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# On the Alleged Locality in the Schrödinger Picture. Comment on Vedral, V. Locality in the Schrödinger Picture of Quantum Mechanics. *Physics* 2024, *6*, 793–800

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**Abstract:** In his recent paper, Vlatko Vedral claims that the Schrödinger picture can describe quantum systems as locally as the Heisenberg picture, relying on a product notation for the density matrix. Here, I refute that claim. I show that the so-called 'local factors' in the product notation do not correspond to individual systems and therefore fail to satisfy Einsteinian locality. Furthermore, the product notation does not track where local gates are applied. Finally, I expose internal inconsistencies in the argument: if, as is also stated, the Schrödinger-picture locality ultimately depends on the explicit bookkeeping of all operations, then the explanatory power of the product notation is de facto undermined.

## 1. Motivation

In the Heisenberg picture, quantum systems can be fully described through local descriptions of their parts, even when the parts are entangled [1–3]. A bipartite system $AB$ admits descriptors $\boldsymbol{q}_A$ and $\boldsymbol{q}_B$ that are

(a) Einstein-local: $\boldsymbol{q}_A$ is independent of actions performed on system $B$, and vice versa;
(b) empirically complete: $\boldsymbol{q}_A$ and $\boldsymbol{q}_B$ encompass sufficient information to calculate the probability distributions associated with any measurement performed on the joint system $AB$.

In the Schrödinger picture, while reduced density matrices $\rho_A$ and $\rho_B$ fulfill property (a) above, they fail to satisfy property (b): the probability distributions of measurement outcomes on $AB$, encompassed in $\rho_{AB}$, cannot be obtained from the local $\rho_A$ and $\rho_B$ when the system is entangled. On the other hand, should one adopt $\rho_{AB}$ as a description of each individual system, empirical completeness would be trivially obtained at the price of losing Einsteinian locality. Thus, the Schrödinger picture appears to impose a dichotomy: a system can be described either locally or completely, but not both. As mentioned, the Heisenberg picture resolves this dichotomy with descriptors that satisfy both (a) and (b).

In his recent paper [4], Vlatko Vedral suggests that the Schrödinger picture can also circumvent this dichotomy. Upon writing the global density matrix in a so-called 'product notation', Vedral claims that

(Q1) "[T]he state in the product notation is as local in the Schrödinger picture as it is in the Heisenberg picture, meaning that the evolution of the whole can be specified by the 'local factors' as defined here."

In the current reply, I demonstrate that this claim does not hold. Even in the product notation, neither the density operator nor its factors simultaneously satisfy (a) and (b), because the so-called 'local factors' are *not* local.

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The central focus of Ref. [4] concerns a related but more specific issue: identifying the location of phase shifts and other local gates. In its common representation, the Schrödinger state does not generically permit this identification. For instance, applying a phase shift (the Pauli $Z$ gate, $\mathrm{diag}(1,-1)$) to the first qubit of a pair in the state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ produces $(|00\rangle - |11\rangle)/\sqrt{2}$, which is the same state that would be obtained if the phase shift were applied to the second qubit.

In the Heisenberg picture of quantum computation, local phase information is encoded in the affected descriptor, and, by virtue of Einsteinian locality, not in other descriptors. (For example, $|\Phi^+\rangle$ can be represented by descriptors $\boldsymbol{q}_1 = (q_{1z}q_{2x}, q_{1x})$ and $\boldsymbol{q}_2 = (q_{2x}, q_{1x}q_{2z})$, assuming that the Heisenberg state is set to $|00\rangle$ (see Ref. [3] for background on descriptors). Upon performing a $Z$ gate on qubit 1, its descriptor becomes $\boldsymbol{q}_1' = (-q_{1z}q_{2x}, q_{1x})$, while $\boldsymbol{q}_2' = \boldsymbol{q}_2$. Had the $Z$ gate been performed on qubit 2, the descriptors would have evolved to $\bar{\boldsymbol{q}}_1 = \boldsymbol{q}_1$ and $\bar{\boldsymbol{q}}_2 = (-q_{2x}, q_{1x}q_{2z})$.) Vedral claims that [4]

(Q2a) "There is also a way of expressing the state in the Schrödinger picture that retains the local knowledge of the phase…"

I show in Section 3 below that this, too, is false. The 'locality in the Schrödinger picture' suggested in the title of Ref. [4] is, therefore, misconceived.

## 2. The Argument

First, I present the calculation of Ref. [4], which, in Vedral's words,

(Q3) "[shows] that the Schrödinger picture is as Einstein local as the Heisenberg one."

Consider an initial state $|+\rangle|+\rangle$, whose density matrix can be expressed in the product notation,

$$\rho(0) = \frac{1}{4}(I + X_1)(I + X_2)\,,$$

where the following notation for Pauli operators is being used:

$$\begin{aligned} X_1 &= X \otimes \mathbb{1}\,, \qquad Z_1 = Z \otimes \mathbb{1}\,, \\ X_2 &= \mathbb{1} \otimes X\,, \qquad Z_2 = \mathbb{1} \otimes Z\,. \end{aligned}$$

Between time 0 and 1, a controlled--$Z$ gate is applied, so

$$U(1,0) = \mathrm{diag}(1,1,1,-1) = (I + Z_1 + Z_2 - Z_1 Z_2)/2\,.$$

This yields

$$\rho(1) = \frac{1}{4}(I + X_1 Z_2)(I + Z_1 X_2)\,. \tag{1}$$

At this stage, let us consider applying a phase shift on qubit 1, so the evolution between time 1 and $2a$ is given by $U(2a,1) = Z_1$. But before proceeding, it is constructive to compare the product notation of $\rho(1)$, given in Equation (1), with its tensor-product form,

$$\rho(1) = \frac{1}{4}(I + X \otimes Z + Z \otimes X + Y \otimes Y).$$

According to Vedral [4],

(Q4a) "It is this form that tricks us into believing that something non-local is occurring in quantum physics. Performing the phase operation on the first qubit here leads to the state $\rho(2a) = \frac{1}{4}(I - X \otimes Z + Z \otimes X - Y \otimes Y)$, and this form just does not tell us which of the two qubits was affected (since [one] could have obtained the same state by a suitable phase kick on the second qubit). In the product notation, on the other hand, the state would become

$$\rho(2a) = \frac{1}{4}(I - X_1 Z_2)(I + Z_1 X_2)\,,$$

which exhibits the minus sign in the state pertaining the first qubit. So, if one looks for a fuller account of what is happening, the product notation is possibly better than the tensor product."

Although not explicitly discussed in Ref. [4], one can verify the effect of applying a phase to the second qubit, i.e., if $U(2b, 1) = Z_2$. The density operator would become

$$\rho(2b) = \frac{1}{4}(I + X_1 Z_2)(I - Z_1 X_2)\,,$$

with the *second* factor affected. As noted in the quote (Q4a), this is seemingly due to the phase being applied to the *second* qubit.

Vedral confirms the point about $\rho(2a)$ "exhibi[ting] the minus sign in the state pertaining to the first qubit" [4]:

(Q5a) "... the product notation just introduced for the Schrödinger picture allows us to formally keep track of the dynamics, just like in the Heisenberg picture. Thus, one can also keep track of where local gates have been applied by checking which factor in the product has been affected."

As explained in Section 3 just below, this is false.

## 3. Why the Product Notation Fails

The example given by Verdral [4], and reexposed in Section 2, is fine-tuned. I present two modifications of the scenario, both making explicit the fact that the factors in Equation (1) do not pertain to any system. Thus, the so-called 'local factors' are not local in any meaningful sense.

Let us go back to time 1, when the state is written as in Equation (1). Instead, consider the effect of applying a local $X$ gate rather than a $Z$ gate: $U(2c, 1) = X_1$. In the product notation, the state would become

$$\rho(2c) = \frac{1}{4}(I + X_1 Z_2)(I - Z_1 X_2)\,,$$

which does *not* affect the first factor in any way, despite it allegedly representing the 'state pertaining to the first qubit'. The action on qubit 1 instead alters the second factor. Unlike with descriptors for which any action on qubit 1 would only alter its corresponding descriptor, there is no such thing in the product notation as a 'state pertaining to the first qubit'. Conversely, $U(2d, 1) = X_2$ would alter the first factor, not the second (and more specifically, it would alter it so as to yield the same state as $\rho(2a)$, with a minus sign in the first factor). Moreover, any generic local gate that commutes with neither $X$ nor $Z$ would affect both factors.

To further illustrate this issue, consider the initial state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, whose density matrix in product notation is given by

$$|\Phi^+\rangle\langle\Phi^+| = \frac{1}{4}(1 + X_1 X_2)(1 + Z_1 Z_2)\,.$$

Whether a $Z$ phase is introduced on qubit 1 or qubit 2, the first factor is the one affected. Here again, any local gate that commutes with neither $X$ nor $Z$ affects both factors.

Thus, the claim that one can determine the location of a gate by inspecting the factors in the product notation—expressed in the quotes (Q2a), (Q4a), and (Q5a)—is untenable. Since these factors do not correspond to individual systems, they fail to satisfy Einsteinian locality. Consequently, the assertion that the Schrödinger picture is as local as the Heisenberg picture—see the quote (Q1) and the statement (Q3)—does not follow from the structure of the product notation.

## 4. Internal Contradiction

The claims quoted so far are contradicted by other claims, which seemingly acknowledge the failure of the product notation.

For example, the argument in the quote (Q4a) culminates, in the penultimate sentence, with the (misconceived) assertion that the first factor pertains to the first qubit. But the sentence that follows the quote (Q4a) is a retraction [4]:

(Q4b) "Even here, naturally, there are operations on the second qubit that would lead to the same state..."

Indeed, as mentioned in Section 3, applying an $X$ gate to the second qubit also yields the state $\rho(2a)$. In particular, this means that an operation on the *second* qubit alters the *first* factor, namely what had just been referred to as 'the state pertaining the first qubit'.

A similar contradiction appears in the quote (Q5a), which explicitly asserts that one can track where local gates have been applied by examining which factor in the product has been affected. The sentence that follows, however, concedes the point [4]:

(Q5b) "Once more, this only means that, in both pictures, states alone do not contain all the relevant information, and one needs to keep track of the dynamics..."

Yes. In general, in the Schrödinger picture, determining where local gates have been applied requires explicitly tracking the dynamics—that is, the gates themselves. The passage continues with the correct observation that Heisenberg-picture descriptions do not face this issue [4]:

(Q5c) "(which, in the Heisenberg picture, is achieved by default by transforming all the algebra of the relevant operators)."

The power of the product notation is also asserted and later retracted in the full version of the quote (Q2), which reads [4]

(Q2) "There is also a way of expressing the state in the Schrödinger picture that retains the local knowledge of the phase by keeping track of all the operations executed on the system."

If retaining local phase information requires explicitly tracking all operations, then what is the contribution of the product notation? After all, there is also a way of using a mathematical constant, for example, $\pi$, to retain the local knowledge of the phase: this is achieved by leaving $\pi$ aside and independently keeping track of all operations executed on the system.

In short, insofar as the paper [4] asserts the locality of the factors in the product notation, it is incorrect. And where it raises the failure of the product notation, it is correct, but self-contradictory.

Importantly, my critique of the 'local factors' has no impact on another key aspect of the paper—namely, the distinction between Einsteinian and Bell locality, as well as between q-number and c-number-based reality. Vedral argues that Bell's theorem poses no tension with Einsteinian locality and should instead be understood as a rejection of an underlying c-number-based reality. This interpretation simultaneously undermines the nonlocal, superdeterministic, and retrocausal attempts to explain Bell correlations with c-valued elements. I fully endorse this position, as I have shown in Ref. [5] how a q-number-based reality enables local violations of Bell inequalities.

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