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# Financial Market Integration and Asset Prices

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*To my mother*

*Nihil Difficile Volenti.*

*Sapere Aude!*

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## *Abstract*

My doctoral thesis examines the relationships among the degree of financial market integration and the pricing of different classes of assets.

The first chapter provides a theoretical framework that uncovers in a model-free way the relationship between international stochastic discount factors (SDFs), stochastic wedges, and financial market structures. Exchange rates are in general different from the ratio of international SDFs in incomplete markets, as captured by a stochastic wedge. Theoretically, this wedge can be zero in incomplete and integrated markets. Market segmentation breaks the strong link between exchange rates and international SDFs, which helps address salient features of international asset returns, while keeping the volatility and cross-country correlation of SDFs at moderate levels.

The second chapter studies the degree of market integration between US corporate bonds and stocks of the corresponding issuing firms, accounting for their characteristics. I document that short-selling constraints are essential restrictions to optimal Sharpe ratio portfolios in order to yield admissible portfolio positions. Moreover, the implied cross-market pricing errors are small and contained within quoted bid-ask spreads. My empirical evidence suggests that markets are more integrated for larger firms, with more liquid corporate bonds and stocks. Similarly, firms that are more leveraged, have a higher asset growth and profitability feature a greater extent of integration between their debt and equity securities.

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# Chapter 1

## Introduction

THE degree of asset market integration is of particular importance for investment and financing decisions, not only for institutional investors and corporations, but also for individual investors. Access to cross-markets, both domestically and internationally, can prove beneficial in terms of portfolio diversification. Specifically, whenever two markets are integrated, identical securities will be assigned identical prices. In reality, however, there are several financial frictions preventing markets from being fully integrated, including financial constraints and limits to arbitrage, transaction costs, or investor heterogeneity, causing inequalities in the marginal rate of substitutions of different agents. Thus, shedding light on the underlying degree of market integration is crucial for refining our insights regarding the pricing of different asset classes.

The most natural framework to study the degree of financial market integration and its implications on asset prices is via an international context. Intuitively, whenever international markets are integrated, domestic investors can trade freely foreign assets and foreign investors can trade freely domestic assets. Within economies that are fully integrated, assets with identical risk should require the same premia, independent of the location. Therefore, in the first chapter of my Ph.D. thesis, using a parsimonious model-free approach, I examine the asset pricing implications of different degrees of international financial market integration, while addressing salient features of exchange rates. Specifically, in international markets, there are different stochastic discount factors (SDFs, hereafter) that are going to price different sets of assets. In particular, whenever international financial markets are fully integrated, the rate of appreciation of the exchange rate is given by the ratio between the foreign and domestic SDFs, a result commonly referred to as the

asset market view. Extant literature has mainly focused on models featuring complete markets in order to accommodate exchange rate anomalies, or puzzles, including the [Backus and Smith \(1993\)](#) puzzle, or disconnect from macroeconomic variables, the low exchange rate volatility, and the currency risk premia, the so-called carry trade. The novelty of the present study is to provide a very general model-free approach, in which international markets are incomplete and likely segmented.

The main theoretical contribution can be summarized as follows. Whenever international markets are integrated, there exists a particular SDF choice, namely the minimum entropy one, for which the asset market view holds even in incomplete markets. This result suggests that one can characterize the evolution of exchange rates by examining the ratio of foreign and domestic SDFs, while matching the three aforementioned puzzles. However, this market structure entails SDFs that are highly volatile and exhibit an almost perfect correlation. These properties are at odds both with typically observed Sharpe ratios and with evidence of home bias, implying that investors prefer to hold local assets. In order to derive more realistic SDFs that are still consistent with exchange rate puzzles, I study international markets that are segmented. This market setting boasts SDFs that feature lower volatility and a lower international co-movement. Additionally, this structure will drive a wedge between the exchange rate and the ratio of SDFs, breaking thus their strong link. I provide an economic interpretation of such deviations in terms of unspanned exchange rate risks, that cannot be replicated using basic assets, such as bonds and stocks.

Guided by the evidence supporting the important role of the underlying degree of market integration for pricing implications, in the second part of my thesis, I investigate more deeply its effects both across and within different asset classes. To this end, I focus on US corporate bonds and stock returns of the corresponding issuing firms and quantify the amount of integration between these two markets.

Canonical models in asset pricing often implicitly assume that markets are integrated, such that there is no arbitrage across markets. For example, the [Merton \(1974\)](#) model posits that the firm value process is divided between debt and equity investors, who hold exposures to the same source of risk. Following a contingent claim approach, debt and equity should be priced using the same stochastic discount factor (SDF). Put differently, this framework implies strict restrictions for the cross-market risk premia and dictates the way bonds and stocks should be priced cross-

sectionally, irrespective of the firm-characteristics driving the returns. In reality, however, there are several financial frictions that may prevent markets from being fully integrated, including financial constraints, such as short-selling restrictions or transaction costs, potentially leading to limits to arbitrage. Thus, shedding light on the underlying extent of market integration is crucial for refining our insights regarding the pricing of different asset classes, as well as for assessing the empirical success of asset pricing models implicitly assuming perfect integration across different markets. Moreover, recent evidence suggests that investors' behaviour may be driven by different firm specific or asset specific characteristics (see, e.g. [Kojien, Richmond, and Yogo \(2019\)](#), among others). Therefore, in the second part of my Ph.D. thesis, I address the question of how integrated corporate bond and stock markets are, by incorporating in the analysis different frictions and considering various firm characteristics.

To study the underlying degree of integration between corporate bonds and stock markets, I adopt a model-free approach, thus overcoming the typical model misspecification critique. Otherwise, it is extremely difficult to disentangle the joint hypothesis test of market integration from correct model specification. To exemplify, in a Capital Asset Pricing Model (CAPM) world, market integration implies that the prices of risk associated with the market factor are identical for corporate bond and stock returns. However, if the underlying factor model is misspecified, one might incorrectly reject market integration, as the prices of risk would tend to differ across various markets. Using instead a model-free approach, I show that the level of integration between corporate bonds and stock markets is higher than previously thought, at least unconditionally.

Under the assumption of frictionless markets, I show that it is always possible to construct empirically a common minimum variance SDF that prices both corporate bonds and stocks. However, upon a closer look at the associated optimal Sharpe ratio portfolio, the latter entails large and often negative positions, suggesting that in practice, if investors face leverage or short-selling constraints, it would be hard to attain. In the real world, markets are not free of frictions and an agent's choice set includes a subset of assets that she takes into consideration or is allowed to hold. Constraints in the choice set may arise due to investment mandates, certain benchmarks, or information frictions that restrain an investor's ability to examine a large universe of assets ([Merton \(1987\)](#)). My contribution is to study the degree of market integration in an extended framework, that incorporates frictions in the form of short-selling constraints. I focus on this type of restriction in order to obtain a portfolio that agents present in the corporate bond and/or

stock market can readily form. Regarding the stock market, there have been instances in which regulators prohibited short-selling all together, such as during the recent financial crisis. On the other hand, trading in the corporate bond market is carried over-the-counter (OTC). Typically, short-selling bonds entails a significant cost, and in illiquid corporate bond markets, it might simply not be possible (see, e.g., [Asquith, Au, Covert, and Pathak \(2013\)](#), [Blanco, Brennan, and Marsh \(2005\)](#) and [Bai and Collin-Dufresne \(2019\)](#)). Moreover, regulators might prevent certain type of institutional investors, such as insurance companies and pension funds, to engage in short-selling activities.

Consequently, I investigate what are the optimal portfolio implications and cross-market pricing relations when short-selling constraints are imposed. When taking into account short-selling constraints, I show that the wealth is typically going to be allocated just in one portfolio of stocks and bonds, that maximizes the Sharpe ratio. The cross-market pricing errors implied by the constrained minimum variance SDFs are between 10 and 50 basis points (bp) per month, and hence within the observed bid-ask spreads. This evidence suggests that even when accounting for short-selling constraints, markets exhibit a high degree of integration, as no profitable arbitrage opportunities arise. Hence, the degree of market integration is better than previously thought. Still, the average pricing errors will differ across portfolio sorts, with bond characteristics such as credit rating and duration yielding smaller stock mispricing, whereas valuation ratios for stock portfolios implying a lower mispricing for bond portfolios. In particular, markets are more integrated for larger firms, with more liquid corporate bonds and stocks. Similarly, firms that are more leveraged, have a higher asset growth and profitability feature a higher extent of integration between their debt and equity securities.

## Chapter 2

# Model-Free International Stochastic Discount Factors

THE financial crisis provided at least two insights into the workings of international financial markets. First, it was a reminder that despite the progressive removal of trade barriers such as capital controls and taxes, cross-border investment activity can be severely disrupted. Recent empirical evidence shows that international market segmentation is a pervasive feature even in the most liquid and developed markets (see [Camanho, Hau, and Rey \(2018\)](#) for international equities and [Maggiori, Neiman, and Schreger \(2019\)](#) for international fixed income markets). Second, the crisis highlighted the importance of financial frictions and has triggered a vast literature documenting the intimate link between asset returns and the health of the financial intermediary sector. In particular, most episodes of disruptions in international financial markets are generally considered to be caused by capital frictions in intermediation. Starting from these observations, this paper develops a model-free theoretical framework for studying the implications of international market segmentation on asset prices and the role of intermediaries in international financial markets.

Canonical models in international finance assume complete and integrated markets, in which stochastic discount factors (SDFs) and exchange rates are pinned down by the marginal utility of domestic and foreign households.<sup>1</sup> It is well known that, under such assumptions, the rate of appreciation of the real exchange rate ( $X$ ) has to be equal to the ratio of foreign ( $M_f$ ) and

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<sup>1</sup> In the following, we consider a market integrated if domestic investors can trade all foreign assets via the exchange rate market and vice versa. In contrast, segmented markets imply that some assets are accessible to investors in one country but not another.

domestic ( $M_d$ ) SDFs, i.e.,  $X = M_f/M_d$ ; an identity referred to as the *asset market view of exchange rates* in the literature. For instance, [Colacito and Croce \(2011, 2013\)](#) rely on recursive preferences and highly correlated long-term SDF components in a complete market setting to address many salient features of international asset returns and macroeconomic quantities.<sup>2</sup>

Despite the success of these models, another strand of the literature questions the completeness assumption of international financial markets and asks whether market incompleteness can help to quantitatively match the data. Incompleteness is appealing because it breaks the link between exchange rate returns and SDF ratios. The resulting deviations from the asset market view can be captured by a stochastic exchange rate wedge ([Backus, Foresi, and Telmer, 2001](#)). However, recent evidence by [Lustig and Verdelhan \(2019\)](#) shows that, in a no-arbitrage setting, some of the constraints imposed on the wedge to jointly address international finance puzzles may be hard to reconcile with the data.

In this paper, we take a different approach, which is motivated by the empirical observation that markets are likely segmented internationally. Notice that market completeness and market integration are economically two distinct concepts. While market completeness concerns the ability of investors to hedge all relevant risks in an economy using portfolios of traded payoffs, international financial market segmentation corresponds to situations in which the sets of traded payoffs differ across currency denominations. We adopt a parsimonious framework that allows us to directly uncover, in a model-free way, the relationship between international SDFs, stochastic wedges, and international market structures.

Using this approach, we establish that, as long as markets are integrated, stochastic wedges are always equal to zero with respect to a certain pair of international SDFs, irrespective of the extent of market incompleteness. This result implies that exchange rate risks can be uniquely determined by the ratio of these SDFs. As such, our results challenge the notion that market incompleteness alone may be instrumental to generate deviations from the asset market view and to our understanding of exchange rate behavior. As part of our empirical analysis, we then show that market segmentation severs the straightjacket of the asset market view and leads to volatile stochastic wedges. This feature not only helps address salient features of international

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<sup>2</sup> Other examples include the habit model of [Heyerdahl-Larsen \(2014\)](#) and [Stathopoulos \(2017\)](#) to generate sizable currency risk premia. [Farhi and Gabaix \(2016\)](#) rely on a complete market economy with time-additive preferences and a time-varying probability of rare consumption disasters. [Gabaix and Maggiori \(2015\)](#) provide a theory of exchange rate determination based on capital flows in segmented financial markets with heterogeneous trading technologies. [Hassan \(2013\)](#), [Hassan and Mano \(2019\)](#) and [Colacito, Croce, Gavazzoni, and Ready \(2018\)](#) emphasize the importance of heterogeneous exposure to global shocks for matching currency risk premia.



asset returns, but also leads to a low volatility and cross-country correlation of international SDFs.

Given the multitude of SDFs pricing returns in incomplete markets, we start our analysis by focusing on the family of minimum dispersion SDFs, each of which minimizes a different notion of variability. This family includes the well-known [Hansen and Jagannathan \(1991\)](#) SDF, which minimizes the SDF variance, as well as the minimum entropy SDF. As our main theoretical contribution, we show that when markets are integrated, minimum entropy SDFs always imply the validity of the asset market view, i.e., the resulting [Backus, Foresi, and Telmer \(2001\)](#)–type stochastic wedge is equal to zero even if markets are incomplete. Hence, in integrated markets, exchange rate risk is pinned down by the dynamics of international minimum entropy SDFs, as is the case in complete markets. This result is a consequence of the specific functional relationship between the minimum entropy SDF and the optimal growth portfolio in each country: the minimum entropy SDF in each country is equal to the reciprocal of the optimal growth portfolio return in that country. As a result, the change in currency denomination, which transforms the return in one country to the other, also transforms the minimum entropy SDF in that country to the other. Taken together, our results indicate that market segmentation is the only way to generate a deviation from the asset market view for the pair of minimum entropy SDFs.

The above findings have important implications for the three asset pricing puzzles that have dominated international finance: (i) the low exchange rate volatility puzzle documented by [Obstfeld and Rogoff \(2001\)](#) and [Brandt, Cochrane, and Santa-Clara \(2006\)](#), (ii) the cyclical puzzle, or the disconnect between exchange rates and macro-economic variables of [Kollmann \(1991\)](#) and [Backus and Smith \(1993\)](#), and (iii) the forward premium anomaly of [Hansen and Hodrick \(1980\)](#) and [Fama \(1984\)](#). Indeed, if one were to ask the question “do stochastic wedges help address exchange rate puzzles in integrated markets?”, then the answer is a resounding *no*. First, when we assume that domestic and foreign investors can trade short-term bonds internationally, the international Euler equations pin down the currency risk premium by construction, irrespective of the properties of stochastic wedges. Second, we show that stochastic wedges are always interpretable as a measure of the unspanned exchange rate risks in international financial markets, i.e., risks not insurable by linear portfolios of basic assets, such as stocks and bonds, in each market. Third, we document that, in the data, stochastic wedges are zero or minuscule for all minimum dispersion SDF pairs when markets are integrated. Therefore,

we extend our analysis to allow for market segmentation instead. In such settings, our model-free SDFs address the three puzzles above and are at the same time consistent with sizable deviations from the asset market view, which lead to appealing properties of international SDFs, such as moderate SDF volatilities and low cross-country SDF correlations.

In our empirical analysis, we study eight benchmark currencies, namely the US dollar, the British pound, the Swiss franc, the Japanese yen, the euro, the Australian dollar, the Canadian dollar, and the New Zealand dollar, over the sample period spanning January 1975 to December 2015. We document several stylized facts about model-free SDFs, which are independent of the dispersion measure used. To this end, we adopt the insights of [Bansal and Lehmann \(1997\)](#), [Alvarez and Jermann \(2005\)](#), and [Hansen and Scheinkman \(2009\)](#) to factorize international SDFs into a permanent and a transient component. First, we find that permanent components of domestic and foreign minimum dispersion SDFs across markets are highly volatile, irrespective of the degree of market segmentation, to the point that they actually dominate the overall SDF variability. The co-movement of permanent SDF components and long-term bond returns is positive, in order to match the typically negative local risk premia of long-term bonds. These features are consistent with previous evidence for the US market in, e.g., [Alvarez and Jermann \(2005\)](#). Second, we find that international SDFs are almost perfectly correlated in integrated markets, due to the nearly perfect correlation of their permanent components. In contrast, this correlation is more than halved in segmented markets.

Further, we ask whether the extracted model-free SDFs are successful at explaining the exchange rate behavior. When equity and long-term bond markets are segmented, but investors are allowed to trade internationally the riskless short-term bonds, i.e., they can invest in the FX carry, we find the ensuing minimum dispersion SDFs to jointly address the three exchange rate puzzles. The low exchange rate volatility is explained by means of a volatile stochastic wedge. The cyclical puzzle is addressed because cross-country differences in transient SDF components are only weakly related to exchange rate returns. And finally, carry trade premia are also in line with the data, because the pricing constraints on risk-free bonds effectively force international SDFs to correctly reproduce the cross-section of currency risk premia. Importantly, in segmented markets, international SDFs exhibit average volatilities that are consistent with the local Sharpe ratios, together with large deviations from the asset market view and low cross-country correlations, ranging from 12% to 65%. This finding is important, because the high

cross-country correlation of marginal utilities implied by canonical models in international finance is hard to explain empirically. When we allow investors to trade all assets internationally, i.e., we assume integrated markets, we show that the corresponding minimum dispersion SDFs again address the three exchange rate puzzles. However, the market integration assumption comes at the cost of significantly larger SDF dispersions and nearly perfect SDF correlations that reflect uniformly small deviations from the asset market view.

Finally, in a quest of linking our model-free SDFs to observable economic variables, we find no relation with international consumption growth proxies. Inspired by the work of [Gabaix and Maggiori \(2015\)](#), who study an economy in which households' asset demand is met by financial intermediaries, we then analyze empirically the links between model-free international SDFs and various proxies of financial intermediaries' risk-bearing capacity. Using linear regressions, we find that a composite measure of the capital ratio in [He, Kelly, and Manela \(2017a\)](#), the broker-dealer leverage in [Adrian, Etula, and Muir \(2014a\)](#) and the expected capital shortfall of systemically important international financial institutions in [Brownlees and Engle \(2017\)](#) and [Acharya, Pedersen, Philippon, and Richardson \(2017\)](#) explains up to 35% of the time-series variation of international SDFs.

A number of recent papers explores the ability of market incompleteness to address various puzzles in international finance. Closest to our paper is [Lustig and Verdelhan \(2019\)](#), who derive a set of arbitrage-free constraints between the moments of stochastic wedges, international SDFs, and exchange rate returns. In particular, these constraints help to identify joint moment conditions that may be difficult to reconcile with the data. For instance, the authors show that while stochastic wedges help lower the volatility of the exchange rate, the no arbitrage conditions imposed on the wedge imply a predictable exchange rate component whenever international SDFs have identical volatilities, which is counterfactual to the data.

Distinct from this paper, we characterize optimal minimum dispersion SDFs for the domestic and foreign numéraires separately, and we provide the necessary conditions to jointly address the three exchange rates puzzles while keeping SDF correlations at moderate levels. Different from these authors, who derive arbitrage-free moment constraints on the wedge that are conditional on a priori assumptions about the moments of exchange rates and international SDFs, we extract minimum dispersion SDFs and the resulting wedges from the data jointly. This allows us to match currency risk premia via the Euler equations alone without relying on a predictable component

to match currency risk premia. We show that in integrated markets, deviations from the asset market view, as captured by stochastic wedges, contribute per se little to our understanding of international finance puzzles more generally. Indeed, as long as markets are integrated, wedges are theoretically zero for minimum-entropy SDFs and minuscule for minimum variance SDFs in the data. Moreover, we document that to address the cyclicity puzzle, SDFs must feature a large permanent component.

[Maurer and Tran \(2016\)](#) study continuous-time economies in incomplete but integrated markets and show that the asset market view holds for the minimum variance SDF pair if and only if there is no jump risk. Our approach is different, as we do not impose distributional assumptions on returns and as minimum variance SDFs are a special case of our family of minimum dispersion SDFs. We prove that the asset market view always holds for minimum entropy SDFs in integrated markets, irrespective of the degree of incompleteness. We further show that the asset market view does not hold in general for any other minimum dispersion SDF pair if the exchange rate return is not constant across some states of nature.

Another strand of the literature studies structural models of exchange rate determination in segmented markets. [Chien, Lustig, and Naknoi \(2019\)](#) show that limited stock market participation can reconcile highly correlated international SDFs with a low correlation in consumption growth. [Alvarez, Atkeson, and Kehoe \(2009\)](#) explain the cyclicity puzzle in a general equilibrium model with financial frictions and endogenous market participation. [Gabaix and Maggiori \(2015\)](#) study the disconnect puzzle and violations of the uncovered interest rate parity condition (UIP), in a setting where specialized financiers intermediate households' asset demands in segmented markets. In the data, [Maggiori, Neiman, and Schreger \(2019\)](#) document extreme segmentation in international lending markets at the currency level. We contribute to this literature by jointly addressing the three exchange rate puzzles under a model-free approach and by quantifying empirically the trade-off between large SDF dispersions and international co-movement in integrated versus segmented international financial markets.

Lastly, our paper contributes to the recent literature on financial intermediaries and their effect on international asset prices. For example, [Haddad and Muir \(2018a\)](#) show that financial intermediaries' wealth matters more for assets that households are less willing to hold directly, such as CDS, sovereign bonds, and FX. [Malamud and Schrimpf \(2018\)](#) posit a theoretical model to explain several no arbitrage violations where intermediaries trade with customers in over-

the-counter FX markets. Hébert (2017) documents the impact of financial intermediaries for international money markets in the presence of externalities. Fang and Liu (2018) study the effect of intermediaries' time-varying leverage constraints on exchange rate behavior. Finally, Maggiori (2017) finds that heterogeneity in financial development can rationalize the special role of the United States as the main risk taker in the global financial architecture. Different from these papers, we document strong empirical links between proxies of the health of the intermediary sector and SDFs which are estimated without making parametric assumptions about investors' utility functions or the economic fundamental processes.

The rest of the paper is organized as follows. Section 2.1 provides the theoretical framework for studying model-free SDFs in international financial markets. Section 2.2 presents our main empirical findings. Section 2.3 explores empirically the relation between international model-free SDFs and proxies of financial intermediaries' wealth. Section 2.4 contains robustness checks. Section 2.5 concludes the paper. An online appendix provides additional extensions and results omitted in the paper.

## 2.1 Model-Free SDFs in International Markets

Consider an economy consisting of two countries, one domestic ( $d$ ) and one foreign ( $f$ ), each with its own currency. Investors in each country can trade in a set of assets denominated in their respective currencies, with  $\mathbf{R}_d = (R_{d0}, \dots, R_{dn_d})'$  and  $\mathbf{R}_f = (R_{f0}, \dots, R_{fn_f})'$  denoting the returns in the domestic and foreign countries, respectively.  $R_{d0}$  and  $R_{f0}$  indicate the corresponding risk-free bond returns in the two markets. Throughout, we assume that the collection of assets  $\mathbf{R}_i$  in each country  $i \in \{d, f\}$  are linearly independent.

Let  $M_d$  and  $M_f$  denote generic domestic and foreign stochastic discount factors (SDFs) that price the vectors of returns  $\mathbf{R}_d$  and  $\mathbf{R}_f$  from the perspective of domestic and foreign investors (i.e., in the domestic and foreign currencies), respectively. The corresponding Euler equations are thus given by

$$\begin{aligned}\mathbb{E}(M_d \mathbf{R}_d) &= \mathbf{1} \\ \mathbb{E}(M_f \mathbf{R}_f) &= \mathbf{1},\end{aligned}\tag{2.1}$$

where  $\mathbf{1}$  denotes the vector of ones of appropriate size. Finally, let  $X$  denote the gross exchange rate return, with the exchange rate defined as the domestic currency price of one unit of the

foreign currency.<sup>3</sup>

**Definition 2.1.1.** International financial markets are *integrated* if

$$\text{span}(\mathbf{R}_d) = \text{span}(\mathbf{R}_f X),$$

where  $\text{span}(\mathbf{R}_d)$  and  $\text{span}(\mathbf{R}_f X)$  denote the linear spans of portfolio payoffs generated by domestic returns and foreign returns converted to the domestic currency, respectively.

According to the above definition, international financial markets are integrated when each foreign return is tradable by domestic investors through exchange rate markets, and vice versa. In contrast, when  $\text{span}(\mathbf{R}_d) \neq \text{span}(\mathbf{R}_f X)$ , international markets are (fully or partially) segmented, in the sense that certain returns are accessible to investors in one country, but not the other. Note that the notion of market integration in Definition 2.1.1 is distinct from market completeness: international markets can be integrated even if the collection of assets available in each country does not span all the uncertainty in that country.

**Definition 2.1.2.** The *asset market view of exchange rates* holds with respect to a pair of SDFs  $(M_d, M_f)$  if

$$X = M_f/M_d. \tag{2.2}$$

It is well-known that whenever domestic and foreign markets are complete and integrated, the asset market view of exchange rates holds with respect to the unique pair of SDFs that satisfies the pricing restrictions (2.1). As a result, one can unambiguously back out the implied changes in exchange rates from the two countries' (unique) stochastic discount factors. However, when markets are incomplete, the multiplicity of international SDFs satisfying (2.1) may lead to violations of (2.2). In this case, deviations from the asset market view can be parameterized by a stochastic wedge  $\eta$  between exchange rate returns and the ratio of foreign and domestic SDFs (see, e.g., Backus, Foresi, and Telmer (2001)):

$$X = \frac{M_f}{M_d} \exp(\eta). \tag{2.3}$$

Crucially, since  $M_f$  and  $M_d$  are not uniquely determined, each choice of SDF pairs may lead to

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<sup>3</sup> Throughout, we work with real SDFs, returns, and exchange rates.

a potentially different wedge.

In the remainder of this section, we study how the international market structure and the choice of SDFs jointly determine the extent and nature of deviations from the asset market view, as measured by the stochastic wedge  $\eta$ .

### 2.1.1 Model-Free International SDFs

Since market incompleteness implies that the SDF pair  $(M_d, M_f)$  satisfying (2.1) is not unique, we start our analysis by constructing a family of model-free SDFs for each country. For country  $i \in \{d, f\}$ , we define the *minimum-dispersion SDF* with parameter  $\alpha \neq 1$  as<sup>4</sup>

$$\begin{aligned} M_i^*(\alpha) = \arg \min_{M_i} \quad & \frac{1}{\alpha(\alpha-1)} \log \mathbb{E}[M_i^\alpha] \\ \text{s.t.} \quad & \mathbb{E}[M_i \mathbf{R}_i] = \mathbf{1} \\ & M_i > 0. \end{aligned} \tag{2.4}$$

The pricing restriction  $\mathbb{E}[M_i \mathbf{R}_i] = \mathbf{1}$  in (2.4) ensures that  $M_i$  prices all assets available to investors in country  $i$ , while the positivity constraint  $M_i > 0$  ensures that it is indeed an SDF. The family of minimum dispersion SDFs defined in (2.4) contains some of the well-known SDFs as special cases. For instance, setting  $\alpha = 2$  leads to the minimum variance SDF underlying the Hansen and Jagannathan (1991) bounds, whereas  $\alpha = 0$  yields the minimum entropy SDF. Clearly, all SDFs in this family coincide with one another when markets are complete.

While SDFs are not directly observable from the data, our first result provides an alternative representation of minimum dispersion SDFs in terms of the distribution of asset returns. To state this result, we consider the following portfolio problem (Orłowski, Sali, and Trojani, 2016):

$$\begin{aligned} R_{\lambda_i}^*(\alpha) = \arg \max_{R_{\lambda_i}} \quad & -\frac{1}{\alpha} \log \mathbb{E}[R_{\lambda_i}^{\alpha/(\alpha-1)}] \\ \text{s.t.} \quad & R_{\lambda_i} = \left(1 - \sum_{k=1}^{n_i} \lambda_{ik}\right) R_{i0} + \sum_{k=1}^{n_i} \lambda_{ik} R_{ik} \\ & R_{\lambda_i} > 0, \end{aligned} \tag{2.5}$$

where  $\lambda_{ik}$  denotes the portfolio weight of asset  $k$ . We have the following result:

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<sup>4</sup> The case  $\alpha = 0$  is defined by continuity:  $\lim_{\alpha \rightarrow 0} \frac{1}{\alpha(\alpha-1)} \log \mathbb{E}[M_i^\alpha] = -\mathbb{E}[\log M_i]$ .

**Proposition 1.** Let  $M_i^*(\alpha)$  denote the minimum dispersion SDF of country  $i$  with parameter  $\alpha$ . Then,

$$M_i^*(\alpha) = \frac{R_i^*(\alpha)^{1/(\alpha-1)}}{\mathbb{E}[R_i^*(\alpha)^{\alpha/(\alpha-1)}]}, \quad (2.6)$$

where  $R_i^*(\alpha)$  is the gross return of the optimal portfolio in (2.5). Furthermore,

$$\frac{1}{\alpha(\alpha-1)} \log \mathbb{E}[M_i^\alpha] \geq -\frac{1}{\alpha} \log \mathbb{E}[R_{\lambda_i}^{\alpha/(\alpha-1)}] \quad (2.7)$$

for any stochastic discount factor  $M_i$  and any portfolio return  $R_{\lambda_i}$  in the form of (2.5).

Equation (2.6) in Proposition 1 provides a characterization of the entire family of minimum dispersion SDFs in terms of the returns of a corresponding family of optimal portfolios. For instance, it implies that the minimum variance and minimum entropy SDFs are equal to  $M_i^*(2) = R_i^*(2)/\mathbb{E}[R_i^*(2)^2]$  and  $M_i^*(0) = 1/R_i^*(0)$ , respectively, thus recovering the established results that  $M_i^*(2)$  coincides with the return of the maximum Sharpe ratio portfolio (Hansen and Jagannathan, 1991) and  $M_i^*(0)$  is the reciprocal of the maximum growth portfolio return (Bansal and Lehmann, 1997). Central for our purposes, equation (2.6) provides us with a closed-form expression for model-free SDFs that can be estimated directly from the data. This relationship will serve as the basis of our empirical analysis in Section 2.2.

We conclude this discussion by noting that Proposition 1 also establishes a family of lower bounds (2.7) for all SDFs  $M_i$  in terms of the returns of any portfolio in country  $i$ , formalizing the idea that each value of  $\alpha$  imposes a restriction on a specific moment of  $M_i$ .<sup>5</sup>

### 2.1.2 The Asset Market View

With the above result in hand, we now turn to our main result in this section, which provides the link between international market structures, the choice of SDFs, and the extent and nature of deviations from the asset market view.

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<sup>5</sup> This family of lower bounds contains some of the well-known bounds in the literature as special cases. For instance, when  $\alpha = 2$ , inequality (2.7) reduces to the well-known variance bound of Hansen and Jagannathan (1991), whereas setting  $\alpha = 0$  leads to the entropy bounds (see e.g., Stutzer (1995), Bansal and Lehmann (1997), and Alvarez and Jermann (2005)).



**Proposition 2.** *Suppose international financial markets are integrated. Then,*

- (a) *The asset market view of exchange rates holds with respect to minimum entropy SDFs.*
- (b) *If the exchange rate return  $X$  takes distinct values in all states, then the asset market view holds with respect to minimum dispersion SDFs with parameter  $\alpha \neq 0$  if and only if markets are complete.*

Statement (a) of the above result establishes that, as long as financial markets are integrated, minimum entropy SDFs are consistent with the asset market view of exchange rates, i.e.,  $X = M_f^*(0)/M_d^*(0)$ . Importantly, this relationship holds without imposing any other restrictions on the distribution of asset returns or the structure of market incompleteness. Statement (b) of Proposition 2 further underscores the tight link between exchange rates and minimum entropy SDFs in integrated markets. It states that, unless markets are complete, the minimum entropy SDF pair is the *only* SDF pair satisfying the asset market view: in general,  $X \neq M_f^*(\alpha)/M_d^*(\alpha)$  for all  $\alpha \neq 0$ . Taken together, the two parts of Proposition 2 imply that the wedge  $\eta$  in (2.3) is always equal to zero for minimum entropy SDFs in integrated markets, regardless of the extent of market incompleteness, whereas  $\eta \neq 0$  in general for any other pair of minimum dispersion SDFs.

To see the mechanics behind Proposition 2 in a transparent manner, it is instructive to focus on the special case in which investors in the domestic and foreign countries each have access to a single asset, with returns denoted by  $R_d$  and  $R_f$ , respectively. Since these are the only returns available to investors in each country, market integration is equivalent to  $R_d = R_f X$ . On the other hand, Proposition 1 implies that the minimum dispersion SDFs in the two countries are given by  $M_d^*(\alpha) = R_d^{1/(\alpha-1)}/\mathbb{E}[R_d^{\alpha/(\alpha-1)}]$  and  $M_f^*(\alpha) = R_f^{1/(\alpha-1)}/\mathbb{E}[R_f^{\alpha/(\alpha-1)}]$ , respectively. Therefore,

$$\frac{M_f^*(\alpha)}{M_d^*(\alpha)} = X^{1/(1-\alpha)} \frac{\mathbb{E}[(R_f X)^{\alpha/(\alpha-1)}]}{\mathbb{E}[R_f^{\alpha/(\alpha-1)}]}. \quad (2.8)$$

It is easy to see that, unless  $\alpha = 0$ , the right-hand side of the above equality is in general distinct from  $X$ , as predicted by the proposition. Though only a special case of Proposition 2, this simple example illustrates why minimum entropy SDFs satisfy the asset market view while all other minimum dispersion SDFs, including minimum variance SDFs, do not: the unique functional relationship between the minimum entropy SDF and the reciprocal return of the

maximum growth portfolio in each country implies that multiplication by the exchange rate return  $X$  — which converts returns from one currency to the other — also converts one SDF to the other:

$$M_d^*(0) = 1/R_d^*(0) = 1/(R_f^*(0)X) = M_f^*(0)/X. \quad (2.9)$$

To summarize, Proposition 2 underscores three important characteristics of minimum entropy SDFs in integrated markets. First, it establishes that changing from one currency denomination to another is just a change in scale: by transforming the growth optimal portfolio in one currency to the other, the change in denomination also transforms the domestic SDF to the foreign SDF while preserving its minimum entropy property. Second, Proposition 2 implies that, even though SDFs in incomplete markets are not uniquely determined, exchange rate risks are always uniquely pinned down by the ratio of minimum entropy SDFs. Finally, and more economically relevant, Proposition 2 has implications for our understanding of Backus, Foresi, and Telmer (2001)-type wedges in equation (2.3): it implies that, as long as international markets are integrated, the stochastic wedge  $\eta$  corresponding to minimum entropy SDFs is equal to zero, even if the domestic and foreign markets are incomplete. Importantly, this observation also indicates that the only way to break the strong link between exchange rates and minimum entropy SDFs is by means of introducing some form of market segmentation.

### 2.1.3 Unspanned Exchange Rate Risks

Our results in the previous subsection emphasize that minimum variance SDFs are in general inconsistent with the asset market view of exchange rates, even when international markets are fully integrated. Our next result, which is a consequence of Proposition 2, characterizes the extent of this inconsistency in terms of unspanned exchange rate risks in international financial markets.

**COROLLARY 1.** *Suppose international financial markets are integrated but incomplete. Then,*

$$X = \frac{M_f^*(2)}{M_d^*(2)} \exp(\eta(2)), \quad (2.10)$$

where the stochastic wedge  $\eta(2)$  corresponding to minimum variance SDFs is given by

$$\eta(2) = \log \left( \frac{1 + [M_f^*(0) - M_f^*(2)]/M_f^*(2)}{1 + [M_d^*(0) - M_d^*(2)]/M_d^*(2)} \right). \quad (2.11)$$

Corollary 1 establishes that deviations from the asset market view with respect to minimum variance SDFs can always be captured by the ratio of the relative projection errors of foreign and domestic minimum entropy SDFs on the space of foreign and domestic returns. The projection error  $M_i^*(0) - M_i^*(2)$  can be interpreted as unspanned risk in country  $i$ , reflecting nonlinear risks that are not replicable by means of linear asset portfolios.<sup>6</sup> Therefore, Corollary 1 suggests that the stochastic wedge  $\eta(2)$  in the Backus, Foresi, and Telmer (2001) decomposition can be interpreted as a measure for the amount of unspanned domestic and foreign exchange rate risks. The expression in equation (2.10) also indicates that  $\eta(2)$  depends on (i) potential non-linearities in each country, as the minimum entropy portfolio is a non-linear transformation of the growth optimal portfolio and (ii) the differences between growth optimal and mean-variance optimal portfolios in domestic and foreign markets.

We can also leverage the relationship established in Proposition 1 to represent the departure from the asset market view in terms of asset returns. More specifically, the juxtaposition of equations (2.6) and (2.11) implies that

$$\eta(2) = \log \left( \frac{R_d^*(2)/\mathbb{E}[R_d^*(2)^2]}{R_f^*(2)/\mathbb{E}[R_f^*(2)^2]} \right) + \log \left( \frac{R_d^*(0)}{R_f^*(0)} \right), \quad (2.12)$$

where  $R_i^*(2)$  and  $R_i^*(0)$  are the returns of maximum Sharpe ratio and maximum growth portfolios in country  $i = \{d, f\}$ , respectively. Equation (2.12) therefore illustrates that the stochastic wedge depends on the mean-variance trade-off in the domestic market relative to the foreign market (as captured by the first term) as well as the relative risk-return trade-offs in domestic and foreign markets due to the higher moments of returns (as captured by the second term).

#### 2.1.4 SDF Decomposition

Before we turn to our empirical analysis, it is useful to decompose the SDFs into their transitory and permanent components. Such a decomposition will shed further light on SDF properties

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<sup>6</sup> The interpretation of  $M_i^*(0) - M_i^*(2)$  as unspanned risk follows from the observation that  $M_i^*(0) - M_i^*(2)$  is orthogonal to the space of traded returns in country  $i$ . In our empirical analysis of Section 2.2, we make use of bonds and stocks as basic assets that investors can trade.

needed to address exchange rate behavior. To this end, we follow [Alvarez and Jermann \(2005\)](#), [Hansen and Scheinkman \(2009\)](#), and [Hansen \(2012\)](#), who show that SDF processes can be factorized into a permanent martingale component — which can be used to characterize pricing over long investment horizons — and a transitory component that is related to the return on a discount bond of (asymptotically) long maturity. Following this approach, we decompose an SDF  $M_i$  in country  $i$  as

$$M_i = M_i^P M_i^T, \quad (2.13)$$

where the permanent component  $M_i^P$  satisfies the martingale normalization  $\mathbb{E}[M_i^P] = 1$  and the transient component  $M_i^T$  is the inverse of the return  $R_{i\infty}$  of the infinite maturity bond, i.e.,  $M_i^T = 1/R_{i\infty}$ .<sup>7</sup> Note that the normalization  $\mathbb{E}[M_i^P] = 1$  of the permanent component is ensured by requiring the return on the infinite maturity bond to be priced by the SDF, i.e.,  $\mathbb{E}[M_i R_{i\infty}] = 1$ .<sup>8</sup>

We conclude this section by noting that the decomposition in (2.13) implies that the exchange rate return, the stochastic wedge  $\eta$ , and the persistent SDF components are related via the following factorization:

$$X = \frac{M_f^P}{M_d^P} \frac{R_{d\infty}}{R_{f\infty}} \exp(\eta). \quad (2.14)$$

In the subsequent sections, we estimate each component of the right-hand side of equation (2.14) in both integrated and segmented markets and study their properties.

## 2.2 Empirical Analysis

In this section, we use the model-free family of minimum-dispersion SDFs and the theoretical results developed in the previous section to characterize the properties of international SDFs, assuming different degrees of segmentation between domestic and foreign financial markets. For

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<sup>7</sup> Two remarks are in order. First, even though the decomposition in [Alvarez and Jermann \(2005\)](#) is not unique in general, there is a unique decomposition under additional stochastic stability assumptions. The complete theoretical framework characterizing existence and uniqueness of the factorization is treated in [Hansen and Scheinkman \(2009\)](#) and [Qin and Linetsky \(2017\)](#), among others. Second, in the data, we do not observe an infinite maturity bond return. While in the main text we approximate this return using a long-term bond of finite maturity, in Section 2.4 we compute non-parametric estimates of permanent components using the sieve approach in [Christensen \(2017\)](#), which is based on the [Hansen and Scheinkman \(2009\)](#) eigenvalue decomposition. We find that both approaches lead to virtually identical results.

<sup>8</sup> Equivalently,  $R_{i\infty}$  is one of the components of return vector  $\mathbf{R}_i$  in problem (2.4). Tradability of  $R_{i\infty}$  obviously impacts the form of minimum dispersion SDFs and increases the SDF variability.

simplicity, we focus on two specific market structures: full integration and segmented markets. When international financial markets are assumed to be fully integrated, investors have access to all assets, both in domestic and foreign markets: the risk-free bond, the aggregate equity return, and the long-term bond return. Market segmentation, on the other hand, arises when international investors do not have access to international long-term bonds and stocks.<sup>9</sup>

This framework allows us to empirically measure the implications of different market structures on stochastic wedges, international SDFs, and unspanned FX risks. We factorize international SDFs into permanent and transient components and single out the ingredients necessary to match international asset returns.

We seek to answer a number of questions. First, how much SDF dispersion is necessary to match unconditional risk premia in integrated versus segmented markets? Second, how are transient and permanent SDF components connected to exchange rate puzzles? Third, how much international SDF co-movement does a given market structure imply? Beyond addressing the well-known exchange rate puzzles, studying these questions is important in order to understand the implications of different financial market structures for the properties of international SDFs.

### 2.2.1 Data

In our empirical analysis, we take the United States (US) as the domestic market and the United Kingdom (UK), Switzerland (CH), Japan (JP), the European Union (EU), Australia (AU), Canada (CA), or New Zealand (NZ) as the foreign markets.<sup>10</sup> We use monthly data between January 1975 and December 2015 from Datastream.<sup>11</sup> The resulting seven exchange rates are expressed with respect to the USD as the domestic currency. We compute equity returns from the corresponding MSCI country index prices and risk-free rates from one-month LIBOR rates. We follow [Alvarez and Jermann \(2005\)](#) and proxy transient SDF components by the inverse of the bond return with the longest maturity available, i.e., the ten-year (government) bond in our

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<sup>9</sup> While many other combinations of segmentation are possible, we focus on these two because they represent the workhorse assumptions in international finance. For example, [Pavlova and Rigobon \(2010\)](#) derive optimal portfolios when investors can access stocks and bonds internationally in integrated markets, whereas [Gabaix and Maggiori \(2015\)](#) allow for trading in short-term bonds only. The latter case represents the most parsimonious setting where investors can trade the FX carry.

<sup>10</sup> We use Germany as a proxy for the EU prior to the introduction of the euro.

<sup>11</sup> The sample period for New Zealand ranges from January 1988 to December 2015 due to data availability on the long-term bonds.

case.<sup>12,13</sup>

We deflate all domestic returns and exchange rates by the corresponding domestic Consumer Price Index in order to obtain real quantities. The results we derive are based on bilateral trading. We report summary statistics of all variables in the online appendix. For brevity, in the main body of the paper, we do not report numbers for each bilateral pair vis-à-vis the US, and instead report averages across all countries. Disaggregated statistics at the country-level are available in the online appendix.

## 2.2.2 Market Structures

**Integrated Markets:** We start our analysis by assuming that markets are fully integrated, in the sense that investors can trade domestic and foreign short- and long-term bonds  $(R_{i0}, R_{i\infty})$  and equity indices  $(R_{i1})$  without restrictions. The tradable vector of returns in the domestic market is therefore given by  $\mathbf{R}_d = (R_{d0}, R_{d\infty}, R_{d1}, \tilde{R}_{d0}, \tilde{R}_{d\infty}, \tilde{R}_{d1})'$ , where  $\tilde{R}_{d0} = R_{f0}X$ ,  $\tilde{R}_{d\infty} = R_{f\infty}X$ , and  $\tilde{R}_{d1} = R_{f1}X$  are the domestic currency returns of the foreign risk-free asset, the foreign long-term bond, and the foreign aggregate equity, respectively. Similarly, the vector of tradable returns in the foreign market is given by  $\mathbf{R}_f = (R_{f0}, R_{f\infty}, R_{f1}, \tilde{R}_{f0}, \tilde{R}_{f\infty}, \tilde{R}_{f1})'$ , where  $\tilde{R}_{f0} = R_{d0}/X$ ,  $\tilde{R}_{f\infty} = R_{d\infty}/X$ , and  $\tilde{R}_{f1} = R_{d1}/X$  are the foreign currency returns of the domestic risk-free asset, the domestic long-term bond, and the domestic aggregate equity, respectively.

Therefore, when international markets are integrated, Proposition 1 implies that the time series of estimated minimum dispersion SDF in country  $i \in \{d, f\}$  is obtained in closed form from equation (2.6):

$$\hat{M}_{i,t}^* = \frac{R_{\hat{\lambda}_i^*,t}^{-1/(1-\alpha)}}{\hat{\mathbb{E}}[R_{\hat{\lambda}_i^*,t}^{-\alpha/(1-\alpha)}]} , \quad (2.15)$$

<sup>12</sup>In order to study whether the ten-year bond return is a valid proxy for the (unobservable) infinite maturity bond return, a model-based approach could also be applied, e.g., based on a family of affine term structure models on countries' yields. Lustig, Stathopoulos, and Verdelhan (2019) do not find significant differences between the yields of a hypothetical infinite maturity bond and a ten-year bond in such a setting.

<sup>13</sup>In Section 2.4, we consider an alternative specification for the transient and permanent components, using a nonparametric approach as in Christensen (2017). The results remain unchanged.

where  $\hat{\mathbb{E}}[\cdot]$  denotes expectation under the empirical return distribution in country  $i$  and

$$\begin{aligned} R_{\hat{\lambda}_i^*, t} &= R_{i0,t} + \hat{\lambda}_{i1}^*(R_{i1,t} - R_{i0,t}) + \hat{\lambda}_{i2}^*(R_{i\infty,t} - R_{i0,t}) \\ &\quad + \hat{\lambda}_{i3}^*(\tilde{R}_{i0,t} - R_{i0,t}) + \hat{\lambda}_{i4}^*(\tilde{R}_{i\infty,t} - R_{i0,t}) + \hat{\lambda}_{i5}^*(\tilde{R}_{i1,t} - R_{i0,t}) \end{aligned} \quad (2.16)$$

is the optimal portfolio return in market  $i$  with the estimated vector of portfolio weights  $\hat{\lambda}_i^*$  in (2.16) given by the unique solution to the exactly identified set of empirical moment conditions:<sup>14</sup>

$$\hat{\mathbb{E}} \left[ R_{\hat{\lambda}_i^*}^{-1/(1-\alpha)} (\mathbf{R}_i - R_{i0} \mathbf{1}), \right] = 0, \quad (2.17)$$

where  $\mathbf{1}$  is a vector of ones of appropriate size. We estimate parameter vector  $\hat{\lambda}_i^*$  in (2.17) using the method of moments for two separate values of  $\alpha$ : (i)  $\alpha = 2$ , which corresponds to the minimum variance SDF and (ii)  $\alpha = 0$ , which corresponds to the minimum entropy SDF.

**Segmented Markets:** In our segmented markets specification, we assume that investors in each country still have access to the risk-free bond, the long-term bond, and the equity return in their home country, but are restricted to trade only the risk-free bond of the other country. Notice that this is the maximal amount of international segmentation we can impose while still allowing investors to trade the FX carry.

Under such a market structure, the vector of tradable returns in the domestic market is given by  $\mathbf{R}_d = (R_{d0}, R_{d1}, R_{d\infty}, \tilde{R}_{d0})'$ , where, once again,  $\tilde{R}_{d0} = R_{f0}X$  is the domestic currency return of the foreign risk-free asset. Similarly, the vector of tradable returns in the foreign market is given by  $\mathbf{R}_f = (R_{f0}, R_{f1}, R_{f\infty}, \tilde{R}_{f0})'$ , where  $\tilde{R}_{f0} = R_{d0}/X$  denotes the foreign currency return of the domestic risk-free asset. Note that, besides matching the risk premia on the domestic returns, minimum dispersion SDFs in country  $i$  are still forced to match the risk premia on  $\tilde{R}_{i0}$ . This means that all minimum dispersion SDFs exactly match the exchange rate risk premium in the data, thus implicitly incorporating the forward premium anomaly.

Estimating the time series of minimum dispersion SDFs in each country  $i$  is akin to that of integrated markets, with equation (2.15) relating  $\hat{M}_{i,t+1}^*$  to the optimal portfolio return solving (2.5). The key difference with the case of integrated markets, however, is that the estimation

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<sup>14</sup>Note that equation (2.17) represents the first-order conditions associated with optimization problem (2.5). Furthermore, recall that when markets are integrated, investors in each country  $i$  have access to six different assets with returns  $\mathbf{R}_i = (R_{i0}, R_{i\infty}, R_{i1}, \tilde{R}_{i0}, \tilde{R}_{i\infty}, \tilde{R}_{i1})'$ . Therefore, the system of equations (2.17) provides five equations to estimate the five portfolio weights in (2.15) for each country.

of minimum dispersion SDFs is based on a reduced number of moment conditions (2.17), as the equity and long-term bonds are no longer traded internationally. Hence, the estimated optimal portfolio return in each market  $i \in \{d, f\}$  is given by:

$$R_{\hat{\lambda}_i^*, t} = R_{i0, t} + \hat{\lambda}_{i1}^*(R_{i1, t} - R_{i0, t}) + \hat{\lambda}_{i2}^*(R_{i\infty, t} - R_{i0, t}) + \hat{\lambda}_{i3}^*(\tilde{R}_{i0, t} - R_{i0, t}),$$

with  $(\hat{\lambda}_{i1}^*, \hat{\lambda}_{i2}^*, \hat{\lambda}_{i3}^*)$  estimated from the three moment conditions in (2.17).

### 2.2.3 Estimating Minimum Dispersion SDFs

We report the summary statistics of estimated minimum dispersion SDFs in Table 2.1 for the average of each bilateral pair vis-à-vis the US. The left two columns in Panel A and B report moments for minimum entropy and minimum variance SDFs, respectively, when we assume that international investors have access to all assets. Recall that since the domestic and foreign risk-free rate, bond return, and equity return are all priced by domestic and foreign minimum dispersion SDFs, the risk premia of these returns are all matched by construction. In particular, the forward premium anomaly is implicitly incorporated by the pricing properties of our minimum dispersion SDFs, meaning that we exactly match currency risk premia.

**Table 2.1:** Properties of International SDFs

This table reports joint sample moments of the average model-free SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ( $\alpha = 0$ ) and Panel B for minimum variance SDFs ( $\alpha = 2$ ),  $i = d, f$ ,  $j = d, f$ ,  $i \neq j$  in integrated and segmented markets. There is a US domestic SDF for each bilateral trade and we report average statistics. Foreign denotes the average across all other countries in our sample. We use monthly data from January 1975 to December 2015.

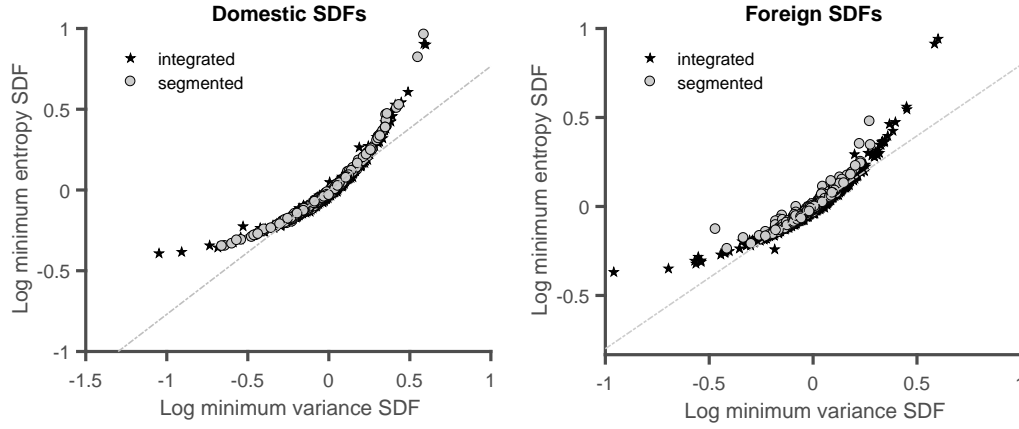
	Panel A: $\alpha = 0$ (minimum entropy)				Panel B: $\alpha = 2$ (minimum variance)			
	Integrated		Segmented		Integrated		Segmented	
	<i>Domestic</i>	<i>Foreign</i>	<i>Domestic</i>	<i>Foreign</i>	<i>Domestic</i>	<i>Foreign</i>	<i>Domestic</i>	<i>Foreign</i>
$\mathbb{E}[M_i]$	0.982	0.975	0.983	0.976	0.982	0.979	0.983	0.975
$\text{Std}(M_i)$	0.791	0.772	0.646	0.566	0.712	0.690	0.596	0.516
$\text{Std}(M_i^T)$	0.120	0.093	0.120	0.093	0.120	0.093	0.120	0.093
$\text{Std}(M_i^P)$	0.869	0.835	0.728	0.639	0.773	0.742	0.667	0.579
$\sqrt{\text{Entropy}(M_i)}$	0.475	0.460	0.399	0.352	–	–	–	–
$\text{Corr}(M_i^T, M_i^P)$	-0.452	-0.466	-0.532	-0.599	-0.504	-0.528	-0.582	-0.665
$\text{Corr}(M_i, M_j)$	–	0.989	–	0.446	–	0.987	–	0.422

When markets are assumed to be integrated, SDF sample volatility is by construction lowest for  $\alpha = 2$  and takes a value of 0.71 for the domestic and 0.69 for the foreign average SDFs. A first important finding is that these volatilities exceed by a large amount the equity Sharpe ratios in each country, which indicates a clearly tightened Hansen and Jagannathan (1991) bound in



integrated markets.<sup>15</sup> While the summary statistics of minimum variance and minimum entropy SDFs are of similar magnitude at first sight, significant differences can be gleaned from Figure 2.1, where we plot both average domestic and foreign minimum variance SDFs against minimum entropy SDFs. We notice that the latter are approximately a non-linear transformation of the former, with differences that become particularly stark in the tails of the SDF distribution.

**Figure 2.1:** Minimum Variance and Minimum Entropy SDFs



The left (right) panel depicts a scatter plot between the average domestic (foreign) log minimum entropy and the log minimum variance SDFs in both integrated and segmented markets. Data is monthly and runs from January 1975 to December 2015.

To understand the properties driving SDF variability in more detail, we next focus on transitory and permanent components. We find that most of the SDF dispersion is generated by the permanent component. This is in line with the US evidence in [Alvarez and Jermann \(2005\)](#). Moreover, the correlation between domestic and foreign SDFs is virtually perfect.<sup>16</sup> Note that while this high co-movement is expected for the minimum entropy SDFs, due to their consistency with the asset market view under integrated markets in Proposition 2 (a), our findings show that it is a general feature of market integration, as it emerges also for the minimum variance SDFs, which do not satisfy the asset market view in general. The properties of international model-free SDF under segmented markets are starkly different. Due to the reduced set of pricing restriction on returns, the variability of minimum dispersion SDF naturally decreases relative to the full integration case. For instance, the variability of minimum variance SDFs drops by 17% and by 26% for domestic and foreign SDFs, respectively. As under integrated markets, permanent SDF components determine the largest fraction of SDF variability and are positively correlated with

<sup>15</sup>The average annualized equity Sharpe ratio is 0.31 for the set of developed countries studied in our paper.

<sup>16</sup>[Lustig, Stathopoulos, and Verdelhan \(2019\)](#) also document high correlations of permanent components internationally, under integrated markets.

the long-term bond return.<sup>17</sup> More importantly, the correlation between US and foreign SDFs is on average much lower than under integrated markets: it is 44% for the minimum entropy and 42% for the minimum variance SDFs, respectively. This is a direct consequence of the more pronounced deviations from the asset market view which arise when markets are segmented. The low correlation is appealing because cross-country correlations of any macroeconomic variables at quarterly or annual frequency are usually found to be moderate.

We can also compare the source of the non-linearities in Figure 2.1 under the two market structures. To this end, we examine the correlation between the optimal growth and mean variance portfolio returns. In integrated markets, the correlation between the two optimal returns is almost perfect (in absolute terms). This correlation drops to around 0.8 in domestic markets and 0.74 in foreign markets (in absolute terms) for the mean variance optimal portfolios under segmented markets. Although the correlation under segmented markets is still substantial, this evidence suggests that such changes in the optimal portfolios are sufficient to generate economically relevant deviations from the asset market view.

#### 2.2.4 International Finance Puzzles

Are estimated model-free SDFs consistent with salient features of exchange rate behavior? We address this question in the context of the three international finance puzzles previously mentioned. Note that since we always allow domestic and foreign investors to trade the short-term bond internationally — i.e., they have access to the FX carry trade — currency risk premia are perfectly matched by construction. In what follows, we therefore focus on the exchange rate volatility puzzle and the cyclical puzzle, i.e., the disconnect between exchange rates and macroeconomic variables.

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<sup>17</sup>Because of the lower dispersion of the permanent SDF components, these correlations are larger in absolute value, in order to match the negative long-term bond risk premia in local currencies. Indeed, the correlation between the transient and permanent SDF components for an SDF  $M$  can be expressed as:

$$\text{Corr}(M^T, M^P) = \frac{\mathbb{E}[M] - \mathbb{E}[M^T]}{\sqrt{\text{Var}(M^T)}\sqrt{\text{Var}(M^P)}} . \quad (2.18)$$

When the distribution of the transient SDF component is fixed by an observable proxy, the numerator is also fixed and the correlation increases in absolute value whenever the volatility of the permanent SDF component decreases.

## Exchange Rate Volatility Puzzle

The high SDF co-movement under integrated markets is related to the volatility puzzle in [Brandt, Cochrane, and Santa-Clara \(2006\)](#). Specifically, whenever the asset market view holds, international SDFs need to be almost perfectly correlated in order to match the low exchange rate volatility. The intuition for this result is typically motivated within the context of complete and integrated markets. However, the evidence in [Table 2.1](#) shows that high SDF correlations arise more broadly also when the asset market view does not hold in incomplete and integrated markets, e.g., for minimum variance SDFs.

To understand this finding, recall that in our integrated market setting, the SDF variability is dominated by the variability of the permanent SDF component. Thus, the almost perfect SDF co-movement can be understood by rearranging terms in [equation \(2.14\)](#), to get the identity:

$$X_t \frac{R_{f\infty,t}}{R_{d\infty,t}} = \frac{M_{f,t}^P}{M_{d,t}^P} e^{\eta_t}. \quad (2.19)$$

As the variability of the left-hand side of [equation \(2.19\)](#) in the data is rather low, identity [\(2.19\)](#) can hold either under a low variability of both the ratio of permanent SDF components and the wedge, under a strong negative co-movement between the ratio of permanent SDF components and the wedge, or under a combination of these effects. An obvious implication is that under the asset market view ( $\eta_{t+1} = 0$ ), permanent components need to be almost perfectly positively related. In contrast, when the market view does not hold, a trade-off between the co-movement of permanent SDF components and the long-run cyclical of exchange rate wedges can emerge.

To study this trade-off in more detail, we estimate the wedge using the closed-form expressions for minimum dispersion SDFs:

$$X_t \exp(-\eta_t) = \frac{R_{\hat{\lambda}_d^*,t}^{1/(1-\alpha)} R_{\hat{\lambda}_f^*,t}^{-1/(1-\alpha)}}{\hat{\mathbb{E}} \left[ R_{\hat{\lambda}_d^*,t}^{-\alpha/(1-\alpha)} \right]^{-1} \hat{\mathbb{E}} \left[ R_{\hat{\lambda}_f^*,t}^{-\alpha/(1-\alpha)} \right]}, \quad (2.20)$$

with the optimal returns defined in [equation \(2.16\)](#). [Table 2.2](#) reports the wedge properties of model-free SDFs in integrated and segmented markets. Panel A reports results for minimum variance SDFs only, as the wedge for the minimum entropy SDF pair vanishes by construction in integrated markets (see [Proposition 2](#)).

Consistent with the above intuition, the wedge variability under integrated markets is an order

**Table 2.2:** Wedge Summary Statistics

This table reports the annualized mean, standard deviation, skewness and kurtosis of the wedge, defined as  $\eta_t = \log \left( \frac{X_t M_{d,t}^*(\alpha)}{M_{f,t}^*(\alpha)} \right)$ , for  $\alpha = 0, 2$ , i.e. for the minimum entropy and variance SDFs. The domestic currency is the US dollar. We report averages across all currency pairs. The SDFs account for the fact that domestic investors can trade any foreign asset (integrated markets, Panel A) or short-term risk-free foreign bonds only (segmented markets, Panel B). We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988.

	$\mathbb{E}[\eta]$	$\text{Std}(\eta)$	$\text{Skew}(\eta)$	$\text{Kurt}(\eta)$
<i>Panel A: Integrated Markets</i>				
$\alpha = 2$	-0.008	0.051	-1.593	4.817
<i>Panel B: Segmented Markets</i>				
$\alpha = 0$	-0.015	0.396	0.176	0.570
$\alpha = 2$	-0.012	0.473	-0.055	0.306

of magnitude smaller than the domestic and foreign SDF variability.<sup>18</sup> Therefore, asset market view deviations under integrated markets are small also for minimum variance SDFs. Moreover, the corresponding permanent SDF components exhibit an almost perfect co-movement in Table 2.3.

In contrast, the asset market view deviations under segmented markets in Panel B of Table 2.2 are large in absolute value, both for minimum entropy and minimum variance SDFs, with wedges that are similarly volatile as minimum dispersion SDFs. Additionally, the correlation between the permanent components is greatly reduced, i.e. they are almost half the one under integrated markets.

In this market setting, the wedge also displays a non-trivial cyclical property with respect to the minimum dispersion SDFs, which is highlighted by the pronounced positive (negative) correlation with the permanent components of domestic (foreign) SDFs in Table 2.4. The wedge co-movement with the transient component is instead typically weaker and of opposite sign for domestic and foreign SDFs.

We summarize this section as follows. Most workhorse models in international finance rely on almost perfectly correlated SDFs to address the exchange rate volatility puzzle. This high correlation, however, is hard to reconcile in the data as cross-country correlations of

<sup>18</sup> Lustig and Verdelhan (2019) show that in order to match the low exchange rate volatility puzzle using equation (2.3), the wedge needs to co-vary positively (negatively) with the domestic (foreign) SDF, i.e., it needs to be pro-cyclical. Given the very low wedge dispersion, these cyclical properties are not particularly insightful under integrated markets and we do not report them here to save space.

**Table 2.3:** Correlation of Permanent SDF Components

This table reports the average correlation between permanent components of domestic and foreign SDFs for  $\alpha = 0$  (minimum entropy) and  $\alpha = 2$  (minimum variance). We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988. Standard errors are computed using a circular block bootstrap of size 10 with 10,000 simulations and reported in square brackets. \*\*\* denotes significance at the 1% level.

	<i>Integrated Markets</i>		<i>Segmented Markets</i>	
	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$
$\text{Corr}(M_d^P, M_f^P)$	0.985***	0.983***	0.580***	0.559***
SE	[0.004]	[0.003]	[0.090]	[0.046]

**Table 2.4:** Correlation Between Wedge and SDFs (Segmented Markets)

This table reports the correlation between the wedge  $\eta$ , the (log) domestic and foreign minimum entropy SDFs ( $\alpha = 0$ ), as well as the log permanent and transient components of minimum entropy SDFs. Log SDFs are denoted by  $m_i = \log M_i$  and log SDF components by  $m_i^U = \log M_i^U$  ( $i = d, f$  and  $U = T, P$ ). We use monthly data from January 1975 to December 2015, except for New Zealand, for which the data starts from January 1988. Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10,000 simulations and reported in square brackets. \*\*\* denotes significance at the 1% level.

	$\text{Corr}(\eta, m_i)$	SE	$\text{Corr}(\eta, m_i^P)$	SE	$\text{Corr}(\eta, m_i^T)$	SE
Domestic	0.552***	[0.046]	0.548***	[0.045]	-0.271***	[0.053]
Foreign	-0.508***	[0.059]	-0.520***	[0.059]	0.385***	[0.052]

macroeconomic fundamentals are orders of magnitude lower. Market incompleteness is attractive in this context as it can break the mechanical link between exchange rates and SDFs. Our results, however, show that even in incomplete integrated markets, this correlation needs to be almost perfect, as deviations from the asset market view are small. In contrast, segmented economies where investors can only trade the risk-free bond internationally feature volatile exchange rate wedges and low correlations between international SDFs, while still addressing the exchange rate volatility puzzle.

### Cyclicalities Puzzle

We now study the exchange rate cyclicalities properties by means of the following regressions:

$$\begin{aligned}
m_{f,t} - m_{d,t} &= \delta + \beta x_t + u_t, \\
m_{f,t}^U - m_{d,t}^U &= \delta^U + \beta^U x_t + u_t^U,
\end{aligned}$$

for  $U = T, P$ , where we regress both the difference between log foreign and domestic SDFs and their log transient and permanent components on the log exchange rate return  $x_t$ .

**Table 2.5:** Cyclicalities Regressions

This table reports the point estimates of a linear regression of the log difference between average foreign and average domestic SDFs on the log real exchange rate return:  $m_{f,t} - m_{d,t} = \delta + \beta x_t + u_t$ , where small-cap letters denote quantities in logs. We additionally report average point estimates of a linear regression of the log difference of each component of the SDF on the log change in the real exchange rate:  $m_{f,t}^U - m_{d,t}^U = \delta^U + \beta^U x_t + u_t^U$ , where  $U = P, T$  for permanent and transitory components, respectively. We use monthly data from January 1975 to December 2015. Standard errors are reported in square brackets. \*\*\* highlights significance at the 1% level.

	<i>Panel A: Integrated</i>		<i>Panel B: Segmented</i>	
	$\alpha = 0$	$\alpha = 2$	$\alpha = 0$	$\alpha = 2$
$\beta$	1.000*** [0.000]	1.097*** [0.079]	0.969*** [0.311]	1.200*** [0.365]
$\beta^P$	0.981*** [0.069]	1.076*** [0.113]	0.950*** [0.353]	1.181*** [0.409]
$\beta^T$	0.019 [0.069]	0.019 [0.069]	0.019 [0.069]	0.019 [0.069]

When the asset market view holds, the population point estimate from these regressions based on the overall SDFs is exactly one, which is the case emerging for minimum entropy SDFs under integrated markets. More generally, [Lustig and Verdelhan \(2019\)](#) show that the same finding holds under incomplete markets when risk-free returns are traded internationally. Therefore, we expect similar results also for minimum variance SDFs. Finally, since the SDF variability in [Table 2.1](#) is dominated by the permanent component, we anticipate analogous implications for regressions using the persistent SDF components.

Panel A of [Table 2.5](#) reports average cyclicalities slope coefficients in integrated markets across the different currency pairs. For both dispersion measures, the regressions with  $m_{f,t} - m_{d,t}$  and  $m_{f,t}^P - m_{d,t}^P$  produce estimated coefficients that are positive, highly significant, and close to one, with estimates based on the permanent component that are almost indistinguishable from the total SDF estimates. Turning to the regressions with transitory SDF components, we obtain estimated coefficients that are statistically not different from zero. Hence, we conclude that also the cyclicalities puzzle can be explained in a setting of integrated markets, by a transitory component that is largely unrelated to exchange rate changes.

Panel B of [Table 2.5](#) reports cyclicalities slope coefficients in segmented markets. The results

are in line with those obtained assuming market integration, although the coefficients display now larger confidence intervals. In particular, the estimates in Panel B show that indeed all point estimates are significantly different from zero and never significantly different from the target value of one, except for the transient component, which is unrelated with exchange rate changes.

Together with the previous results, we conclude that the model-free SDFs of both integrated and segmented market settings are consistent with the three exchange rate puzzles. However, the large SDF dispersions and correlations under integrated market may potentially present a challenge for asset pricing models that assume the validity of the asset market view. One effective way to lower the dispersion and correlation of international SDFs is by means of some form of market segmentation. In this setup, martingale SDF components across countries are less volatile and only weakly correlated, while differences in transient SDF components are disconnected from exchange rate variations.

### 2.2.5 Unspanned Exchange Rate Risks

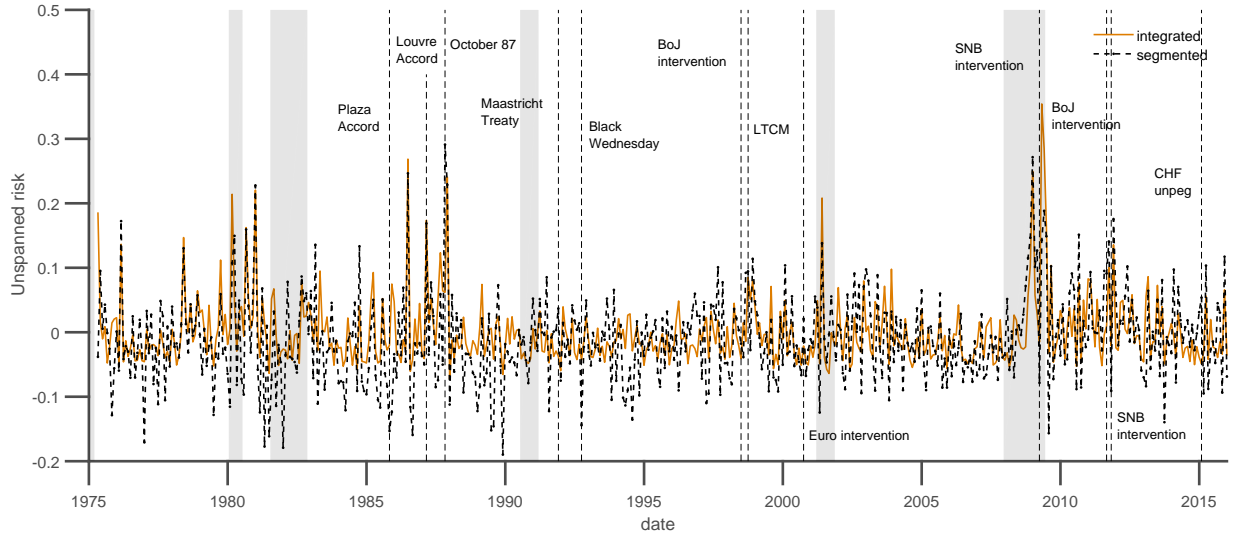
Recall from Corollary 1 that the stochastic wedge in incomplete markets measures the amount of unspanned or uninsurable exchange rate risks. While the asset market view holds in integrated markets for minimum entropy SDFs, the latter cannot be traded using only basic assets. The minimum variance SDF, however, can be traded directly, since it is a linear function of the available returns. In the following, we quantify the amount of unspanned exchange rate risk both under integrated and under segmented long-term bond and equity markets.

To uncover the time-series behavior of unspanned risks, we plot in Figure 2.2 the average unspanned risk across the different currency pairs vis-à-vis the USD in segmented and integrated markets.<sup>19</sup> There are two noteworthy observations. First, substantial spikes in unspanned risks occur during major financial market events in both market settings. Second, the time series behavior of unspanned risks contrasts sharply across different market structures: while in integrated markets the largest spikes are positive, in segmented markets they tend to be both negative and positive. This evidence is confirmed in Figure 2.3, where we plot the empirical

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<sup>19</sup>Note that the interpretation of unspanned exchange rate risks can be extended to minimum variance SDFs obtained in segmented markets, because we can always embed a segmented international market into a nesting integrated market. We make use of this insight to identify unspanned exchange rate risks in segmented markets in our empirical analysis.

**Figure 2.2:** Time-Series of Unspanned Risks

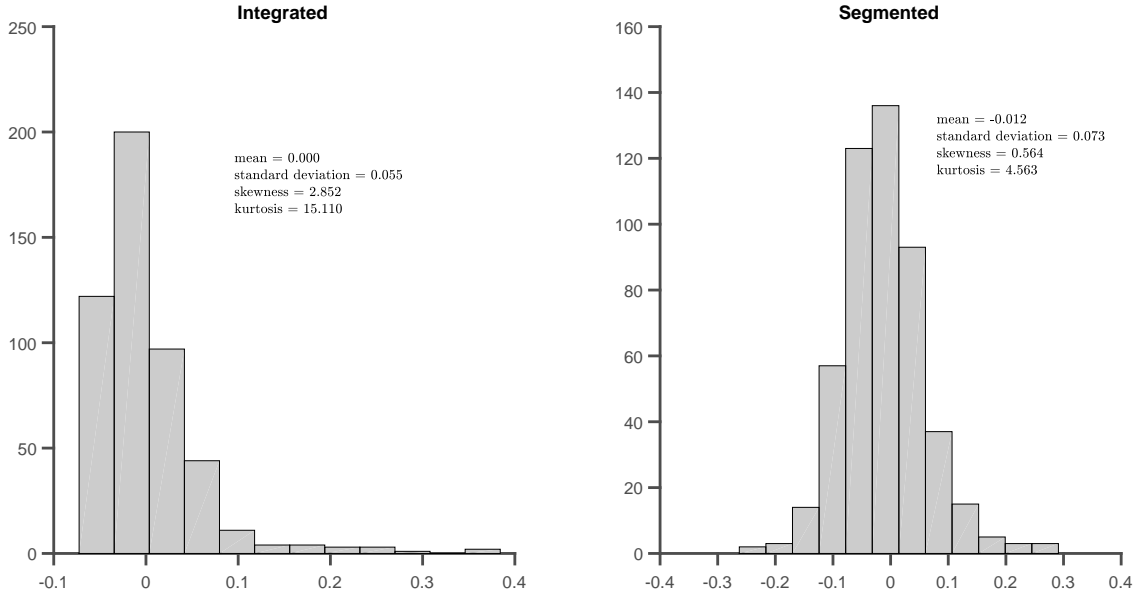


The figure plots the average US unspanned component, computed as  $M_d(0) - M_d(2)$ , of all currency pairs vis-à-vis the USD, both in segmented and integrated markets. Data is monthly and runs from January 1975 to December 2015. Gray shaded areas highlight recessions as defined by the NBER.

distribution of the average US unspanned risks under integrated (left panel) and segmented (right panel) markets. Indeed, the distribution of unspanned risks in integrated markets is clearly more positively skewed. This positive skewness of unspanned risks reflects times in which the minimum entropy SDFs considerably exceed the minimum variance SDFs in instances where asset returns are particularly low, emphasizing the role of higher-order moments. The differential behavior of unspanned risks under various market structures has implications also for the type of assets that are needed to replicate or hedge these risks. For example, the unspanned risk under integrated markets resembles a typical “insurance”-like contract, with payoffs that spike in bad times, similar to the payoffs of out-of-the-money FX put options. In contrast, unspanned risks under segmented markets feature a less pronounced positive skewness and a higher probability of negative risk realizations. Such risks are therefore better replicated by the so-called FX risk reversal strategies, which are long short portfolios of out-of-the-money FX call and put options, or FX butterfly spreads, which are portfolios long out-of-the-money FX calls and puts and short at-the-money FX calls and puts; see, e.g., [Carr and Wu \(2007\)](#) and [Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan \(2015\)](#).



**Figure 2.3:** Distribution of Unspanned Risk



The left panel plots the empirical distribution of the average unspanned US risk in integrated markets. The right panel plots the distribution of the average unspanned US risk in segmented markets. Data is monthly and runs from January 1975 to December 2015.

### 2.2.6 Cross-Country SDF Correlations

In order to better understand the impact of market structures on SDF properties, it is useful to contrast our approach to the one in [Bakshi, Cerrato, and Crosby \(2018\)](#). In their paper, the authors minimize international SDF co-movement subject to a “good-deal bound” in a hypothetical economy with integrated markets and a common numéraire. Low cross-country SDF correlations are achieved by means of sufficiently volatile unspanned exchange rate risks, which are orthogonal to the spaces of domestic and foreign returns, and perfectly negatively correlated. The latter feature seems hard to reconcile with empirical evidence in international finance. For instance, [Farhi et al. \(2015\)](#) document that out-of-the-money put options of high interest rate currencies jointly become more expensive after 2008. Similarly, [Mueller et al. \(2017\)](#) document high option-implied correlations during recession periods.

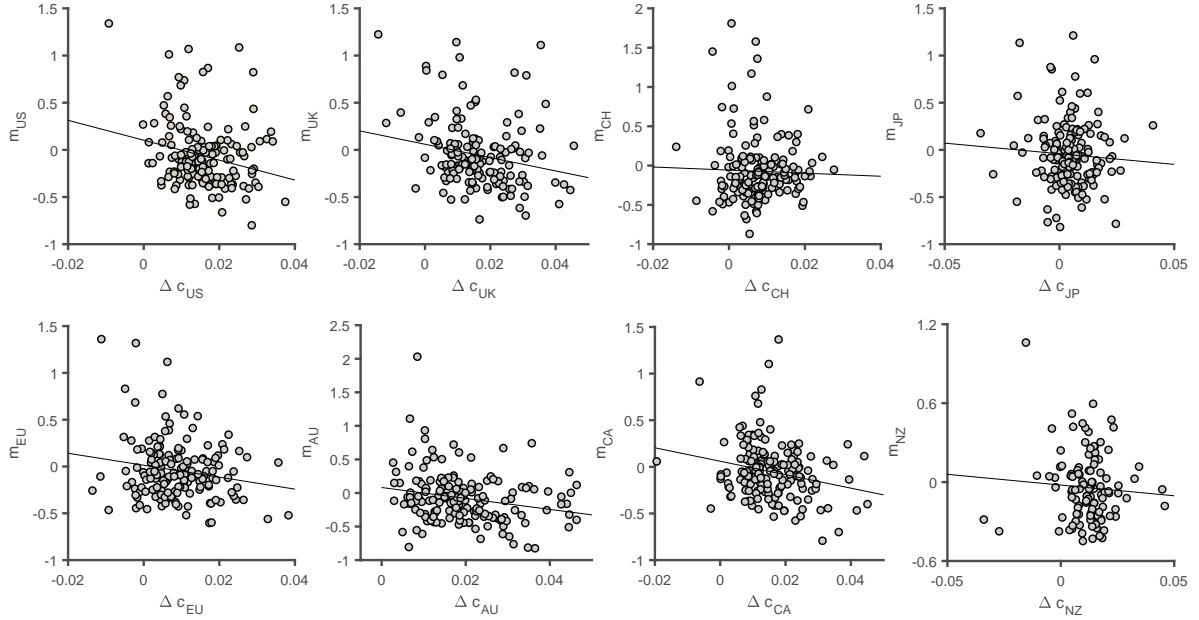
The approach in [Bakshi, Cerrato, and Crosby \(2018\)](#) is conceptually and economically quite different from ours. Recall from [Proposition 1](#), that domestic and foreign minimum dispersion SDFs are equivalent to the solutions of an optimal portfolio problem for domestic and foreign investors. Hence, in our setting the low correlation under segmented markets is not the result of an optimization that imposes ex-ante a low SDF co-movement.

In summary, while it is always possible to generate weakly correlated SDFs in integrated incomplete international financial markets, the resulting restrictions imposed on the stochastic wedge may imply empirically counter-factual properties. We document that a more natural way to obtain a low SDF correlation is to assume some degree of market segmentation.<sup>20</sup>

## 2.3 Intermediaries in International Financial Markets

Most canonical models in international finance rely on a consumption-based SDF. As a first illustration, we depict in Figure 2.4, scatter plots of log minimum entropy SDFs and log consumption growth, for every country in our sample. Not surprisingly, the correlation between these series is close to zero or mildly negative.

**Figure 2.4:** International SDFs and Consumption Growth



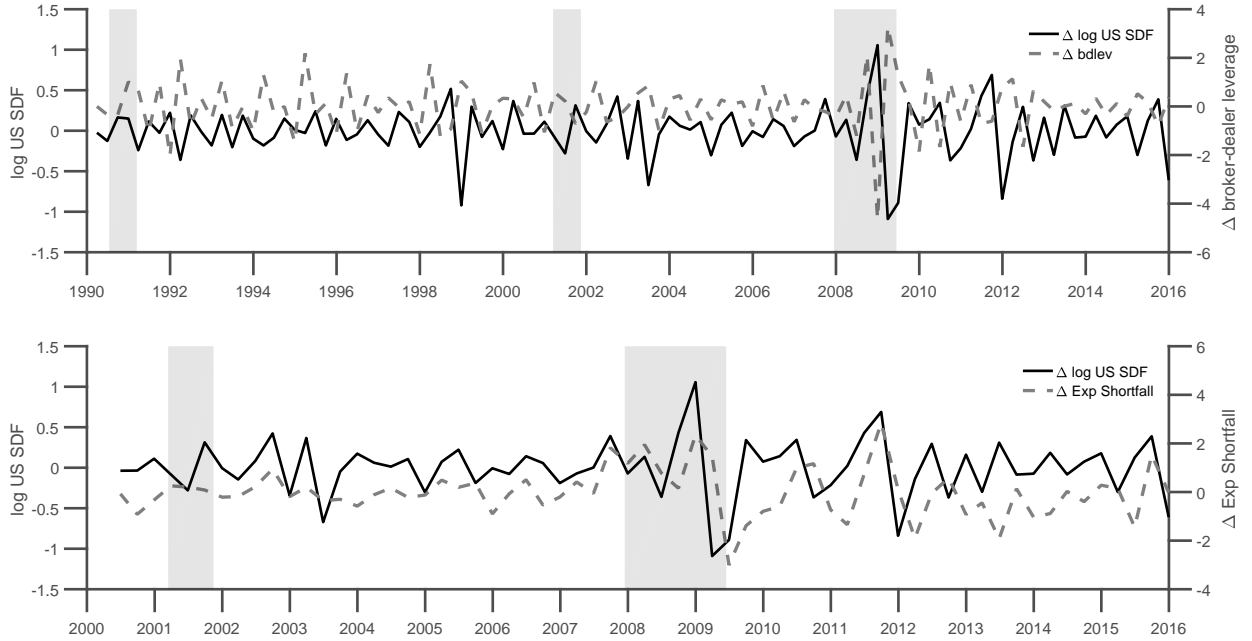
The figure plots (log) minimum entropy SDFs for the eight countries (y-axis) together with (log) consumption growth (x-axis) and the fitted least-square line. Data is quarterly and runs from January 1975 to December 2015, except for New Zealand, for which the sample starts in January 1988.

A broader look at other economic variables related to model-free SDFs is motivated by the growing literature documenting the importance of financial intermediaries for asset prices, particularly in markets with complex financial assets, such as credit default swaps, sovereign

<sup>20</sup>Moreover, in integrated markets, domestic and foreign unspanned risks feature an almost perfect correlation, whereas under segmented markets, this correlation is significantly reduced to around 0.37.

bonds, and FX; see, e.g., [Haddad and Muir \(2018a\)](#).<sup>21</sup> Typically, the role of these institutions is to intermediate financial transactions between two groups of investors in otherwise segmented markets. Segmentation is usually assumed exogenously, arising because of regulation, agency problems, or lack of specialized knowledge.<sup>22</sup>

**Figure 2.5: US SDF and Intermediary Constraints**



The figure plots the average (log) minimum entropy US SDF together with changes in broker dealer leverage (upper panel) and intermediaries' expected shortfall. Broker-dealer leverage is a composite measure from [Adrian, Etula, and Muir \(2014a\)](#) and, the intermediary equity measure of [He, Kelly, and Manela \(2017a\)](#), while expected shortfall is the average expected shortfall of 15 US and foreign intermediaries from [Brownlees and Engle \(2017\)](#) and [Acharya, Pedersen, Philippon, and Richardson \(2017\)](#). Data is quarterly and runs from January 1990 to December 2015 (upper panel) and from June 2000 to December 2015 (lower panel). Gray shaded areas highlight recessions as defined by the NBER.

We next explore in more detail the links between model-free SDFs and empirical measures of financial intermediaries' constraints. We rely on two established proxies of intermediary constraints: First, we construct a composite measure of the broker-dealer measure of [Adrian, Etula, and Muir \(2014a\)](#) and the intermediary equity measure of [He, Kelly, and Manela \(2017a\)](#).<sup>23</sup>

<sup>21</sup> [He and Krishnamurthy \(2018\)](#) review several settings in which financial intermediaries matter for FX markets.

<sup>22</sup> [Malamud and Schrimpf \(2018\)](#), on the other hand, study a model with endogenous international market segmentation due to intermediaries' markups.

<sup>23</sup> The former measure is defined as the ratio between broker dealers' total financial assets and the difference between total financial assets and liabilities available from the Federal Reserve Flow of Funds tables. The latter is defined as the aggregate value of market equity divided by the sum of the aggregate market equity and the aggregate book debt of primary dealers who serve as counterparties of the Federal Reserve Bank of New York. Following [Haddad and Muir \(2018a\)](#), we standardize both measures and take their average, to reflect the mean

Notice that the intermediary equity measure of [He, Kelly, and Manela \(2017a\)](#) is based on information on primary dealers who are large and sophisticated international financial institutions that operate in virtually the entire universe of international capital markets. In particular, only around 1/3 of all institutions are headquartered in the US, the rest being mainly from Europe, Japan, and Canada.<sup>24</sup> Second, we also use measures of expected shortfall for 15 US and foreign financial institutions from [Brownlees and Engle \(2017\)](#) and [Acharya, Pedersen, Philippon, and Richardson \(2017\)](#).<sup>25</sup>

The top panel of Figure 2.5 plots changes in the average log US minimum entropy SDF and changes in broker-dealer leverage. Interestingly, we find a strong negative co-movement between model-free SDFs and broker-dealer leverage. In the lower panel of Figure 2.5, we depict changes in the average log US minimum entropy SDF together with changes in the average expected shortfall of large financial intermediaries in the FX market. Again, we find a strikingly high (positive) correlation between the two series.

To study the relationship more formally, we present in Table 2.6 estimated coefficients from regressing changes in domestic (Panel A) and foreign (Panel B) SDFs onto changes in broker-dealer leverage or capital shortfall:<sup>26</sup>

$$\Delta \ln M_{i,t} = \beta_l \Delta \text{bdlev}_t + \beta_s \Delta \text{CS}_t + \beta_v \Delta \text{VIX}_t + \beta_b \Delta \text{BS}_t + \epsilon_{i,t}. \quad (2.21)$$

In addition to these two measures, we also consider changes in the VIX and changes in the (inverse) butterfly spread from currency options. We include these two measures for two reasons. First, VIX is used as a proxy of global intermediaries' leverage constraints in the works of [Adrian and Shin \(2014\)](#), [Bruno and Shin \(2015\)](#), or [Miranda-Agrippino and Rey \(2018\)](#), among many others. Second, market segmentation can arise for various reasons, such as the existence of

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of the risk bearing capacity measures used in the literature.

<sup>24</sup> A current list of primary dealers can be found here <http://www.newyorkfed.org/markets/primarydealers>

<sup>25</sup> The simulation-based measure in [Brownlees and Engle \(2017\)](#) merges balance-sheet together with market data to estimate a conditional capital shortfall of a firm. More specifically, it is calculated from the size of the firm, its degree of leverage, and its expected equity loss conditional on a market decline. As prudential capital ratio, the authors assume a 8% capital ratio in the US and 5.5% elsewhere. To decide which institutions to include in our sample, we rely on Euromoney, which conducts an annual survey of the largest market participants in the FX market, see, e.g., <http://www.euromoney.com/article/b18bzd2g51lqkn/fx-survey-2018-overall-results>. From this list, we use measures of expected capital shortfall for Bank of America, JP Morgan, Goldman Sachs, Citibank, State Street, Morgan Stanley, representing US institutions, and UBS, Deutsche Bank, HSBC, Standard Chartered, Barclays, BNP Paribas, Commerzbank, Credit Suisse, and Nomura representing foreign institutions. We then sum over all expected shortfall measures to get an aggregate proxy. Data on expected shortfall was graciously provided by the V-Lab at New York University.

<sup>26</sup> To make estimated coefficients comparable across different regressors, we demean and divide all variables by their respective standard deviations.

frictions in international financial markets or limited participation of some investors, as trading complex assets requires some expertise. Recall that the empirical properties of the unspanned risks presented in Section 2.2.5 strongly resemble payoffs of out-of-the-money FX put options or FX butterfly spreads. Since replicating non-linear payoffs requires some degree of financial sophistication, we assume that financial intermediaries are uniquely positioned in this respect.<sup>27</sup>

**Table 2.6:** Model-Free SDFs and Intermediary Constraints

The table reports estimated coefficients from regressing the average US and foreign minimum entropy SDFs on changes in intermediaries' leverage (bdlev), expected capital shortfall (CS), changes in VIX, and (inverse of) the FX butterfly spread:  $\Delta \ln M_{i,t} = \beta_l \Delta \text{bdlev}_t + \beta_s \Delta \text{CS}_t + \beta_v \Delta \text{VIX}_t + \beta_b \Delta \text{BS}_t + \epsilon_{i,t}$ . All variables are standardized, i.e., for each variable we subtract the mean and divide by the standard deviation.  $t$ -statistics are calculated according to Newey and West (1987) and reported in parenthesis. Data runs from January 1990 to December 2015, except for capital shortfall (butterfly spread) for which the data runs from June 2000 (January 1996) to December 2015 (December 2013).

<i>Panel A: Domestic</i>										
	Integrated				Segmented					
$\Delta \text{bdlev}$	-0.458				-0.482	-0.447				-0.485
$t$ -statistic	(-4.07)				(-3.33)	(-3.59)				(-3.45)
$\Delta \text{CS}$		0.284			0.212		0.319			0.253
$t$ -statistic		(2.56)			(2.50)		(2.51)			(3.09)
$\Delta \text{VIX}$			0.553					0.506		
$t$ -statistic			(6.77)					(5.75)		
$\Delta \text{BS}$				0.579					0.580	
$t$ -statistic				(8.69)					(6.83)	
$R^2$	21.01%	8.06%	30.60%	33.52%	32.05%	19.95%	10.17%	25.60%	33.66%	35.41 %
<i>Panel B: Foreign</i>										
	Integrated				Segmented					
$\Delta \text{bdlev}$	-0.437				-0.465	-0.394				-0.394
$t$ -statistic	(-3.79)				(-3.13)	(-4.33)				(-2.73)
$\Delta \text{CS}$		0.283			0.222		0.212			0.174
$t$ -statistic		(2.59)			(2.75)		(2.01)			(1.74)
$\Delta \text{VIX}$			0.560					0.507		
$t$ -statistic			(7.38)					(4.78)		
$\Delta \text{BS}$				0.570					0.468	
$t$ -statistic				(8.36)					(7.21)	
$R^2$	19.12%	7.98%	31.35%	32.44%	30.95%	15.53%	4.50%	25.70%	21.88%	20.74%

We present regression results in Table 2.6. We notice that all estimated coefficients are highly statistically significant and carry the expected signs. For example, as the expected short-fall goes up in both domestic and foreign countries, changes in SDFs increase. Similarly, increases in VIX, which indicate a tightening of value-at-risk constraints, lead to increases in international SDFs

<sup>27</sup>This intuition is in line with Gromb and Vayanos (2018) who argue that segmentation may arise because less sophisticated investors lack specialized knowledge or trading infrastructure to access more complex products such as derivatives. See also Eisfeldt, Lustig, and Zhang (2018) for an equilibrium model in a market for complex assets where segmentation arises endogenously due to expertise.

both in segmented and integrated markets. Moreover, there is a strong negative relation with the broker dealer leverage: as the risk-bearing capacity of intermediaries shrinks, international SDFs increase. The  $R^2$ s of univariate regressions range between 5% and 34%, whereas in the multivariate regressions, the broker dealer leverage and expected shortfall explain between 21% and 35% of the variation in model-free SDFs.

## 2.4 Robustness

In this section, we perform four robustness checks to our main analysis. First, we study the effects of including emerging market economies to our sample. Second, we assess the impact of different subsamples on our results. Third, we explore non-parametric estimates for the long-run factorization of international SDFs and fourth, we explore differences across US and foreign-based intermediary proxies.

### 2.4.1 Emerging Markets

Our main analysis covers developed countries, which are likely to be less segmented from the US than emerging market economies; see, e.g., [Bekaert and Harvey \(2017\)](#), among many others. Therefore, we reproduce our main results using the following emerging countries: Brazil, Czech Republic, Hungary, Indonesia, Malaysia, Mexico, Philippines, Poland, Singapore, South Africa, South Korea, and Thailand. Since data quality (especially on long-term bonds) is not very reliable in the early sample, we start our analysis in 2005 instead.

We first estimate minimum-dispersion SDFs on the emerging markets data. For brevity, we do not report all results here, as we find summary statistics of emerging markets SDFs to be very similar to those obtained for developed markets.<sup>28</sup> We then address the properties of unspanned FX risks for emerging market economies and report the corresponding summary statistics in [Table 2.7](#). For comparison, we also report the summary statistics for average unspanned FX risk vis-à-vis the US in developed countries, for an overlapping sample. Also in this case, we notice a high similarity between the developed and emerging markets results, as all moments of unspanned risks are almost identical. The last row of [Table 2.7](#) focuses on a comparison of the comovement between domestic and foreign unspanned risks in developed and in emerging markets, under the same market segmentation structures introduced earlier. Interestingly, we

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<sup>28</sup>For example, the null of an  $F$ -test for equal variances across developed and emerging market SDFs cannot be rejected. We therefore conclude that the main marginal properties of emerging market SDFs are not significantly different from those in developed markets, at least with regards to their first and second moments.

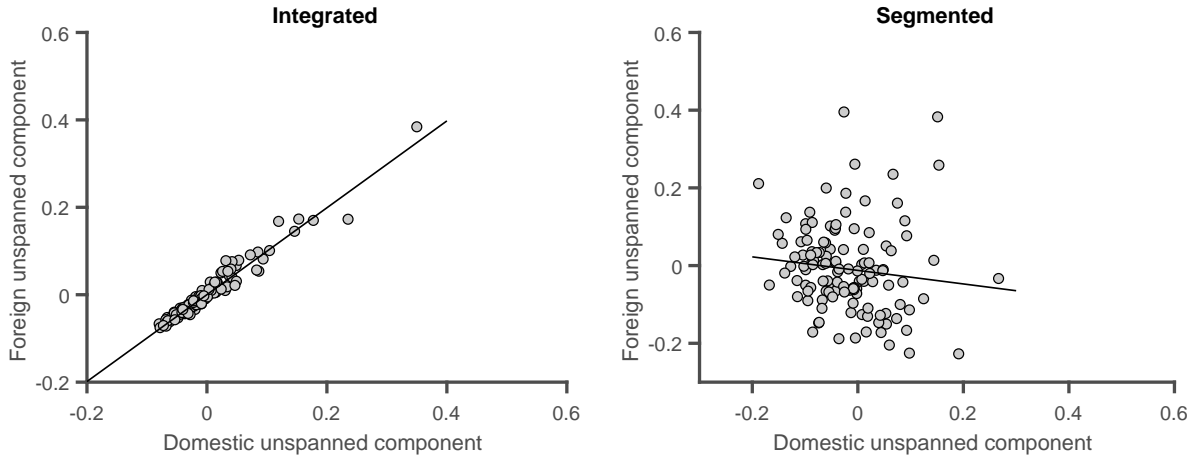
find a very significant drop in the correlations of unspanned domestic and foreign risks from integrated to segmented market settings. Finally, we depict in Figure 2.6 the scatter plot of domestic against foreign unspanned components for emerging markets. Again, we notice a strong correlation between unspanned components under integrated markets and a weak negative (albeit insignificant) correlation under segmented markets. Overall, we conclude that the properties of emerging market model-free SDFs are not very different from those of developed markets model-free SDFs, once the comparison is based on identical assumptions regarding the degree of international market segmentation.

**Table 2.7:** Summary Statistics Unspanned Components

The table reports summary statistics (mean, standard deviation, skewness, and kurtosis) for the average US unspanned exchange rate risk vis-à-vis either developed or emerging market countries.  $\bar{\rho}$  indicates the average correlation among all developed or emerging market domestic against foreign unspanned components in either integrated or segmented markets. The last line reports the corresponding  $p$ -value of a test of no significant correlation. Data runs from August 2005 to December 2015.

	Developed markets		Emerging markets	
	<i>Integrated</i>	<i>Segmented</i>	<i>Integrated</i>	<i>Segmented</i>
Mean	0.006	0.004	0.000	-0.019
Std	0.064	0.073	0.061	0.077
Skewness	2.694	0.750	2.525	0.695
Kurtosis	12.842	3.649	12.657	3.931
$\bar{\rho}$	0.99	-0.13	0.98	-0.22
	[0.000]	[0.136]	[0.000]	[0.016]

**Figure 2.6:** Unspanned Components in Emerging Markets



The figure shows the scatter plot of domestic against foreign unspanned components across all emerging market currency pairs vis-à-vis the US, computed as  $M_i^*(0) - M_i^*(2)$ , assuming market integration (left panel) and assuming market segmentation (right panel). Data runs from August 2005 to December 2015.

### 2.4.2 Subsamples

One might be worried that our results are unduly affected by the failure of covered interest rate parity (CIP) in our sample (see, e.g., [Du, Tepper, and Verdelhan \(2018\)](#)). However, it is unlikely that there is any material impact for the following reasons. First, our approach requires no arbitrage to hold for equities, short- and long-term bonds only, and we do not impose the correct pricing of other assets (such as FX forward contracts). Second, a violation of the no-arbitrage condition (2.4) is equivalent to non-existence of a strictly positive SDF. In our data sample, estimated SDFs are always strongly strictly positive (i.e., far from zero), therefore ruling out the possibility of arbitrage opportunities for the set of traded assets we consider. Third, CIP violations are usually documented in money market rates other than the government bond market data that we employ, e.g., interbank offer rates, overnight-index swap rates, or general collateral rates; see [Rime, Schrimpf, and Syrtstad \(2017\)](#) and [Du, Tepper, and Verdelhan \(2018\)](#), among others. Nevertheless, to check the potential impact of the sample choice, we reproduce our main tables based on a period ending in 2008 and find that our results remain qualitatively and quantitatively the same. For brevity, we do not report these results here.

Another concern may be that our results in Table 2.6 are affected by outliers, as for example during the 2008 crisis period. To address this issue, we run the same regressions but exclude the 2008 crisis, which corresponds to the largest change in proxy of intermediary constraints. We find the estimated coefficients to be highly statistically significant, as in the full sample period, however, the average  $R^2$  drops by 2% to 16%. To save space, we report these results in the online appendix.

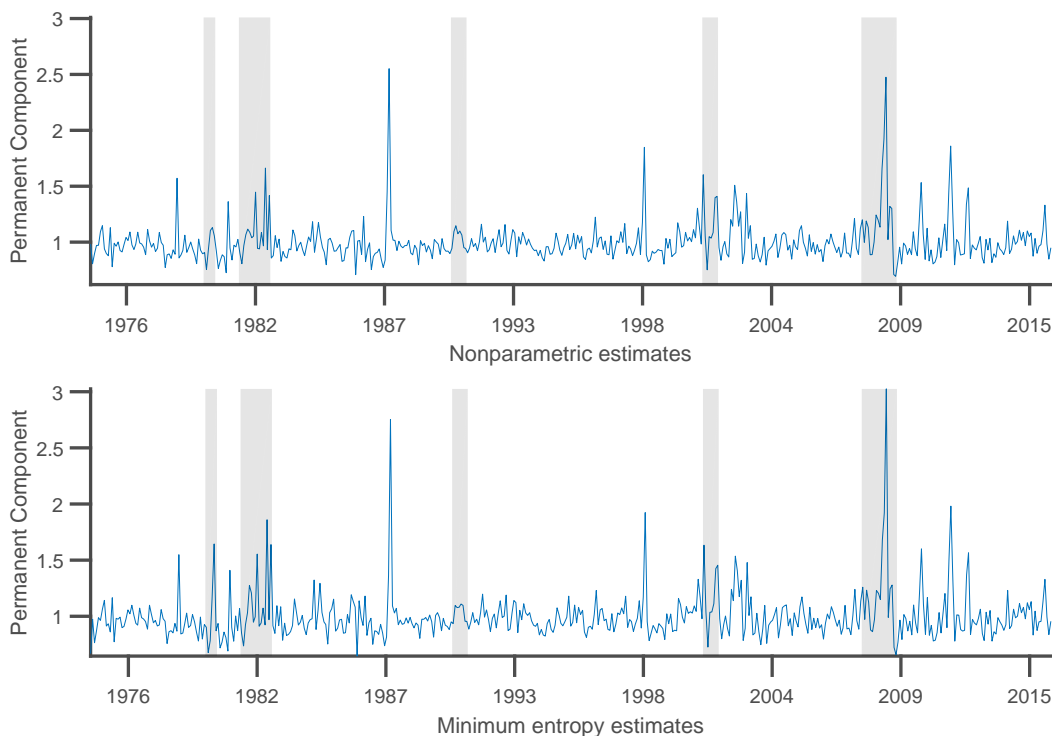
### 2.4.3 Long-Run SDF Factorizations

In our empirical analysis, we factorize minimum dispersion SDFs into transitory and permanent components using as a proxy for the unobservable return of infinite maturity bonds the long, but finite-horizon bond return. While this is of course an approximation, note that this SDF decomposition can impact our main analysis only in terms of the exchange rate cyclical patterns and the corresponding cyclical regression results. Intuitively, given the dominating role of martingale components in minimum dispersion SDFs, it is unlikely that this approximation error can substantially change our findings. Indeed, model-based evidence from estimated



affine term structure models tends to support this intuition.<sup>29</sup> However, to formally address these concerns, we follow [Christensen \(2017\)](#) and estimate non-parametrically the solution of the Perron-Frobenius eigenfunction problem that uniquely characterizes the long-run SDF factorization in [Hansen and Scheinkman \(2009\)](#). We defer exhaustive estimation details to the online appendix and report in [Figure 2.7](#) the time-series of estimated average permanent SDF components, based on both the approach of [Section 2.2.2](#) and the non-parametric methodology described in the online appendix.

**Figure 2.7:** Minimum Entropy and Nonparametric Permanent Components



The figure plots the time-series of non-parametric estimates of the permanent component (upper panel) and the minimum entropy permanent component (lower panel) averaged across different currencies vis-à-vis the USD. Data is monthly and runs from January 1975 to December 2015. Gray shaded areas highlight recessions as defined by the NBER.

The two series exhibit similar time series properties, with large common spikes during crises and recession periods, and are almost perfectly correlated. While the unconditional dispersion of the two permanent components is similar and generates in both cases the largest fraction of the overall SDF volatility, the non-parametric component has a consistently slightly lower volatility

<sup>29</sup>See again [Lustig, Stathopoulos, and Verdelhan \(2019\)](#), who do not obtain large differences between the yields of a hypothetical infinite maturity bond and a ten-year bond in such settings.

than the overall SDF volatility. This implies a positive risk premium for the infinite maturity bond return resulting from the non-parametric SDF factorization.<sup>30</sup> Given the dominating role of permanent SDF components under both approaches, these findings imply overall unchanged exchange rate cyclical patterns and cyclical regression results, using either of the two proxies for the permanent SDF components.<sup>31</sup> In particular, the cyclical puzzle is addressed by means of the transitory component, which is unrelated with changes in exchange rates.

#### 2.4.4 Intermediary Constraints

The main proxy for intermediary constraints is based on the primary dealers' equity measure of He, Kelly, and Manela (2017a). One question is whether these primary dealers are also the relevant players in the FX market. Data on dealer identities are in general hard to obtain for the FX market since the vast majority of trades happens over-the-counter. However, perhaps not very surprisingly, survey results indicate that there is a large overlap between the list of primary dealers and major dealers in the FX market.<sup>32</sup>

For robustness, we construct leverage ratios for domestic and foreign intermediaries from the survey using data from Compustat, CRSP, Bloomberg, and Bankscope. More specifically, following the approach in He, Kelly, and Manela (2017a), we construct a leverage ratio for domestic and foreign financial institutions as follows:

$$\text{leverage}_{i,t} = \frac{\sum_i \text{market equity}_{i,t}}{\sum_i (\text{market equity}_{i,t} + \text{book debt}_{i,t})}, \quad i = d, f,$$

where we sum over either all US or foreign institutions.<sup>33</sup> We find that the correlation between foreign and domestic leverage ratios is high (94%).<sup>34</sup> The correlation of foreign and domestic leverage ratios with the He, Kelly, and Manela (2017a) measure is also large (96%). To check whether there is any material difference between the results using the various measures, we run

<sup>30</sup>This finding for the sign of the risk premium of infinite maturity bond returns is consistent with the empirical evidence in Bakshi and Chabi-Yo (2012). The volatilities of the non-parametric permanent SDF components are reported in the online appendix.

<sup>31</sup>To save space, we do not report detailed results here.

<sup>32</sup>See, e.g., <http://www.euromoney.com/article/b18bzd2g51lqkn/fx-survey-2018-overall-results>

<sup>33</sup>We define as domestic institutions: Bank of America, JPMorgan, Goldman Sachs, Citibank, State Street, Morgan Stanley. As foreign banks we define: UBS, Deutsche Bank, HSBC, Standard Chartered, Barclays, BNP Paribas, Commerzbank, Credit Suisse, and Nomura.

<sup>34</sup>He, Kelly, and Manela (2017a) define the capital ratio as the inverse of market leverage, we therefore also take the inverse to make results comparable.

univariate regressions akin to equation (2.21). In interest of space, we do not report regression results here but relegate them to the online appendix. We find the estimated coefficient on these leverage ratios to be highly statistically different from zero and similar in size to those reported in Table 2.6.

## 2.5 Conclusions

In this paper, we provide a parsimonious model-free framework to study the asset pricing implications of different degrees of international financial market segmentation. Our main contributions can be summarized as follows.

First, we theoretically show that, when international financial markets are integrated, the minimum entropy SDF pair always satisfies the asset market view of exchange rates, irrespective of the extent of market completeness. This means that, as long as markets are integrated, exchange rate risks can always be uniquely pinned down by the ratio of international SDFs (as is the case in complete markets). Second, we establish that a stochastic wedge arises for any other pair of minimum dispersion SDFs, including the minimum variance SDFs. Third, we argue that the stochastic wedge with respect to the minimum variance SDF pair is interpretable as a measure of unspanned exchange rate risks in international financial markets.

We then use our characterization results to estimate the minimum entropy and minimum variance SDFs from the data, assuming various market structures. We find that, to explain puzzles in international macro finance, SDFs need to satisfy two key properties: (i) large permanent SDF components that are correlated across countries and (ii) internationally traded risk-free bonds. When markets are integrated, we obtain highly volatile minimum dispersion SDFs that are almost perfectly correlated internationally. Such high correlations reflect non-existent and minuscule deviations from the asset market view for minimum entropy and minimum variance SDF pairs, respectively. Market segmentation, on the other hand, lowers both the variability and the co-movement of minimum dispersion SDFs, by generating larger deviations from the asset market view. Given the strong support for market segmentation in the data (e.g., [Maggiori, Neiman, and Schreger \(2019\)](#)), this finding offers a promising characterization of salient properties of international SDFs.

Our findings speak to a growing intermediary asset pricing literature that emphasizes the important role played by financiers in international financial markets. In this context, we did

not explicitly model the presence of financial frictions, such as transaction costs or margin and collateral constraints, on the intermediary SDF. Studying the implications of financial frictions for international SDFs in such settings is an exciting topic that we leave for future research.

# Appendix A

## Appendix

### A.1 Proofs and Derivations

**Proof of Proposition 1:** We first prove the result for the case of  $\alpha \neq 0$ . When  $\alpha \neq 0$ , the definition of minimum dispersion SDF in (2.4) can be rewritten as

$$\begin{aligned}
 M_i^* = \arg \min_{M_i} \quad & \frac{1}{\alpha(\alpha - 1)} \mathbb{E}[M_i^\alpha] \\
 \text{s.t.} \quad & \mathbb{E}[M_i(R_{ik} - R_{i0})] = 0 \quad k = 1, \dots, n_i \\
 & \mathbb{E}[M_i R_{i0}] = 1 \\
 & M_i > 0
 \end{aligned} \tag{A.1}$$

following a monotone transformation of the objective function and rearranging of the constraints. It is immediate to verify that the objective function in (A.1) is convex in  $M_i$  for all  $\alpha \neq 0$ , thus implying that first-order conditions are necessary and sufficient for optimality. As a result, any strictly positive solution  $M_i^*$  of (A.1) has to satisfy the following first-order conditions:

$$\frac{1}{\alpha - 1} M_i^{*\alpha-1} = \mu_{i0} R_{i0} + \sum_{k=1}^{n_i} \mu_{ik} (R_{ik} - R_{i0}), \tag{A.2}$$

where  $\mu_{i0}$  and  $\mu_{ik}$  are the Lagrange multipliers corresponding to the pricing constraints  $\mathbb{E}[M_i R_{i0}] = 1$  and  $\mathbb{E}[M_i(R_{ik} - R_{i0})] = 0$ , respectively. Multiplying both sides of the above

equation by  $M_i^*$  and taking expectations implies that

$$\mathbb{E}[M_i^{*\alpha}] = (\alpha - 1)\mu_{i0}, \quad (\text{A.3})$$

where we are using the pricing constraint  $\mathbb{E}[M_i^*(R_{ik} - R_{i0})] = 0$ . Therefore, equations (A.2) and (A.3) imply that

$$M_i^* = \left( (\alpha - 1)\mu_{i0} \left( R_{i0} + \sum_{k=1}^{n_i} \lambda_{ik}^* (R_{ik} - R_{i0}) \right) \right)^{1/(\alpha-1)} = (\mathbb{E}[M_i^{*\alpha}] R_{\lambda_i^*})^{1/(\alpha-1)}, \quad (\text{A.4})$$

where  $\lambda_{ik}^* = \mu_{ik}/\mu_{i0}$  and  $R_{\lambda_i^*} = R_{i0} + \sum_{k=1}^{n_i} \lambda_{ik}^* (R_{ik} - R_{i0})$ . Raising both sides of the above equation to the power  $\alpha$  and taking expectations, we obtain  $\mathbb{E}[M_i^{*\alpha}] = (\mathbb{E}[R_{\lambda_i^*}^{\alpha/(\alpha-1)}])^{1-\alpha}$ . Plugging this expression back into equation (A.4) thus implies that the minimum dispersion SDF corresponding to parameter  $\alpha$  is given by

$$M_i^* = R_{\lambda_i^*}^{1/(\alpha-1)} / \mathbb{E}[R_{\lambda_i^*}^{-\alpha/(1-\alpha)}]. \quad (\text{A.5})$$

Replacing for  $M_i^*$  in the pricing constraints of (A.1) with the expression in (A.5) implies that

$$\mathbb{E}[R_{\lambda_i^*}^{1/(\alpha-1)} (R_{ik} - R_{i0})] = 0$$

for all  $k$ . Note that this is the first-order condition corresponding to problem (2.5), where we are using the fact that, by (A.5),  $M_i^* > 0$  implies that  $R_{\lambda_i^*} > 0$ . Therefore, it must be the case that  $R_{\lambda_i^*} = R_i^*(\alpha)$ . The juxtaposition of this observation with equation (A.5) thus establishes (2.6). Now, to establish (2.7), note that  $M_i^*(\alpha)$  and  $R_i^*(\alpha)$  in (2.6) are the optimizers of (2.4) and (2.5), respectively. Therefore, inequality (2.7) has to hold for any pair of SDF and portfolio returns  $M_i$  and  $R_{\lambda_i}$ .

We now turn to the case of  $\alpha = 0$ . The optimization problem (2.4) can be rewritten as

$$\begin{aligned} M_i^* &= \arg \min_{M_i} -\mathbb{E}[\log M_i] \\ \text{s.t. } &\mathbb{E}[M_i(R_{ik} - R_{i0})] = 0 \quad k = 1, \dots, n_i \\ &\mathbb{E}[M_i R_{i0}] = 1 \\ &M_i > 0. \end{aligned}$$

Once again, the convexity of the objective function implies that first-order conditions are necessary and sufficient for optimality. Hence, when the optimizer  $M_i^*$  is strictly positive, we have

$$-M_i^{*-1} = \mu_{i0}R_{i0} + \sum_{k=1}^{n_i} \mu_{ik}(R_{ik} - R_{i0}),$$

where the Lagrange multipliers  $\mu_{i0}$  and  $\mu_{ik}$  are defined analogously. But note that the above equation coincides with (A.2) by setting  $\alpha = 0$ . Thus, following steps identical to those that led to equations (A.3)–(A.5) establishes (2.6) and (2.7) for  $\alpha = 0$ .  $\square$

**Proof of Proposition 2:**

**Proof of part (a):** Let  $M_d^*(0)$  denote the minimum entropy SDF for the domestic country. Equation (2.6) in Proposition 1 implies that

$$M_d^*(0) = 1/R_d^*(0) = \left( \sum_{k=0}^{n_d} \lambda_{dk}^* R_{dk} \right)^{-1}, \quad (\text{A.6})$$

where the portfolio weights  $(\lambda_{d0}^*, \dots, \lambda_{dn_d}^*)$  add up to 1 and satisfy the pricing equation  $\mathbb{E}[M_d^*(0)\mathbf{R}_d] = \mathbf{1}$  in the form of

$$\mathbb{E} \left[ \frac{R_{dk}}{\sum_{k=0}^{n_d} \lambda_{dk}^* R_{dk}} \right] = 1 \quad (\text{A.7})$$

for all  $k = 0, 1, \dots, n_d$ , where  $n_d = n_f$ . On the other hand, the assumption that international financial markets are integrated (i.e.,  $\text{span}(\mathbf{R}_f X) = \text{span}(\mathbf{R}_d)$ ) implies that, for any  $s = 0, 1, \dots, n_f$ , there exists a collection of portfolio weights  $(\mu_{s0}, \dots, \mu_{sn_d})$  adding up to one such that  $R_{fs} = \sum_{k=0}^{n_d} \mu_{sk} R_{dk} / X$ . Thus, multiplying both sides of equation (A.7) by  $\mu_{sk}$  and summing over all  $k$  implies that

$$\mathbb{E} \left[ \frac{\sum_{k=0}^{n_d} \mu_{sk} R_{dk} / X}{\sum_{k=0}^{n_d} \lambda_{dk}^* R_{dk} / X} \right] = \mathbb{E} \left[ \frac{R_{fs}}{\sum_{k=0}^{n_d} \lambda_{dk}^* R_{dk} / X} \right] = 1 \quad (\text{A.8})$$

for all  $s = 0, 1, \dots, n_f$ . We now use the assumption that markets are integrated one more time. In particular, this assumption guarantees that, for any  $k = 1, \dots, n_d$ , there exists a collection of portfolio weights  $(\zeta_{k0}, \dots, \zeta_{kn_f})$  adding up to one such that  $R_{dk} = \sum_{r=0}^{n_f} \zeta_{kr} R_{fr} X$ . Consequently,

equation (A.8) implies that

$$\mathbb{E} \left[ \frac{R_{fs}}{\sum_{r=0}^{n_f} \lambda_{fr}^* R_{fr}} \right] = 1 \quad (\text{A.9})$$

for all  $s = 0, 1, \dots, n_f$ , where  $\lambda_{fr}^* = \sum_{k=0}^{n_d} \lambda_{dk}^* \zeta_{kr}$ . But note that the above equation is nothing but the first-order condition corresponding to the optimal portfolio problem (2.5) in the foreign country. Thus, by Proposition 1,  $M_f^*(0) = (\sum_{r=0}^{n_f} \lambda_{fr}^* R_{fr})^{-1}$ . Consequently,

$$M_f^*(0) = \left( \sum_{k=0}^{n_d} \lambda_{dk}^* \sum_{r=0}^{n_f} \zeta_{kr} R_{fr} \right)^{-1} = \left( \sum_{k=0}^{n_d} \lambda_{dk}^* R_{dk} / X \right)^{-1} = X M_d^*(0),$$

where the first equality is a consequence of the definition  $\lambda_{fr}^* = \sum_{k=0}^{n_d} \lambda_{dk}^* \zeta_{kr}$ , whereas the last equality is a consequence of equation (A.6).  $\square$

**Proof of part (b):** First, suppose that international markets are complete. In this case, all minimum dispersion SDFs in country  $i$  coincide with the corresponding minimum entropy SDF, that is,  $M_i^*(\alpha) = M_i^*(0)$  for all  $\alpha$ . As a result, statement (a) of Proposition 2 guarantees that the pair  $M_d^*(\alpha)$  and  $M_f^*(\alpha)$  satisfies the asset market view for all values of  $\alpha$ .

To prove the reverse implications, suppose to the contrary that markets are incomplete but the asset market view holds with respect to a pair of minimum dispersion SDFs satisfying (2.4) with dispersion parameter  $\alpha \neq 0$ , that is,  $M_f^*(\alpha) = X M_d^*(\alpha)$ . By Proposition 1,

$$\frac{R_f^*(\alpha)}{\mathbb{E}^{\alpha-1}[R_f^*(\alpha)^{\alpha/(\alpha-1)}]} = X^{\alpha-1} \frac{R_d^*(\alpha)}{\mathbb{E}^{\alpha-1}[R_d^*(\alpha)^{\alpha/(\alpha-1)}]},$$

where  $R_i^*(\alpha)$  is the gross return of the optimal portfolio in (2.5) corresponding to country  $i$ . This means that there exist portfolio weights  $\lambda_f^* = (\lambda_{f0}^*, \dots, \lambda_{fn}^*)'$  and  $\lambda_d^* = (\lambda_{d0}^*, \dots, \lambda_{dn}^*)'$  such that<sup>1</sup>

$$\frac{1}{\mathbb{E}^{\alpha-1}[R_f^*(\alpha)^{\alpha/(\alpha-1)}]} (\lambda_f^*)' \mathbf{R}_f = \frac{1}{\mathbb{E}^{\alpha-1}[R_d^*(\alpha)^{\alpha/(\alpha-1)}]} (\lambda_d^*)' \mathbf{R}_d X^{\alpha-1}.$$

On the other hand, the assumption that international financial markets are integrated implies

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<sup>1</sup> Here we are using the fact that the number of assets in the two countries are identical, i.e.,  $n_f = n_d = n$ . This is a consequence of market integration and the assumptions that the collection of assets  $\mathbf{R}_i$  in each country  $i \in \{d, f\}$  are linearly independent.



that there exists a square and invertible matrix  $\Omega$  such that  $\mathbf{R}_d = \Omega \mathbf{R}_f X$ . As a result,

$$\gamma' \mathbf{R}_f = \zeta' \mathbf{R}_f X^\alpha, \quad (\text{A.10})$$

where  $\gamma = \lambda_f^* / \mathbb{E}^{\alpha-1}[R_f^*(\alpha)^{\alpha/(\alpha-1)}]$  and  $\zeta = \Omega' \lambda_d^* / \mathbb{E}^{\alpha-1}[R_d^*(\alpha)^{\alpha/(\alpha-1)}]$ . In the Proof of Proposition 1 we established that the weights  $(\lambda_{f0}^*, \dots, \lambda_{fn}^*)$  are proportional to the Lagrange multipliers of the convex optimization problem (A.1) with  $n+1$  linearly independent constraints, thus implying that all elements of  $\lambda_f^*$  and hence those of  $\gamma$  in (A.10) are different from zero.

We now show that equation (A.10) implies that

$$\text{span}(\mathbf{R}_f) = \text{span}(\mathbf{R}_f X^\alpha). \quad (\text{A.11})$$

We first show that  $\text{span}(\mathbf{R}_f) \subseteq \text{span}(\mathbf{R}_f X^\alpha)$ . Suppose to the contrary that  $\text{span}(\mathbf{R}_f) \not\subseteq \text{span}(\mathbf{R}_f X^\alpha)$ . This implies that there exists a basis for  $\text{span}(\mathbf{R}_f X^\alpha \cup \mathbf{R}_f)$  consisting of  $\mathbf{R}_f X^\alpha$  and a non-empty subset of the components of  $\mathbf{R}_f$ , which we denote by  $\mathcal{S}$ . That is,  $\text{span}(\mathbf{R}_f X^\alpha \cup \mathbf{R}_f) = \text{span}(\mathbf{R}_f X^\alpha \cup \mathcal{S})$  and  $\text{span}(\mathcal{S}^c) \subseteq \text{span}(\mathbf{R}_f X^\alpha)$ . We can thus rewrite the left-hand side of (A.10) as

$$\gamma' \mathbf{R}_f = \sum_{R_{fk} \in \mathcal{S}} \gamma_k R_{fk} + \sum_{R_{fk} \in \mathcal{S}^c} \gamma_k R_{fk} = \sum_{R_{fk} \in \mathcal{S}} \gamma_k R_{fk} + \delta' \mathbf{R}_f X^\alpha \quad (\text{A.12})$$

for a uniquely determined vector  $\delta$ , where we are using the fact that  $\text{span}(\mathcal{S}^c) \subseteq \text{span}(\mathbf{R}_f X^\alpha)$  and that the returns in  $\mathbf{R}_f X^\alpha$  are linearly independent. The right-hand side of the above expression therefore gives the unique representation of  $\gamma' \mathbf{R}_f \in \text{span}(\mathbf{R}_f X^\alpha \cup \mathbf{R}_f)$  in terms of vectors in  $\mathbf{R}_f X^\alpha \cup \mathcal{S}$ . But recall that  $\gamma$  is element-wise different from zero. In particular,  $\gamma_k \neq 0$  for all  $k$  such that  $R_{fk} \in \mathcal{S}$ . Therefore, equation (A.12) implies that  $\gamma' \mathbf{R}_f \notin \text{span}(\mathbf{R}_f X^\alpha)$ , which contradicts (A.10). As a result, it must be the case that  $\text{span}(\mathbf{R}_f) \subseteq \text{span}(\mathbf{R}_f X^\alpha)$ . But note that, since both  $\mathbf{R}_f$  and  $\mathbf{R}_f X^\alpha$  are full rank,  $\text{span}(\mathbf{R}_f) \subseteq \text{span}(\mathbf{R}_f X^\alpha)$  only if  $\text{span}(\mathbf{R}_f) = \text{span}(\mathbf{R}_f X^\alpha)$ , thus establishing (A.11).

With (A.11) in hand, we now proceed to completing the proof. The assumption that financial markets are integrated but incomplete implies that the number of states exceed the number of assets in each country, i.e.,  $s > n + 1$ . As a result, the matrix of returns  $\mathbf{R}_f \in \mathbb{R}^{(n+1) \times s}$  can be

expressed as

$$\mathbf{R}_f = [\underline{\mathbf{R}}_f \quad \bar{\mathbf{R}}_f],$$

where  $\underline{\mathbf{R}}_f \in \mathbb{R}^{(n+1) \times (n+1)}$  is a square and invertible matrix and  $\bar{\mathbf{R}}_f \in \mathbb{R}^{(n+1) \times (s-n-1)}$ .<sup>2</sup> On the other hand, (A.11) implies that there exists a matrix  $Q$  such that  $\mathbf{R}_f X^\alpha = Q \mathbf{R}_f$ . Therefore,

$$[\underline{\mathbf{R}}_f \quad \bar{\mathbf{R}}_f] \begin{bmatrix} \underline{X}^\alpha & 0 \\ 0 & \bar{X}^\alpha \end{bmatrix} = Q [\underline{\mathbf{R}}_f \quad \bar{\mathbf{R}}_f], \quad (\text{A.13})$$

where  $\underline{X}$  and  $\bar{X}$  are diagonal matrices of dimensions  $n+1$  and  $s-n-1$ , respectively. Consequently,  $\underline{\mathbf{R}}_f \underline{X}^\alpha = Q \underline{\mathbf{R}}_f$ , and as a result,  $Q = \underline{\mathbf{R}}_f \underline{X}^\alpha \underline{\mathbf{R}}_f^{-1}$ . Plugging back the expression for  $Q$  into (A.13) thus implies that  $\bar{\mathbf{R}}_f \bar{X}^\alpha = \underline{\mathbf{R}}_f \underline{X}^\alpha \underline{\mathbf{R}}_f^{-1} \bar{\mathbf{R}}_f$ , which can be rewritten as

$$(\underline{\mathbf{R}}_f^{-1} \bar{\mathbf{R}}_f) \bar{X}^\alpha = \underline{X}^\alpha (\underline{\mathbf{R}}_f^{-1} \bar{\mathbf{R}}_f).$$

Since  $\alpha \neq 0$  and all elements of  $X$  are distinct, the above equality can hold only if  $\underline{\mathbf{R}}_f^{-1} \bar{\mathbf{R}}_f = 0$ , which in turn implies that  $\bar{\mathbf{R}}_f = 0$ . In other words, all assets in the foreign country have zero returns in  $s - n - 1$  many states of the world. But in view of equation (2.6), this also implies that  $M_f^*(\alpha) = 0$  on all such states, which contradicts the positivity constraint imposed in (2.4). Thus, if the markets are incomplete, the pair of minimum dispersion SDFs  $M_d^*(\alpha)$  and  $M_f^*(\alpha)$  does not satisfy the asset market view for any  $\alpha \neq 0$ .  $\square$

**Proof of Corollary 1:** Recall from Proposition 2 that  $X = M_f^*(0)/M_d^*(0)$ . On the other hand, by definition, the stochastic wedge  $\eta(2)$  satisfies (2.10). Therefore,

$$\exp(\eta(2)) = \frac{M_d^*(2)M_f^*(0)}{M_f^*(2)M_d^*(0)}.$$

Taking logarithms from both sides of the above equation establishes (2.11).  $\square$

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<sup>2</sup> The existence of an invertible block  $\underline{\mathbf{R}}_f$  follows from the linear independence of assets in  $\mathbf{R}_f$ .

## Chapter 3

# How Integrated Are Corporate Bonds and Stock Markets?

### 3.1 Introduction

CANONICAL models in asset pricing often implicitly assume that markets are integrated, such that there is no arbitrage across markets. For example, the [Merton \(1974\)](#) model posits that the firm value process is divided between debt and equity investors, who hold exposures to the same source of risk. Following a contingent claim approach, debt and equity should be priced using the same stochastic discount factor (SDF). Put differently, this framework implies strict restrictions for the cross-market risk premia and dictates the way bonds and stocks should be priced cross-sectionally, irrespective of the firm-characteristics driving the returns. In reality, however, there are several financial frictions that may prevent markets from being fully integrated, including financial constraints, such as short-selling restrictions or transaction costs, potentially leading to limits to arbitrage. Thus, shedding light on the underlying extent of market integration is crucial for refining our insights regarding the pricing of different asset classes, as well as for assessing the empirical success of asset pricing models implicitly assuming perfect integration across different markets. Moreover, recent evidence suggests that investors' behaviour may be driven by different firm specific or asset specific characteristics (see, e.g. [Koijen, Richmond, and Yogo \(2019\)](#), among others). Therefore, in this paper, I address the question of how integrated corporate bond and stock markets are, by incorporating in the analysis different frictions and considering various firm characteristics.

The extent of asset market integration is of particular importance for investment and financing decisions, not only for institutional investors and corporations, but also for individual investors. In absence of perfect risk sharing, access to cross-markets, both domestically and internationally, can prove beneficial in terms of portfolio diversification and can entail a more efficient allocation of resources. Additionally, if debt and equity markets are integrated, investors can hedge across these markets. Still, studying and quantifying market integration is hindered by the lack of a well-accepted measure.<sup>1</sup> There are two different dimensions of market integration that are important to distinguish. First, there can be evidence of segmentation within an asset class. For instance, a test of market integration for a particular stock which is cross-listed is equivalent to examining whether the law of one price holds across the two exchanges. Second, markets can be segmented across different asset classes. In this case, a test of market integration implies absence of arbitrage opportunities, or the existence of a common SDF for the two markets. This follows from the fact that assets entailing identical risks should have prices that yield the same expected returns. In this paper, the focus is directed towards the second instance, since the analysis involves two different classes of assets: corporate bonds and the corresponding stock of the issuing firm.

To study the underlying degree of integration between corporate bonds and stock markets, I adopt a model-free approach, thus overcoming the typical model misspecification critique. Otherwise, it is extremely difficult to disentangle the joint hypothesis test of market integration from correct model specification. To exemplify, in a Capital Asset Pricing Model (CAPM) world, market integration implies that the prices of risk associated with the market factor are identical for corporate bond and stock returns. However, if the underlying factor model is misspecified, one might incorrectly reject market integration, as the prices of risk would tend to differ across various markets. Using instead a model-free approach, I show that the level of integration between corporate bonds and stock markets is higher than previously thought, at least unconditionally.

A direct test of integration requires to observe investors present in both corporate bond and stock markets. In fact, a sufficient condition for integration is that there exists at least one agent trading across markets. Since data regarding portfolio positions of investors is not readily available, I am going to study a necessary condition for integration instead: the existence of a

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<sup>1</sup> Several notions of market integration have been proposed in the literature. They can be summarized as pertaining to either a parametric, or to a nonparametric framework. I review these different approaches in the related literature section.

common pricing rule for the two markets. Intuitively, the idea is to take the stance of investors who are Sharpe ratio maximizers. The metric used to quantify the degree of integration is given by the cross-market pricing implications of the optimal Sharpe ratio portfolios in the respective market. Whenever investors have access to both bonds and stocks, and they can freely form portfolios, the corresponding markets are going to be perfectly integrated. Consequently, there should be no cross-market pricing discrepancies, if markets are arbitrage-free. On the other hand, differences in cross-market pricing, or pricing errors, capture departures from perfect market integration. However, in order to assess whether these pricing discrepancies lead to actual arbitrage opportunities, trading costs need to be taken into account. For this reason, in the empirical exercise, I am going to compare the size of the pricing errors implied by optimal Sharpe ratio portfolios with a measure of transaction costs, given by the quoted bid-ask spreads in the corporate bonds and stocks markets. Using the universe of US firms, for both corporate bonds and stock returns, I form portfolios based on firm level characteristics, for the following reasons. First, portfolios reduce idiosyncratic volatility. Second, it is relevant to assess whether investors specialize in different types of assets, or are more interested in the firm itself, and have a preference for a particular firm. Specifically, for both corporate bond and stock returns, I form portfolios based on size, value, leverage, credit rating, duration, stock momentum, asset growth, profitability and liquidity. More importantly, analyzing the optimal portfolio positions associated with different firm characteristics sheds light on the underlying trading motifs of investors.

[Hansen and Jagannathan \(1991\)](#) have showed that constructing optimal Sharpe ratio portfolios is equivalent to minimizing the variance of the SDF. The reason I focus on this type of SDF is twofold. First, the minimum variance SDF bounds the maximal attainable Sharpe ratio in the economy. Second, due to the equivalent (dual) relation between minimum variance SDF and the mean-variance portfolio frontier, it admits a closed-form representation in the return space. Hence, the minimum variance SDF is going to be a linear combination of the traded returns and can be easily interpreted as the optimal Sharpe ratio portfolio. Finally, this SDF and its associated optimal portfolio are the relevant objects of study to measure cross-market pricing relations and thus quantify the underlying degree of integration, especially in presence of frictions. Essentially, I am measuring the pricing error of a return in one market, implied by the pricing rule of the other market. Notice that this SDF choice is going to yield the most conservative results, as accounting for higher order moments generally improves (cross) pricing.

Under the assumption of frictionless markets, I show that it is always possible to construct empirically a common minimum variance SDF that prices both corporate bonds and stocks. However, upon a closer look at the associated optimal Sharpe ratio portfolio, the latter entails large and often negative positions, suggesting that in practice, if investors face leverage or short-selling constraints, it would be hard to attain. In the real world, markets are not free of frictions and an agent's choice set includes a subset of assets that she takes into consideration or is allowed to hold. Constraints in the choice set may arise due to investment mandates, certain benchmarks, or information frictions that restrain an investor's ability to examine a large universe of assets ([Merton \(1987\)](#)). My contribution is to study the degree of market integration in an extended framework, that incorporates frictions in the form of short-selling constraints. I focus on this type of restriction in order to obtain a portfolio that agents present in the corporate bond and/or stock market can readily form. Regarding the stock market, there have been instances in which regulators prohibited short-selling all together, such as during the recent financial crisis. On the other hand, trading in the corporate bond market is carried over-the-counter (OTC). Typically, short-selling bonds entails a significant cost, and in illiquid corporate bond markets, it might simply not be possible (see, e.g., [Asquith, Au, Covert, and Pathak \(2013\)](#), [Blanco, Brennan, and Marsh \(2005\)](#) and [Bai and Collin-Dufresne \(2019\)](#)). Moreover, regulators might prevent certain type of institutional investors, such as insurance companies and pension funds, to engage in short-selling activities.

Consequently, I investigate what are the optimal portfolio implications and cross-market pricing relations when short-selling constraints are imposed. When taking into account short-selling constraints, I show that the wealth is typically going to be allocated just in one portfolio of stocks and bonds, that maximizes the Sharpe ratio. The cross-market pricing errors implied by the constrained minimum variance SDFs are between 10 and 50 basis points (bp) per month, and hence within the observed bid-ask spreads. This evidence suggests that even when accounting for short-selling constraints, markets exhibit a high degree of integration, as no profitable arbitrage opportunities arise. Hence, the degree of market integration is better than previously thought. Still, the average pricing errors will differ across portfolio sorts, with bond characteristics such as credit rating and duration yielding smaller stock mispricing, whereas valuation ratios for stock portfolios implying a lower mispricing for bond portfolios. In particular, larger firms, with more liquid corporate bonds and stocks, appear to be more integrated. Similarly, firms that are more

leveraged, have a higher asset growth and profitability feature a higher extent of integration between their debt and equity securities.

A natural question arising is whether the constrained model-free extracted SDFs perform better than existing factor models for bonds and stocks. As competing models, I consider the [Fama and French \(1993\)](#) 3 Factor Model, proxying for market, size and value and the bond factor model of [Bai, Bali, and Wen \(2019\)](#) capturing exposure to the market, downside risk, credit and liquidity factors, as well as a combination of the two models. To discriminate between the various models, I use two criteria: (i) the size of the implied pricing errors and (ii) the R-Square of a generalized least squared (GLS) regression of returns on the proposed factors. The reason I focus on GLS R-Square rather than the OLS one follows from the critique of [Lewellen, Nagel, and Shanken \(2010\)](#), that apparently strong explanatory power in fact provides rather weak support for a model, especially if the test assets are the ones formed on size and book-to-market portfolios. A horse race between factor models and the SDFs suggests the latter yield on average smaller pricing errors, and can have a higher GLS R-Square. Therefore, the constrained SDFs not only exhibit a better pricing performance, but can be interpreted economically as optimal portfolios of traded assets. Finally, to rationalize the risk factors embedded in the model-free SDFs, I show that the SDFs incorporating short-selling constraints appear to be closely linked to the market factor and the intermediary risk factor of [He, Kelly, and Manela \(2017b\)](#).

### 3.1.1 Related Literature

This paper contributes to two strands of literature and fills the gap between market integration and financial frictions. Extant literature has focused on two approaches when studying market integration. The first relies on a parametric specification for asset pricing models, generally expressed in terms of factor models. In this framework, only a small number of priced factors drive the behavior of risk premia. Market integration implies that the corresponding prices of risk of each factor are equal across markets. An extensive body of literature studies the cross-sectional drivers of stock returns. Corporate bonds have received limited attention, even though they represent a significant portion of the financial markets.<sup>2</sup> Recent advances suggest that a factor model accounting for market risk, downside risk, credit, and liquidity risk performs well in explaining the cross-section of bond returns ([Bai, Bali, and Wen \(2019\)](#)). Still, existing factor

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<sup>2</sup> According to the Federal Reserve Flow of Funds, as of the fourth quarter of 2018, the total amount outstanding of corporate bonds amounts to \$13.5 trillion, compared to the US equity market of \$35 trillion.

models explaining stock returns do not perform well in fitting the cross-section of corporate bond returns. Possible explanations for these discrepancies seem to suggest different risk factors for the two asset classes ([Chordia, Goyal, Nozawa, Subrahmanyam, and Tong \(2017\)](#)), or potential market frictions leading to differences in prices, such as financial constraints and limits to arbitrage ([Shleifer and Vishny \(1997\)](#), [He and Krishnamurthy \(2013\)](#), [Kapadia and Pu \(2012\)](#)). In general, the main limitation arising when using parametric tests lies in specifying the correct asset pricing model, as a rejection of market integration can simply occur from potential misspecifications.<sup>3</sup>

The second is a nonparametric approach, pioneered by [Chen and Knez \(1995\)](#), which tests the degree of integration by minimizing a distance between the SDFs of the two markets under scrutiny. Whenever this distance is zero, the two SDFs coincide and thus markets are integrated. Departures from zero reflect deviations from perfect integration and quantify the associated mispricing. This methodology is better suited for testing the law of one price across markets, rather than absence of arbitrage opportunities. In this paper, I adopt a nonparametric approach in order to study the degree of market integration between corporate bonds and stocks, by examining the optimal portfolio implications and cross-market pricing relations of Sharpe ratio investors. I contribute to this literature by studying the degree of market integration in an extended framework, that incorporates frictions in the form of short-selling constraints when deriving the SDFs. I focus on this type of restriction in order to obtain a portfolio that agents present in the corporate bond and/or stock market can readily form.

Market integration has been initially studied in an international framework, by testing whether the prices of global risk are identical across countries. Following an international CAPM model, the only priced risk should be the systematic risk relative to the world market ([Stehle \(1977\)](#), [Errunza and Losq \(1985\)](#), [Jorion and Schwartz \(1986\)](#)). International markets appear to be mildly segmented, as different countries have different prices of risk associated with the global factor. [Dumas and Solnik \(1995\)](#) test a conditional version of the international CAPM, while including an additional factor capturing foreign-exchange risk premia and find

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<sup>3</sup> [Gagliardini, Ossola, and Scaillet \(2019\)](#) propose a diagnostic criterion for approximate factor structure in large cross-sectional equity datasets. An asset pricing factor model is correctly specified for a set of returns whenever the associated error terms are only weakly cross-sectionally correlated. Otherwise, the criterion indicates the number of potential omitted factors. This methodology is constructed for individual equities, i.e. for a large number of test assets, and it may yield imprecise results when considering portfolios as test assets. Moreover, even if the model is correctly specified from a statistical point of view, mispricing might not be entirely ruled out.



no evidence of segmentation between currency markets and stock markets. [Bekaert and Harvey \(1995\)](#) introduce a measure of time-varying market integration stemming from a regime-switching model. Their approach allows conditionally expected returns in any country to be driven by their covariance with a world market portfolio and by the variance of the market portfolio in the respective country. [De Roon, Nijman, and Werker \(2001\)](#) provide evidence that incorporating short-sale constraints and/or transaction costs leads to vanishing diversification benefits of US investors from emerging markets, highlighting the importance of accounting for trading costs when computing the risk premia.

More recently, [Pukthuanthong and Roll \(2009\)](#) propose as a measure of global market integration the R-Squared from time-series regressions of country aggregate equity indices on common global factors. Accordingly, an R-Squared equal to one implies perfect integration. Translated into an SDF approach, there exists a common SDF, correctly pricing the market portfolio in each country. [Sandulescu, Trojani, and Vedolin \(2019\)](#) show that market segmentation is a key ingredient to address salient features of international asset returns, while keeping cross-country correlations at moderate levels. [Koijen, Richmond, and Yogo \(2019\)](#) document that a small set of six firm characteristics related to risk, productivity, and profitability explains most of the variation in a panel of firm-level valuation ratios across countries and determines investors' demand functions.

Overall, the evidence suggests that despite the progressive removals of trade and legal barriers, and the fact that international integration has improved over time, markets are still fairly segmented. I contribute to this literature by using a model-free approach to quantify the extent of market integration. Intuitively, I am measuring the pricing error of a return in one market, implied by the pricing rule of the other market. Furthermore, the methodology employed is general and can accommodate conditioning information in order to allow for the degree of market integration to change through time. The focus of my study is directed towards local, rather than international, market integration. Quantifying the extent of integration at the local level is a natural starting point, since if these markets exhibit segmentation, it is likely that it will spill over to international markets. In this context, understanding what type of firm features leads to more integrated markets is key, as it may explain investors' incentives to trade certain classes of assets.

A second recent stream of literature is more directly related to the goal of my analysis.

The existence of differences in expected returns for the two asset classes is typically attributed to different risk factors (Fama and French (1993), Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017)), or the presence of heterogeneous investors (Dumas (1989), Chien, Cole, and Lustig (2011)). In particular, different classes of investors hold these two assets: stocks are readily available and mainly held by households, whereas institutional investors, such as insurance companies, pension funds or mutual funds dominate the corporate bond market. Despite the potential existence of different risk factors or different types of investors, I argue that this does not necessarily imply that stock and bond markets are segmented. In fact, if there exists an agent trading in the two markets, it suffices to render them integrated. For instance, Ma (2018) documents that non-financial firms act as cross-market arbitrageurs in their debt and equity securities.

In related work, Choi and Kim (2018) test market integration between corporate bonds and stock markets by examining the relationship between risk premia in the two markets. Fitting different factor models to both corporate bond and stock returns, they find that the prices of risk tend to differ across the two markets, suggesting a limited degree of market integration. I extend this study using a model-independent approach and by examining portfolios of corporate bonds and stocks formed on a larger set of anomaly variables. To have a higher cross-sectional dispersion, I consider additionally portfolios derived based on double sorts. The empirical findings suggest that corporate bonds and stock markets exhibit a high degree of integration within characteristic sorts, even after incorporating short-selling constraints. My approach circumvents the issue of possible misspecification. Moreover, the derived SDFs admit an economic interpretation in terms of optimal Sharpe ratio portfolios.

Finally, an important literature emphasizes that several market frictions might segment corporate bonds and stock markets, including financial constraints and limits to arbitrage (Shleifer and Vishny (1997), Duarte, Longstaff, and Yu (2006), He and Krishnamurthy (2013)), or slow moving capital (Duffie (2010), Greenwood, Hanson, and Liao (2018)). Kapadia and Pu (2012) document short-horizon pricing discrepancies across firms' credit and equity markets, pointing to a lack of integration between the two markets, stemming from limits to arbitrage. Analyzing the credit default swap (CDS)-equity markets, the authors propose a statistical measure of market integration, using the concordance of price changes in the two markets. Specifically, their definition of market integration is based on the frequency of instances when

price discrepancies, i.e. arbitrage opportunities, arise for a pair of stock prices and CDS spreads. Different from this study, I focus on returns for corporate bonds, which cover a larger part of the universe of US firms, since CDS do not span all corporate bonds. I quantify the level of market integration by analyzing the cross-market pricing relations of optimal Sharpe ratio portfolios of corporate bonds and stocks formed based on firm-level characteristics. Accounting for short-selling constraints, I find that the pricing discrepancies across portfolios sorted on the same characteristics are typically contained inside the quoted bid-ask spreads, suggesting an absence of profitable arbitrage opportunity.

The rest of the paper is organized as follows. Section 2 provides the theoretical framework for studying market integration and model-free SDFs. Section 3 describes the potential determinants driving corporate bond and stock returns. Section 4 presents the data and the main empirical findings. Section 5 contains robustness checks. Section 6 concludes the paper.

## 3.2 Market Integration and Stochastic Discount Factors

Using a model-free approach, this paper studies the degree of market integration across and within different asset classes, with an emphasis on corporate bonds and stocks of the same firm. Additionally, an analysis of the properties of minimum variance stochastic discount factors (SDFs) existing in these markets is provided. The goal is to document the extent of market integration and pricing performance implied by different SDFs in presence of market frictions, such as short-selling constraints.

Consider an economy populated by square integrable random variables  $X$  defined on an  $L^2$  space, with corresponding norm  $\|X\| \equiv \sqrt{\mathbf{E}[X^2]}$ . In this economy, investors can trade in a set of  $N$  assets, consisting of corporate bonds and stocks of the issuing firms. The corresponding gross returns are gathered in a vector  $\mathbf{R}$  of length  $N$ . Let  $M$  denote a generic SDF pricing return vector  $\mathbf{R}$ . Investors can construct portfolios of the form:

$$\mathcal{P} = \{R_p : R_p = \mathbf{R}'\theta, \quad \theta \in C \subseteq \mathbb{R}^N\}, \quad (3.1)$$

with  $\theta$  being the vector of portfolio weights.

### 3.2.1 The Case of Frictionless Markets Assumption

Under the assumption of *frictionless* markets, the no-arbitrage condition implies the following Euler equation:

$$\mathbb{E}[M\mathbf{R}] = \mathbf{1}, \quad (3.2)$$

with  $\mathbf{1}$  denoting the vector of ones of appropriate size.

The set of admissible SDFs is defined as:

$$\mathcal{M} = \{M \in L^2 : M \geq 0, \quad \mathbb{E}[M\mathbf{R}] = 1\}. \quad (3.3)$$

Accordingly, an admissible SDF is a random variable with finite second moment, that has to be nonnegative and has to price correctly the set of available returns. Moreover, investors can form portfolios freely, i.e.  $C = \mathbb{R}^N$ .

To define a market integration measure, assume there are two markets, denoted by  $B$  and  $S$ , with corresponding SDFs  $M_B$  and  $M_S$ . This assumption does not require pricing consistency across the two markets, but whenever two markets are perfectly integrated, they will assign identical prices to identical returns. Different asset classes, or assets belonging to the same class but divided into different characteristics can form the two markets. For the purpose of the present study, the two markets considered are the one pertaining to the corporate bonds and the one belonging to the stocks of the issuing firms.

Let  $\mathcal{M}_B$  and  $\mathcal{M}_S$  be the sets of admissible SDFs for markets  $B$  and  $S$ , pricing returns  $\mathbf{R}_B$  and  $\mathbf{R}_S$ , respectively. Since integration implies that pricing across markets should be as close as possible, a natural way to define a measure of integration is to minimize the distance between the SDFs in the two markets. To this end, I build upon the approach of [Hansen and Jagannathan \(1997\)](#) (HJ, henceforth). The idea is to minimize the distance between a set of admissible SDFs  $\mathcal{M}_S$  and a candidate SDF  $y \in \mathcal{M}_B$ . Specifically, the problem reads:

$$\delta(y, S) = \min_{M_S \in \mathcal{M}_S} \|y - M_S\|^2, \quad (3.4)$$

with  $\|\cdot\|^2$  denoting the squared  $L^2$  norm. Intuitively, this measure is going to reflect how differently the two markets assign prices to assets. Naturally, markets are going to be integrated if the two SDFs coincide. The greater the cross-market pricing differences, the stronger the evidence of market segmentation between the two markets considered. In the data, integration between two markets can be tested by estimating this distance directly. Notice that this test

is independent of any parametric specifications for the underlying asset pricing models. In summary, the integration measure  $\delta(\cdot, \cdot) = 0$  can be interpreted as a benchmark cross-market relation. In all other circumstances, it provides an estimate of the maximal pricing error between the two markets, given by  $\sqrt{\delta(\cdot, \cdot)}$ . Put differently, it represents the maximal arbitrage opportunity from buying a return in one market and simultaneously selling it in the other.

There are two interconnected issues arising: (i) the SDF is not observable and (ii) how to optimally pick the candidate SDF  $y$ . Since the objective is not to rely on any parametric specifications, I derive a well-established SDF in the literature, the minimum variance SDF of [Hansen and Jagannathan \(1991\)](#), which bounds the maximal attainable Sharpe ratio in the economy. It follows that due to the equivalent representation of minimum variance SDFs in the return space, in terms of optimal portfolios, one can derive the SDFs directly from observable prices. In other words, the minimum variance SDF problem can be recast equivalently through ([Korsaye, Quaini, and Trojani \(2018\)](#)):<sup>4</sup>

$$\begin{aligned} M^* &= \min_{M \in \mathcal{M}} \frac{1}{2} \mathbb{E}[M^2] \\ R_p^* &= \max_{\theta \in \mathbb{R}^N} \mathbb{E}[\mathbf{1}'\theta - (\mathbf{R}'\theta)^2/2] \quad s.t. \quad \mathbf{R}'\theta \geq 0. \end{aligned} \tag{3.5}$$

Accordingly, solving the optimization problem expressed in terms of returns and portfolio weights yields closed-form solutions for model-free SDFs that can be estimated directly from the data. This powerful insight will set the ground for the empirical analysis section of this study.

Specifically, the minimum variance SDF reads:

$$M^* = \mathbf{R}'\theta^* = R_p^*, \tag{3.6}$$

with  $\theta^* \in \mathbb{R}^N$  being the optimal vector of portfolio weights that delivers the maximal attainable Sharpe ratio in the economy. Since  $M^*$  is linear in the set of returns  $\mathbf{R}$  available to investors, it is also the only tradable SDF in the set  $\mathcal{M}$ , i.e.  $M^*$  is itself a portfolio of returns.

Finally, the problem in (3.4) quantifies the maximal pricing error across the two markets. Using the pricing rule of one market to price the returns in the other market yields the actual cross-market pricing performance. To determine how well do bond minimum variance SDFs price

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<sup>4</sup> The approach can be generalized to accommodate a large family of dispersion measures, such that higher-order moments of returns are taken into account.

stock returns and vice versa, I compute the following pricing error:

$$PE_j = \mathbb{E}[M_i^* \mathbf{R}_j] - \mathbf{1}, \quad (3.7)$$

with  $PE_j$  denoting the pricing error for returns in market  $j = B, S$  implied by the SDF in market  $i = B, S$  and  $i \neq j$ . Whenever the implied pricing errors are zero, it follows that the minimum variance SDF from one market can be used to price both sets of returns, consistent with absence of arbitrage opportunities across markets. Notice however that in practice, differences in cross-market pricing, or pricing errors, should be compared with trading costs in order to assess whether they lead to profitable arbitrage opportunities.

### 3.2.2 The Case of Markets with Frictions Assumption

In presence of market *frictions*, the constrained SDF, denoted by  $\tilde{M}$ , will yield an error when pricing gross returns  $\mathbf{R}$ . In particular, the pricing relation is going to be sublinear (see, e.g. [Luttmer \(1996\)](#)):

$$\mathbb{E}[\tilde{M}\mathbf{R}] \leq \mathbf{1}, \quad (3.8)$$

with the inequality holding componentwise. Market frictions, such as short-selling constraints, that might be enforced by some regulatory institution, or transaction costs, can be incorporated by adjusting either the set of possible weights, or the returns. In particular, in presence of short-selling constraints,  $\theta \in C = \mathbb{R}_+^N$ , the nonnegative part of  $\mathbb{R}^N$ . With transaction costs, the set of returns will be augmented to reflect the long and short positions at the corresponding bid and ask prices, i.e. the dimension of the return vector becomes  $2N$ .

The set of admissible SDFs in presence of market frictions is given by:

$$\tilde{\mathcal{M}} = \{\tilde{M} \in L^2 : \tilde{M} \geq 0, \quad \mathbb{E}[\tilde{M}\mathbf{R}] \leq \mathbf{1}\}. \quad (3.9)$$

The duality relation between optimal portfolio of returns and minimum variance SDFs holds also in presence of different market frictions. The solution is obtained by including the appropriate restrictions in Equation (3.5), i.e.  $M \in \tilde{\mathcal{M}}$ , or equivalently,  $\theta \in \mathbb{R}_+^N$ . Specifically, the minimum variance SDF is still going to be a linear function of the returns for which the short selling constraints are not binding.

To study the degree of market integration in presence of financial frictions, while being

consistent with a model-free approach, I employ the HJ distance previously described. Put differently, my goal is to identify whether there is a common SDF for corporate bond and stock returns, after accounting for market frictions such as short-selling constraints. To this end, I quantify the distance between a constrained SDF in one market and the set of admissible SDFs in the other market, according to Equation (3.4). In this case, the candidate SDF  $y$  will be derived consistently with the frictions and the HJ distance will provide an estimate of the maximal pricing error made when pricing returns in the other market. Moreover, cross-market pricing relation should satisfy the relation in (3.8). Intuitively, since market integration implies that similar returns should be assigned similar prices, the pricing error implied by the restricted common SDF should be economically small.

**Definition 3.2.1.** In presence of frictions, if the following relations hold, markets are integrated:

$$\begin{aligned}\sqrt{\delta(\tilde{M}_B, S)} &\leq \tau_S \\ \sqrt{\delta(B, \tilde{M}_S)} &\leq \tau_B,\end{aligned}\tag{3.10}$$

with  $\tau_S$  ( $\tau_B$ ) denoting the corresponding bid-ask spread in market  $S$  ( $B$ ).

Hence, in the empirical application, I compare the entailed pricing errors with the size of the bid-ask spreads. The view is that if the pricing errors are within the quoted bid-ask spreads, there are no arbitrage opportunities across markets.

Since returns on corporate bonds and stocks can be driven by different firm-level characteristics, which can affect investors' trading motifs, I outline in the subsequent section what are the potential determinants.

### 3.3 Potential Determinants of Corporate Bond and Stock Returns

In the following, I describe what are the potential factors driving expected returns for corporate bonds and stocks. The factors are chosen to reflect both firm-level and asset specific characteristics. In addition, I consider well-established variables, or anomalies, documented in the literature to price the cross section of stock returns. The aim is to examine whether they have the same implications also for bond returns, since a contingent claim approach would imply

a strong link between the risk premia on the two asset classes. Moreover, these variables will constitute the building blocks of different portfolio sorts for stocks and bonds, which will represent the test assets for constructing the SDFs and quantifying the underlying degree of integration. The sorting is particularly relevant in presence of different market frictions, such as short-selling constraints, as it shows what firm characteristics matter for investors when forming their optimal portfolios. I consider the following variables.

**Credit Ratings.** The underlying credit riskiness of firms reflect their ability to pay back their debt and provide an estimate of the probability of default. Depending on their credit riskiness profile, different classes of investors will hold different assets ([Collin-Dufresne and Goldstein \(2001\)](#)). For the corporate bond market in particular, asset-class-sensitive institutional investors tend to segment the markets between investment-grade (safe) and high-yield (junk) bonds ([Chen, Lookman, Schürhoff, and Seppi \(2014\)](#)). Consequently, different market participants can interpret these two types of bonds as distinct asset classes, including passive and active asset managers, asset owners in general and even regulators, who might prohibit certain types of institutional investors from holding high-yield bonds and prevent them from engaging in short-selling activities. As a typical example, pension funds and insurance companies hold mainly investment-grade bonds, whereas hedge funds have a preference for riskier high-yield bonds. Portfolio managers usually view high-yield corporate bonds as exhibiting a behavior similar to stocks, due to their sensitivity to the market conditions, especially during bad times. For instance, using high-yield bond data, [Ronen and Hotchkiss \(1999\)](#) provide evidence that bond and stock returns react jointly to common factors. Moreover, in frictionless markets, the high-yield bond can be replicated using the risk-free interest rate and a put option on the firm value ([Merton \(1974\)](#)). It is worthwhile to mention that the focus of this study is on credit ratings, as opposed to credit default swaps (CDSs), because the latter do not span all corporate bonds and will bias the selection towards large issuing companies. Hence, it is interesting to examine the implications regarding the underlying degree of market integration of sorting corporate bonds and stocks into portfolios based on credit ratings. Notice that in practice, it suffices that one investor is able to trade both corporate bonds and stocks in order for the markets to be integrated. If adjusting potential price discrepancies across the two markets would incur high transaction costs, then these differences might persist.

**Duration.** The average period of time until the cash flows are received can represent another



dimension along which the extent of market integration might vary. In particular, according to the preferred-habitat theory, introduced by [Culbertson \(1957\)](#) and refined by [Vayanos and Vila \(2009\)](#), which is a variant of the market segmentation theory, different investors prefer one maturity length over another. For instance, pension funds have a preference for longer maturities than mutual funds, in order to match their obligations. In general, an investor will buy a bond with a different maturity than his preferred one, only for a premium. Regarding the stock side, [Lettau and Wachter \(2007\)](#) develop a dynamic model in which firms are perceived as long-lived assets discriminated by the timing of their cash-flows. Using the corresponding book-to-market (BM) ratio, the authors show that growth firms are high-duration assets, similar to long-term bonds, whereas value firms are low-duration assets. Analyzing bond and stock portfolios of different duration represents thus an appealing feature to study the level of market integration.

**Size.** Smaller firms require on average a higher expected return than larger firms ([Banz \(1981\)](#), [Fama and French \(1992\)](#)). Market capitalization of firms might thus drive the behavior of investors for trading. Since larger firms entail a lower expected return on their stocks and are less likely to default, in the data their bonds and stocks might be more closely integrated than those of small firms.

**Value.** Value firms are generally trading at a lower price than their fundamentals, as measured by their large BM ratios. Consequently, they have a higher expected return than growth firms ([Fama and French \(1992\)](#)). On the other hand, enhancing firm growth prospects may imply a lower likelihood of defaulting. Following [Pastor and Veronesi \(2003\)](#), the BM ratio can be used to proxy for future profitability. Therefore, the BM influences the underlying value of the firm, driving the trading incentives of investors and might have an impact on the degree of integration between corporate bond and stock markets.

**Leverage.** Firms that exhibit higher leverage ratios are more indebted and typically closer to default ([Merton \(1974\)](#)). As such, for highly leveraged firms, the expected return on bonds should be larger. Since leverage affects the firm value, it might also influence the extent of market integration between corporate bonds and stocks. Leverage is computed as the ratio between book value of debt and the sum between the book value of debt and the market value of equity.

**Stock Momentum.** [Jegadeesh and Titman \(1993\)](#) document that past stock winners tend to outperform past losers over short to medium horizons, a phenomenon referred to as momentum. Higher stock momentum thus imply a higher valuation of the firm, which could potentially

decrease the probability of default. In addition, [Avramov, Chordia, Jostova, and Philipov \(2007\)](#) provide strong evidence of momentum among low credit quality firms, suggesting that high-yield bonds are likely more affected. The return stock momentum is computed as the cumulative 12 months return skipping the most recent one.

**Asset Growth.** Firms with improved asset growth are farther away from default. The findings of [Cooper, Gulen, and Schill \(2008\)](#) suggest a strong relation between asset growth and stock returns. In line with the aforementioned study, asset growth is measured as the percentage change in total assets.

**Profitability.** Firms with higher profitability are likely to be distant from the boundary triggering the default and to have on average higher expected returns ([Fama and French \(2008\)](#)). Profitability is derived as the ratio of equity income to book equity.

**Liquidity.** [Pástor and Stambaugh \(2003\)](#) provide evidence that liquidity is an important factor explaining the cross section of stock returns. Similarly, liquidity appears to drive also the returns on corporate bonds ([Chen, Lesmond, and Wei \(2007\)](#)). Liquidity is thus meaningful not only to characterize the behavior of returns, but also the implied degree of market integration. In particular, the two classes of assets trade on different platforms, and corporate bonds are more expensive to trade than stocks, i.e. on average an order of magnitude higher, as measured by their bid-ask spreads. Accounting for trading restrictions will be particularly relevant for illiquid bonds, as these are precisely the ones difficult to short-sell ([Blanco, Brennan, and Marsh \(2005\)](#)). In the empirical analysis, I measure liquidity using the percent quoted spread, computed as the ratio between the bid-ask spread and the mid price (see, e.g. [Fong, Holden, and Trzcinka \(2017\)](#) for a comparison of liquidity proxies).

### 3.4 Empirical Analysis

In this section, I describe the data sources and the construction of the variables of interest. Further, I estimate the degree of integration between corporate bond and stock markets by considering different portfolios sorted on firm-level characteristics. The reason I focus on portfolio sorts is twofold. First, deriving SDFs that correctly price a large cross-section of returns might not be feasible using standard techniques. For instance, to provide a comparison between the performance of model-free SDFs and existing factor models, I will employ the Fama MacBeth procedure, which cannot handle a large number of test assets. Second, constructing sorts

is beneficial because it provides a simple picture of how average returns vary across a given characteristic. Moreover, aggregating returns into portfolios eliminates noise and delivers more precise estimates, while improving efficiency. In addition, I use value-weighted returns when sorting stocks or corporate bonds on variables of interest, in order to avoid the estimates to be driven by small companies. Next, I provide comparative statistics for the derived minimum variance SDFs introduced in the theory section and examine their time series properties. Further, I supply a horse race between the pricing performance of model-free extracted SDFs in presence of market frictions in the form of short-selling constraints and existing stock and bond factor models. Finally, to rationalize the risk factors embedded in model-free extracted SDFs, I explore their link with the market factor and an intermediary risk factor.

### 3.4.1 Data

The data for corporate bonds and the associated stock of the issuing firms is obtained by merging four different databases. The sample comprises the entire universe of US corporate bonds satisfying selection criteria commonly used in the literature. FINRA’s TRACE (Trade Reporting and Compliance Engine) database reports the most comprehensive data regarding corporate bonds transactions, including the price and the trading volume. In particular, it provides information regarding over-the-counter (OTC) secondary market transactions by all market participants, who are mainly large institutional investors, such as insurance companies, pension funds and mutual funds.

Using the TRACE database to study corporate bond returns is appealing because it reports actual transaction prices from dealers, which enables a natural framework for examining the economic forces of the market. On the other hand, databases that report matrix-based quotes, i.e. a price derived by a rule of thumb based on bond characteristics in absence of a dealer quote, although they might have a longer sample period, they can obscure the market mechanisms. These serve more as indicative quotations, and not as an obligation on price and quantity (Elton, Gruber, Agrawal, and Mann (2001)).

The sample period spans January 2005 to December 2017, with a monthly frequency. Since corporate bonds are not as liquid as stocks, it is very likely that for many of them no transactions are encountered at a daily or higher frequency. The beginning of the sample period corresponds to the date when reporting to TRACE became mandatory. This circumvents issues with regards

to possible biases arising in the preceding period, dating back to July 2002 (the launch date), when companies were reporting to TRACE based on a voluntary basis. Corporate bond issue characteristics, such as bond type, issue and maturity date, the offering amount, the coupon rate, as well as information about the issuing firm, including the credit ratings, are retrieved from Mergent FISD. Moreover, the WRDS Bond Returns database merges the two aforementioned databases and provides information about corporate bonds monthly returns, as well as the link with equity issues for every firm, through the CRSP database.

The filters used follow the standard literature (see, e.g., [Bai, Bali, and Wen \(2019\)](#)). As a first step, cleaning the data requires eliminating the three most common errors encountered in the trade reports filled by dealers, i.e. cancellations, corrections and reversals ([Dick-Nielsen \(2009\)](#)). The bond type has to fulfill the following criteria: (i) coupon type is fixed or zero, (ii) not under Rule 144A, (iii) the bond type is senior, either debenture, medium term note, or medium term note zero, i.e. no convertible or assets backed bonds are considered. Further minor corrections include filtering for missing data, such as missing price, volume or data ranges.

Additionally, the following filters are considered: (i) remove bonds maturing in less than a year, since these are the ones that are going to be delisted from indices,<sup>5</sup> (ii) remove bonds that have missing credit ratings or duration, (iii) focus on bonds having a trading volume higher than \$10,000<sup>6</sup> and (iv) remove bonds that do not have corresponding stock data in CRSP and Compustat. Finally, to avoid bonds that show up just for several months and then disappear from TRACE, I require the bonds to be in the TRACE data for at least one full year.

Lastly, equity data for the issuing firms regarding price and shares outstanding is retrieved from CRSP, whereas firm fundamentals are obtained from Compustat.

## Variable Definitions

In this subsection, I illustrate how the variables of interest used in the empirical analysis are constructed.

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<sup>5</sup> Since trading corporate bonds is mainly performed over-the-counter (OTC), the main class of investors are institutional ones, such as insurance companies, pension funds, mutual funds and to a lesser extent, hedge funds. Insurance companies and pension funds typically follow long-term, buy-and-hold strategies, so they do not rebalance their portfolios often. As such, they tend to discard the bonds having a maturity less than a year.

<sup>6</sup> [Bessembinder, Maxwell, and Venkataraman \(2006\)](#) remove transactions with trading volume less than \$100,000, to reflect trades from institutional investors.

For each corporate bond  $i = 1, \dots, I$ , the monthly gross return is computed as follows:

$$R_{i,t+1} = \frac{P_{i,t+1} + C_{i,t+1} + AI_{i,t+1}}{P_{i,t} + AI_{i,t}},$$

where  $P_{i,t+1}$  is the price,  $C_{i,t+1}$  is the coupon payment and  $AI_{i,t+1}$  the accrued interest of bond  $i$  at time  $t + 1$ .

Similarly, the gross return of stock  $j = 1, \dots, J$  reads:

$$R_{j,t+1} = \frac{P_{j,t+1} + D_{j,t+1}}{P_{j,t}},$$

where  $P_{j,t+1}$  is the price and  $D_{j,t+1}$  is the dividend payment of stock  $j$  at time  $t + 1$ .

The liquidity proxy, or percent quoted spread, for stock  $j = 1, \dots, J$  is derived as:

$$L_{j,t+1} = \frac{P_{j,t+1}^A - P_{j,t+1}^B}{(P_{j,t+1}^A + P_{j,t+1}^B)/2},$$

where  $P^A$  ( $P^B$ ) denotes the ask (bid) price.

The stock return momentum used to construct sorts is computed as:

$$R_{j,t}^{MOM} = \prod_{s=13}^{s-1} R_{j,s}.$$

After applying the corresponding filters, the data consists of 443,206 bond-month observations and 93,253 unique stock-month observations. Provided that a company can have multiple bonds outstanding at a given point in time, the number of bond observations is naturally higher. To illustrate, throughout the sample period considered, there are in total 1,636 firms and 11,743 bonds. The construction of portfolio sorts proceeds as follows. For deciles based on size, book-to-market, leverage, asset growth and profitability, the sorts are done once a year in June and monthly value-weighted returns are calculated from July through June the following year (Fama and French (1993)). For momentum, the deciles sorts are constructed based on the cumulative 12-month returns, excluding the most recent month. For liquidity, the deciles sorts are derived based on the previous month transaction cost incurred, measured as the percent quoted bid-ask spread. For credit rating and duration, the quintile sorts are formed based on observations in the preceding month. A higher numerical score for the credit rating indicates a riskier corporate bond. To proxy for the overall credit rating of the stock, I take the average

credit ratings of the issuing firms' outstanding bonds.

Finally, the methodology yields 10 portfolios of corporate bonds and of stocks using value-weighted returns sorted on size, value, momentum, leverage, asset growth, profitability, liquidity and 5 portfolios sorted on credit rating and duration. The reason I use quintiles for credit rating and duration rather than a higher granularity is because bonds are highly concentrated across this two dimensions. In particular, 80% of the bonds in the sample are investment grade, the rest being high-yield bonds.

## Summary Statistics

The summary statistics regarding corporate bond and stock returns, as well as firm-level characteristics are reported in Table 3.1. It follows that, on average, the representative bond has a monthly return of 0.544% and is an investment grade bond, having a numerical rating of 8.37 (BBB) and a duration of 6.48 years. The average firm is a growth firm, mid-cap, with a market capitalization of about 13 billion, a leverage of 0.34 and a book-to-market ratio of 0.7. Moreover, it has on average a profitability of 12% and an annual asset percentage growth of 2.8%. The corresponding momentum return over the past 12 months yields about 13% on average, however it exhibits substantial cross-section variation, with a median of 6%. The average percent quoted bid-ask spread for individual stocks is about 8 bp, whereas the one for bonds is one order of magnitude higher, around 0.7%.

Tables 3.2 and 3.3 report the average value-weighted portfolio returns for different characteristics sorts considered. Panel A outlines the bond returns, whereas Panel B the stock returns. Specifically, Table 3.2 reports the portfolio returns formed on quintile credit rating and duration sorts. The safest bonds are in the lowest quintile, as measured by their credit rating number.<sup>7</sup> Naturally, they exhibit a lower return. High-yield bonds in the top quintile require a return double in size the one in the bottom quintile. A similar picture emerges for the stock returns, which increase in the credit ratings. Finally, bonds in the bottom quintile duration have lower returns than the ones in the top quintile, suggesting that investors typically require a higher return to hold longer maturity bonds. The same evidence is encountered also for the stock returns, which, interestingly, exhibit a U-shaped pattern.

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<sup>7</sup> The credit rating is expressed in conventional numerical scores, and can take values between 1 (AAA rating) and 21 (C rating). Higher numerical score means higher credit risk. Numerical ratings of 10 or below (BBB- or better) are considered investment grade, and ratings of 11 or higher (BB + or worse) are labeled high yield.

**Table 3.1:** Summary Statistics

This table reports the mean, median and the standard deviation of the returns, numerical rating and duration for bonds and stocks. The credit rating is expressed in conventional numerical scores, and can take values between 1 (AAA rating) and 21 (C rating). Higher numerical score means higher credit risk. Numerical ratings of 10 or below (BBB- or better) are considered investment grade, and ratings of 11 or higher (BB + or worse) are labeled high yield. Duration is expressed in years. Size is computed as the natural logarithm of market capitalization. Value is the book-to-market equity value. Leverage is the ratio between book value of debt and the book value of debt plus common equity. Momentum is the cumulative return over the previous 12 months skipping the most recent month. Asset growth represents the percentage change in total assets. Profitability is the ratio between operating profit and book equity. Stock (bond) bid-ask spread is the ratio between the bid-ask spread and the mid price. Data is monthly and runs from January 2005 to December 2017.

	Mean	Median	Std
Bond Return (%)	0.544	0.421	0.293
Stock Return (%)	0.976	0.968	10.23
Credit Rating	8.367	8	3.334
Duration	6.484	5.295	4.036
Size	9.477	9.653	1.797
Value	0.703	0.550	1.533
Leverage	0.337	0.312	0.179
Momentum	0.134	0.059	0.686
Asset Growth (%)	2.8	0.5	11.2
Profitability (%)	12.2	6.4	18.43
Stock Bid-Ask Spread (bp)	8	3	3.4
Bond Bid-Ask Spread (bp)	69	48	74

**Table 3.2:** Portfolio Returns Based on Quintile Sorts

This table reports the value-weighted returns of portfolios formed on credit rating and duration. Panel A reports bond returns, whereas Panel B reports stock returns. Data is monthly and runs from January 2005 to December 2017.

Panel A: Bond Returns					
	Low	2	3	4	High
Credit rating	0.440%	0.560%	0.530%	0.530%	0.880%
Duration	0.290%	0.380%	0.480%	0.570%	0.600%
Panel B: Stock Returns					
	Low	2	3	4	High
Credit rating	0.470%	0.640%	0.760%	0.620%	0.840%
Duration	0.710%	0.490%	0.520%	0.580%	0.740%

From Table 3.3, both bonds and stocks portfolios formed on decile size sorts yield a higher return for small firms (in the Low portfolio), than for large firms (High portfolio). Growth firms exhibit a lower return for bonds, consistent with the idea that their corresponding probability of default is lower. They also have a higher average stock return, congruent with the fact that value stocks have been underperforming after the crisis. Leverage appears to be negatively related with stock returns, as the portfolios in the bottom decile have a higher average return than the one in the top decile. The average bond returns are very stable across the leverage deciles, exhibiting thus a low cross sectional dispersion. This may arise as a result of other relevant firm or asset characteristics inside the bins. To explore this possibility in a more systematic way, I will consider double sorted portfolios as a robustness check. Momentum portfolio sorts deliver low stock returns in the bottom decile and high returns in the top decile, strengthening the idea that past winners tend to outperform past losers. The opposite effect is encountered for the bond returns, where the portfolio in the bottom momentum decile generates a higher return than the one in the top decile. Firms with higher asset growth entail a lower return both for stocks and bonds, in line with the findings of [Hou, Xue, and Zhang \(2015\)](#). At the same time, whereas the stock returns exhibit significant cross-sectional variation, the average return on the bonds vary less across the deciles. Similar to the leverage sorts, some other firm or bond characteristics may be obscured, such as duration or credit riskiness. This will be addressed when constructing double sorts. Profitability sorted portfolios display a higher stock return when moving from the bottom to the top decile, consistent with the idea that more profitable firms earn higher equity returns (see, e.g., [Fama and French \(2015\)](#), [Novy-Marx \(2013\)](#)). However, since more profitable firms are likely further away from the default boundary and thus safer, they should entail a lower bond return, as the results in Table 3.3 suggest. Bonds that are more liquid, i.e. they exhibit lower bid-ask spreads, have lower average returns. The stock average returns for liquidity decile sorts display significant cross-sectional variation. The absence of a monotone relationship might be due to the fact that both the liquidity measure and the holding period return are computed at a monthly frequency, in line with evidence provided by [Liu \(2006\)](#). Moreover, since returns are value-weighted inside the deciles, the returns of small, illiquid stocks will be assigned a tighter share.

In summary, forming portfolios of corporate bonds and the stock of the corresponding issuer based on firm characteristics sorts should not be arbitrary, as firm features can have different effects on the returns of the two classes of assets. Specifically, when sorting on size, leverage,



**Table 3.3:** Portfolio Returns Based on Decile Sorts

This table reports the value-weighted returns of portfolios formed on different sorts. Panel A reports the bond returns, whereas Panel B reports the stock returns. Data is monthly and runs from January 2005 to December 2017.

<b>Panel A: Bond Returns</b>										
	Low	2	3	4	5	6	7	8	9	High
Size	0.729%	0.730%	0.696%	0.701%	0.614%	0.563%	0.558%	0.557%	0.526%	0.475%
Value	0.489%	0.466%	0.484%	0.445%	0.522%	0.486%	0.505%	0.450%	0.486%	0.650%
Leverage	0.460%	0.490%	0.480%	0.460%	0.510%	0.500%	0.480%	0.460%	0.490%	0.480%
Momentum	0.614%	0.436%	0.549%	0.482%	0.455%	0.530%	0.519%	0.540%	0.495%	0.395%
Asset Growth	0.480%	0.481%	0.469%	0.486%	0.503%	0.537%	0.545%	0.512%	0.504%	0.464%
Profitability	0.546%	0.549%	0.510%	0.489%	0.443%	0.521%	0.489%	0.485%	0.471%	0.465%
Liquidity	0.310%	0.350%	0.370%	0.420%	0.480%	0.550%	0.540%	0.590%	0.570%	0.710%

<b>Panel B: Stock Returns</b>										
	Low	2	3	4	5	6	7	8	9	High
Size	1.014%	0.834%	1.004%	0.943%	0.987%	0.879%	0.900%	0.886%	0.884%	0.739%
Value	1.005%	0.874%	0.768%	1.036%	0.608%	0.737%	0.763%	0.729%	0.800%	0.690%
Leverage	0.750%	0.920%	0.730%	0.810%	0.900%	0.790%	0.650%	0.580%	0.810%	0.640%
Momentum	0.409%	0.618%	0.736%	0.858%	0.813%	0.997%	0.996%	1.086%	0.939%	1.301%
Asset Growth	0.866%	0.915%	0.875%	0.552%	0.706%	0.844%	0.987%	0.991%	0.846%	0.706%
Profitability	0.337%	0.719%	0.824%	0.933%	0.791%	0.730%	0.878%	0.805%	1.029%	0.845%
Liquidity	1.080%	0.740%	0.800%	0.330%	0.740%	0.600%	0.610%	0.270%	0.290%	0.380%

asset growth, credit rating and duration, average bond and stock returns tend to follow similar patterns. Value, momentum, profitability and liquidity, on the other hand, induce opposite effects for average bond and stock returns. Moreover, the dispersion across decile portfolios varies depending on the characteristic considered, and the relationship might not be always monotonic.

### 3.4.2 Market Integration and Minimum Variance SDFs

In this section, I estimate the degree of integration and the minimum variance SDFs introduced in the theory. I start by describing the results derived under the assumption of frictionless markets and document why this setting yields unrealistic implications. Then, I provide findings obtained in presence of different market frictions. The frictionless markets case can be regarded as a benchmark and the corresponding optimal Sharpe ratio portfolio is key in order to infer whether in the actual markets, investors would be able to attain it.

## I. The Case of Frictionless Markets Assumption

### Minimum Variance SDFs

Testing for market integration in the data is equivalent to testing whether there exists a common SDF pricing corporate bonds and stock returns. In the following, I derive the minimum variance SDFs that price exactly the portfolio returns of bonds, stocks, or both, sorted on different firm-level characteristics. The SDFs are estimated using the sample counterpart of the quantities in Equation (3.5). Notice that the solution in this case admits an analytical form. The annualized volatility of these SDFs are reported in Table 3.4. The minimum variance bond SDFs entail a higher annualized volatility than the minimum variance stock SDFs, for portfolios sorted on credit rating, duration, size, and momentum, suggesting that they offer a higher Sharpe ratio. In contrast, stock portfolios sorted on value, leverage, asset growth and profitability yield a higher Sharpe ratio than the bond portfolios. For some characteristics sorts, especially in the case of bond portfolios, some might argue that not every type of investor would be able to generate a Sharpe ratio as high as 1.6, particularly if it involves short positions. Perhaps a hedge fund will have no issues to form such an optimal portfolio, but insurance companies or pension funds might be unable to do so.

**Table 3.4:** Volatility of SDFs

This table reports the annualized volatility of the various SDFs derived that correctly price the portfolios of returns sorted on different characteristics. Data is monthly and runs from January 2005 to December 2017.

	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
Minimum Variance Bond SDF	1.124	1.039	0.721	0.805	0.524	0.708	0.672	0.824	1.632
Minimum Variance Stock SDF	0.366	0.363	0.689	0.897	0.836	0.640	0.919	0.840	1.017
Minimum Variance Bond & Stock SDF	1.167	1.160	1.040	1.290	1.088	1.252	1.272	1.390	2.383

To have a better understanding regarding the dynamics between marginal minimum variance SDFs pricing either bonds or stocks, I report in Table 3.5 their correlations. Several aspects are worth mentioning. First, most of the correlations between the bond and stock SDFs are not statistically different from zero. Second, firm-level characteristics play an important role, as they have different implications for the sign of the correlation, consistent with the average returns reported in Tables 3.2 and 3.3. For instance, the minimum variance SDF that prices portfolios of bonds sorted on credit ratings positively co-moves with the one pricing stocks sorted on credit rating. The opposite is observed for the SDFs pricing portfolios of returns sorted on profitability, as the minimum variance SDFs exhibit a negative correlation. In summary, different

firm characteristics may entail contrasting effects for the average returns of corporate bonds and stocks. In particular, firm-specific information that is mainly related to the mean (variance) of the firm's underlying assets induce a positive (negative) contemporaneous correlation between corporate bonds and stocks (see, e.g. [Kwan \(1996\)](#)).

**Table 3.5:** Correlations between Bond and Stock Minimum Variance SDFs

This table reports the correlations between minimum variance bond and stock SDFs for different portfolio sorts considered. Data is monthly and runs from January 2005 to December 2017. \*\*\*, \*\* and \* denotes significance at the 1%, 5% and 10% level, respectively.

		Minimum Variance Bond SDF								
Stock SDF		Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
	Credit rating	0.296***	-0.199**	0.260***	0.009	0.051	0.059	0.169**	-0.050	-0.018
	Duration	0.163**	-0.115	0.105	-0.019	0.062	0.111	-0.006	-0.125	-0.085
	Size	0.020	-0.014	0.137*	-0.040	0.028	0.059	0.046	0.013	-0.070
	Value	0.063	-0.016	0.040	-0.073	0.152*	-0.158**	0.050	0.127	-0.031
	Leverage	-0.156*	-0.027	0.008	0.124	0.143*	0.131	-0.208***	-0.129	-0.042
	Momentum	0.095	-0.228***	0.079	-0.130	0.102	0.068	-0.068	0.030	-0.013
	Asset Growth	0.187**	0.067	0.088	-0.113	0.066	0.128	0.089	-0.106	0.040
	Profitability	0.068	-0.018	0.033	-0.018	0.209***	0.050	0.087	-0.250***	-0.070
	Liquidity	0.095	0.003	0.005	-0.133*	0.019	0.146*	0.068	-0.062	-0.182**

Having established a rather low correlation between the minimum variance bond and stock SDFs, it is more relevant to examine the pricing performance implied by one type of SDF when applied to the other market. To this end, I report in Panel A of [Table 3.6](#) the average pricing errors implied by minimum variance SDFs, computed as  $\mathbb{E}[M_B R_S] - 1$  and  $\mathbb{E}[M_S R_B] - 1$ , with  $S$  denoting the stocks and  $B$  the bonds.<sup>8</sup> For illustrative purposes, the results are reported for the minimum variance SDFs that correctly price portfolios sorted on the same characteristic, i.e. the first column outlines the pricing error made by the bond SDF correctly pricing bond portfolios constructed on credit rating sorts when applied to stock portfolios constructed on credit rating sorts and so on. The pricing performance varies when fixing one SDF and applying it to all the portfolio sorts considered in the other market, and in most of the cases, the fit worsens. Ultimately, the fit is mechanically related to how difficult it is to price a cross-section of returns. For the characteristics examined, the minimum variance bond SDFs yield a pricing error that varies between 5.2 bp and 44 bp. The lowest pricing errors are the ones associated with credit rating portfolio sorts, whereas the highest ones are encountered for momentum and liquidity. Overall, bond valuations tend to overprice stock portfolios. In contrast, stock valuations tend to underprice bond portfolios, as the average pricing error is negative. Additionally, the pricing

<sup>8</sup> In interest of space, I report the pricing errors corresponding to each quintile or decile portfolio in [Table B.1](#) in the Appendix.

errors implied by stock SDFs for bond portfolios are higher on average (in absolute terms). It follows that under the assumption of frictionless markets, if one uses the marginal SDF derived in one market to price the assets in the remaining market, the associated pricing errors exhibit significant cross-sectional dispersion and in some cases, the mispricing is as high as 72 bp per month.

**Table 3.6:** Pricing Errors and HJ Distance of Minimum Variance SDFs

This table reports in Panel A the average pricing errors implied by minimum variance SDFs, computed as  $E[M_B R_S] - 1$  and  $E[M_S R_B] - 1$ , with  $S$  denoting the stocks and  $B$  the bonds. Pricing errors are the ones implied by the bond (stock) SDF that correctly prices the portfolios sorted on the corresponding characteristic. Panel B reports the [Hansen and Jagannathan \(1997\)](#) (HJ) distance between the minimum variance bond SDF, denoted by  $\hat{M}_B$  and the set of admissible SDFs for the stock market  $S$  (first row), as well as the HJ distance between the minimum variance stock SDF, denoted by  $\hat{M}_S$  and the set of admissible SDFs for the bond market  $B$ . All measures are reported in basis points (bp). Data is monthly and runs from January 2005 to December 2017.

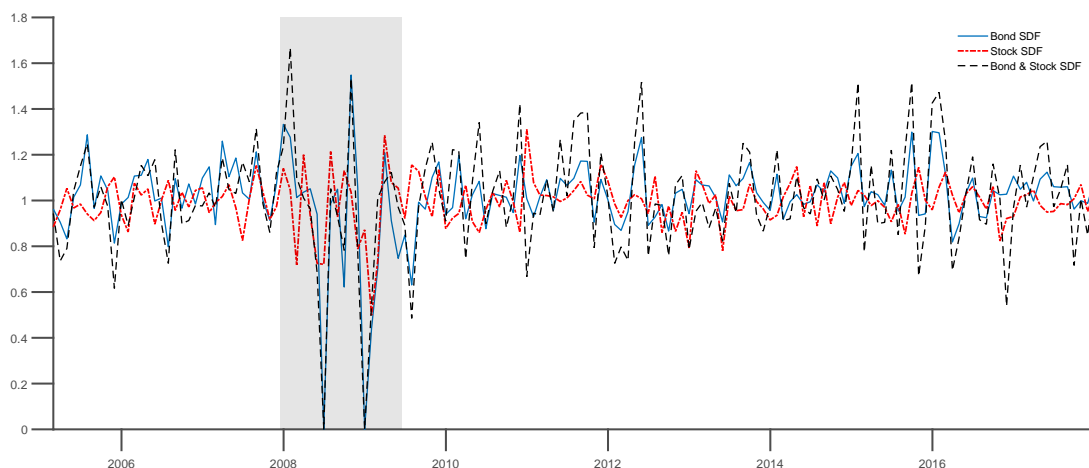
Panel A: Pricing Errors									
	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$E[M_B R_S] - 1$	5.20	26.40	20.10	20.70	18.10	44.00	28.50	16.80	29.50
$E[M_S R_B] - 1$	-24.20	-30.20	-28.70	-37.40	-56.60	-66.30	-57.20	-71.40	-60.60
Panel B: HJ Distance of Minimum Variance SDFs									
	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$\sqrt{\delta(\hat{M}_B, S)}$	6.10	12.28	18.54	27.70	22.31	19.13	25.47	30.97	39.58
$\sqrt{\delta(B, \hat{M}_S)}$	25.84	31.25	18.39	29.36	15.01	28.87	18.48	30.28	51.40

Panel B reports the [Hansen and Jagannathan \(1997\)](#) distance between the minimum variance bond SDFs and the set of admissible stock SDFs, as well as the distance between the minimum variance stock SDFs and the set of admissible bond SDFs. The maximal implied pricing errors obtained by fixing the minimum variance bond SDF vary between 6 bp for credit rating sorts and 40 bp for liquidity. Using the stock SDF to price bond portfolios leads on average to higher maximal pricing errors, with the exception of leverage and asset growth sorts. Specifically, the pricing errors vary between 15 bp for leverage sorts and 51 bp for liquidity sorts. Notice however that if in reality pricing errors have to exist because of various frictions, then linear pricing rules are only approximate. This insight changes the interpretation of results and opens still the possibility for market integration.

Figure 3.1 plots the time-series of minimum variance SDFs pricing the portfolios of bonds sorted on credit rating (solid line), along with the minimum variance SDF pricing stock returns sorted on credit rating (dashed-dotted line) and the minimum variance SDF pricing both bond and stock returns sorted on credit rating (dashed line). All SDFs spike during the financial crisis and the higher volatility of the bond SDF compared to the stock SDF is now apparent. Moreover,

the SDF pricing both classes of assets appears to follow more the dynamics of the bond SDF. Indeed, taking a closer look at the associated optimal mean-variance portfolio, the positions are higher in the bond portfolios, than in the stock portfolios. Importantly, the characteristics on which portfolios are constructed are going to impact the dynamics of the global minimum variance SDF differently.<sup>9</sup>

**Figure 3.1:** Time-Series of Minimum Variance SDFs

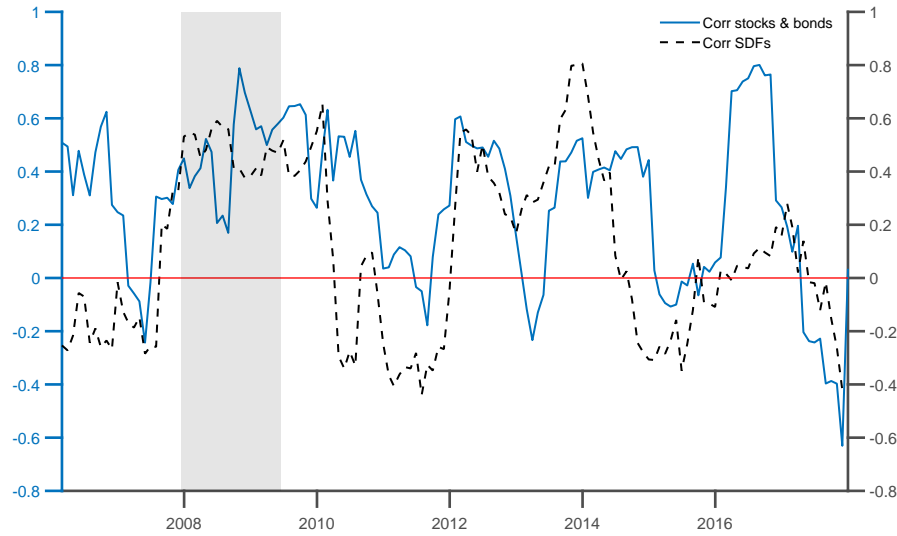


This figure plots the time-series of minimum variance SDFs pricing portfolios of bonds sorted on credit rating (solid line), along with the minimum variance SDF pricing stock returns sorted on credit rating (dashed-dot line) and the minimum variance SDF pricing both bond and stock returns sorted on credit rating (dashed line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

Finally, to have a better sense of the evolution between the bond and stock returns sorted into credit rating quintiles, on the one hand, and between the minimum variance SDFs pricing separately the corporate bonds and the stocks, I report in Figure 3.2 their rolling correlations. The rolling window size is 12 months, yielding annual correlations estimates. Both rolling correlations fluctuate during the period considered, exhibiting large swings. As expected, there is on average a positive correlation between stock and bond returns sorted on credit rating, as well as a positive co-movement between their respective minimum variance SDFs. Both correlations increase during the recent financial crisis.

<sup>9</sup> To provide an additional example, for portfolios formed on firm size, the global minimum variance SDF is going to resemble more the SDF pricing stocks only, as Figure B.1 illustrates. The SDFs also spike during episodes when the Federal Reserve raised the interest rates, such as in 2016 and 2017.

**Figure 3.2: Rolling Correlations**



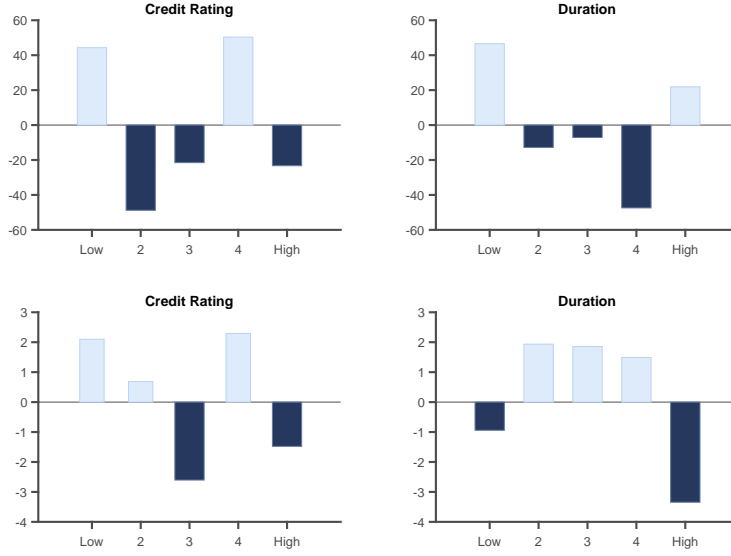
This figure plots the moving correlations between the average stock returns and the average bond returns sorted on credit rating (left axis). The right axis plots the moving correlations between the minimum variance SDFs pricing the stock returns, and the minimum variance SDF pricing bond returns, sorted on credit rating (dashed line). Data is monthly and runs from January 2005 to December 2017. The window size for the moving correlations is 12 months, producing trailing annual correlation estimates.

## Portfolio Implications

In this section, I investigate the portfolio implications of the minimum variance SDFs derived under the assumption of frictionless markets. This analysis is essential in order to determine whether, in practice, the average investor would be able to form such portfolios.

Figures 3.3 and 3.4 plot the portfolio weights associated with the minimum variance SDFs that price portfolios of bond returns for the various characteristics considered (top panels) and the weights corresponding to stock portfolios of the issuing firms (bottom panels). There are two noteworthy observations. First, the weights in the different bond portfolios are significantly higher than in the stock portfolios. This is a consequence of large bond Sharpe ratios coupled with low bond return volatility. In practice, this would imply that only investors able to take excessive leveraged positions could construct such a portfolio. Second, many of the bond weights carry a negative sign, yielding different short positions along some quintile or decile portfolios. In reality however, it is difficult to short-sell corporate bonds, as this activity is not costless, and in some cases, especially for highly illiquid bonds, it might simply not be possible (see, e.g., [Asquith, Au, Covert, and Pathak \(2013\)](#) and [Blanco, Brennan, and Marsh \(2005\)](#), among

**Figure 3.3: Portfolio Weights (Quintiles)**



This figure plots the portfolio weights implied by the minimum variance SDFs correctly pricing portfolios of bonds sorted on credit ratings and duration (top panel) and portfolios of the corresponding stocks of the issuing firms (bottom panel). Data is monthly and runs from January 2005 to December 2017.

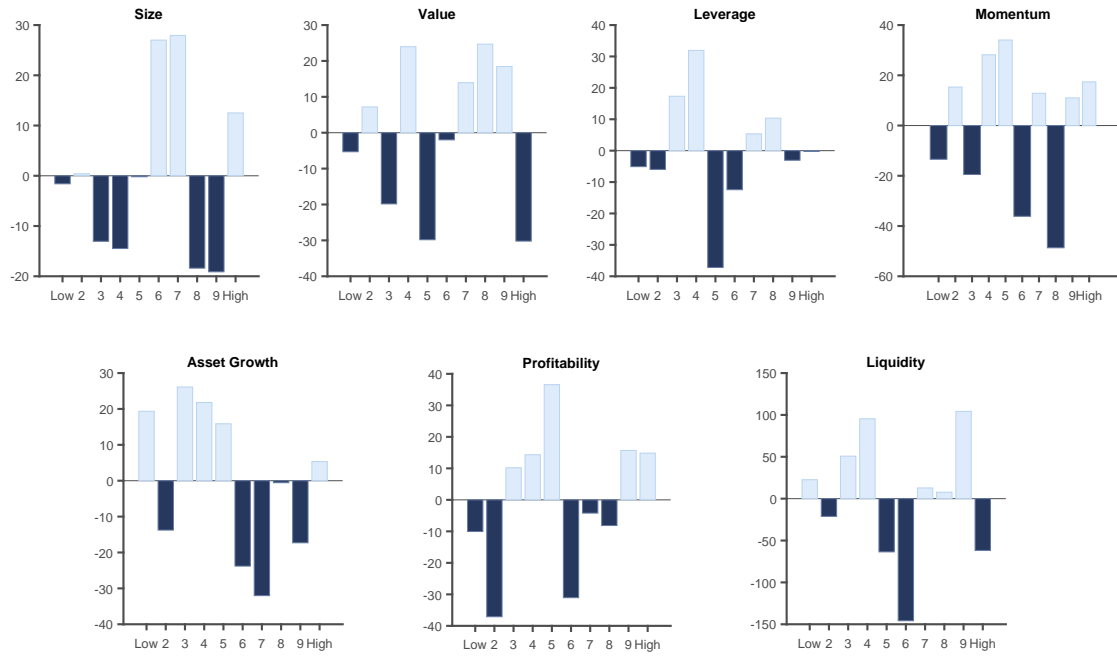
others). Hence, there are significant costs associated with short-selling the bonds, and for some classes of investors, such as pension funds or insurance companies for example, regulators might even not allow them to engage in short-selling activities. To examine the cross-market pricing relation and the associated degree of market integration in a more realistic framework, I derive next the results of minimum variance SDFs incorporating different financial frictions, such as short-selling constraints.

## *II. The Case of Market Frictions Assumption*

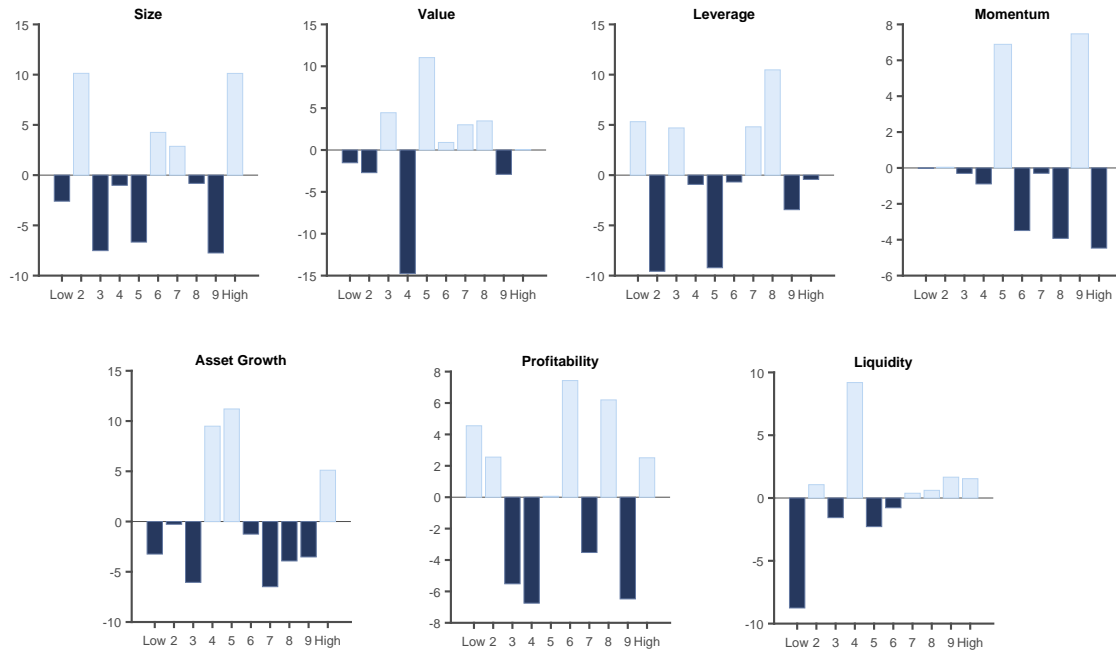
### **Minimum Variance SDFs**

Since financial markets are not free of frictions, and investors face different constraints when trading, I derive the optimal solution while restricting short-selling in bonds, stocks, or both. Notice however that the methodology I employ is general and can accommodate various types of constraints, including leverage and financial constraints. Hence, even though it is not limited to short-selling restrictions, the latter represent appropriate constraints that investors encounter, especially in the corporate bond market. The estimation procedure is similar, while imposing nonnegative weights and having pricing constraints that are less restrictive. The minimum

**Figure 3.4: Portfolio Weights (Deciles)**



**Panel A: Bond Positions**



**Panel B: Stock Positions**

This figure plots the portfolio weights implied by the minimum variance SDFs correctly pricing portfolios of bonds sorted on different characteristics (Panel A) and portfolios of the corresponding stocks of the issuing firms (Panel B). Data is monthly and runs from January 2005 to December 2017.



variance SDFs are still going to be a linear function of the returns, but only the ones for which the short-selling constraints are not binding. Naturally, since these returns will be a subset of all the available returns, the SDFs will have a lower volatility. Moreover, as in most cases the majority of the wealth will be allocated only to one characteristic sorted portfolio, maximizing the Sharpe ratio, the resulting SDFs will exhibit a higher correlation. In order to study the extent of market integration, i.e. whether a common SDF for portfolios of bonds and stocks exists in presence of short selling constraints, I examine the cross-market pricing performance.

**Table 3.7:** Pricing Errors and HJ Distance in Presence of Short-Selling Constraints

This table reports the pricing errors and the HJ distance implied by minimum variance SDFs which account for short-selling constraints. Panel A reports the pricing errors implied by the bond (stock) SDF for bond (stock) portfolios, computed as  $E[M_B R_B] - 1$ , or  $E[M_S R_S] - 1$ , with  $B$  denoting the bonds and  $S$  the stocks. Panel B reports the cross-market pricing errors, computed as  $E[M_B R_S] - 1$  and  $E[M_S R_B] - 1$ . Pricing errors are the ones implied by the bond (stock) SDF that prices the portfolios sorted on the corresponding characteristic. Panel C reports the [Hansen and Jagannathan \(1997\)](#) (HJ) distance between the minimum variance bond SDF, denoted by  $\hat{M}_B$  and the set of admissible SDFs for the stock market  $S$  (first row), as well as the HJ distance between the minimum variance stock SDF, denoted by  $\hat{M}_S$  and the set of admissible SDFs for the bond market  $B$ . All measures are reported in basis points (bp). Data is monthly and runs from January 2005 to December 2017.

Panel A: Pricing Errors									
	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$E[M_B R_B] - 1$	14.8	17.6	13.3	5.3	1.9	9.8	3.2	5.1	17.8
$E[M_S R_S] - 1$	12	10	20.2	18.9	16.3	8.5	25.1	27.8	27.2
Panel B: Cross-Market Pricing Errors									
	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$E[M_B R_S] - 1$	22.2	32.2	42.3	34.4	29.6	47.4	35.5	33.4	23.9
$E[M_S R_B] - 1$	-11	-18.4	-23.5	-26.3	-26.6	-56.7	-23.7	-24.5	-10.6
Panel C: HJ Distance of Minimum Variance SDFs									
	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$\sqrt{\delta(\hat{M}_B, S)}$	9.81	9.98	19.52	25.93	18.39	17.99	26.25	24.05	29.20
$\sqrt{\delta(B, \hat{M}_S)}$	28.57	30.57	21.13	27.33	14.49	27.97	19.77	26.30	46.64

The average pricing errors implied by the restricted SDFs are reported in Table 3.7.<sup>10</sup> Notice that in presence of short-selling constraints, the constrained bond (stock) SDFs are not ensured to correctly price the bond (stock) portfolios (Panel A). Still, the implied pricing errors are small, varying from 2 to 18 bp for bond SDFs. Stock SDFs also imply a small pricing error, albeit slightly larger than the bond, varying from 9 to 27 bp. It is more relevant to analyze the cross-market implied pricing errors, especially in order to determine whether a common SDF exists when short-selling constraints are accounted for. Panel B reports the pricing errors made when using the constrained bond SDF to price stock portfolios. Again, the bond SDF employed is

<sup>10</sup>Tables B.2 and B.3 in the Appendix report individual results for each characteristic sorted portfolio.

formed on the same characteristics as the stock portfolios. The constrained bond SDFs perform well in pricing the stock portfolios, as the implied pricing errors vary from 22 bp to 47 bp. It follows that, in absence of short-selling, minimum variance bond SDFs based on portfolios that are rebalanced monthly, yield an average monthly pricing error of 30 bp, across all firm characteristics. Provided that these implied pricing error are within the quoted bid-ask spreads for stock portfolios, the constrained bond SDF can be used to price the returns in the stock market. For instance, analyzing a series of anomalies, [Novy-Marx \(2013\)](#) estimate transaction costs between 20 and 65 bp for stock portfolios.<sup>11</sup> Turning to the performance of stock SDFs with short-selling constraints for bond portfolios, they also appear to do well in terms of pricing. In particular, the implied pricing errors are always within quoted bid-ask spreads (in absolute terms), especially since bonds have higher transaction costs than stocks.<sup>12</sup> Stock short-selling constraints lead to lower corporate bond prices (since the pricing errors are always negative), in line with the model predictions and findings of [Atmaz and Basak \(2019\)](#).

Overall, the evidence suggests that using the constrained SDF in one market performs reasonably well in pricing the portfolio of returns in the other market, at least in the sense that the implied pricing errors are similar in magnitude with typically observed bid-ask spreads. Lastly, Panel C reports the HJ distance, i.e. an estimate of the maximal implied pricing errors across markets in presence of short-selling constraints. Accordingly, the associated pricing errors fluctuate between 10 and 30 bp for bond constrained SDFs, implying that the cross-pricing can be actually improved by incorporating some type of constraints. It follows that constraining the set of possible portfolio weights achieves some form of regularization. Intuitively, this is equivalent with avoiding overfitting in sample (for the bond portfolios), while enhancing the fit out-of-sample (for stock portfolios). The constrained stock portfolios yield pricing errors that vary between 15 bp for leverage sorts and 47 bp for liquidity sorts.

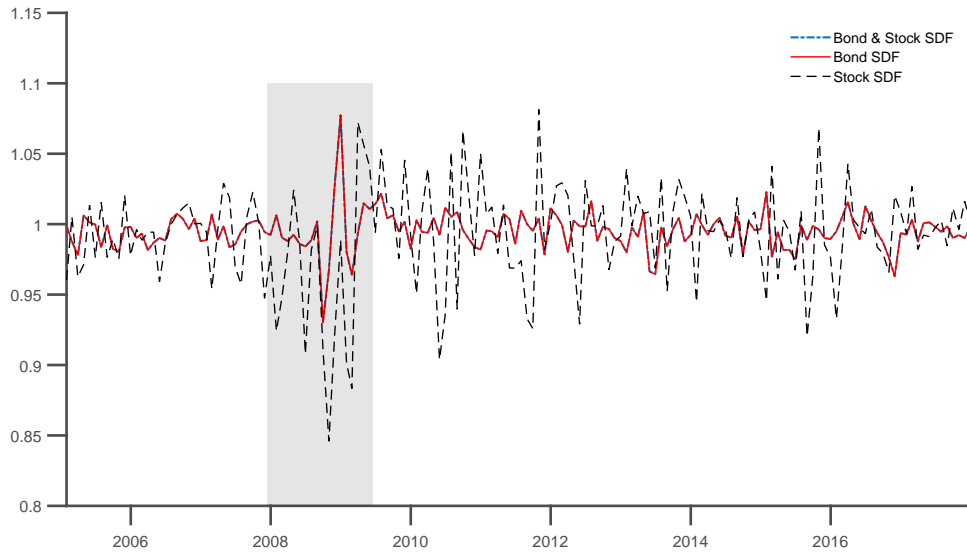
Ultimately, to illustrate the properties of the constrained SDFs, I plot in [Figure 3.5](#) the time-series of the minimum variance SDF pricing both stock and bond portfolios sorted on firm size, while restricting short-selling in bonds (dashed-dotted line), the minimum variance bond SDF with short-selling constraints (solid line) and the minimum variance stock SDF with short-selling constraints (dashed line). The following observations are worth emphasizing. First,

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<sup>11</sup>In reality, the actual transaction costs incurred by individual investors can be even higher, especially when taking into account monitoring costs.

<sup>12</sup>Similarly, in practice, the costs associated with trading bonds can be even higher, considering the potential price impact.

**Figure 3.5:** Time-Series of Minimum Variance SDFs in Presence of Short-Selling Constraints



This figure plots the time-series of minimum variance SDFs pricing portfolios of both bond and stock returns sorted on firm size (dashed-dotted line), along with the marginal minimum variance SDFs, in presence of short-selling constraints. The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

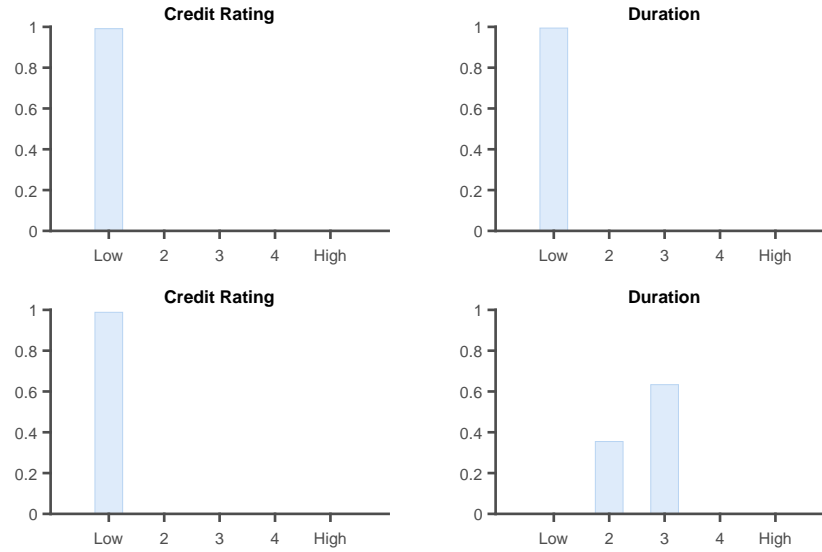
the minimum variance SDFs in presence of short-selling constraints exhibit a significantly lower dispersion. Second, the minimum variance SDF that prices both stock and bond portfolios sorted on size, consistently with no short-selling allowed, follows closely the dynamics of the marginal bond SDF.

### Portfolio Implications

Since financial constraints impact the form of the optimal SDF in the underlying market, I examine next the portfolio implications that emerge in such cases. The corresponding portfolio weights are reported in Figures 3.6 and 3.7. In presence of bond short-selling constraints, all resources will be allocated only in the quintile or decile portfolio that maximizes the Sharpe ratio. With respect to the stock short-selling constraints, there are instances when the resources will be divided across two or three portfolios. Overall, the portfolio implications derived when imposing bond and/or stock short-selling constraints appear to be more realistic, in the sense that investors would be able to readily form such portfolios in practice.

The evidence provided in the case of markets with frictions in the form of short-selling constraints suggests that it is still possible to construct a common SDF pricing both corporate

**Figure 3.6:** Portfolio Weights in Presence of Short-Selling Constraints (Quintiles)



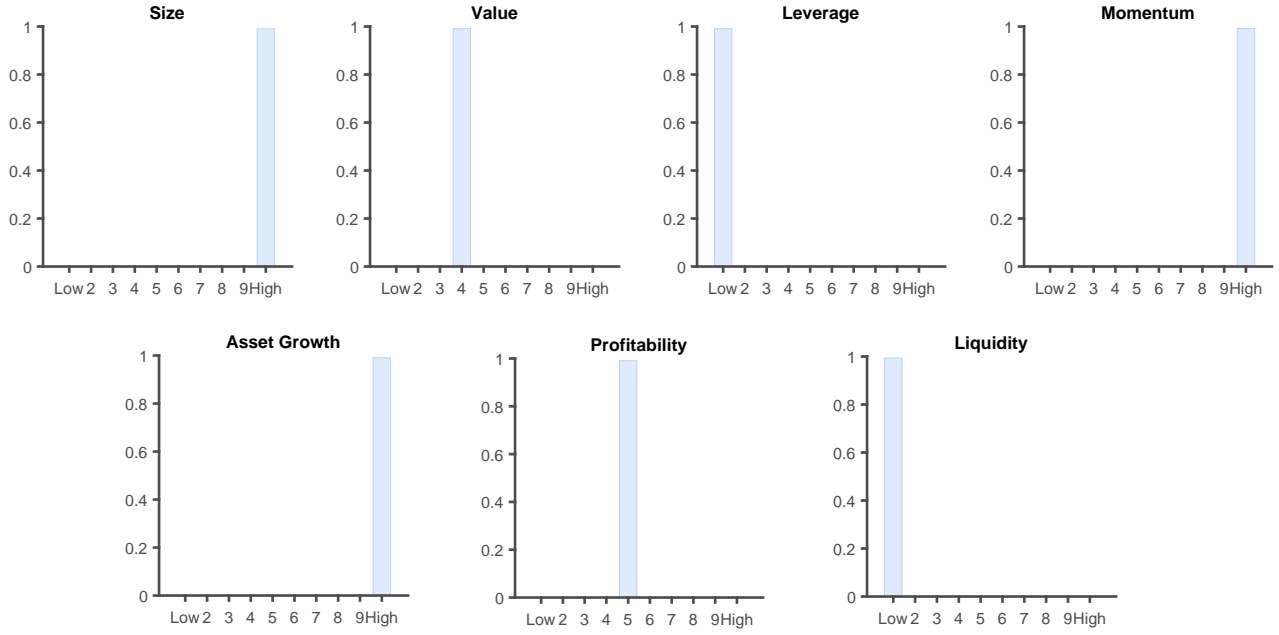
This figure plots the portfolio weights implied by the minimum variance SDFs pricing portfolios of bonds sorted on credit ratings and duration (top panel) and portfolios of the corresponding stocks of the issuing firms (bottom panel), accounting for short-selling constraints. Data is monthly and runs from January 2005 to December 2017.

bonds and stocks reasonably, at least consistently with the typical bid-ask spreads. Moreover, taking a closer look at the associated optimal portfolio, it seems plausible to argue that agents can devise such an object. However, a potential concern regarding the cross-market pricing performance stems from the fact that some of the decile portfolios, especially for the corporate bond returns, do not exhibit sufficient cross-sectional dispersion. In other words, the good pricing fit may be due to the fact that average returns are similar across deciles. I systematically address this in the robustness section, where I construct double sorted portfolios based on firm-level characteristics. I show that, despite the increased cross-sectional variation, results are consistent if I use single or double sorts.

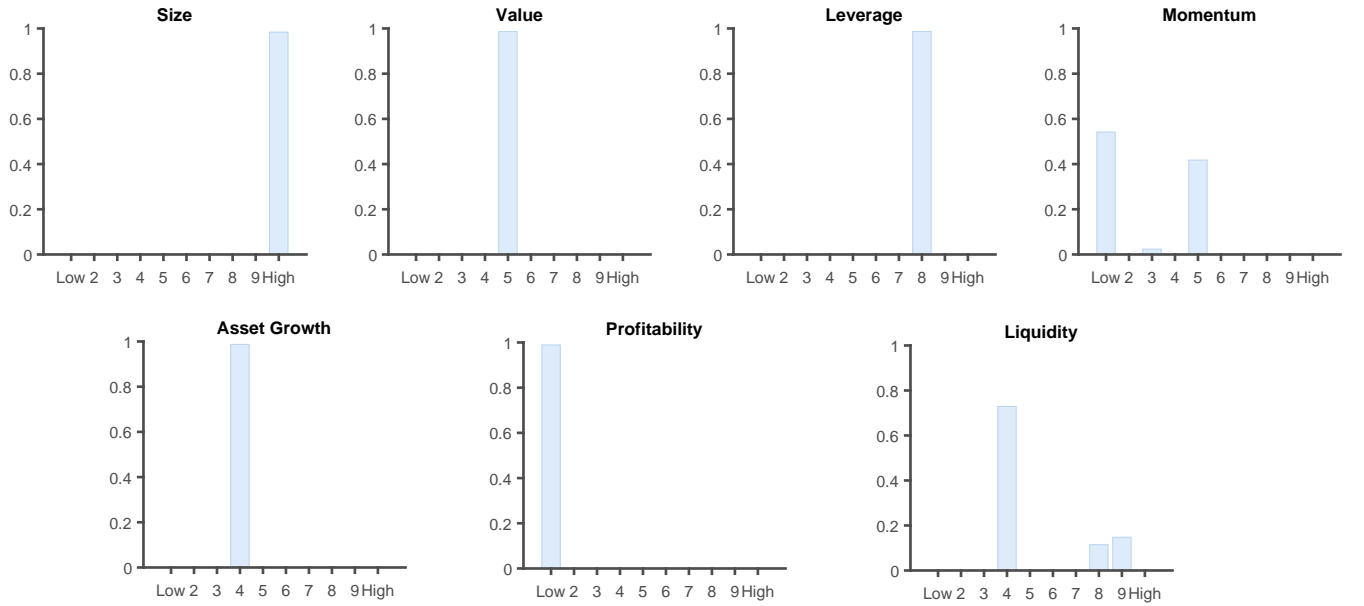
### 3.4.3 Constrained Minimum Variance SDFs vs. Factor Models

A natural question is whether the constrained model-free extracted SDFs perform better than existing factor models for bonds and stocks. Therefore, in this subsection, I evaluate the pricing performance of existing factor models and provide a comparison with respect to the minimum variance constrained SDFs. As competing models, I consider the [Fama and French \(1993\)](#) 3 Factor Model, proxying for market, size and value and the bond factor model of [Bai, Bali, and](#)

**Figure 3.7:** Portfolio Weights in Presence of Short-Selling Constraints (Deciles)



Panel A: Bond Positions



Panel B: Stock Positions

This figure plots the portfolio weights implied by the minimum variance SDFs pricing portfolios sorted on different firm-level characteristics in presence of short-selling constraints. Panel A (B) reports results for the bond (stock) portfolios. Data is monthly and runs from January 2005 to December 2017.

Wen (2019) capturing exposure to the market, downside risk, credit and liquidity factors, as well as a combination of the two models.<sup>13</sup> As test assets, I use double sorted portfolios based on quintile firm characteristics, for the following reasons: (i) double sorting will potentially deliver more cross-sectional variation between the portfolios and (ii) this approach will yield 25 test assets, which is the benchmark used in testing asset pricing models.

For the factor models, I consider the following empirical specification:

$$R_{it} = \alpha_i + \beta_i' F_t + \varepsilon_{it}, \quad (3.11)$$

where  $R_{it}$  is the return on the test assets  $i = 1, \dots, 25$  at time  $t$  and  $F_t$  is the vector containing the corresponding factors at time  $t$ . For the Fama French 3 factor model,  $F = [MKT \quad SMB \quad HML]'$ , capturing the market, size and value factors. For the bond factor model,  $F = [MKT \quad DRF \quad CRF \quad LRF]'$ , proxying for market, downside risk, credit risk and liquidity risk. I also consider the case where the stock and bond factors are stacked. For the model-free extracted global (for both stocks and bonds) SDF accounting for short selling constraints,  $F = [\hat{M}]'$ .

I estimate the factor models using the Fama MacBeth approach. The methodology proceeds in two steps. First, I run time-series regressions to obtain the betas, or loadings, on the corresponding factors. Second, I run cross-sectional regressions on the estimated betas from the first step in order to retrieve the price of risk of the factors. When performing this two-stage regression, I adjust the standard errors to account for errors-in-variables, since the betas are estimated in the first step, for heteroskedasticity, as the variance of residuals is not constant, and for potential autocorrelation in error terms.

To discriminate between the various models, I use two criteria: (i) the size of the implied pricing errors and (ii) the R-Square of a generalized least squared (GLS) regression of returns on the proposed factors. The reason I focus on GLS R-Square rather than the OLS one follows from the critique of Lewellen, Nagel, and Shanken (2010), that apparently strong explanatory power in fact provides quite weak support for a model, especially if the test assets are the ones formed on size and book-to-market portfolios. Another advantage is that the GLS R-Square has a meaningful economic interpretation in terms of the relative mean-variance efficiency of

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<sup>13</sup> Results are similar for the stock returns if I employ instead the four factor model, including the momentum factor, or the five factor model of Fama and French (2008), accounting additionally for profitability and investment factors.

a model’s factor-mimicking portfolios. In other words, the GLS R-Square reflects a factor’s proximity to the mean-variance boundary, whereas the OLS R-Square does not have in general a direct relation to the factor’s location in mean-variance space.

The size of the implied pricing errors for the various models considered, as measured by the HJ distance is reported in Table 3.8. Specifically, for the corresponding bond test assets, I compute the pricing errors implied by the minimum variance SDF with short-selling constraints, as well as the pricing errors implied by the bond factor model. All the estimates of the HJ distance are statistically significant at the 1% level. The model-free SDF entails lower pricing errors on average than the bond factor model. When considering stocks as test assets, I compare the performance of the minimum variance SDF with short-selling constraints with the Fama French 3 factor model. Again, all the estimates of the HJ distance are strongly statistically significant at conventional levels. Summarizing, the SDFs accounting for short-selling constraints can deliver an enhanced pricing performance relative to (unconstrained) factor models.

**Table 3.8:** Pricing Errors Implied by Factor Models

This table reports the pricing error implied by factor models, as well as the one implied by the minimum variance SDF with short-selling constraints, computed using the [Hansen and Jagannathan \(1997\)](#) (HJ) distance. The measure is reported in basis points. Heterokedasticity robust standard errors are reported in brackets. Data is monthly and runs from January 2005 to December 2017.

	Bonds		Stocks	
	SDF	BF	SDF	FF3
Credit rating & Duration	71.73 [12.9]	75.54 [14.6]	51.76 [7.32]	60.42 [8.52]
Credit rating & Liquidity	51.60 [9.6]	50.99 [10.7]	57.92 [7.45]	56.55 [6.42]
Credit rating & Size	52.36 [10.3]	54.07 [11.5]	45.72 [8.8]	44.21 [8.2]
Duration & Value	57.82 [11.1]	61.50 [11.1]	40.88 [9.4]	43.47 [10.2]
Duration & Liquidity	51.99 [10.7]	49.14 [10.9]	44.80 [9.7]	45.29 [9.7]
Leverage & Profitability	35.65 [9.2]	35.06 [8.9]	45.54 [7.9]	45.73 [7.9]
Size & Leverage	44.14 [9.9]	43.05 [9.6]	49.98 [7.9]	51.04 [7.6]
Size & Liquidity	37.52 [10.1]	33.43 [10.2]	58.78 [8.9]	61.21 [10.2]
Size & Profitability	43.55 [7.5]	49.89 [7.8]	36.63 [7.8]	30.70 [7.7]
Size & Value	36.06 [9.3]	36.94 [8.9]	32.63 [7.4]	30.22 [7.8]

The associated GLS R-Square from fitting the Fama French 3 factor model, the bond factor model, or a combination of both, along with fitting the global minimum variance constrained SDF are reported in Table 3.9. The test assets considered are the 25 portfolios obtained from double sorts, for stocks, bonds, or both. None of the factors considered provide a perfect fit, as measured by the GLS R-Square. Nevertheless, the best fit on average is obtained for portfolios of stocks. The Fama French 3 factor model delivers GLS R-Square ranging from 4% to 35%. Importantly, the constrained global minimum variance SDF provides a better fit than the Fama French factor, bond factor, or a combination of both for some instances, as highlighted in Table 3.9. This evidence is meaningful especially in the view that the SDF is basically a one factor model. For the bond portfolios, the best fit tends to be given by the bond factor model. Still, it is relevant to stress that stock and bond factor models are constructed by assuming that investors can build long and short positions in some decile sorted portfolios, i.e. the weights are unrestricted. Besides the potential short-selling constraints, it is likely that some of these portfolios are not tradable in practice, especially if they contain small illiquid stocks or bonds. Finally, when considering instead both stocks and bonds portfolios as test assets, the fit is rather poor, regardless of the factors used.

**Table 3.9:** GLS R-Square

This table reports the GLS R-Square obtained by running the following regression using Fama MacBeth procedure:

$$R_{it} = \alpha_i + \beta_i' F_t + \varepsilon_{it},$$

where  $R_{it}$  is the return on the test assets  $i$  at time  $t$  and  $F_t$  is the vector containing the corresponding factors at time  $t$ . For the Fama French 3 factor model (FF3),  $F = [MKT \text{ } SMB \text{ } HML]'$ , capturing the market, size and value factors. For the bond factor model (BF),  $F = [MKT \text{ } DRF \text{ } CRF \text{ } LRF]'$ , proxying for market, downside risk, credit risk and liquidity risk. For the model-free extracted global SDF accounting for short selling constraints,  $F = [\hat{M}]$ . Results are reported for stock, bond and stock and bond portfolios. Data is monthly and runs from January 2005 to December 2017.

	Stocks				Bonds				Stocks & Bonds			
	FF3	BF	FF3 + BF	SDF	FF3	BF	FF3 + BF	SDF	FF3	BF	FF3 + BF	SDF
Credit rating & Duration	12.4%	18.6%	28.1%	14.6%	2.4%	9.3%	14.1%	1.1%	1.2%	2.9%	3.5%	0.0%
Credit rating & Liquidity	22.4%	13.7%	33.0%	<b>34.5%</b>	5.7%	9.9%	17.0%	0.0%	1.4%	1.7%	3.6%	0.2%
Credit rating & Size	10.1%	5.9%	34.0%	8.5%	5.0%	20.4%	11.2%	0.1%	0.9%	4.6%	5.7%	0.2%
Duration & Value	10.5%	19.3%	23.1%	6.4%	5.1%	11.4%	27.7%	0.5%	3.4%	4.6%	7.8%	0.2%
Duration & Liquidity	7.3%	8.1%	18.9%	<b>13.7%</b>	5.7%	7.1%	31.4%	0.0%	8.6%	5.7%	13.1%	0.0%
Leverage & Profitability	5.3%	5.8%	18.4%	4.2%	4.7%	26.1%	15.0%	0.6%	5.0%	1.8%	5.5%	0.6%
Size & Leverage	4.0%	2.3%	7.8%	<b>4.2%</b>	3.3%	8.5%	7.6%	0.1%	1.4%	2.0%	3.4%	0.7%
Size & Liquidity	13.8%	13.9%	30.6%	<b>14.8%</b>	12.7%	8.4%	31.1%	0.1%	4.2%	0.9%	5.1%	0.5%
Size & Profitability	35.1%	3.9%	47.4%	12.1%	5.7%	6.8%	24.0%	0.2%	6.9%	1.3%	7.7%	0.3%
Size & Value	15.4%	2.0%	25.7%	5.3%	20.3%	18.3%	22.7%	0.3%	2.6%	1.3%	2.7%	0.1%

In summary, a horse race between factor models and the SDFs suggest the latter yield on average smaller pricing errors, as measured by the corresponding HJ distance and can yield a

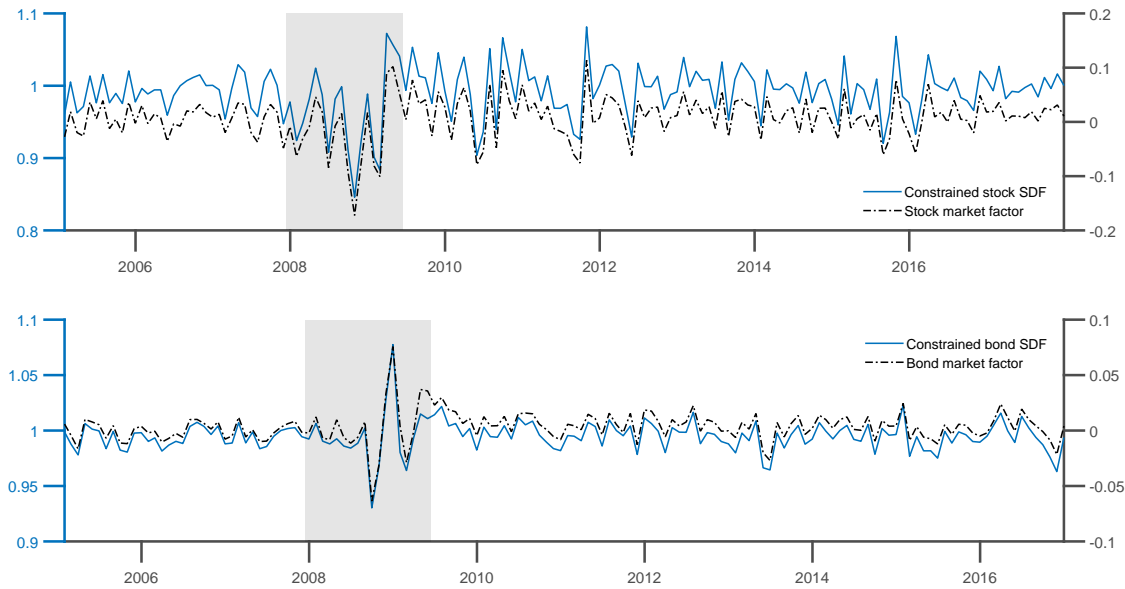


higher GLS R-Square in some cases. Overall, the constrained SDFs not only exhibit a better pricing performance, but can be interpreted economically as optimal Sharpe ratio portfolios of traded assets.

### 3.4.4 Minimum Variance SDFs and Risk Factors

To grasp what are the risk factors embedded in the model-free extracted SDFs, I investigate their relationship with the factors from existing models. The unconstrained minimum variance SDFs seem to be largely unrelated with the Fama French 3 factor model or the bond factor model.<sup>14</sup> The SDFs accounting for short selling constraints, on the other hand, are typically explained by factor models, with most of the contribution coming from the market factor. This result is intuitive, as the market portfolio itself consists only of long positions and is mean-variance efficient. In fact, as Figure 3.8 illustrates, both the constrained minimum variance stock SDF (top panel) and the constrained bond SDF (bottom panel) strongly positively co-move with the market factor.

**Figure 3.8:** Constrained Minimum Variance SDFs and Market Factor



This figure plots the time-series of short-selling constrained minimum variance SDFs pricing portfolios of stock (bond) returns sorted on firm size in top (bottom) panel, along with the market factor of [Fama and French \(1993\)](#) and the bond market factor of [Bai, Bali, and Wen \(2019\)](#) (dashed-dotted line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017 (2016) for stocks (bonds).

<sup>14</sup>In interest of space, I do not report the results.

Still, there remains a part which is unexplained by these factors and amounts on average to 40%. Motivated by the growing literature documenting the substantial role of financial institutions in determining equilibrium asset prices, I explore additionally the relationship between the minimum variance SDFs and intermediaries. The intermediary asset pricing literature posits that the SDF of the financial intermediary, rather than the one of the household, prices financial assets, in particular sophisticated securities that are harder to trade in practice (see, e.g., [Adrian, Etula, and Muir \(2014b\)](#) and [Haddad and Muir \(2018b\)](#), among others). Since the model-free extracted SDFs contain information about the relevant risk factors, it is natural to compare them to the intermediary risk factor. To this end, I use the intermediary capital risk factor of [He, Kelly, and Manela \(2017b\)](#), which captures the shocks to the equity capital ratio of financial intermediaries (Primary Dealer counterparties of the New York Federal Reserve). Intuitively, whenever an intermediary faces a negative shock to its equity capital, its risk-bearing capacity is affected. Hence, we should expect a strong link between constrained, rather than unconstrained, minimum variance SDF and the intermediary risk factor. The top panel of Figure 3.9 plots the time-series of short-selling constrained minimum variance stock SDFs, along with the intermediary risk factor.<sup>15</sup> When short-selling in stocks is restricted, the associated SDF strongly positively co-moves with the intermediary risk factor (with a correlation of 75%), especially during the recent financial crisis. This result is intuitive, as according to the portfolio weights in Figure 3.7, all the wealth is allocated to the portfolio containing the largest firms. Indeed, upon a closer inspection, the firms in the top decile are also highly leveraged and they are among the primary dealers of the NY FED.<sup>16</sup> The short-selling constrained bond SDF, on the other hand, features a weak co-movement with the intermediary risk factor (bottom panel), having a correlation of  $-3\%$ .

To test the relationship between minimum variance SDFs and intermediaries more systematically, I run the following regressions:

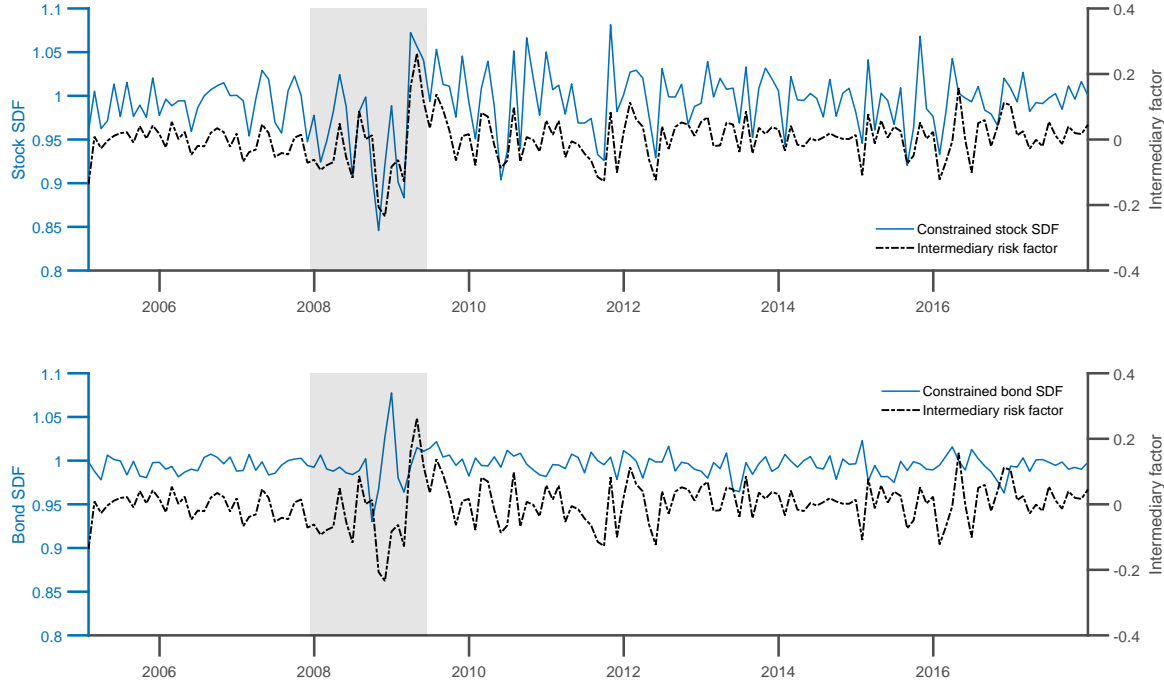
$$M_{it} = \alpha_i + \beta_i \kappa_t + \epsilon_{it}, \quad (3.12)$$

with  $i = B, S$  denoting the bond or stock and  $\kappa$  the intermediary risk factor. Results are reported

<sup>15</sup>For illustrative purposes, Figure 3.9 plots the SDF pricing portfolios of returns sorted on firm size. Plots are similar for other characteristics-sorts.

<sup>16</sup>The list of companies present in the sample that coincide with the primary dealers of the NY FED is as follows: Citigroup, Goldman Sachs, Morgan Stanley, Bank of America, J.P. Morgan, Wells Fargo.

**Figure 3.9:** Constrained Minimum Variance SDFs and Intermediary Risk Factor



This figure plots the time-series of short-selling constrained minimum variance SDFs pricing portfolios of stock (bond) returns sorted on firm size in top (bottom) panel, along with the intermediary risk factor of [He, Kelly, and Manela \(2017b\)](#) (dashed-dotted line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

in Table 3.10. The test assets on which the SDFs are constructed are the double-sorted portfolios. The constrained bond SDF exhibits no apparent relation with the intermediary risk factor. Only the SDFs pricing portfolios sorted on duration and value, and leverage and profitability returns load significantly and positively on the intermediary risk factor. Interestingly, the unconstrained bond SDF loads instead negatively on the intermediary risk factor, and significantly for portfolios sorted on credit rating, size, leverage and value. Overall, the explanatory power is rather low. A different picture emerges for the stock SDF, especially the constrained one: the estimates for the intermediary risk factor are positive and statistically significant at the 1% level. Moreover, the associated  $R^2$  is high, ranging from 42% to 61%. The unconstrained stock SDFs also have a positive loading on the intermediary risk factor, but the explanatory power is lower. Altogether, the SDFs accounting for stock short-selling constraints appear to capture to a great extent the intermediary risk factor. It is important to notice that these findings stem from time-series regressions. Indeed, consistent with previous evidence, if running cross-section regressions

**Table 3.10:** Minimum Variance SDFs and Intermediary Risk Factor

This table reports the estimated coefficients from regressing the minimum variance SDFs on the intermediary risk factor ( $\kappa$ ):  $M_{it} = \alpha_i + \beta_i \kappa_t + \epsilon_{it}$ , with  $i = B, S$  denoting the bond or stock. The constrained SDFs account for short-selling constraints, whereas the unconstrained one is unrestricted. Both types of SDFs price different double-sorted portfolios. Data is monthly and runs from January 2005 to December 2017. Standard errors are reported in brackets. \*\*\*, \*\* and \* denotes significance at the 1%, 5% and 10% level, respectively.

		Bond SDF		Stock SDF	
		Constrained	Unconstrained	Constrained	Unconstrained
Credit rating & Duration	$\alpha$	0.997*** [0.000]	0.999*** [0.057]	0.994*** [0.002]	0.987*** [0.002]
	$\beta$	0.003 [0.007]	-1.595* [0.873]	0.599*** [0.040]	1.169* [0.040]
	$R^2$	0.07%	1.49%	58.80%	1.45%
Credit rating & Liquidity	$\alpha$	0.995*** [0.001]	0.996*** [0.040]	0.995*** [0.002]	0.977*** [0.044]
	$\beta$	-0.021 [0.002]	-0.887 [0.615]	0.586*** [0.040]	3.807*** [0.675]
	$R^2$	0.09%	0.70%	58.20%	16.70%
Credit rating & Size	$\alpha$	0.995*** [0.001]	0.997*** [0.038]	0.994*** [0.002]	0.991*** [0.040]
	$\beta$	0.001 [0.015]	-1.484** [0.589]	0.452*** [0.042]	0.273 [0.575]
	$R^2$	0.00%	3.35%	41.70%	0.15%
Duration & Value	$\alpha$	0.997*** [0.001]	0.997*** [0.046]	0.992*** [0.002]	0.988*** [0.033]
	$\beta$	0.022** [0.010]	0.06 [0.709]	0.65*** [0.041]	0.916* [0.510]
	$R^2$	2.40%	0.00%	61.30%	1.42%
Duration & Liquidity	$\alpha$	0.997*** [0.001]	0.998*** [0.041]	0.994*** [0.003]	0.987*** [0.037]
	$\beta$	0.001 [0.006]	-0.627 [0.630]	0.628*** [0.049]	1.244** [0.564]
	$R^2$	0.0%	0.6%	51.4%	2.5%
Leverage & Profitability	$\alpha$	0.995*** [0.001]	0.995*** [0.029]	0.994*** [0.003]	0.991*** [0.037]
	$\beta$	0.051*** [0.019]	-0.434 [0.443]	0.811*** [0.058]	0.921 [0.560]
	$R^2$	3.64%	0.62%	55.20%	1.09%
Size & Leverage	$\alpha$	0.995*** [0.001]	0.995*** [0.034]	0.993*** [0.003]	0.989*** [0.040]
	$\beta$	0.001 [0.018]	-0.89* [0.513]	0.862*** [0.057]	1.224** [0.613]
	$R^2$	0.0%	1.3%	59.3%	1.9%
Size & Liquidity	$\alpha$	0.995*** [0.001]	0.994*** [0.029]	0.995*** [0.003]	0.984*** [0.046]
	$\beta$	0.021 [0.018]	-0.487 [0.436]	0.534*** [0.048]	2.797*** [0.699]
	$R^2$	0.23%	0.16%	43.30%	8.81%
Size & Liquidity	$\alpha$	0.995*** [0.001]	0.995*** [0.035]	0.993*** [0.003]	0.988*** [0.029]
	$\beta$	-0.01 [0.016]	-0.59 [0.536]	0.714*** [0.054]	1.489*** [0.450]
	$R^2$	0.2%	0.1%	52.9%	6.0%
Size & Value	$\alpha$	0.995*** [0.001]	0.995*** [0.027]	0.994*** [0.002]	0.991*** [0.026]
	$\beta$	0.017 [0.016]	-0.696* [0.411]	0.488*** [0.037]	0.758* [0.391]
	$R^2$	0.10%	1.19%	51.90%	1.74%

instead, the intermediary risk factor will be more relevant for corporate bond returns, than for stocks.<sup>17</sup>

## 3.5 Robustness

In this section, I perform two robustness checks to my main analysis. First, I study the implications of constructing double sorted portfolios, in addition to univariate sorts. Second, I assess the impact of introducing an additional type of friction, where the attainable Sharpe ratio in the economy is restricted.

### 3.5.1 Double Sorting

In this subsection, I redo the analysis for double sorted portfolios of corporate bond and stock returns, based on different firm or asset characteristics quintiles. The motivation is twofold. First, double sorting can potentially deliver a larger cross-sectional variation between the return portfolios. This is particularly relevant for the bond returns, for which some other characteristic might be obscured when building univariate sorts. For instance, when sorting bond returns based on a firm characteristic, such as profitability, one might obtain flat average returns across portfolios only because a bond feature, such as credit rating or duration, is not accounted for. Double sorting will explicitly rule out such instances. Second, to provide a comparison in terms of pricing performance of my model-free extracted SDFs and existing asset pricing models, it is appropriate to have a larger cross-section. Consequently, by forming double sorts based on quintile firm characteristics, the number of test assets will equal 25, the benchmark when testing asset pricing models.

Previous evidence from univariate sorts guides the way in which double sorts should be constructed. In particular, since firm-level characteristics appear to matter distinctly for average bond and stock returns, the sorting should not be arbitrary. I consider thus the following double sorts. Credit rating and duration, since riskier bonds and those with a longer maturity require a higher expected returns. Similarly, firms with higher credit risk have a larger average stock return. Credit rating and liquidity, since low credit quality firms are likely to be more affected by illiquidity. Credit rating and size, as smaller firms are more likely to default. Duration and value, because they both capture investors' preferences in terms of maturity length. Specifically,

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<sup>17</sup>In interest of space, I do not report the regression results in the main text.

growth firms are high-duration assets, similar to long-term bonds. Duration and liquidity, as short-term bonds are traded more frequently than long-term ones, as the latter are typically buy-and-hold strategies. Leverage and profitability, since they both drive the underlying value of the firm. In particular, highly leveraged firms that are also profitable, might require a lower expected return. Size and leverage, for the reason that larger firms which are also significantly leveraged can entail a higher expected return. Size and liquidity, to grasp whether investors have a preference for different firm sizes, once liquidity is accounted for. Size and profitability, as larger profitable firms can also command a higher expected return. Size and value, as proxy for the typical test assets used in evaluating different asset pricing models.

**Table 3.11:** Average Pricing Errors for Double Sorted Portfolios

This table reports the average pricing errors implied by minimum variance SDFs for double sorted portfolios accounting for short-selling constraints, computed as  $E[M_i R_i] - 1$ , with  $i = B, S$  denoting the bonds and stocks. The last two columns report the [Hansen and Jagannathan \(1997\)](#) (HJ) distance between the minimum variance bond SDF with short-selling constraints, denoted by  $\hat{M}_B$  and the set of admissible SDFs for the stock market  $S$ , as well as the HJ distance between the minimum variance stock SDF with short-selling constraints, denoted by  $\hat{M}_S$  and the set of admissible SDFs for the bond market  $B$ , for double sorted portfolios. The measures are reported in basis points. Data is monthly and runs from January 2005 to December 2017.

	Bond SDF		Stock SDF		HJ distance	
	Bond Portfolios	Stock Portfolios	Bond Portfolios	Stock Portfolios	$\sqrt{\delta(\hat{M}_B, S)}$	$\sqrt{\delta(B, \hat{M}_S)}$
Credit rating & Duration	32.42	47.46	16.15	42.63	51.40	68.45
Credit rating & Liquidity	18.22	30.85	21.30	53.00	58.10	48.72
Credit rating & Size	17.18	42.67	13.02	51.97	46.64	46.78
Duration & Value	20.67	35.18	-20.41	17.31	41.13	56.45
Duration & Liquidity	21.33	34.12	12.98	43.60	45.89	51.12
Leverage & Profitability	6.97	30.37	-5.71	40.77	46.02	36.66
Size & Leverage	10.37	36.63	-7.76	44.73	50.83	43.30
Size & Liquidity	10.65	34.21	14.70	54.90	59.36	36.38
Size & Profitability	9.37	34.68	-6.79	42.31	38.38	44.42
Size & Value	9.87	37.05	-7.15	37.39	32.67	35.13

Since in the real world markets are not free of frictions, I derive the minimum variance SDFs pricing double sorted portfolios of bonds and/or stocks, in presence of short-selling constraints. In interest of space, I report in Table 3.11 the average pricing error implied by the constrained SDFs.

The general conclusion following this evidence is that even after increasing the level of granularity of the portfolios, the pricing performance of minimum variance SDF is satisfactory, in the sense that it can be sustained by the usual bid-ask spreads observed in the corporate bond and stock markets. However, it is worthwhile to mention that the pricing errors for the double sorted portfolios exhibit a higher cross-sectional dispersion and it is very likely that if one would study individual bonds and stocks rather than portfolios, the associated pricing errors

will be too high to be motivated by transaction costs. Put differently, the size and the similarity of the portfolios can also improve the degree of integration between the two markets. Indeed, computing the maximal implied pricing errors as measured by the HJ Distance suggest that they naturally increase when considering double sorts rather than univariate sorts, fluctuating between 30 and 70 basis points per month, as in the last two columns Table 3.11.

### 3.5.2 Market Frictions Restricting the Attainable Sharpe Ratio

An additional friction consists of restricting the attainable Sharpe ratio in the economy. Following the seminal work of Cochrane and Saá-Requejo (2000), who introduce good-deal bounds in order to rule out extremely high Sharpe ratios, I impose an upper bound on the volatility of the SDF. Intuitively, this SDF constraint implies that returns featuring low variance must have small prices. This is especially the case for bond returns. For consistency, I examine the asset pricing implications also when restricting the stocks' Sharpe ratios.

A natural question arising is what is an economic reasonable restriction for the Sharpe ratio available in the economy? Moreover, there is also a trade-off with respect to the Sharpe ratio that investors actually desire, in order to induce them to trade. For instance, it is likely that investors will require a higher Sharpe ratio than the historical market one, of 0.5 on an annual basis. To be consistent with market valuations, I use an upper volatility bound for the stock and bond portfolios that is equal to 70% of the Sharpe ratio of the global minimum variance SDF.<sup>18</sup>

To study the cross-market pricing implications between corporate bonds and stocks when the attainable Sharpe ratio is restricted, I report in Table 3.12 the corresponding HJ distance. When an upper volatility bound is imposed for the various bond SDFs, the maximal implied pricing error with respect to the stock market varies from 9.8 basis points for credit rating portfolios to 38.87 basis points for liquidity portfolios. When instead the Sharpe ratio of stock portfolios is constrained, the pricing error is higher, ranging from 14.31 basis points for leverage portfolios to 51.49 for liquidity sorted portfolios. Overall, the pricing errors implied by the SDFs preventing good-deal bounds appear to be within the typical incurred bid-ask spreads when trading corporate bonds and stocks.

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<sup>18</sup>Results are similar when varying the proportion considered for the global Sharpe ratio.

**Table 3.12:** HJ Distance in Presence of Good-Deal Bounds

This table reports the HJ distance between the minimum variance bond SDF derived when good-deal bounds are imposed, denoted by  $\hat{M}_B$  and the set of admissible SDFs for the stock market  $S$ , as well as the HJ distance between the minimum variance stock SDF derived when good-deal bounds are imposed, denoted by  $\hat{M}_S$  and the set of admissible SDFs for the bond market  $B$ . The restricted SDFs reflect an upper volatility bound for the stock and bond portfolios that is equal to 70% of the Sharpe ratio of the global minimum variance SDF. All the integration measures are reported in basis points. Data is monthly and runs from January 2005 to December 2017.

	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$\delta(\hat{M}_B, S)$	9.82	10.88	18.45	26.53	22.35	18.33	25.49	27.56	38.87
$\delta(B, \hat{M}_S)$	25.84	31.25	19.02	27.77	14.31	28.67	18.44	28.18	51.49

### 3.6 Conclusion

Shedding light on the underlying extent of market integration is crucial for refining our insights regarding the pricing of different asset classes, as well as for assessing the empirical success of canonical asset pricing models implicitly assuming perfect integration across different markets. There are two aspects worth emphasizing. First, there can be evidence of market segmentation not only across different asset classes, but also within an asset class. Second, considering different asset characteristics can give rise to different magnitudes of cross-market pricing errors.

Using a model-free approach that circumvents potential issues arising from the joint hypothesis test of market integration and correct model specification, I show that under the assumption of frictionless markets, empirically, there always exists a common SDF jointly pricing corporate bonds and stocks in the US market. However, the optimal portfolio associated with this SDF entails large and often negative positions, suggesting that it may be costly to construct in practice, especially in presence of financial or short-selling constraints. Since in the real world investors face various restrictions when designing their optimal portfolios, I study the implications of different market frictions. For instance, I find that introducing short-selling constraints generates cross-market implied pricing errors sustainable with quoted bid-ask spreads. The results suggest that markets are more integrated for larger firms, with more liquid corporate bonds and stocks. Similarly, firms that are more leveraged, have a higher asset growth and profitability feature a higher degree of integration between their debt and equity securities.



# Appendix B

## Appendix

This section reports additional tables and figures omitted from the main body of the paper.

### B.1 Omitted Tables

**Table B.1: Pricing Errors**

This table reports the pricing errors implied by minimum variance SDFs, computed as  $E[M_B R_S] - 1$  and  $E[M_S R_B] - 1$ , with  $S$  denoting the stocks and  $B$  the bonds. Panel A (B) reports the pricing errors implied by the bond (stock) SDF that correctly prices the portfolios sorted on the corresponding characteristic. Data is monthly and runs from January 2005 to December 2017.

Panel A: Minimum Variance Bond SDF											
		Low		2	3	4	High				
Credit rating		0.000		0.002	0.001	0.000	-0.001				
Duration		0.005		0.002	0.001	0.002	0.004				
		Low	2	3	4	5	6	7	8	9	High
Stock portfolios	Size	0.001	0.000	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	Value	0.005	0.003	0.002	0.004	-0.001	0.001	0.002	0.001	0.001	-0.002
	Leverage	0.002	0.004	0.001	0.002	0.004	0.002	0.001	0.000	0.002	-0.000
	Momentum	0.000	0.003	0.003	0.004	0.004	0.006	0.005	0.006	0.005	0.009
	Asset Growth	0.003	0.004	0.004	0.000	0.002	0.002	0.004	0.004	0.003	0.002
	Profitability	-0.006	-0.002	0.000	0.002	0.000	0.000	0.001	0.001	0.004	0.001
	Liquidity	0.008	0.003	0.004	-0.002	0.002	-0.001	0.000	-0.002	-0.006	-0.002
Panel B: Minimum Variance Stock SDF											
		Low		2	3	4	High				
Credit rating		-0.004		-0.003	-0.003	-0.003	-0.000				
Duration		-0.005		-0.004	-0.003	-0.002	-0.002				
		Low	2	3	4	5	6	7	8	9	High
Bond portfolios	Size	-0.002	-0.002	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.004	-0.004
	Value	-0.004	-0.004	-0.004	-0.005	-0.003	-0.004	-0.004	-0.004	-0.004	-0.002
	Leverage	-0.006	-0.006	-0.006	-0.006	-0.005	-0.006	-0.006	-0.006	-0.006	-0.006
	Momentum	-0.005	-0.007	-0.006	-0.007	-0.007	-0.006	-0.007	-0.007	-0.007	-0.008
	Asset Growth	-0.006	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.006	-0.006	-0.006
	Profitability	-0.006	-0.007	-0.007	-0.007	-0.008	-0.007	-0.007	-0.008	-0.007	-0.008
	Liquidity	-0.008	-0.007	-0.007	-0.007	-0.006	-0.006	-0.006	-0.005	-0.005	-0.004

**Table B.2:** Pricing Errors in Presence of Short-Selling Constraints

This table reports the pricing errors implied by minimum variance SDFs accounting for short-selling constraints, computed as  $E[M_B R_B] - 1$ , or  $E[M_S R_S] - 1$ , with  $B$  denoting the bonds and  $S$  the stocks. Panel A (B) reports the pricing errors implied by the bond (stock) SDF for bond (stock) portfolios. Data is monthly and runs from January 2005 to December 2017.

Panel A: Minimum Variance Bond SDF											
			Low	2	3	4	High				
Credit rating			0.000	0.001	0.001	0.001	0.004				
Duration			0.000	0.001	0.002	0.003	0.003				
Bond portfolios		Low	2	3	4	5	6	7	8	9	High
	Size	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.000
	Value	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.002
	Leverage	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	Momentum	0.002	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	Asset Growth	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000
	Profitability	0.001	0.001	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000
	Liquidity	0.000	0.000	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.004
Panel B: Minimum Variance Stock SDF											
			Low	2	3	4	High				
Credit rating			0.000	0.001	0.002	0.001	0.003				
Duration			0.003	0.000	0.000	0.001	0.002				
Stock portfolios		Low	2	3	4	5	6	7	8	9	High
	Size	0.004	0.002	0.003	0.002	0.003	0.002	0.002	0.002	0.002	0.000
	Value	0.004	0.002	0.001	0.004	0.000	0.001	0.001	0.001	0.002	0.001
	Leverage	0.001	0.003	0.001	0.002	0.003	0.002	0.000	0.000	0.002	0.001
	Momentum	0.000	0.001	0.000	0.001	0.000	0.001	0.001	0.002	0.000	0.003
	Asset Growth	0.003	0.003	0.003	0.000	0.001	0.003	0.004	0.004	0.003	0.001
	Profitability	0.000	0.002	0.004	0.004	0.003	0.002	0.004	0.002	0.005	0.003
	Liquidity	0.007	0.004	0.004	0.000	0.004	0.003	0.003	0.000	0.000	0.000

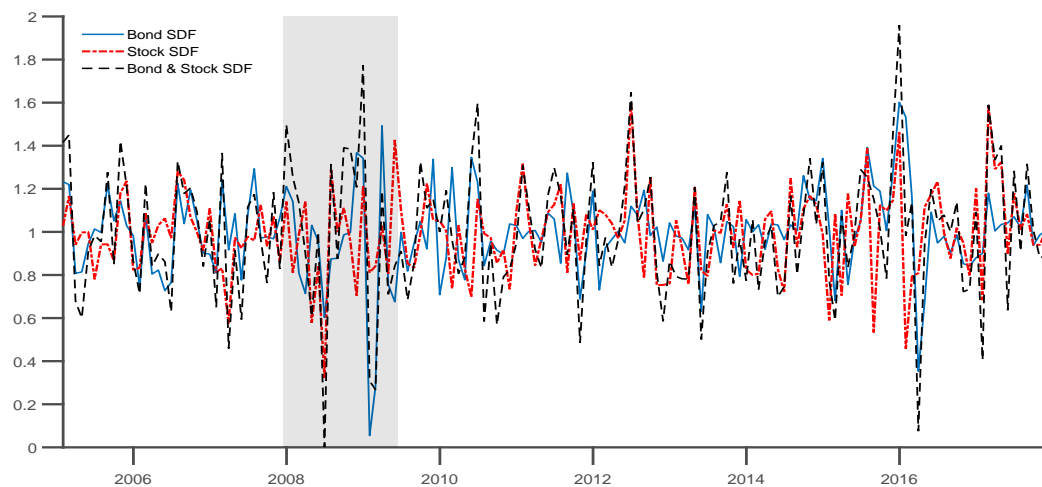
**Table B.3:** Cross-Market Pricing Errors in Presence of Short-Selling Constraints

This table reports the pricing errors implied by minimum variance SDFs accounting for short-selling constraints, computed as  $E[M_B R_S] - 1$  and  $E[M_S R_B] - 1$ , with  $S$  denoting the stocks and  $B$  the bonds. Panel A (B) reports the pricing errors implied by the bond (stock) SDF for stock (bond) portfolios. Data is monthly and runs from January 2005 to December 2017.

Panel A: Minimum Variance Bond SDF											
		<div>Low234High</div>									
Credit rating		0.0000.0020.0030.0020.004									
Duration		0.0040.0020.0020.0030.005									
		Low	2	3	4	5	6	7	8	9	High
Stock portfolios	Size	0.005	0.004	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.003
	Value	0.006	0.004	0.003	0.006	0.002	0.003	0.003	0.003	0.003	0.002
	Leverage	0.003	0.005	0.003	0.004	0.004	0.003	0.002	0.001	0.004	0.002
	Momentum	0.000	0.002	0.003	0.005	0.004	0.006	0.006	0.007	0.005	0.009
	Asset Growth	0.004	0.004	0.004	0.001	0.002	0.004	0.005	0.005	0.004	0.002
	Profitability	-0.001	0.003	0.004	0.005	0.003	0.003	0.004	0.004	0.006	0.004
	Liquidity	0.008	0.004	0.005	0.000	0.004	0.003	0.003	0.000	0.000	0.001
Panel B: Minimum Variance Stock SDF											
		<div>Low234High</div>									
Credit rating		-0.003-0.001-0.002-0.0020.002									
Duration		-0.004-0.003-0.002-0.001-0.001									
		Low	2	3	4	5	6	7	8	9	High
Bond portfolios	Size	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.004
	Value	-0.003	-0.003	-0.003	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003	-0.001
	Leverage	-0.003	-0.003	-0.003	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003
	Momentum	-0.004	-0.006	-0.005	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.007
	Asset Growth	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	-0.003
	Profitability	-0.002	-0.002	-0.002	-0.003	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003
	Liquidity	-0.002	-0.002	-0.002	-0.001	-0.001	0.000	0.000	0.001	0.001	0.002

## B.2 Omitted Figures

**Figure B.1:** Time-Series of Minimum Variance SDFs



This figure plots the time-series of minimum variance SDFs pricing portfolios of bonds sorted on firm size (solid line), along with the minimum variance SDF pricing stock returns sorted on firm size (dashed-dot line) and the minimum variance SDF pricing both bond and stock returns sorted on firm size (dashed line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

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