere (R. Ishikawa et al. 2021; H. Li et al. 2023). Successfully launched on 2021 October 8, CLASP2.1 acquired 2D spectropolarimetric data (16 slit positions) in an active region, whose inversion provided information on the thermal, dynamic, and magnetic structure of its atmosphere (H. Li et al. 2024).



Modeling the polarization signals of the Mg II h and k lines presents severe challenges, especially from the computational point of view. The relatively low plasma density in their region of formation, the upper chromosphere, has several consequences in the generation of their spectral line polarization. First, they form far from local thermodynamic equilibrium (hereafter, NLTE; e.g., D. Mihalas 1978). Second, the relatively high degree of radiation anisotropy induces a substantial degree of atomic level polarization (population imbalances and coherence among the magnetic sublevels), which in turn produces scattering polarization (J. Trujillo Bueno 2001; E. Landi Degl’Innocenti & M. Landolfi 2004). Third, the frequency correlation between absorbed and reemitted photons in scattering events gives rise to significant partial frequency redistribution (PRD) effects (e.g., R. W. Milkey & D. Mihalas 1974; H. Uitenbroek 1997).

The formation of the intensity spectrum of the Mg II h and k lines has been studied in depth, including the impact of multidimensional radiation transfer together with PRD effects (e.g., J. Leenaarts et al. 2013a, 2013b; A. V. Sukhorukov & J. Leenaarts 2017). Regarding their polarization, it has been demonstrated that PRD effects and quantum interference between the upper levels of both lines (hereafter J -state interference) play a key role in shaping their polarization profiles (L. Belluzzi & J. Trujillo Bueno 2012; E. Alsina Ballester et al. 2016, 2022; T. del Pino Alemán et al. 2016; T. del Pino Alemán et al. 2020). Most of the abovementioned studies took advantage of the so-called angle-average (AA) approximation for PRD. In this approximation, the angular and frequency parts of the redistribution function are decoupled by (angular-)averaging the radiation field before introducing it in the (angular-)averaged redistribution expressions (e.g., D. Mihalas 1978; L. Belluzzi & J. Trujillo Bueno 2014). This significantly reduces the dimensionality of the redistribution integral and, in turn, the amount of redistribution functions to compute. Considering the unpolarized case, J. Leenaarts et al. (2012) generalized the AA approximation to the dynamic case, that is, to the presence of plasma bulk velocity motions. Their approximation can, in turn, be generalized to the polarized case. While the AA approximation gives highly satisfactory results when applied to the modeling of the intensity spectrum (see J. Leenaarts et al. 2012; A. V. Sukhorukov & J. Leenaarts 2017), this is not, in general, the case for the polarization profiles. Considering an academic setting, M. Sampooran et al. (2017) showed that the AA approximation can significantly impact the linear polarization profiles in the spectral region closer to the line core. Focusing on the Ca I line at 422.7 nm, G. Janett et al. (2021) showed that the AA approximation produces an artificial trough in the line-core peak of the linear polarization profiles, while L. Belluzzi et al. (2024) highlighted the inaccuracies that it can introduce in the polarization signals produced by the forward-scattering Hanle effect. The impact of this approximation is even more conspicuous for the Na I D₁ line at 589.6 nm, leading to a completely different shape of the linear polarization profiles (G. Janett et al. 2023). By contrast, F. Riva et al. (2024) found that the AA approximation can be suitably applied to model the Stokes profiles of the He II Ly α at 30.4 nm.

The complexity of including angle-dependent (AD) PRD effects in the modeling of the Mg II doublet is even more critical when performing spectropolarimetric inversions, which require a significant amount of spectral synthesis in order to minimize a certain merit function. An inversion that accurately

accounts for AD PRD is nowadays prohibitive in terms of computing time (e.g., H. Li et al. 2022). Therefore, it is of interest to assess the suitability of the AA approximation in the modeling of the Mg II resonant doublet.

In this paper, we study the impact of assuming the AA approximation in the modeling of the scattering polarization of the Mg II h and k lines. In Section 2 we briefly describe the atomic and atmospheric models, as well as the radiative transfer code used in this work. In Section 3 we show the results of this analysis in semiempirical 1D model atmospheres (Section 3.1), in a 3D magnetohydrodynamic (MHD) model (using the 1.5D approximation) while mimicking the instrumental degradation of CLASP2 (Section 3.2), and in 1D models resulting from spectropolarimetric inversions of the CLASP2 data (Section 3.3). Finally, we present our main conclusions in Section 4.

2. Modeling Details

In this section, we describe the necessary actors in the modeling of the spectral line polarization of the Mg II h and k lines, namely the atomic and atmospheric models and the method of solution.

2.1. Atomic and Atmospheric Models

We use the same atomic model for the magnesium atom as in T. del Pino Alemán et al. (2020), but removing the 2D levels (see their Figure 1), that is, we include the ground and first excited terms of Mg II and the ground level of Mg III. The ground term has a single atomic level, $^2S_{1/2}$, and the upper term has the upper levels of the Mg II h and k transitions, $^2P_{1/2}^\circ$ and $^2P_{3/2}^\circ$, respectively. Note that both the upper and lower levels of the Mg II h transition have angular momenta $J = 1/2$ and thus they cannot carry atomic alignment (see, e.g., E. Landi Degl’Innocenti & M. Landolfi 2004). J. Leenaarts et al. (2013a) showed that this model atom does an excellent job when solving the radiative transfer (RT) problem for unpolarized radiation in an atmospheric model such as the C model of J. M. Fontenla et al. (1993), but does not perform as well when using a column of a 3D model atmosphere resulting from an MHD simulation, where one can incur in a significant error in the emergent intensity. Our goal being the assessment of the AA approximation, we thus opted for the smallest reasonable atomic model to drastically reduce the computation time. This is indeed crucial when considering hundreds of atmospheric models.

We first solve the RT problem for polarized radiation in a series of 1D model atmospheres. The height scale, temperature, microturbulent velocity, and electron and hydrogen number densities are those corresponding to the C model of J. M. Fontenla et al. (1993; hereafter FAL-C). Using FAL-C as a base, we build two additional atmospheric models. The first variation (hereafter FAL-C_v) is built by interpolating, in continuum (at 500 nm) optical depth, the vertical velocity component from one of the snapshots of the time series of the hydrodynamical simulation by M. Carlsson & R. F. Stein (1997). We chose a snapshot (the 100th) with a velocity gradient which produces a significant “distortion” of the emergent intensity profile with the idea of having an extreme scenario of velocity gradient. The second variation (hereafter FAL-C_v⁺) adds a horizontal velocity component, which is 0.7 times the vertical component with a height-dependent azimuth, as shown in Figure 1. Note that the resulting atmospheric models are not realistic, but they are

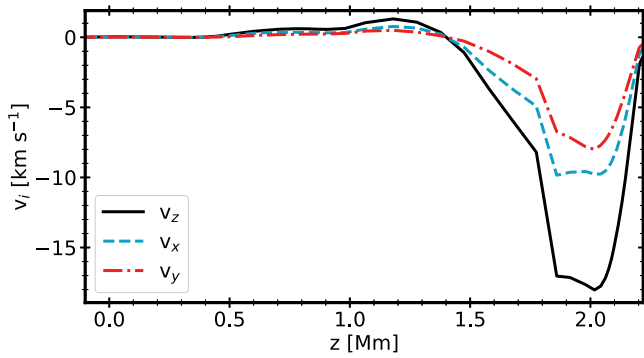


Figure 1. Velocity field component along the vertical z -axis (solid black curve), along the x -axis (azimuth zero, blue dashed curve), and along the y -axis (red dashed-dotted curve) for the 1D models considered in this work. Our first 1D model has no velocity, the second has only the v_z component, and the third has all of them. The v_z stratification results from the interpolation, in the τ_{500} scale, of the velocity in the 100th snapshot of the time series by M. Carlsson & R. F. Stein (1997) with a particularly strong gradient. The horizontal component is a factor 0.7 of the vertical component, with an azimuth that changes with height.

suited to assess the AA approximation when modeling the polarization of the Mg II h and k lines in the presence of bulk velocity gradients. These bulk velocity gradients have a significant impact on the emergent scattering polarization, enhancing the anisotropy (e.g., E. S. Carlin et al. 2012) and contributing to the lack of axial symmetry (e.g., T. del Pino Alemán et al. 2018). For each of these three atmospheric models, we solve the RT problem for a variety of height-independent magnetic fields, with strengths in the range $B \in [0.1 \text{ G}, 5000 \text{ G}]$ (in logarithmic steps) and inclinations with respect to the vertical direction $\theta_B \in \{0^\circ, 10^\circ, 30^\circ, 60^\circ, 90^\circ\}$, always at azimuth $\chi_B = 0^\circ$. For each of the 1176 resulting model atmospheres, we solve the problem considering PRD effects both in their general AD formulation and exploiting the AA approximation. The idea is to cover as best as possible the relevant space of parameters in terms of the magnetic field to ensure the generality of our conclusions. Considering only one azimuth for the magnetic field does not deduct in generality, as we calculate the emergent profiles for several azimuths of the line of sight (LOS; see Section 3.1).

We then perform the pixel-by-pixel solution in a 3D MHD model of an enhanced network region (M. Carlsson et al. 2016). By degrading the synthetic profiles mimicking the CLASP2 instrumentation we can determine what is the impact of the AD PRD effects given the spatial, temporal, and spectral resolution of the CLASP2 and CLASP2.1 spectropolarimetric observations of the Mg II h and k lines. Due to the significant computational cost, we have selected a small square region around the center of the field of view, i.e., a 1.71^2 Mm^2 region centered at approximately (12, 12) Mm. We reduced the horizontal resolution to 143 km, resulting in a sample of 13×13 (169) LOSs. We kept the full height resolution, but we removed the region at the top of the model where the dominant species is Mg III (ionization fraction over 99.9%) and the region at the bottom below the height where the continuum optical depth at 500 nm is larger than ~ 1.8 (larger than 0.25 in logarithmic scale). The solution of the RT problem in individual columns of the 3D MHD model results in excessively narrow Stokes profiles (see, e.g., J. E. Vernazza et al. 1976). Consequently, we have added to each column the corresponding FAL-C microturbulent velocity stratification through an interpolation in optical depth at 500 nm.

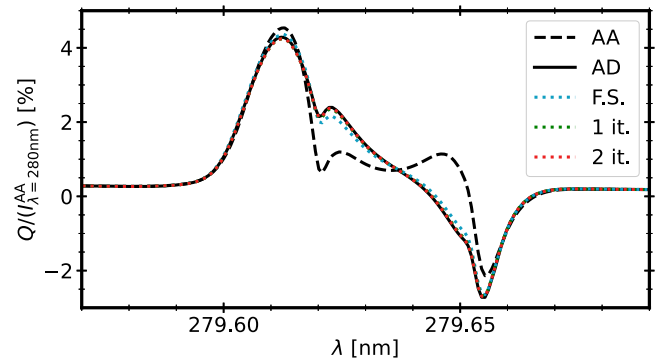


Figure 2. Linear polarization Q spectrum of the Mg II k line, normalized to the AA intensity at $\lambda = 280 \text{ nm}$, in FAL- C_{v_z} for a LOS with $\mu = 0.1$ and $\chi = 0^\circ$, with $|B| = 20 \text{ G}$ and $\theta_B = 60^\circ$. The black solid (dotted) curves show the Stokes parameters calculated with the AD (AA) redistribution. The colored dashed curves show the AD solution with three different simplifications described in the main text, with “F.S.” meaning formal solution and “it.” iteration. Note that the red and green dashed curves almost fully overlap and, in turn, almost coincide with the black solid curve, indicating that the emergent profile converges very quickly toward the full solution. In all figures, the reference direction for positive Q is parallel to the limb.

Finally, we solve the RT problem in the four models resulting from the inversion of spectropolarimetric profiles from the CLASP2 mission in H. Li et al. (2024).

2.2. Method of Solution

We solve the RT problem for polarized radiation with the synthesis modules of the HanleRT-TIC⁸ code, which can account for atomic and scattering polarization, quantum interference, and PRD effects (both in the general AD case and in the AA approximation). The code and the implemented theoretical approach are described in T. del Pino Alemán et al. (2016) and T. del Pino Alemán et al. (2020). The solution of the full AD problem is very computationally demanding, especially when solving the problem in the many columns extracted from the 3D MHD model, with relatively fine spacing between spatial grid points and a relatively complex stratification of the velocity and magnetic field vectors.

For the sake of computational efficiency, we also explored the possibility of accelerating the AD calculations by exploiting the previously computed AA solution as an initial guess and then performing only one or two (AD) iterations.⁹ This approach to solving the RT problem with AD redistribution was applied by K. N. Nagendra et al. (2002) to solve this problem for the first time in an isothermal and homogeneous slab for an academic two-level atom. In Figure 2 we show the emergent Stokes Q profile of the Mg II k line resulting from different solution strategies for the FAL- C_{v_z} model with a magnetic field with $|B| = 20 \text{ G}$ and $\theta_B = 60^\circ$, and a LOS with $\mu = 0.1$ and $\chi = 0^\circ$. The Q profiles of the model chosen for this figure show significant differences between the AA (dotted curve) and AD (solid curves) solutions, which will be studied in detail in Section 3.1. By simply taking the solution (density matrix elements and Stokes parameters at each frequency, height, and propagation direction) from the AA problem and

⁸ The code is publicly available at <https://gitlab.com/TdPA/hanlert-tic>.

⁹ An iteration consists of the solution of the radiative transfer equations in every direction using the radiation field and density matrix from the last iteration, followed by the solution of the statistical equilibrium equations using the new radiation field.

performing the last formal solution¹⁰ for the desired LOS with the AD redistribution, we get the dashed blue curve. While there are some evident differences with the full AD solution at the plot level, the fact that both profiles are so close indicates that the self-consistent solution itself does not change excessively when considering the AA approximation. The green curve shows the emergent linear polarization when performing a single iteration with the AD redistribution starting from the solution of the problem with the AA approximation. The agreement significantly improves, meaning that the radiation field quickly relaxes to that of the full AD solution. Finally, the red curve shows the result of performing a second AD iteration, which is in good agreement with the full solution. This result of the behavior of the AD problem is very promising, as it could open the possibility of also including AD redistribution effects when solving the inversion problem. The results shown in Sections 3.2 and 3.3 for the AD solution are calculated by performing just two iterations exploiting the AA solution as the initial guess.

3. Results

In this section, we summarize the results of our RT calculations in the atmospheric models described in Section 2.1. We first study the simpler 1D models, providing an insight into the general impact of the AA approximation. We then study the (pixel-by-pixel) solution in the 3D model atmosphere resulting from an MHD simulation, focusing on how the instrumental degradation (to a spatial, temporal, and spectral resolution similar to that of CLASP2) affects the difference between AA and AD calculations. Finally, we show the comparison between the AA and AD calculations for model atmospheres resulting from the inversion of spectropolarimetric profiles observed by CLASP2 to evaluate what impact the AA approximation could have in the inferred model atmospheres.

3.1. The Impact of the Angle-average Approximation in 1D Models

As mentioned in Section 2.1, we have solved the RT problem for polarized radiation in a total of 1176 1D model atmospheres presenting three varieties of velocity vector fields and a number of height-independent magnetic fields with different strengths and inclinations. For each of these models, we synthesize the emerging Stokes spectra for 10 inclinations, with $\mu = \cos(\theta) \in \{0.1, 0.2, \dots, 1.0\}$ being θ the heliocentric angle, and with 13 azimuths $\chi \in \{0^\circ, 15^\circ, \dots, 180^\circ\}$, resulting in a total of 130 directions. This large amount of settings guarantees a very wide and properly sampled space of parameters, to avoid drawing conclusions from potential particular cases. However, this amount of synthesis results in 10^5 Stokes profiles. In order to study these results, we define the merit function,

$$\chi^2(j, \Omega_k) = \sum_{i=1}^{N_\nu} \frac{\gamma |S_j^{\text{AA}}(\nu_i, \Omega_k) - S_j^{\text{AD}}(\nu_i, \Omega_k)|}{\max[S_j^{\text{AA}}(\nu_i, \Omega_k), S_j^{\text{AD}}(\nu_i, \Omega_k)]}, \quad (1)$$

with $S_j(\nu_i, \Omega_k)$ the Stokes parameters ($I(\nu_i, \Omega_k)$, $Q(\nu_i, \Omega_k)$, $U(\nu_i, \Omega_k)$, and $V(\nu_i, \Omega_k)$ for $j = 0, 1, 2$, and 3 , respectively), for each LOS Ω_k and each of the N_ν frequencies ν_i . The super-indexes distinguish between the solutions with the AA approximation

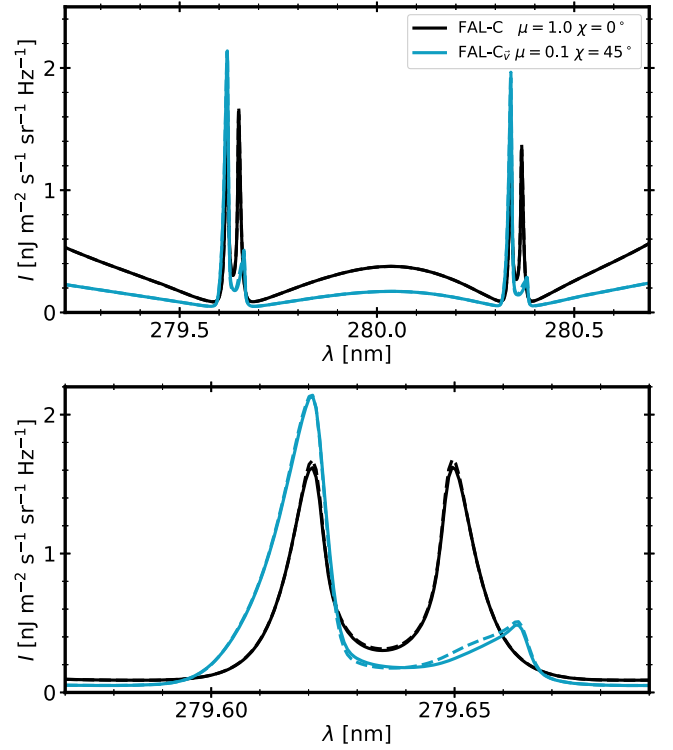


Figure 3. Intensity spectrum in FAL-C for a LOS with $\mu = 1.0$ (black curves), and in FAL-C_v for a LOS with $\mu = 0.1$, $\chi = 45^\circ$, and the magnetic field for which we get the largest $\chi^2(\nu_j, \Omega_k)$ in Equation (1) ($|B| = 2000$ G and $\theta_B = 10^\circ$; blue curves). The solid (dashed) curves show the Stokes parameters calculated with AD (AA) redistribution. The bottom panel shows a zoom around the k line in the top panel.

or with the general AD redistribution. The quantity γ is a constant that is zero when the polarization is small, that is, below 10^{-2} of the minimum intensity at the same LOS. We use the resulting $\chi^2(j, \Omega_k)$ to decide which Stokes profiles to check and show in detail. Note that this merit function is a test on the magnetic field for each of the combinations of indexes “ j ” and “ k ”.

For the intensity I , the resulting $\chi^2(0, \Omega_k)$ is typically 2 orders of magnitude smaller (and its maximum at least 1 order of magnitude smaller) than for the linear polarization. In Figure 3 we compare the AA and AD solutions for the intensity profile for two cases: (a) FAL-C model without a magnetic field at disk center ($\mu = 1.0$), which showcases the typical profile in a semiempirical model, and (b) FAL-C_v model with $|B|$ and $\theta_B = 10^\circ$ for a LOS with $\mu = 0.1$ and $\chi = 45^\circ$, which gives the maximum value for $\chi^2(0, \Omega_k)$. As already shown by A. V. Sukhorukov & J. Leenaarts (2017), the AA approximation is very suitable for modeling the intensity profiles of the Mg II doublet. This result also corroborates the effectiveness of the unpolarized AA preconditioning technique for the AD modeling of the Mg II doublet proposed by G. Janett et al. (2024), which reduces the number of iterations to convergence by a factor 40.

For the circular polarization V , the larger values of $\chi^2(3, \Omega_k)$ are typically found for those combinations of magnetic field and LOS that minimize the V signal. For a given magnetic field strength and LOS, the absolute error (the numerator in Equation (1) without the γ factor) tends to vary less than the

¹⁰ The formal solution consists of solving the radiative transfer equations for a specific propagation direction for a given radiation field and density matrix.

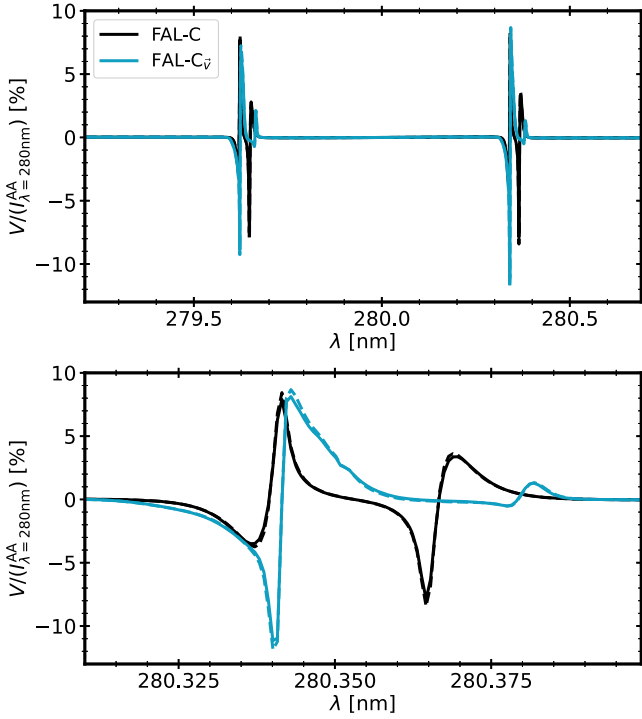


Figure 4. Circular polarization V spectrum, normalized to the AA intensity at $\lambda = 280$ nm, in FAL-C (black curves) and FAL-C $_{\nu}$ (blue curves) for a LOS with $\mu = 1.0$ and $\chi = 0^\circ$, and a vertical magnetic field of 200 G. The solid (dashed) curves show the Stokes parameters calculated with the AD (AA) redistribution. The bottom panel shows a zoom around the h line in the top panel.

signal itself for different magnetic field inclinations and, therefore, the relative error is enhanced for the less favorable magnetic field inclinations (i.e., those that minimize its longitudinal component). It is important to point out that these differences (and the smallest signals) are of the order of the noise level in state-of-the-art polarimetric observations (in general, they are clearly below the noise level for the most recent observations at this wavelength). Figure 4 compares AD and AA calculations at $\mu = 1.0$ in two FAL-C models in the presence of a vertical magnetic field of $|B| = 200$ G, which is representative of the typical differences between the AA and AD solutions for circular polarization signals above the noise level. In conclusion, we find that the AA approximation is suitable for modeling the circular polarization if the polarization signal is large enough. When the circular polarization signal is small, the relative difference between AD and AA calculations is enhanced. However, such small polarization signals are below the polarimetric accuracy available nowadays and in the near future at these wavelengths.

For the linear polarization, the situation is more complex and it is thus necessary to explicitly distinguish the cases with and without magnetic field. We first focus our attention on the nonmagnetized and static case presented in Figure 5. The AA approximation seems to do an excellent job in the whole spectrum except for the k line-center region, where it introduces two small “peaks” at either side of the line center. This behavior is similar to that found in the Ca I line at 422.7 nm by G. Janett et al. (2021), although for that line, the difference in polarization at exactly the line center was significantly larger, likely due to the different location and

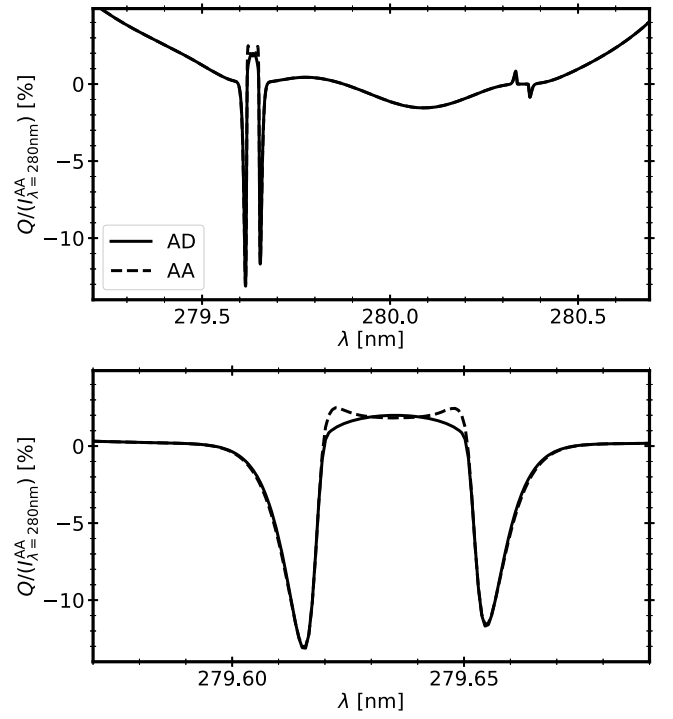


Figure 5. Linear polarization Q spectrum, normalized to the AA intensity at $\lambda = 280$ nm, in FAL-C for a LOS with $\mu = 0.1$ and $\chi = 0^\circ$, with $|B| = 0$ G. The solid (dashed) curves show the Stokes parameters calculated with the AD (AA) redistribution. The bottom panel shows a zoom around the k line in the left panel.

extension of its formation region. Note that this behavior of the linear polarization Q is not seen when, instead of Q , one represents the fractional linear polarization Q/I , as the particular shape of the intensity profile hides such peaks when taking the ratio (e.g., E. Alsina Ballester et al. 2016; T. del Pino Alemán et al. 2016). Overall, the AA approximation does a relatively good job for the nonmagnetized and static scenarios.

In the presence of a velocity field gradient, the position in wavelength of the line core changes differentially with height. It is thus expected that the error of the AA approximation (which is significantly close to the line center) contaminates the nearby wings, as shown in Figure 6. Because the differences between the AA and AD solutions are mostly limited to the k line, we only include its spectral range in the figure. We note that the velocity gradients in these models are significant (see Figure 1), which favors the impact of the AA approximation on the wings in these calculations, due to the Doppler shift of the line core.

For the magnetized and static models, the behavior of the AA approximation is similar to that of the nonmagnetic case in most of our parameter space of magnetic fields and LOSs. However, the largest $\chi^2(1, \Omega_k)$ or $\chi^2(2, \Omega_k)$ reveal some particular combinations, such as those shown in Figure 7. In those cases, we can see that for magnetic fields close to the Hanle critical field (~ 20 G) or in the Zeeman regime (several kilogauss), the behavior of the AA approximation is still similar to the nonmagnetic case. However, in the regime in between (tens and hundreds of Gauss) the shape of the Q profile can be altered in such a way that the sign of the polarization changes (see green curves in Figure 7). Another particular case that is worth pointing out is that corresponding to a vertical magnetic field. The complete frequency

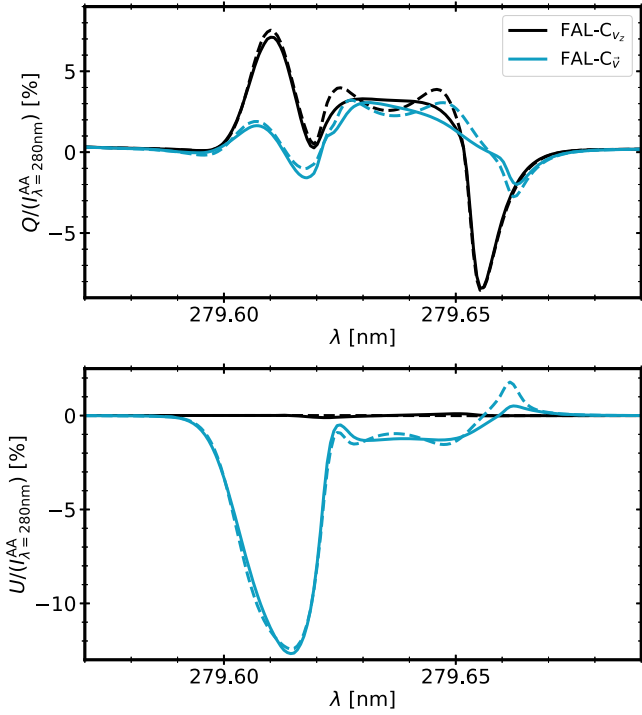


Figure 6. Linear polarization Q (top panel) and U (bottom panel) spectrum, around the Mg II k line, normalized to the AA intensity at $\lambda = 280$ nm, in FAL- C_{v_z} (black curves) and in FAL- $C_{\vec{v}}$ (blue curves) for a LOS with $\mu = 0.1$ and $\chi = 0^\circ$, with $|B| = 0$ G. The solid (dashed) curves show the Stokes parameters calculated with the AD (AA) redistribution. Note that the FAL- C_{v_z} profile does not present any U signal for the AA redistribution due to the axial symmetry.

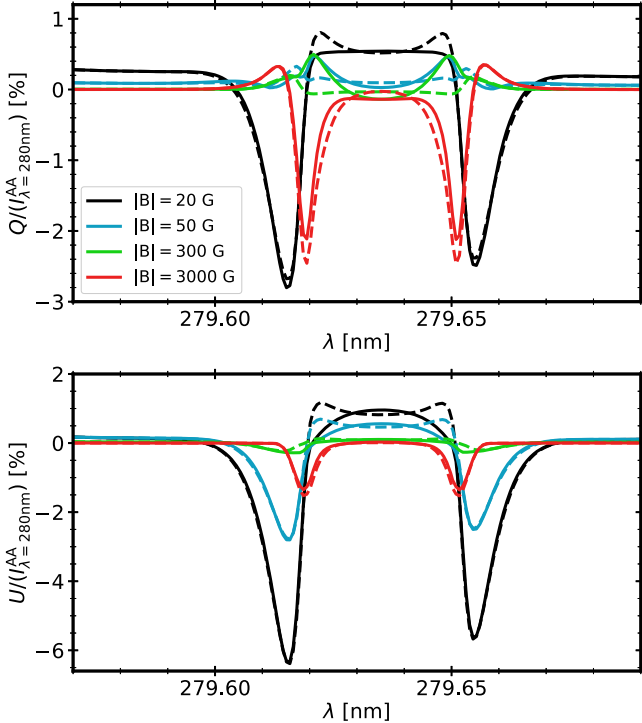


Figure 7. Linear polarization Q (top panel) and U (bottom panel) spectrum, around the Mg II k line, normalized to the AA intensity at $\lambda = 280$ nm, in FAL-C for a LOS with $\mu = 0.1$ and $\chi = 0^\circ$, with $|B| = 20$ (black curves), 50 (blue curves), 300 (green curves), and 3000 G (red curves) and $\theta_B = 60^\circ$. The solid (dashed) curves show the Stokes parameters calculated with the AD (AA) redistribution.

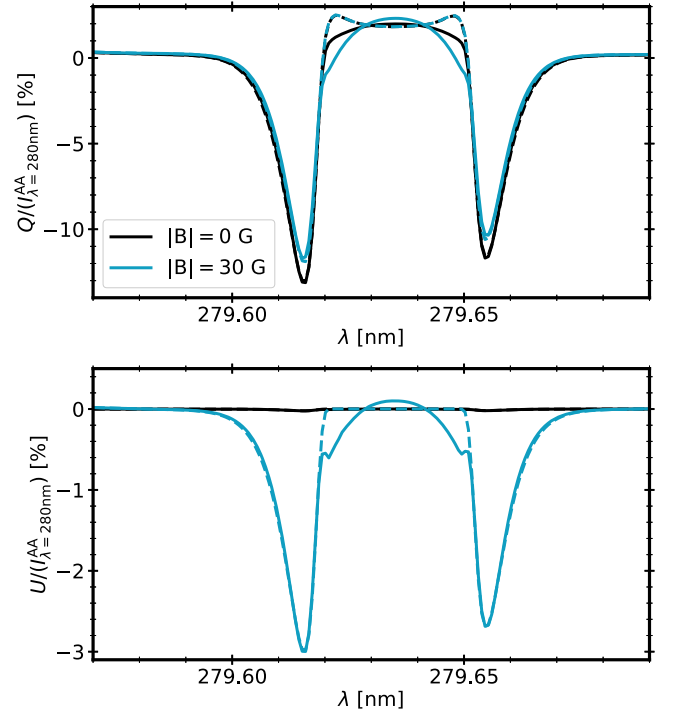


Figure 8. Linear polarization Q (top panel) and U (bottom panel) spectrum, around the Mg II k line, normalized to the AA intensity at $\lambda = 280$ nm, in FAL-C for a LOS with $\mu = 0.1$ and $\chi = 0^\circ$, for a vertical magnetic field with $|B| = 0$ (black curves) and 30 G (blue curves). The solid (dashed) curves show the Stokes parameters calculated with the AD (AA) redistribution.

redistribution theory of spectral line polarization (see E. Landi Degl’Innocenti & M. Landolfi 2004) shows that if there is a clear symmetry axis (the vertical in our physical scenario), the Hanle effect can only act on the polarization if the direction of the magnetic field is different from this symmetry axis. However, the general AD solution shows that the Hanle effect acts in the line core even in the presence of a vertical magnetic field, as shown in Figure 8, although the impact is rather small. By contrast, both Q and U signals remain unchanged in the line core (in particular $U=0$) for the AA solution. This peculiar behavior had also been found when solving the RT problem for an academic atom in a slab model accounting for AD redistribution (K. N. Nagendra et al. 2002; H. D. Supriya et al. 2013) and, as expected, it also takes place in a stratified model atmosphere. Finally, as in the nonmagnetic case, the inclusion of velocity gradients shifts the position of the line core differentially in height, propagating the error introduced by the AA approximation farther from the line core, or enhancing such a difference, as shown in Figure 2.

In summary, the AA approximation is suitable for the modeling of the intensity and circular polarization profiles of the Mg II h and k lines, but it can introduce significant errors in the linear polarization profile, especially in the presence of both velocity and magnetic fields (which is the case in the real Sun).

We note that, based on previous studies, the AA approximation seems to depart even more from the more general AD solution in spectral lines forming deeper in the solar atmosphere, such as the Ca I line at 422.7 nm (G. Janett et al. 2021) and the Na I D₁ line at 589.6 nm (G. Janett et al. 2023). In both cases, the AA approximation has a very significant impact even in 1D semiempirical atmospheric models without any magnetic field. There are also spectral

lines in which the impact of the AA approximation is even less pronounced than in the Mg II k line, such as the He II Ly α at 30.4 nm (F. Riva et al. 2024).

3.2. The Impact of the Angle-average Approximation with Limited Resolution

We then solved the RT problem, pixel-by-pixel, in the 3D model resulting from an MHD simulation described in Section 2.1, synthesizing the emergent Stokes profiles for the LOS with $\mu = 1.0$. We have then degraded the emerging spectra to mimic the CLASP2 observations. We have considered the following degradation contributions: (1) diffraction in the spectral dimension, modeled with a Gaussian spectral point-spread function (PSF) with FWHM = 110 mÅ (D. Song et al. 2018; T. Tsuzuki et al. 2020); (2) finite width of the spectrograph slit, modeled with the square PSF of ~ 0.98 in the direction perpendicular to the slit (“x”-direction; slit width of 7 μm ; D. Song et al. 2018); and (3) finite pixel size, modeled with a square PSF of 0.527 in the direction parallel to the slit (“y”-direction) and of 49.8 mÅ in the spectrum (CLASP2 plate-scale; L. A. Rachmeler et al. 2022). We have not considered the spatial convolution with the Airy disk because of the reduced horizontal extension of our selected region, as only the pixels close to the center of the field of view would be free of boundary effects.

The AA approximation being suitable for Stokes I and V (what we have also confirmed in the 3D model), we only focus on the linear polarization profiles. In Figure 9 we show the emergent linear polarization profiles for two pixels in the field of view, with and without accounting for instrumental degradation. We show the degraded profiles with the wavelength sampling of the CLASP2 data, with error bars corresponding to the typical photon noise in the quiet-Sun observations in CLASP2 (typically $\sigma/I \approx 10^{-3}$). The differences between the AA and AD results at the 3D MHD model’s resolution are even more evident than in the 1D case presented in Section 3.1. This is likely due either to the complex stratification of the magnetic field vector in individual columns of the 3D MHD model (while in Section 3.1 we only considered height-independent magnetic fields) or to the more intricate stratification of the velocity vector. However, Figure 9 shows that this striking difference is significantly reduced when accounting for instrumental degradation, especially when accounting for polarization noise. Although measurable differences between the AA and AD results in the peak amplitudes of the polarization profiles are still present, most of the details in the spectral structure are buried within the noise or are not captured due to the limited spectral sampling. At this level of resolution and noise, the largest differences are between 1σ and 2σ and the AA and AD solutions are close to compatible.

3.3. The Potential Impact of the Angle-average Approximation in 1D Inversions

Finally, we solved the RT problem for polarized radiation in the 1D model atmospheres resulting from full Stokes inversions of CLASP2 data performed with the HanleRT-TIC code (H. Li et al. 2024). Some of these inversions required the addition of ad hoc parameters, namely the radiation field tensor components characterizing the lack of axial symmetry due to horizontal RT effects. Since these ad hoc radiation field tensor

components cannot be considered in our AD calculations, they were neglected in this work (see Section 2.1). The emergent profiles shown below could thus differ from those shown in H. Li et al. (2024), but the comparison between the AA and AD solutions is still suitable to evaluate the impact of the AA approximation.

In Figure 10 we show the emergent linear polarization profiles for the four atmospheric models shown in Figures 2–5 of H. Li et al. (2024). These models correspond to the inversion of one pixel at the edge of a plage region, at $\mu = 0.712$ (Figure 2 in H. Li et al. 2024), and three pixels in the quiet Sun at $\mu = 0.563$, 0.101, and 0.417 (Figures 3, 4, and 5 in H. Li et al. 2024, respectively). We show both the syntheses with full spectral resolution (no spectral degradation and spectral sampling from the forward solver) and for the CLASP2 wavelength sampling after applying a Gaussian spectral PSF with FWHM = 110 mÅ (D. Song et al. 2018; T. Tsuzuki et al. 2020). For the degraded profiles, we include the error bars corresponding to the photon noise associated to each pixel (typically $\sigma/I \approx 10^{-3}$). In Figure 10 (second to fourth rows), we see that the differences between the AA and AD calculations in the quiet-Sun models are contained and essentially in agreement with what we found with the nonmagnetic semiempirical 1D models (see Section 3.1). By contrast, the plage profile (first row) shows larger differences, which is expected due to the larger magnetic field strength. However, when taking into account the spectral PSF and the polarization noise, most of the AA and AD profiles become compatible. In the worst cases (Q profile of the plage pixel, first row, and the red wing of the U profile of the second row), the difference is approximately of 2σ .

4. Conclusions

To test the suitability of the AA approximation in the modeling of the spectropolarimetric profiles of the Mg II h and k lines, we have solved the RT problem with AA and AD PRD in a number of different atmospheric models. We found that, in order to suitably include AD effects in 1D atmospheric models, it is sufficient to initialize the problem with the AA solution and then perform one or two AD iterations, dramatically reducing the computational cost. This strategy, which is similar to the one proposed by K. N. Nagendra et al. (2002), who considered academic settings, allows for an extreme acceleration of a highly computationally demanding problem. Note that, while this holds true for the Mg II h and k lines, the applicability of this solution strategy has to be studied on a line-by-line basis. Moreover, it will be worthwhile to assess if the same solution strategy can be reliably applied to full 3D RT problems.

Moreover, we found that the AA approximation is suitable to model both the intensity and circular polarization profiles of the Mg II h and k lines in 1D semiempirical models. By contrast, the AA approximation has an impact on the shape of the linear polarization profiles, especially close to the k line center, and this is enhanced when velocity gradients are in place. The difference between the AA and AD solutions can be further enhanced in the presence of a magnetic field. Indeed, if the field is aligned with the symmetry axis of the problem, the Hanle effect would not operate under the AA approximation (in which symmetry is fully preserved), but would still modify the line core in the general AD case.

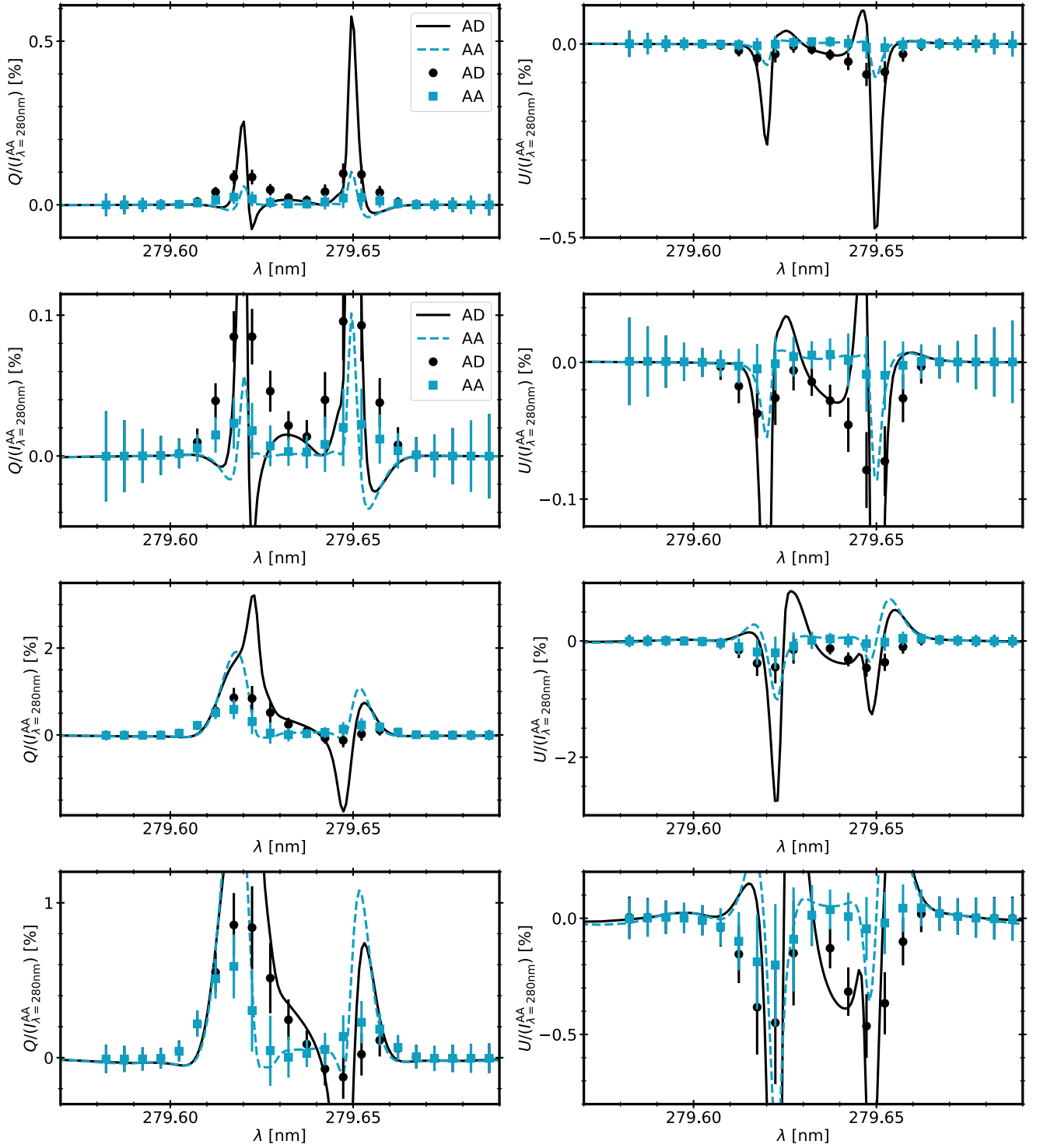


Figure 9. Linear polarization Q (left column) and U (right column) spectrum, around the Mg II k line, normalized to the AA intensity at $\lambda = 280$ nm for the 3D MHD atmospheric model in the central pixel of the field of view (first row) and in the pixel of the field of view showing the maximum $|Q|$ signals in the degraded profiles (third row). The filled circles and filled squares show the degraded profiles, mimicking a CLASP2 observation, for the AD and AA solutions, respectively, and their error bars correspond to the typical noise in the quiet-Sun profiles in the CLASP2 observations, between $\sigma \approx 5 \times 10^{-4} I$ and $1 \times 10^{-3} I$, with I the intensity profile. The panels in the second (fourth) row show a zoom in the polarization scale of the panels in the first (third) row to better visualize the differences between the degraded profiles.

Furthermore, we confirmed that the AA approximation is excellent for the intensity and circular polarization profiles when considering single columns of a 3D MHD model atmosphere. By contrast, large differences between AA and AD results appear in the linear polarization profiles. However, such differences

significantly reduce when accounting for instrumental degradation (of an instrument like CLASP2) and polarization noise. Nevertheless, when the resolution and polarimetric accuracy of the observations improve, the suitability of the AA approximation could become questionable.

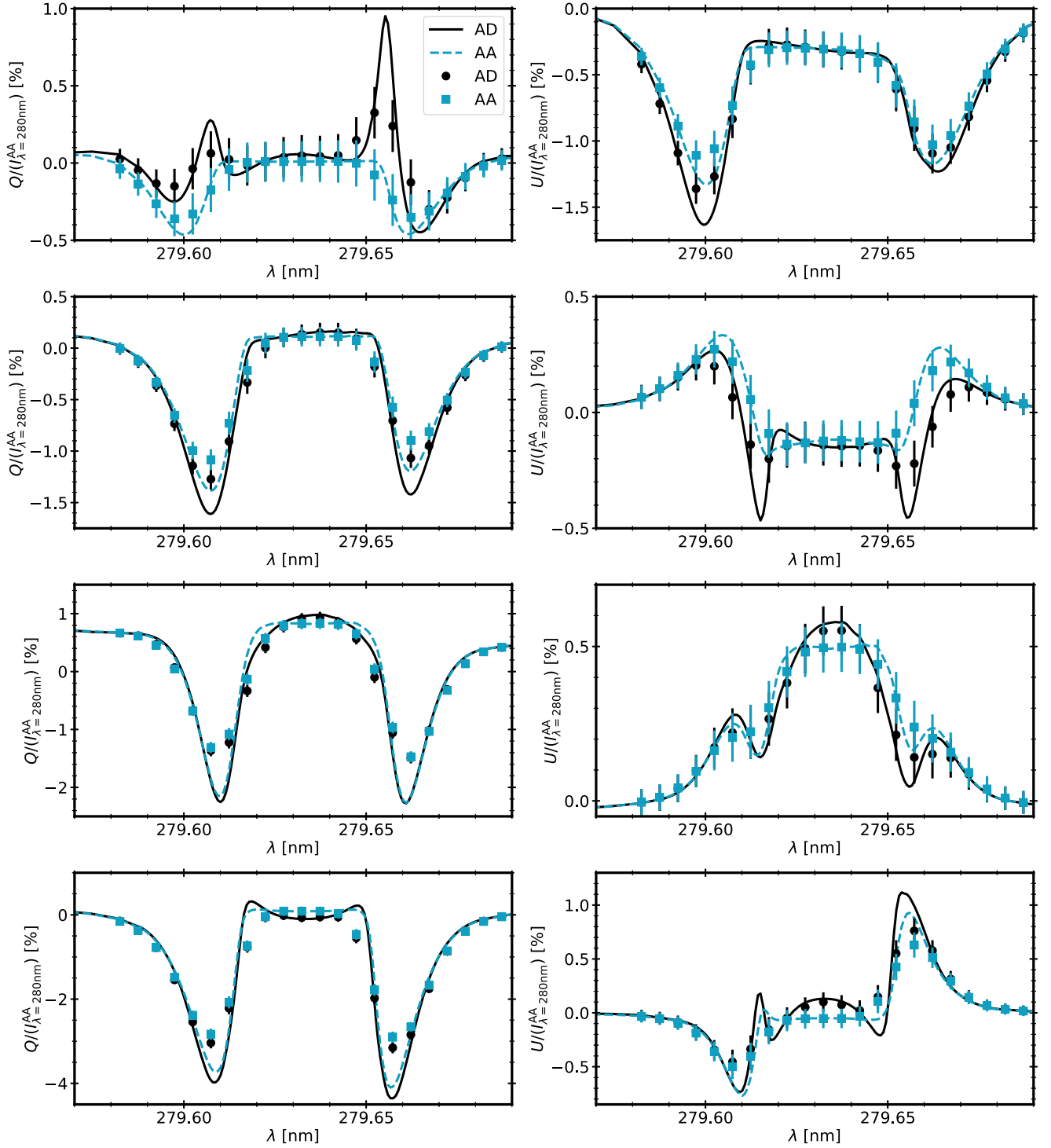


Figure 10. Linear polarization Q (left column) and U (right column) spectrum, around the Mg II k line, normalized to the AA intensity at $\lambda = 280$ nm, calculated in the model atmospheres resulting from the inversion of four pixels in the CLASP2 observations by H. Li et al. (2024). From top to bottom, each row corresponds to Figures 2, 3, 4, and 5 in H. Li et al. (2024), respectively. The filled circles and filled squares show the degraded profiles, mimicking a CLASP2 observation, for the AD and AA solutions, respectively, and their error bars indicate the corresponding noise in the quiet-Sun profiles in the CLASP2 observation, between $\sigma \approx 5 \times 10^{-4}I$ and $1 \times 10^{-3}I$, with I the intensity profile.

Finally, the AA and AD results are also compatible with most of the Stokes profiles computed in the 1D atmospheric models resulting from the full Stokes inversion of CLASP2 observations (H. Li et al. 2024), after degradation, except for the considered plage pixel. From these results, we do not expect a relevant impact of AD redistribution on the quiet-Sun

magnetic field inferred by H. Li et al. (2024) through inversions based on AA redistribution.

In conclusion, there is a clear impact of the AA approximation in the scattering linear polarization profiles of the Mg II h and k lines, which could, in principle, significantly affect the magnetic field diagnostics. However, for the state-of-the-art

spectropolarimetric observations, the resolution and polarimetric accuracy (spatial resolution of $\sim 1''$ and 0.1% polarimetric accuracy at 3σ level; R. Ishikawa et al. 2021; D. Song et al. 2022) are such that the impact of the AA approximation is only marginal.

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