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Doctoral Thesis

Essays on Liquidity and Asset Pricing

Author:
Volodymyr Vovchak

Thesis Committee:

Prof. Francesco Franzoni (Supervisor) - Swiss Finance Institute, USI
Prof. Angelo Ranaldo (External member) - University of St.Gallen
Prof. Giovanni Barone-Adesi - Swiss Finance Institute, USI
Prof. Alberto Plazzi - Swiss Finance Institute, USI

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Contents

List of Figures

List of Tables

To brave people of my country
Хоробрим людям України

Summary Introduction

The research consists of two parts. The first is related to the stock market liquidity, clarifies difference between liquidity risk and liquidity level, and studies relationship among them. Further, relation of liquidity and the investment holding horizon is studied. Specifically, I investigate the liquidity preferences of long- and short term investors. The second part, considers the question of the shape of the pricing kernel. The empirical pricing kernel is estimated and compared to its theoretical counterpart. The following three chapters will be part of the thesis.

Liquidity and Liquidity Risk in a Cross-Section of Stock Returns

Finance literature advocates the importance of liquidity in both dimensions: as a characteristic (trading cost, trading activity, price impacts etc.) and as a separate pricing factor. Liquidity is the degree to which an asset or security can be traded in the market without affecting the prices of assets or how quickly an asset can be converted to cash without loss of its value. In academia, liquidity risk is understood as the sensitivity of the stock return to the aggregate market liquidity. Investors should consider both liquidity level and liquidity risk when analyzing stock returns and asset pricing. The importance of discriminating between liquidity level and liquidity risk is emphasized by Lou/Sadka (2011). One question I answer in my thesis is relative impact of liquidity and liquidity risk on the stock returns.

In this paper I investigate which dimension of liquidity is more helpful in explaining stock returns. I use a portfolio approach to analyze impacts of liquidity risk and liquidity level on stock returns, both separately and jointly. This is widely used in asset pricing literature; papers on liquidity and liquidity risk include Pástor/Stambaugh (2003) and Lou/Sadka (2011). I create liquidity variable as in Amihud (2002) and liquidity risk variable as liquidity betas in Acharya/Pedersen (2005); Pástor/Stambaugh (2003); Watanabe/Watanabe (2008) and Sadka (2006). Then, by creating portfolios sorted on these variables, I investigate joint effects of liquidity level and liquidity risk. Portfolios based on Amihud's liquidity measure and different types of liquidity risk are formed, when alphas of these portfolios are estimated for three pricing models and significance of alphas are used for making judgments about explanatory power of each indicator.

I find that liquidity level has greater impact on common stock returns than liquidity risk does. Results are somewhat stronger for the early subsample, though they are also valid for more recent data. This finding is of particular importance for investors who might consider taking into account the liquidity level variables for portfolio allocation, asset pricing and risk management.

Liquidity and Investment Horizon

Researchers agree that less liquid stocks grant higher returns, the so called liquidity premium and that there is a strong commonality in liquidity. Further, understanding this results leads to necessity to acknowledge their importance and use them in practice. One of the questions following liquidity matters is the question of relationship of liquidity of a stock and the investment horizon of its typical holder (Amihud/Mendelson, 1986). Long term investors, who trade very infrequently, and by doing so, can get profits holding illiquid stocks. The second chapter of my thesis is devoted to relationships between liquidity of a stock (both liquidity and liquidity risk) and the holding horizon of the average investor holding this stock in his portfolio.

This chapter deals with relationship between liquidity and investment horizon. The question has two fairly independent tasks, which are natural outcomes of the previous article, the first, relating holding horizon with trading cost, price impact of trade and trading activity, and the second, investigation of linkages between holding horizon and liquidity risk. The theoretical background for the former question is provided in Amihud/ Mendelson (1986). In their framework the authors prove that assets with higher spreads are held by long term investors in equilibrium. My paper implements empirical test of this statement for a few liquidity measures and its idea is similar to the paper of Atkins/ Dyl (1997). Motivation for the second question comes from the paper by Beber et al. (2011), whose finding suggest existence of linkages between liquidity risk and holding horizon.

The analysis is conducted with cross-sectional regressions. The holding horizon is measured as the turnover of institutional investors holding particular stock (so called "churn ratio", see for example Cella et al., 2013). This measure is considered as a good proxy for the investment horizon. To overcome possible endogeneity issues I propose the dividend-to-price ratio as an instrument for the holding horizon. Results are robust to this and other specifications and tests.

This research finds the following: concerning the relationship between liquidity and holding horizon I find that illiquidity (relative spread, Amihud measure) is positively related to the investment holding horizon. More liquid stocks are traded more often, by the short term investors. Secondly, liquidity risk is negatively related to the holding horizon. Specifically, I find that stocks that are held by the short term investors are, in general, more disposed to aggregate liquidity shocks. Short term investors are willing to take on liquidity risk, which might be in line with liquidity provision by hedge funds.

Is the Price Kernel Monotone?

The last chapter of the thesis (which was written earlier than two previous) deals with the shape of the pricing kernel and the pricing kernel puzzle. According to economic theory, the shape of the state price density (SPD) per unit probability (also known as the asset pricing kernel, Rosenberg/Engle (2002) or stochastic discount factor (SDF), Campbell et al. (1997)) is a decreasing function in wealth. There is a number of papers trying to support or reject this statement. We estimate and show that the pricing kernel is monotonically decreasing.

In this paper we compute the kernel price both in a single day and as an average of kernel prices over a period of time, holding maturity constant. In order to do so one needs to estimate the risk neutral and physical distribution of the price changes. In order to estimate the risk neutral distribution, we use the well-known result in Breeden/ Litzenberger (1978). The difference with previous works is in the options we use. Instead of creating option prices through nonparametric or parametric models (all the previous research use artificial price of options and this could introduce a bias in the methodology), we use only the options available on the market.

We then construct the historical density using the GJR GARCH model with Filtered Historical Simulation already presented in Barone-Adesi et al. (2008). As discussed in Rosenberg/Engle (2002), among the several GARCH models, the GJR GARCH with FHS has the flexibility to capture the leverage effect and the ability to fit daily S&P500

index returns. Then, the set of innovations estimated from historical returns and scaled by their volatility gives an empirical density function that incorporates excess skewness, kurtosis, and other extreme return behavior that is not captured in a normal density function. These features avoid several problems in the estimation of the kernel price. For example, using a simple GARCH model where the innovations are standard normal (0; 1) leads to a misspecification of the return distribution of the underlying index.

Once we have the two probabilities, we take the ratio between the two densities, discounted by the risk-free rate, in a particular day, to compute the kernel price for a fixed maturity. We repeat the same procedure for all the days in the time series which have options with the same maturity and then we take the average of the kernel price through the sample. At the same time we apply kernel smoothing on the estimated values of the price kernel to confirm our result. We take our estimated average price kernel, consider its monotone version and then compare the monotone version with the estimated version by means of Kolmogorov-Smirnov test.

Our tests support the pricing kernel monotonicity. We show that the ratio between the two probabilities, is monotonically decreasing in agreement with economic theory.

Chapter 1.

Liquidity and Liquidity Risk in the Cross-Section of Stock Returns

—*by Volodymyr* Vovchak—

1.1 Introduction

There is extensive literature concerning liquidity impact on asset prices. Early literature takes a closer look at liquidity as a stock characteristic which matters for pricing (Amihud/Mendelson, 1986; Cooper et al., 1985; Datar et al., 1998). More recent papers concentrate on liquidity as a risk factor (Pástor/Stambaugh, 2003; Sadka, 2006) or as a risk factor in the CAPM framework (Acharya/Pedersen, 2005; Holmström/Tirole, 2001; Liu, 2006). Although researchers acknowledge the importance of liquidity risk, liquidity level is still used to explain asset prices. For example, Amihud (2002) shows that liquidity level is important for both cross-section and time series of stock prices. The importance of liquidity for explaining expected returns in emerging markets has been studied by Bekaert et al. (2007) and Jun et al. (2003) among others. Jun et al. (2003) find that stock returns in emerging countries are positively correlated with aggregate market liquidity. Overall, there is a clear recognition of the importance of liquidity for asset pricing models, but there is no clear understanding about what is more important for an investor while making investment decisions: the liquidity level or the liquidity risk.

Two aspects of liquidity can be distinguished, one related to market ability to operate despite of trading frictions and cost, and the second connected to the fact that liquidity is a priced factor. Chordia et al. (2000, p. 6) presume that liquidity potentially has two channels to influence asset prices. They attribute liquidity level to a static channel, while liquidity risk impacts asset prices through a dynamic channel. Therefore, investors should consider both liquidity level and liquidity risk when analyzing stock returns and asset pricing. The importance of discriminating between liquidity level and liquidity risk is emphasized by Lou/Sadka (2011). They find that during the 2008 crisis, liquid stocks suffered as much as or sometimes even more than illiquid, and stocks with low liquidity risk performed better than those with high liquidity risk, irrespective of the liquidity level. That is, if both liquidity level and liquidity risk matter for asset returns, then liquidity risk tends to lower or change direction of the liquidity effect on asset returns. Liquidity level alone has been proven in the literature to have a negative impact on stock returns, i.e. investors require higher returns to invest into illiquid stock. However, once liquidity risk is taken into account, it might be the case that the impact of liquidity switches to positive (this partially is shown in Lou/Sadka (2011)). This may be due to links between liquidity level and liquidity risk. For example more liquid stocks may carry less liquidity risk or vice versa.

All the above considerations bring forward the interesting task of disentangling liquidity level and liquidity risk impacts on the stock returns. This problem was partially addressed by Acharya/Pedersen (2005). In their liquidity adjusted CAPM, they use expected liquidity in their pricing equation, and find that the level effect is rather weak in comparison with liquidity betas[1]. One explanation for their result may be that liquidity betas are estimated using the liquidity level, or another explanation might be that betas are estimated with noise. Furthermore betas in their paper are highly correlated which could explain their findings.

In this paper I investigate which dimension of liquidity is more helpful in explaining stock returns. It may seem that liquidity risk has much importance, but for investors, estimating liquidity risk factor may be time consuming, complicated or imprecise, while liquidity level is relatively easy to estimate, and thus, easier to use for investment purposes. My research question may somewhat resemble that of Acharya/Pedersen (2005), although there are a few distinctions. First, this is

[1] They find liquidity level to be significant only when regressed together with “net beta”. “Net beta” is the sum of different liquidity betas and market beta.

entirely empirical work and, second, Acharya/Pedersen conclude more importance of liquidity risk "over and above [...] the level of liquidity" while results reported here suggest that liquidity risk impact vanishes when liquidity level is taken into account. Further, the Acharya/Pedersen paper is more about commonality in liquidity than liquidity risk.

I use a portfolio approach to analyze impacts of liquidity risk and liquidity level on stock returns, both separately and jointly. This is widely used in asset pricing literature; papers on liquidity and liquidity risk include Pástor/Stambaugh (2003) and Lou/Sadka (2011). I create liquidity variable as in Amihud (2002) and liquidity risk variable as liquidity betas in Acharya/Pedersen (2005); Pástor/Stambaugh (2003); Watanabe/Watanabe (2008) and Sadka (2006). Then, by creating portfolios sorted on these variables, I investigate joint effects of liquidity level and liquidity risk. Portfolios based on Amihud's liquidity measure and different types of liquidity risk are formed, when alphas of these portfolios are estimated for three pricing models and significance of alphas are used for making judgments about explanatory power of each indicator. I find that liquidity level has greater impact on common stock returns than liquidity risk does. Results are somewhat stronger for the early subsample, though they are also valid for more recent data. This finding is of particular importance for investors who might consider taking into account the liquidity level variables for portfolio allocation, asset pricing and risk management.

The paper is organized as follows. Section 1.2 provides a brief overview of the asset pricing literature devoted to liquidity. In section 1.3, variables which are used for analysis are presented. The data and methodology is discussed in section 1.4. In section 1.5, the main results are presented and discussed. Further, section 1.6 conducts robustness checks, presents results for alternative measures of liquidity risk, and describes results of subsample analysis. Section 1.7 offers concluding remarks.

1.2 Literature review

Liquidity is an important indicator in modern financial markets. There is a growing body of literature on the impact of liquidity on assets' returns, market returns, returns volatility, etc. There is documented evidences on importance of different liquidity dimensions: trading cost dimension (bid-ask spread – Amihud/Mendelson, 1986, 1989; Jones, 2002), trading quantity dimension (volume – Campbell et al., 1993; Llorente et al., 2002, turnover – Datar et al. 1998, trading activity – Avramov et al., 2006a; Chordia et al., 2001a), price reaction to trading volume (illiquidity measure introduced in Amihud, 2002) and trading speed dimension (Liu, 2006). Baker/Stein (2004) have found that liquidity is the outcome of investors' sentiment. Liquidity become so important that there are papers discussing different properties of liquidity, for example commonality in liquidity studied by Chordia et al., 2000; Mancini et al., 2013, or intraday patterns of liquidity Ranaldo, 2001.Amihud et al. (2005) give an extensive overview of theoretical and empirical literature concerning liquidity implications for asset pricing.

There are two streams of literature on liquidity, one is concerned with ability of liquidity to influence financial indicators of stocks, while another stream of literature looks into systematic properties of liquidity, and advocates the existence of liquidity risk factors. The first branch of the liquidity literature includes papers by Amihud (2002); Amihud/Mendelson (1986); Brennan/Subrahmanyam (1996); Brennan et al. (1998); Chordia et al. (2001b) and many others.

These studies consider liquidity and related indicators as variables which can explain time series variability of the stock return and/or return volatility. Brennan/ Subrahmanyam (1996) find significant relations between several of intra day liquidity measures and required rate of returns. These relations hold even after controlling for Fama/French's risk factors. Chordia et al. (2001b) report negative cross-sectional relationship between stock returns and the second moment of trading volume and stock turnover. Campbell/Grossman/Wang (1993) find that, for stock indexes and individual large stocks, daily return autocorrelation declines with trading volume. Avramov/Chordia/Goyal (2006a) show that liquidity has an impact on return autocorrelation even after controlling for trading volume. In another paper Avramov/Chordia/Goyal (2006b) show how trading activity explains the asymmetric effect in daily volatility of individual stock return. Jones (2002) shows that liquidity is a good stock return predictor for low frequency market return data (he uses annual data from 1900 through 2000). He finds that the quoted spread and the turnover of Dow Jones stocks predicts market return even after controlling for the dividend price ratio, a well-known returns predictor. Brennan et al. (1998) examine relationships between stock return, measures of risk and some non-risk characteristics. The authors find that trading volume negatively affects returns and this relation is stable in the principal component as well as in the Fama/French (1993) framework.

Another stream of literature treats liquidity as a pricing factor in the CAPM framework. Pástor/Stambaugh (2003) find that market wide liquidity is a pricing factor for common stocks. Acharya/Pedersen (2005) introduce the liquidity adjusted CAPM with three types of liquidity betas and show that liquidity is a priced factor in a cross-section of stock returns. Yet another paper, by Liu (2006), considers a similar model with a new liquidity measure addressing another dimension of liquidity – the speed of trading. Support of liquidity as a pricing factor rather than as a characteristic is confirmed in Jones (2002). He tests if return predictability by liquidity is due to pure transaction cost effects or to liquidity being the priced factor. Jones finds the pricing factor story to be more plausible. Mancini et al. (2013) develop a liquidity measure tailored to foreign exchange market and show that liquidity risk based on this measure is heavily priced.

This paper is aimed to bring together all the literature discussed above. It enhances understanding of liquidity for the asset pricing, specifies and clarifies channels, through which liquidity is influencing stock prices. I find which type of liquidity is of more use for assessing and predicting the asset prices. In this respect this work is similar to that of Lou/Sadka (2011) and Acharya/Pedersen (2005). Lou/Sadka (2011) show that for the financial crisis of 2008, seemingly safe liquid stocks underperformed illiquid stocks. They show that liquidity beta, the correlation of the stock return with unexpected changes in aggregate market liquidity, is able to explain asset prices better than liquidity level. Here I do the same analysis for the longer time period, and find that result of their analysis does not hold for bigger sample. Another work akin to this paper is Acharya/Pedersen (2005). The authors consider the liquidity adjusted CAPM, where liquidity is represented at both level and risk dimensions. Their findings also support liquidity risk, although they have some problems with correlation of different types of liquidity risk. To overcome this weaknesses I implement portfolio approach and use simpler definition of liquidity risk.

1.3 Constructing variables

As mentioned above, liquidity has several different channels to influence asset prices. These may include price impact (price pressures), as in micro structure literature, or trading costs, or an asset's return sensitivity to liquidity. When considering the liquidity level dimension, I concentrate on the price impact measure of Amihud. This measure is widely used because of its simplicity and its ability to explain stock returns adequately as a liquidity measure. I construct liquidity as follows

$$Illiq_{im} = \frac{1}{D_{im}} \sum_{t=1}^{D_{im}} \frac{|r_{itm}|}{DVol_{itm}} \tag{1.1}$$

where r_{itm} and $DVol_{itm}$ are return and dollar volume traded of the stock i on day t of month m. Because illiquidity is changing considerably over time, I scale *Illiq* by the average market illiquidity in a given month, i.e. as the liquidity of an individual stock I use

$$Illiq^S_{im} = \frac{Illiq_{im}}{Illiq^M_m} \tag{1.2}$$

where $Illiq_{im}$ is from equation (1.1) and

$$Illiq_m^M = \frac{1}{N_m} \sum_{i=1}^{N_m} Illiq_{im}, \tag{1.3}$$

here N_m – is the number of stocks in month m.

Amihud (2002) argues that this variable is positively correlated to other liquidity variables, such as Kyle's price impact measure λ, and fixed-cost measure, ψ, connected to bid-ask spread[2]. Further, he shows that his variable does a good job of explaining stock returns both cross-sectionally and over time.

Next, I describe the construction of the liquidity risk variable. Liquidity risk is estimated as beta from the time-series regression[3]:

$$r_{it}^e = \beta_i^0 + \beta_i^M \cdot mkt_t + \beta_i^S \cdot smb_t + \beta_i^{BM} \cdot hml_t + \beta_i^L \cdot Mliq_t + \varepsilon_{it}, \tag{1.4}$$

where r_{it}^e is an excess return of stock i (stock return minus the risk-free rate), mkt is the excess return of the market index and smb and hmb are returns of long-short spreads obtained by sorting stocks according to their market value and book-to-market ratio. More details can be found in Fama/French (1993). $Mliq$ is a measure of aggregate market liquidity. Its construction is discussed below. Liquidity beta obtained in this way captures an asset's co-movement with market wide liquidity. Including other Fama-French factors assures that β^L measures only market liquidity effects, not size or book-to-market effects. β^L is allowed to vary over time.

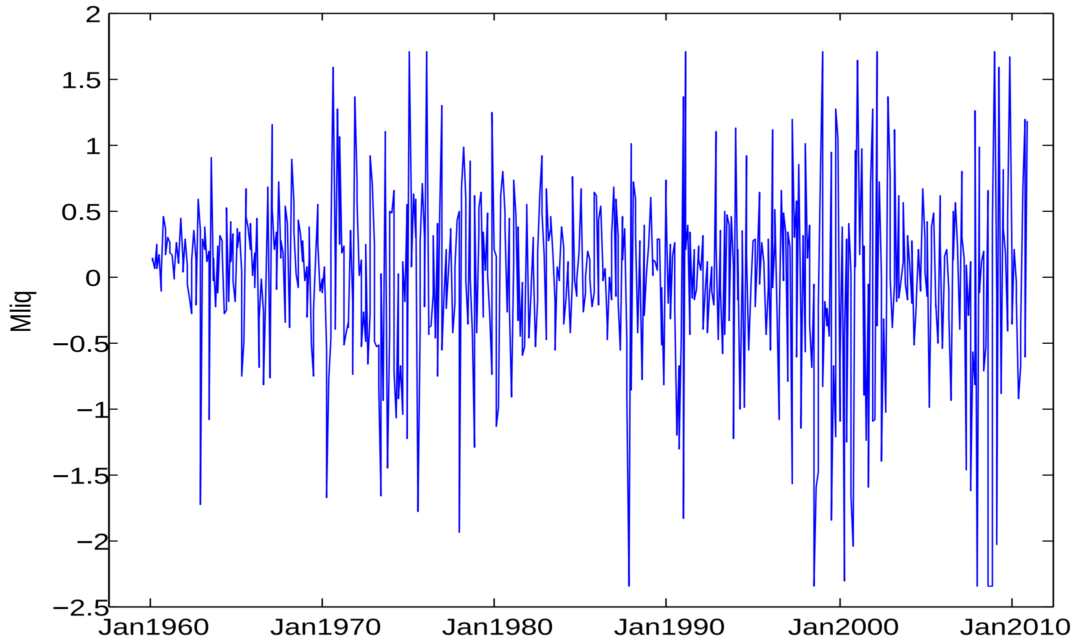
Now, I describe the construction of market liquidity, $Mliq$. Similarly to Watanabe/Watanabe (2008) and Acharya/Pedersen (2005). Equation (1.3) defines the average market illiquidity. Further, I construct innovations in $Illiq^M$ to avoid problems which can arise from strong persistence in the market illiquidity[4]. To do so I estimate a modified AR(2) model on the whole sample of the average market

[2] Both measures were used in Brennan/Subrahmanyam (1996) to investigate relations between return premium and different components of cost.

[3] The equation (1.4) is estimated on monthly data with a moving window of three years.

[4] The autocorrelation of the monthly market illiquidity is over 86%.

FIGURE 1.1: **Aggregate Market Liquidity Shocks.**



illiquidity. The model is

$$\frac{MkCap_{t-1}}{MkCap_1} Illiq_t^M = \alpha + \beta_1 \frac{MkCap_{t-1}}{MkCap_1} Illiq_{t-1}^M + \beta_2 \frac{MkCap_{t-1}}{MkCap_1} Illiq_{t-2}^M + \varepsilon_t, \quad (1.5)$$

where $MkCap_{t-1}$ is the market capitalization on month $t-1$ (stocks used to calculate $MkCap_{t-1}$ are those admitted in month t), and $MkCap_1$ is the market capitalization at the beginning of my sample. Ratio $\frac{MkCap_{t-1}}{MkCap_1}$ is included as a detrending factor which controls for the time trend in $Illiq^M$. This factor is the same for contemporaneous and lagged $Illiq^M$ in order to obtain innovations in illiquidity only. Including a different time factor may cause the innovations to contain changes streaming from a change in market capitalization. For further discussion see Acharya/Pedersen (2005); Watanabe/Watanabe (2008), and Pástor/Stambaugh (2003).

The unexpected illiquidity shocks are errors from equation (1.5). In fact I use $-\hat{\varepsilon}_t$ as the innovation of expected liquidity, because it is better to have $Mliq$ reflecting innovations in liquidity and $\hat{\varepsilon}_t$ represent illiquidity innovations. Figure 1.1 shows a graph of the market liquidity innovations series from January 1960 through December 2010. It is noticeable that plunges of $Mliq$ match with major economic crashes.

TABLE 1.1: **Summary Statistics of Liquidity Risk and Liquidity Level Portfolios**

The table reports summary statistics of value weighted portfolios. Summary statistics is provided for the following variables average logarithm of the stock capitalization (*Ln(size)*), average of the ratio of the book value to the market value (*Book-to-Market*), average of the dividend to price ratio (*Dividend-Price ratio*), and *average (minimum and maximum) number of stocks* in a given portfolio. Portfolios are sorted on the basis of liquidity level (Panel A), and liquidity risk (Panel B).

	1	2	3	4	5
Panel A. Liquidity level portfolios					
Ln(size)	16.301	14.066	13.181	12.395	11.444
Book-to-Market	.48435	.51459	.54611	.60347	.607
Dividend-Price ratio	.032281	.029734	.027874	.02718	.025968
Average number of stocks	272	266	260	251	231
Minimum number of stocks	196	190	190	188	149
Maximum number of stocks	338	326	321	320	304
Panel B. Liquidity risk portfolios					
Ln(size)	15.172	15.841	15.992	15.769	15.014
Book-to-Market	.50083	.47353	.50292	.49763	.48537
Dividend-Price ratio	.025572	.031599	.035184	.033359	.02417
Average number of stocks	250	261	261	259	249
Minimum number of stocks	187	187	188	188	178
Maximum number of stocks	314	321	326	323	311

This section described the construction of the main sorting variables: liquidity risk and liquidity level. The next section discusses data used in this paper, portfolio construction and portfolio characteristics, such as size, book-to-market etc.

1.4 Data and empirical methodology

The data used in this paper includes returns on NYSE and AMEX common stocks (CRSP share codes 10 and 11) from CRSP, from 1964 through 2010. The daily CRSP datafile is used to calculate liquidity variables described in Section 1.3 for each month. The monthly CRSP datafile is used to calculate size and the dividend-price ratio. To calculate liquidity beta, β^L, using equation (1.4) series smb, hml and mkt are retrieved from Kenneth French's web page[5].

To implement my analysis, I form 5 portfolios on liquidity level, 5 portfolios on liquidity risk and 9 independently double sorted portfolios (3 on liquidity level and 3 on liquidity risk), along with spread portfolios. As noted the liquidity level is

[5]Kenneth French's Data library is here: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

measured with (1.2) and liquidity risk is β_i^L from equation (1.4). Portfolios are formed as follows. For each stock I estimate $Illiq_i^S$ and/or β_i^L for the year $t-1$ and calculate returns of each portfolio in a year t. Stocks used in constructing portfolios are those satisfying the following conditions: (i) the average price of the stock in a month $t-1$ is higher than \$5 and lower than \$1,000, and (ii) the stock has at least 200 days of data in year $y-1$. These conditions are imposed in order to make estimations more reliable and also allow me to avoid the problem of the minimum tick of \$1/8 for the low-price stocks.

Summary statistics of portfolios are provided in Table 1.1 and Table 1.2. Table 1.1 reports average log of size, average book-to-market, average dividend-price ratio and average number of stocks in each portfolio for liquidity level sorted (Panel A) and liquidity risk sorted (Panel B) portfolios. On average, there are 250 stocks in each liquidity risk sorted portfolio. The number of stocks in risk sorted portfolios is never less than 179, while the maximum is 326. For level sorted portfolios, the numbers are: the average 253, the minimum is 150 and the maximum is 338. For double sorted portfolios (see Table 1.2), the average number of stocks is 140, the lowest number of stocks in portfolio is 78, the highest 230. The liquidity risk sorted portfolios have a fairly even distribution of parameters. Only slightly lower size and dividend-price ratios for extreme portfolios can be noticed. In contrast, for liquidity level sorted portfolios some monotonic patterns can be observed; stocks with lower liquidity are small and value stocks. This is in line with the well-known fact that liquidity is negatively correlated with size. Similar patterns are conspicuous in double sorted portfolios.

1.5 Empirical results

In this section I present the results of the portfolio analysis. As mentioned above, I form 5 portfolios on liquidity level and on liquidity risk, and for testing joint impact 9 (3×3) double sort portfolios, and consider spread portfolios. When returns of each portfolio are regressed on different pricing factors, the significance of alphas is then analyzed. Results reported here are for value weighted returns, but they also hold for equally weighted returns.

Table 1.2: **Summary Statistics of Double-Sorted Portfolios**

The table reports summary statistics of value weighted independently double sorted portfolios. Summary statistics is provided for the following variables average logarithm of the stock capitalization (*Ln(size)*), average of the ratio of the book value to the market value (*Book-to-Market*), average of the dividend to price ratio (*Dividend-Price ratio*). For each row liquidity level is fixed, while for each column liquidity risk is fixed.

	1	2	3	1	2	3
	Average number of stocks			Ln(size)		
1	144	177	129	15.927	16.204	15.86
2	141	144	148	13.191	13.381	13.243
3	138	114	145	11.873	12.064	11.917
	Book-to-Market			Dividend-Price ratio		
1	.48127	.48926	.4881	.028899	.034843	.030358
2	.53681	.55034	.5455	.026407	.031952	.026752
3	.60727	.60659	.61177	.022079	.029364	.026949

1.5.1 Single sorted portfolios

Liquidity level sorted portfolios I start by analyzing the level impact on stock excess returns. Five level sorted portfolios are created and portfolios consisting of long position in illiquid portfolio and short in liquid portfolio[6]. To obtain the liquidity impact, I run regressions of excess returns of each portfolio on excess market return (CAPM setup), on the three Fama-French factors, and on the Fama-French factors plus momentum factor (the four factor model, see Carhart, 1997). If the liquidity level has some impact on excess returns then alphas of these regressions must be different from zero.

Panel A of Table 1.3 reports the alphas of regressions along with t-statistics. One can see from the table that alphas of liquidity portfolios change significance considerably. For the CAPM regression, significance and magnitude of alphas increase with illiquidity. The portfolio of liquid stocks has alpha of only 0.05% and it is insignificant, with its t-statistic being below one. Further, alphas increase up to 0.79% for the most illiquid portfolio and is highly significant with a t-statistic equal to 5.83. The "5-1" spread portfolio produces an alpha of 0.74% and a t-statistic above 5. These numbers confirm results broadly documented in the literature, that liquidity level has negative impact on returns (see Amihud, 2002; Jones, 2002 or Amihud et al.'s (2005) survey). When controlling for size and book-to-market

[6]Note that level measure given in equation (1.2) measures illiquidity not liquidity.

Table 1.3: **Alphas of Liquidity Risk and Liquidity Level Portfolios**

The table reports the alphas the quintile portfolios, in percentages per month. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). Portfolios are sorted on the basis of liquidity level (Panel A), and liquidity risk (Panel B). The t-statistics are in parentheses.

	1	2	3	4	5	5-1
	Panel A. Liquidity level sorts					
CAPM alpha	0.0500	0.1902	0.3162	0.4887	0.7918	0.7417
	(0.8321)	(2.2503)	(3.0777)	(3.9713)	(5.8299)	(5.5776)
Fama-French alpha	-0.0439	-0.0407	0.0100	0.0960	0.3809	0.4248
	(-0.8079)	(-0.5832)	(0.1389)	(1.4178)	(4.7620)	(5.6832)
Four-factor alpha	0.0358	0.0496	0.0975	0.1642	0.4312	0.3954
	(0.6714)	(0.6847)	(1.3687)	(2.3139)	(5.2135)	(4.9263)
	Panel B. Liquidity risk sorts					
CAPM alpha	0.3804	0.3584	0.3300	0.3236	0.3707	-0.0097
	(3.5187)	(4.1954)	(3.9366)	(3.8125)	(3.5428)	(-0.1504)
Fama-French alpha	0.0658	0.0938	0.0664	0.0628	0.0651	-0.0008
	(0.9081)	(1.5400)	(1.0697)	(1.0351)	(0.9178)	(-0.0119)
Four-factor alpha	0.1272	0.1633	0.1355	0.1318	0.1800	0.0528
	(1.6643)	(2.6197)	(2.1485)	(2.1784)	(2.6418)	(0.7404)

effects (Fama-French setup), individual portfolios produce lower alphas, ranging from insignificant -0.04% for a portfolio of liquid stocks to significant 0.38% for a portfolio of illiquid stocks (t-statistics are -0.81 and 4.76 respectively). Although the "5-1" spread alpha drops in magnitude to 0.425%, its significance remains high. Adding the momentum factor does not change results; the "5-1" spread portfolio has almost 0.4% alpha, which is still highly significant with a t-statistic equal to 4.93.

Overall, results obtained from level sorted portfolio supports the idea that investors require additional returns in order to hold illiquid stocks.

Liquidity risk sorted portfolios Now consider portfolios sorted by liquidity risk. The methodology is the same as above. Results are reported in Panel B of Table 1.3. The CAPM alphas of quintile portfolios are significant, with t-statistics higher than 3.5. The alphas are higher at extremes, 0.38% for portfolio with the lowest risk and 0.37% for the portfolio which carries the most of liquidity risk. The "5-1" spread portfolio has a negative alpha which is essentially zero, both statistically and economically. The negative sign might be attributed to CAPM inability to

Table 1.4: **Alphas of Liquidity Risk-Level Sorted Portfolios**

The table reports the alphas the independently double sorted portfolios, in percentages per month. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). For each row liquidity level is fixed, while for each column liquidity risk is fixed. The t-statistics are in parentheses.

	1	2	3	Level Controlled
		Panel A. CAPM		
1	0.1274	0.1027	0.0735	-0.0539
	(1.6181)	(1.4361)	(1.0289)	(-0.7427)
2	0.3186	0.3276	0.2834	-0.0352
	(2.8998)	(3.3200)	(2.5762)	(-0.5564)
3	0.7161	0.6405	0.6757	-0.0403
	(5.2618)	(5.0551)	(4.9563)	(-0.6391)
Risk Controlled	0.5887	0.5378	0.6022	
	(4.8529)	(4.4688)	(4.9840)	
		Panel B. Fama-French		
1	-0.0341	-0.0442	-0.0485	-0.0144
	(-0.4910)	(-0.7131)	(-0.6901)	(-0.1949)
2	0.0051	0.0289	-0.0276	-0.0328
	(0.0660)	(0.3966)	(-0.3461)	(-0.5070)
3	0.3030	0.2442	0.2633	-0.0397
	(4.0890)	(3.2317)	(3.2967)	(-0.6443)
Risk Controlled	0.3372	0.2884	0.3118	
	(4.4175)	(3.9805)	(3.8556)	
		Panel C. Four-factor		
1	0.0341	0.0384	0.0633	0.0292
	(0.4777)	(0.6005)	(0.9167)	(0.3550)
2	0.0777	0.0997	0.0690	-0.0087
	(0.9481)	(1.3252)	(0.8763)	(-0.1293)
3	0.3499	0.2957	0.3473	-0.0027
	(4.4910)	(3.7220)	(4.1804)	(-0.0405)
Risk Controlled	0.3158	0.2573	0.2840	
	(3.9167)	(3.2767)	(3.2427)	

price stocks adequately. The Fama-French alphas drop significantly in magnitude and lose significance; the highest t-statistic is around 1.5. Alphas range from 0.06% to 0.09%. The "5-1" spread portfolio alpha is virtually zero and insignificant. The four factor alphas increase with respect to the Fama-French alphas both in magnitude and in significance. They do not exhibit any patterns and range from 0.12% for portfolio with the lowest liquidity risk to 0.18% for the highest liquidity risk portfolio. They are all significant except the lowest liquidity risk portfolio alpha which has a t-statistic equal to 1.6. However, the "5-1" spread portfolio has an alpha of 0.05% with a low t-statistic of 0.76.

We can take from this analysis that liquidity risk does not have the same effect as liquidity level during the time span from 1964 through 2010. This contradicts results reported in Lou/Sadka (2011). However, one should remember that they analyzed only the 2008 financial crisis, which is reputed to have been caused by liquidity issues. Looking into the past, one should admit that there was not much evidence of liquidity problems, thus liquidity risk is not better than liquidity itself when it comes to pricing stocks.

1.5.2 Double-sorted portfolios

This section reports results for double-sorted portfolios, and answers the main question of the paper: Is it liquidity risk or liquidity level which matters for asset pricing? Here I again report results for value-weighted portfolio returns. Results with equally weighted portfolio returns are a bit weaker but they are in line with those reported here.

In the Table 1.4 results of the double sort portfolio analysis are presented. Results shown include alphas and their t-statistics from regressions of the portfolio excess returns on the market excess return (Panel A, the CAPM setup), on the Fama-French factors (Panel B), and on the Fama-French factors and momentum factor (Panel C, the four factor setup). Sorting is done independently on liquidity risk and level. The horizontal sorting in the table is on the basis of liquidity risk, the vertical sorting is on the basis of liquidity level.

Individual portfolios' alphas in the CAPM setup range from 0.07% for the liquid-high liquidity risk portfolio to 0.72% for the illiquid low liquidity risk portfolio. Alphas increase in magnitude and in significance with illiquidity for all liquidity risk levels. For low liquidity risk portfolios they are higher than for high liquidity risk portfolios. Further consider liquidity risk controlled spread portfolios, which are given below individual alphas. For low liquidity risk spread portfolio alpha is equal to 0.59% and is significant with t-statistic equal to 4.85. For the high liquidity risk portfolio, alpha is equal to 0.6% and its t-statistic is equal to 4.98. Liquidity level controlled portfolios have alphas which are insignificant, negative, and close to zero. They range in magnitude from -0.054% to -0.035%, and have t-statistics which never exceed 0.075 in absolute value.

Panel B of Table 1.4 reports Fama-French alphas and their t-statistics. In this case, alphas drop in magnitude significantly, although their significance remains about the same as in case of the CAPM framework. The alphas of individual portfolios now range from -0.03% for the liquid-low liquidity risk portfolio to 0.3% for illiquid low liquidity risk portfolio. Patterns are similar to those in CAPM, i.e. alphas increase in significance and magnitude with illiquidity for every liquidity risk level. The only difference now is that they are significant only for illiquid portfolios. Spread portfolios controlled for liquidity risk have significant alphas which are, nevertheless, lower than they are in the CAPM case. The alpha for the low liquidity risk spread portfolio is 0.34% and its t-statistic is 4.42. The alpha for the medium risk portfolio is 0.29% with t-statistic 3.98. For high liquidity risk spread portfolio, alpha is significant (t-statistic is 3.86) and is equal to 0.31%. Liquidity level controlled spread portfolios are still insignificant and negative. Liquid spread portfolio alpha is -0.01% and is highly insignificant. Medium liquidity spread portfolio has alpha -0.03% (t-statistic is -0.51), and illiquid spread portfolio alpha is -0.04% and is insignificant.

Results reported for the Fama-French setup hold also for the four factor setup. Magnitudes and t-statistics are very close to that of Fama-French alphas and t-statistics.

In summary, the results presented in this section suggest that liquidity level influences asset prices more than liquidity risk. This means that investors are more concerned with costs of trading than with exposures to the market-wide liquidity. My result is opposite to that of Lou/Sadka (2011). This may be attributed to high trading costs and frictions from 1964 to 2010. During the 2008 financial crisis the market was highly sensitive to liquidity, giving more explanatory power to liquidity risk.

1.6 Robustness checks

One may argue that liquidity risk used in previous sections[7] is not a priced factor, and it is quite noisy. That is why liquidity risk defined this way cannot be a fair

[7] It is liquidity risk which is defined as in Acharya/Pedersen (2005).

Table 1.5: **Alphas of Liquidity Risk Portfolios**

The table reports the alphas of the quintile portfolios, in percentages per month. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). Portfolios are sorted on the basis of Pastor and Stambaugh liquidity risk (Panel A), and Sadka liquidity risk (Panel B). The t-statistics are in parentheses.

	1	2	3	4	5	5-1
	Panel A. Pastor and Stambaugh liquidity risk sorts					
CAPM alpha	-0.1288	0.0920	0.0164	0.0792	0.1458	0.2746
	(-1.5780)	(1.3267)	(0.2488)	(1.4042)	(2.0530)	(2.5601)
Fama-French alpha	-0.1549	0.0959	-0.0431	0.0439	0.0606	0.2156
	(-1.8222)	(1.6337)	(-0.8693)	(0.8643)	(0.8294)	(1.8809)
Four-factor alpha	-0.1592	0.0544	-0.0311	0.0407	0.1317	0.2908
	(-1.8420)	(0.8714)	(-0.5845)	(0.7550)	(1.8238)	(2.4882)
	Panel B. Sadka liquidity risk sorts					
CAPM alpha	-0.0164	0.0426	0.1787	0.0723	0.0880	0.1044
	(-0.1216)	(0.3573)	(1.6841)	(0.7088)	(0.8573)	(0.6962)
Fama-French alpha	-0.1289	-0.0236	0.1181	0.0021	0.0250	0.1539
	(-1.1304)	(-0.2321)	(1.3825)	(0.0270)	(0.2729)	(1.0240)
Four-factor alpha	-0.1663	0.0400	0.1159	-0.0213	0.1056	0.2719
	(-1.4295)	(0.3746)	(1.2463)	(-0.2534)	(1.1847)	(1.8403)

alternative for liquidity level. This section presents results for alternative liquidity risk measures. I present Pástor/Stambaugh's liquidity risk and Sadka's variable component of liquidity risk. Both are confirmed to be priced factors, which make them good competitors for liquidity level. Results for subsample analysis are also reported.

1.6.1 Alternative liquidity risk measures

Pástor/Stambaugh's liquidity risk Pástor/Stambaugh (2003) introduce a liquidity risk factor based on the following principle: order flow induces greater return reversals when liquidity is low. They calculate the market liquidity measure, then liquidity risk is measured as individual stock return's sensitivity to this market liquidity, i.e. they estimate liquidity betas, controlling for other factors (size and book-to-market). Pástor/Stambaugh find that returns of stocks with high liquidity betas are about 7.5% higher than returns of stocks with low liquidity beta.

I calculate liquidity risk as in equation (1.4) except in place of $Mliq_t$ I use Pástor/ Stambaugh's innovation in liquidity, $\mathcal{L}_t$[8]. The next five portfolios are formed as above and alphas of these portfolios are analyzed. Panel A of Table 1.5 reports results of the estimation. CAPM alphas are monotonically increasing from -0.13% for the lowest liquidity risk portfolio to 0.15% for high liquidity risk portfolio. Only the highest risk portfolio alpha is significant. The "5-1" spread portfolio alpha is equal to 0.27% and is significant with t-statistic equal to 2.56. When the Fama-French setup is used, alphas of individual portfolios are still increasing monotonically, but only the lowest portfolio alpha is significant at 10% confidence level. The "5-1" spread portfolio alpha is equal to 0.21% and has t-statistic 1.88 (i.e. it is significant at 10% confidence level). Alphas from the four factor setup are similar to those from the Fama-French model, and range from -0.16% for the lowest liquidity risk portfolio to 0.13% for the highest liquidity risk portfolio. The spread portfolio alpha is equal to 0.29% and is significant at the 2.5% confidence level (t-statistic is 2.48).

Clearly, Pástor/Stambaugh's liquidity risk performs much better than the liquidity risk measure from Section 1.5. Next I check how Pástor/Stambaugh's liquidity risk compares to liquidity level. As before, 9 independently double sorted portfolios are created, and Table 1.6 reports their alphas and t-statistics.

Panel A of Table 1.6 presents CAPM alphas of double sorted portfolios. Individual portfolio alphas change from -0.06% for liquid low liquidity risk portfolio to 0.57% for illiquid medium liquidity risk portfolio. They are significant for medium liquidity and illiquid portfolios. Risk controlled portfolios have alphas of 0.52%, 0.55% and 0.42% for low, medium and high liquidity risk portfolios, and are all significant at 1% confidence level. Level controlled portfolios are insignificant (the highest t-statistic is 1.19). This agrees with results from Section 1.5.

Panel B of Table 1.6 reports Fama-French alphas, which for individual portfolios change from -0.033% for medium liquidity level medium liquidity risk portfolio to 0.24% for illiquid medium liquidity risk portfolio, most of this alphas are insignificant. Risk controlled spread portfolios have returns of 0.15%, 0.24% and 0.09% for

[8]See Pástor/Stambaugh (2003) for details on construction of $\mathcal{L}_t$. I download $\mathcal{L}_t$ series from Lubos Pastor's web page: http://www.chicagobooth.edu/faculty/directory/p/lubos-pastor.

Table 1.6: **Alphas of Pastor and Stambaugh Liquidity Risk and Liquidity Level Sorted Portfolios**

The table reports the alphas of the independently double sorted portfolios, in percentages per month. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). For each row liquidity level is fixed, while for each column Pastor and Stambaugh liquidity risk is fixed. The t-statistics are in parentheses.

	1	2	3	Level Controlled
		Panel A. CAPM		
1	-0.0606	0.0221	0.0510	0.1116
	(-0.8208)	(0.3682)	(0.8534)	(1.1902)
2	0.2641	0.2134	0.2493	-0.0148
	(2.6861)	(2.3800)	(2.6063)	(-0.2067)
3	0.4582	0.5701	0.4706	0.0124
	(3.5747)	(4.7296)	(3.6554)	(0.1460)
Risk Controlled	0.5188	0.5480	0.4197	
	(3.4231)	(4.0582)	(2.9212)	
		Panel B. Fama-French		
1	-0.0307	-0.0017	0.0250	0.0558
	(-0.4366)	(-0.0410)	(0.4462)	(0.5686)
2	0.0487	-0.0330	0.0076	-0.0411
	(0.6105)	(-0.4635)	(0.1012)	(-0.5759)
3	0.1191	0.2395	0.1177	-0.0015
	(1.3949)	(2.8731)	(1.5402)	(-0.0177)
Risk Controlled	0.1499	0.2412	0.0926	
	(1.5944)	(2.9988)	(1.0817)	
		Panel C. Four-factor		
1	-0.0538	-0.0073	0.0535	0.1073
	(-0.7515)	(-0.1577)	(0.9360)	(1.0668)
2	0.0208	-0.0100	0.0494	0.0286
	(0.2485)	(-0.1340)	(0.6331)	(0.3820)
3	0.0981	0.2665	0.1199	0.0218
	(1.1306)	(3.1799)	(1.5351)	(0.2441)
Risk Controlled	0.1519	0.2738	0.0664	
	(1.5743)	(3.3508)	(0.7187)	

low, medium and high liquidity risk portfolios. Only the medium risk spread portfolio is significant. Level controlled spread portfolios have alphas of 0.06%, -0.04% and -0.001% for liquid, medium liquidity and illiquid portfolios. All are insignificant with very low t-statistics. It is important to note that t-statistics of level controled portfolios are lower than t-statistics of risk controlled spread portfolios. The results of the four factor model are similar to those from the Fama-French model. Supporting the fact that t-statistics from risk controlled spread portfolios are higher than liquidity controlled ones.

Summarizing, this section provides further evidence of liquidity level being more reliable pricing variable than liquidity risk. Next I move to Sadka's liquidity risk.

Sadka's liquidity risk In his paper, Sadka (2006) shows that unexpected market-wide changes of the variable component of liquidity rather than fixed component are priced. In this section, Sadka's variable component of liquidity risk is compared with to liquidity level. I retrieve Sadka's variable component of liquidity from Wharton Research Data Services (WRDS). Data is available from 1983 through 2008. Further, Sadka's liquidity beta is estimated from a 3 year rolling window regression of equation (1.4), with Sadka's variable component of liquidity in place of $Mliq_t$. Next, portfolio analysis is conducted.

Panel B of Table 1.5 reports results of portfolios sorted according to Sadka's liquidity risk. Alphas for the CAPM model range from -0.02% for the least risky, to 0.18% for medium risk portfolio. Only the medium risk portfolio alpha is significant at the 10% confidence level. The "5-1" spread portfolio alpha has an alpha of 0.1%, which however is insignificant with t-statistic of only 0.7. Fama-French alphas demonstrate similar patterns. They are negative for low risk portfolios, positive high for medium risk portfolios and positive low (almost zero) for high risk portfolios. The "5-1" spread portfolio alpha increases in magnitude and significance with respect to the CAPM spread alpha. It is 0.15% and its t-statistic is 1.02. Portfolio alphas within the four factor setup range from -0.16% for the lowest risk portfolio to 0.11% for medium risk portfolio. None of them is significant. The "5-1" spread portfolio has alpha 0.27%, which is significant at the 10% confidence level.

Table 1.7 presents alphas of independently double sorted portfolios. Panel A reports CAPM alphas. They range from 0.06% to 0.4%, and are significant only for illiquid portfolios. Spread portfolio alphas, controlled for liquidity risk, are quite close to each other and have t-statistics a little below the 10% critical value. Liquidity level controlled spread portfolio alphas are insignificant, with t-statistics below 1.

Panel B reports Fama-French alphas, which range from -0.08% to 0.16% for individual portfolios and are insignificant. Analyzing risk controlled and level controlled spread portfolio alphas, one can infer that spread portfolios controlled for risk have on average higher alphas and t-statistics, although all are insignificant

Table 1.7: **Alphas of Sadka Liquidity Risk and Liquidity Level Sorted Portfolios**

The table reports the alphas the independently double sorted portfolios, in percentages per month. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). For each row liquidity level is fixed, while for each column Sadka liquidity risk is fixed. The t-statistics are in parentheses.

	1	2	3	Level Controlled
		Panel A. CAPM		
1	0.0059	0.1340	0.0583	0.0524
	(0.0504)	(1.3819)	(0.6122)	(0.4278)
2	0.1340	0.1746	0.2069	0.0729
	(0.8047)	(1.2463)	(1.4632)	(0.7413)
3	0.3007	0.4001	0.3931	0.0923
	(1.6200)	(2.2262)	(2.0811)	(0.7416)
Risk Controlled	0.2948	0.2662	0.3348	
	(1.5007)	(1.3667)	(1.5719)	
		Panel B. Fama-French		
1	-0.0768	0.0935	0.0012	0.0781
	(-0.8159)	(1.3495)	(0.0179)	(0.6272)
2	-0.0761	-0.0190	0.0191	0.0953
	(-0.5687)	(-0.1736)	(0.1766)	(0.9559)
3	0.0641	0.1691	0.1445	0.0804
	(0.5218)	(1.2315)	(1.1697)	(0.6228)
Risk Controlled	0.1409	0.0756	0.1433	
	(1.1745)	(0.5962)	(1.0938)	
		Panel C. Four-factor		
1	-0.0860	0.1049	0.0529	0.1389
	(-0.8745)	(1.3956)	(0.7504)	(1.0947)
2	-0.0813	-0.0138	0.0572	0.1386
	(-0.5534)	(-0.1246)	(0.4961)	(1.3249)
3	0.0454	0.1965	0.1421	0.0967
	(0.3621)	(1.3778)	(1.1347)	(0.7799)
Risk Controlled	0.1314	0.0916	0.0891	
	(1.1267)	(0.7206)	(0.6813)	

both for individual and spread portfolios. Four factor alphas results are similar to those of Fama-French.

In summary, analysis of portfolios created on the basis of the variable component of liquidity overall supports findings from earlier sections: liquidity as a characteristic has more power in explaining asset returns for the time period from 1965 through 2010 than liquidity as a risk, as measured by different liquidity risk measures.

Table 1.8: **Alphas of Liquidity Risk and Liquidity Level Portfolios for the 1964-1986 Subsample**

The table reports the alphas quintile portfolios, in percentages per month for the 1964-1986 subsample. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). Portfolios are sorted on the basis of liquidity level (Panel A), and liquidity risk (Panel B). The t-statistics are in parentheses.

	1	2	3	4	5	5-1
	Panel A. Liquidity risk					
CAPM alpha	-0.0666	0.1383	-0.0566	-0.0260	0.0754	0.1420
	(-0.6889)	(2.0896)	(-0.8632)	(-0.3952)	(0.8334)	(0.9891)
Fama-French alpha	-0.0089	0.2435	-0.0592	-0.0489	-0.0269	-0.0180
	(-0.0965)	(4.0831)	(-1.0488)	(-0.7367)	(-0.3034)	(-0.1269)
Four-factor alpha	-0.0575	0.1710	-0.0346	0.0208	0.0424	0.0999
	(-0.5783)	(2.7163)	(-0.5670)	(0.3061)	(0.4490)	(0.6608)
	Panel B. Liquidity level					
CAPM alpha	-0.0760	0.1633	0.3657	0.5314	0.7537	0.8297
	(-1.7360)	(2.1793)	(3.2834)	(3.7127)	(4.0689)	(3.7495)
Fama-French alpha	0.0071	0.0072	0.1042	0.1610	0.3006	0.2936
	(0.3109)	(0.1224)	(1.5046)	(2.5451)	(3.5379)	(3.1986)
Four-factor alpha	0.0129	0.0489	0.1331	0.1745	0.2603	0.2474
	(0.5268)	(0.8016)	(1.8419)	(2.5589)	(2.9767)	(2.5946)

1.6.2 Time subsamples

Up to this point, my results have been in contrast to those found in Lou/Sadka (2011). This section aims to explain such contradictory findings as a consequence of changes in pricing over time. The important thing here is that the 2008 financial crisis was a liquidity crisis and hence, during the 2008 financial crisis and after, liquidity risk became important. This may explain the findings in Lou/Sadka (2011). In financial history, liquidity crises are rare, though the one of 2008 was not unique. Thus, liquidity risk has no such a great impact on returns over the 1965-2010 sample. To determine if this explanation is indeed correct, I divide my sample into two subsamples and compare performance of liquidity risk and level in these two subsamples. It turns out that liquidity risk gains more importance in the later subsample, supporting the hypothesis proposed above.

Results of the portfolio analysis are provided in Tables 1.8-1.10. The original sample is split into two: 1964-1986 and 1987-2010. Table 1.8 reports single sort portfolios for the 1964-1986 subsample. Sorts are done as before, on the basis of liquidity risk and liquidity level. From t-statistics of “5-1” spread portfolio alphas,

Table 1.9: **Alphas of Liquidity Risk and Liquidity Level Portfolios for the 1987-2010 Subsample**

The table reports the alphas quintile portfolios, in percentages per month for the 1987-2010 subsample. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). Portfolios are sorted on the basis of liquidity level (Panel A), and liquidity risk (Panel B). The t-statistics are in parentheses.

	1	2	3	4	5	5-1
	Panel A. Liquidity risk					
CAPM alpha	-0.1555	0.0736	0.0907	0.1776	0.2171	0.3726
	(-1.2769)	(0.6453)	(0.8270)	(1.9949)	(2.0352)	(2.3932)
Fama-French alpha	-0.2164	0.0259	0.0047	0.1352	0.1476	0.3639
	(-1.8246)	(0.3050)	(0.0624)	(1.8020)	(1.3824)	(2.1907)
Four-factor alpha	-0.1988	0.0038	0.0125	0.1034	0.2143	0.4131
	(-1.6250)	(0.0422)	(0.1556)	(1.3154)	(2.0191)	(2.4628)
	Panel B. Liquidity level					
CAPM alpha	0.0310	0.1889	0.2283	0.2932	0.4651	0.4341
	(0.3810)	(1.6067)	(1.6452)	(1.7894)	(2.6021)	(2.2421)
Fama-French alpha	0.0009	0.0304	0.0280	0.0440	0.2136	0.2126
	(0.0195)	(0.3345)	(0.2718)	(0.4250)	(1.8328)	(2.0651)
Four-factor alpha	0.0006	0.0323	0.0316	0.0219	0.2267	0.2261
	(0.0124)	(0.3369)	(0.3014)	(0.2052)	(1.8197)	(2.0525)

one can see that liquidity risk has little impact on the profitability of stocks. On the other hand, liquidity level sorted spread portfolios have highly significant alphas, proving liquidity level impact on stock returns.

Inspecting results for the later sample (see Table 1.9), one can see that spread portfolio alphas sorted by liquidity risk become significant and stable over models (alphas for CAPM, Fama-French, and four factor models are almost the same). The alphas of spread portfolio sorted by liquidity level are still significant, although their t-statistics decrease to little above 2. Also, the alphas of the Fama-French and the four factor models are twice as low as the CAPM alphas. Results discussed here indeed support the hypothesis that liquidity risk is much more important in recent years. Notice that the liquidity level remains significant across the recent time period.

Table 1.10 reports the results of double sorting portfolio analysis for both subsamples. Results for liquidity risk and level controlled portfolios confirm the conclusions made for single sort portfolios. Risk controlled alphas are above 0.6% and highly significant for the 1964-1986 sample, while for the 1987-2010 sample

TABLE 1.10: **Alphas of Liquidity Risk-Level Sorted Portfolios**

The table reports the alphas the independently double sorted portfolios, in percentages per month for both subsamples. Left panel reports the 1964-1986 subsample, right – the 1987-2010 subsample. The alphas are estimated as intercepts from the regressions of excess portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). For each row liquidity level is fixed, while for each column liquidity risk is fixed. The t-statistics are in parentheses.

	1	2	3	Level Controlled	1	2	3	Level Controlled
	The 1964-1986 subsample				**The 1987-2010 subsample**			
				Panel A. CAPM alphas				
1	-0.0087	-0.0304	-0.1044	-0.0957	-0.0832	0.0793	0.1924	0.2755
	(-0.1042)	(-0.5034)	(-1.3771)	(-0.7286)	(-0.7248)	(0.7971)	(2.1461)	(2.1054)
2	0.2980	0.2957	0.3595	0.0615	0.2649	0.1573	0.1702	-0.0947
	(2.4112)	(2.7182)	(3.0161)	(0.6228)	(1.8323)	(1.1223)	(1.1613)	(-0.9329)
3	0.5998	0.6818	0.6341	0.0343	0.3600	0.4970	0.3417	-0.0183
	(3.2864)	(4.1020)	(3.6630)	(0.2973)	(2.0104)	(2.8407)	(1.7906)	(-0.1470)
Risk	0.6085	0.7122	0.7385		0.4432	0.4177	0.1493	
Controlled	(2.7034)	(3.5733)	(3.6469)		(2.1361)	(2.2567)	(0.7343)	
				Panel B. Fama-French alphas				
1	0.1459	0.0020	-0.1129	-0.2589	-0.1176	0.0250	0.1550	0.2726
	(1.9123)	(0.0444)	(-1.5033)	(-1.9581)	(-1.1818)	(0.3876)	(1.9358)	(2.0182)
2	0.0801	0.0255	0.0851	0.0050	0.0821	-0.0460	-0.0213	-0.1033
	(0.9974)	(0.3077)	(1.1411)	(0.0521)	(0.7221)	(-0.4365)	(-0.1812)	(-1.0210)
3	0.1815	0.2691	0.2189	0.0374	0.1110	0.2549	0.0742	-0.0367
	(2.0162)	(2.8539)	(2.5627)	(0.3228)	(0.9001)	(2.0395)	(0.6421)	(-0.3079)
Risk	0.0355	0.2670	0.3318		0.2286	0.2299	-0.0807	
Controlled	(0.3196)	(2.5084)	(2.7577)		(1.6061)	(1.9881)	(-0.6866)	
				Panel C. Four-factor alphas				
1	0.0778	0.0208	-0.0467	-0.1245	-0.1163	0.0130	0.1668	0.2831
	(0.9794)	(0.4440)	(-0.5819)	(-0.8796)	(-1.1181)	(0.1863)	(2.0464)	(2.0493)
2	0.0678	0.0768	0.1427	0.0749	0.0477	-0.0367	0.0108	-0.0369
	(0.8304)	(0.8738)	(1.8230)	(0.7342)	(0.4030)	(-0.3311)	(0.0883)	(-0.3534)
3	0.1971	0.3106	0.1801	-0.0169	0.0709	0.2748	0.0869	0.0160
	(2.1358)	(3.1323)	(1.9671)	(-0.1415)	(0.5684)	(2.1183)	(0.7326)	(0.1302)
Risk	0.1193	0.2897	0.2269		0.1872	0.2618	-0.0799	
Controlled	(1.0420)	(2.6501)	(1.7074)		(1.2905)	(2.1824)	(-0.6425)	

alphas decline to 0.44%, 0.42% and 0.15% for low, medium and high liquidity risk portfolios and the alpha of the spread portfolio with high liquidity risk actually becomes insignificant. This implies that liquidity level has lost some explanatory power in a later sample. Level controlled spread portfolios support the hypothesis of enhancement of liquidity risk explanatory power in recent times. Videlicet, for the early subsample, none of the alphas is significant, and for the 1987-2010 subsample one can see that the alpha becomes significant and increases in magnitude for the liquid spread portfolio.

In the Fama-French framework, the difference is even more evident. Spread portfolio controlled for liquidity risk alphas turn from significant to marginally significant for medium risk portfolio only. The alphas of spread portfolios, controlled for the level, have high t-statistics. For the 1987-2010 subsample, alphas are 0.27%, -0.1% and -0.04% for liquid, medium liquidity and illiquid spread portfolios. Only the alpha of liquid spread portfolio is significant at a 10% confidence level. Alphas obtained from the four factor model are similar to those from the Fama-French model. It is worth noting that in the later subsample alphas are stable across models, especially for the liquid spread portfolio. This is a good signal for the robustness of results.

Overall, findings of this section help to contextualize our findings with findings of the other authors, and elicit a more general story of liquidity in asset pricing. My analysis finds that, in general, liquidity level or/and trading cost have stronger effect on stock returns, and systematic patterns streaming from liquidity may emerge due to special market conditions, such as conditions of the 2008 financial crisis.

1.6.2.1 Analyzing recent data in detail

From the subsample analysis it must be admitted that although liquidity level is of higher importance for the cross-section of common stocks, the impact of liquidity become stronger in more recent times. There are two possible explanations for this fact, which are not exclusive nor exhaustive. The first is connected to the special market condition of the 2008 financial crisis, which could have been strong enough to drive results for the later subsample, due to the crucial role of liquidity in the 2008 crisis. The market was suffering severely from the luck of liquidity and thus stocks with higher sensitivity to the aggregate liquidity did perform poorly. This led to liquidity betas becoming crucial for stock prices and is confirmed by findings of Lou/Sadka (2011). An alternative explanation may be that liquidity risk and liquidity level represent the same general concept – liquidity. It might be the case that liquidity risk measures the second moment of liquidity, while liquidity level captures the first moment of the liquidity. Up to recent times, trading costs (liquidity level) were high, implying dominance of the first moment effects, but after decrease of the minimum tick from \$1/8 to \$0.01, the second order approximation may come to stage (See Chordia et al., 2001a for more details on the minimum tick).

Table 1.11: **Comparing the Spread Portfolio Alphas for 2000-2010 and 2008 Subsamples.**

The table reports the alphas thespread portfolios, in percentage points per month. The alphas are estimated as intercepts from theregressions of spread portfolio returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). Spread portfolios are sorted on the basis of liquidity level, Acharya and Pedersen (AP), Pastor and Stambaugh (PS) and Sadka (S) liquidity risk. The t-statistics are in parentheses.

	The 2000-2010 subsample				The 2008 subsample			
	Level	AP Risk	PS Risk	S Risk	Level	AP Risk	PS Risk	S Risk
CAPM	1.0377	-0.2404	0.8073	0.2616	1.0320	-0.1657	1.7293	0.3171
alpha	(3.5055)	(-0.6258)	(2.9164)	(1.0475)	(1.3608)	(-0.1495)	(2.1104)	(0.3904)
Fama-French	0.4430	-0.0730	0.7754	0.4836	0.5127	-0.9779	1.6035	0.0132
alpha	(2.5083)	(-0.1804)	(2.3794)	(1.9396)	(1.0591)	(-1.1849)	(2.0870)	(0.0155)
Four-factor	0.4429	-0.0122	0.7884	0.5118	0.5166	0.0055	1.6201	0.5844
alpha	(2.4772)	(-0.0332)	(2.4527)	(2.1627)	(1.0243)	(0.0086)	(1.9583)	(0.7482)

To test which explanation is more plausible, I estimate spread portfolio alphas for the 2000 through 2010 subsample and for the crisis subsample[9].

Table 1.11 reports the alphas of the spread portfolios formed on liquidity level and liquidity risk (Acharya and Pedersen – AP, Pastor and Stambaugh – PS and Sadka – S) for both subsamples. Alphas of spread portfolios formed on the basis of liquidity level are highly significant for the 2000-2010 subsample (the lowest t-statistic is 2.47). For the crisis subsample, magnitude of alphas does not change much, but they become insignificant. Alphas of the spread portfolios formed on the basis of the AP liquidity risk are insignificant and negative in both subsamples, and thus, can not tell anything about tested hypotheses. For the Sadka liquidity factor, the significance of spread alphas drops, meaning that Sadka's variable component of the liquidity loses its explanatory power during the crisis. Hence, results on the Acharya and Pedersen's and Sadka's liquidity risk do not provide any insights into the relative importance of level vs risk during recent sample.

An interesting result is produced by Pastor and Stambaugh's liquidity risk sorted portfolios. From Table 1.11, it is evident that the alphas of spread portfolios during the crisis double in magnitude, and more importantly, they stay statistically significant, although at a higher confidence level. This finding is in line with Lou/ Sadka (2011) and supports the idea that the 2008 crisis impacts results obtained for the later subsample. First, the alphas of Pastor and Stambaugh's liquidity risk

[9] I estimate portfolio alphas for time span from August of 2007 through March of 2009. This is generally accepted time period of the crisis.

sorted spread portfolios are higher during the crisis (compared to the 2000-2010 subsample and to the whole sample; see table 1.5), meaning that liquidity risk gains more explanatory power during the crisis, and the second significance of liquidity level declines dramatically in the same period.

Overall, analysis in this section supports the hypothesis of the impact of the crisis on the results of the later subsample. Indeed, stock sensitivity to the aggregate liquidity has become crucial over the past five years mostly due to severe market conditions, and specifically, to low market liquidity during the 2008 crisis.

1.7 Conclusion

This paper investigates the relative importance of liquidity risk as compared to liquidity level for the 1964-2010 sample of NYSE and AMEX common stock returns. I find that the liquidity level explains the cross-section of stock returns better than liquidity risk. The analysis suggests that results are much stronger for the early subsample. In the more recent subsample, the importance of liquidity risk increases, substituting part of the liquidity level component. This is in line with the findings of Lou/Sadka (2011), who find that during the 2008 crisis liquidity risk is more important than liquidity level for explaining returns. For the 1987-2010 subsample, liquidity level still remains a better explanatory variable than liquidity risk. This implies that trading cost and market friction maintain their role in financial markets.

Nevertheless, one need to keep in mind that liquidity level and liquidity risk are components of one general concept: the liquidity. It might be the case that liquidity risk measures the second moment of the liquidity while liquidity level captures the first moment of liquidity. Up to recent times, trading costs (liquidity level) were high, implying dominance of the first moment effects, but after the decrease of the minimum tick from $1/8 to $0.01, the second order approximation may come to stage. The analysis carried in previous section though discard this explanation in favor of stringent market conditions observed during the 2008 financial crisis.

The results of this paper are of particular importance for asset pricing, portfolio allocation and risk management, and suggest that investors still need to take liquidity into account more than sensitivity to aggregate liquidity.

Chapter 2.

Liquidity and Investment Horizon

—by Volodymyr Vovchak—

2.1 Introduction

The importance of the investment horizon has been acknowledged since the late 1970s. Levhari/Levy (1977) argue that taking wrong investment horizon in empirical tests of asset pricing models causes bias in the systematic risk and other econometric issues. Levy (1984) shows that estimation of the market beta changes with the length of the horizon. Later, the investment horizon was examined in the stream of liquidity literature. The importance of the investment horizon arises from the importance of liquidity, which has received renewed attention in recent years.

Finance literature advocates the importance of liquidity in both dimensions: as a characteristic (trading cost, trading activity, price impacts etc.) and as a separate pricing factor. It is worth noting the difference between liquidity and liquidity risk. Liquidity is the degree to which an asset or security can be traded in the market without affecting the prices of assets or how quickly an asset can be converted to cash without loss of its value. In academia, liquidity risk is referred to as the sensitivity of the stock return to the aggregate market liquidity. Researchers agree that less liquid stocks grant higher returns, the so called liquidity premium and that there is a strong commonality in liquidity. A question, following all universe of liquidity results, is: do these results help to make profit? At first glance, the answer appears to be no, since higher returns are observed for less liquid stocks, i.e. stocks with high transaction costs. These costs restrict investors from achieving the profits promised by theoretic conclusions. Thus, transaction costs serve to limit arbitrages established due to liquidity matters. But what about long term investors, who trade very infrequently, and by doing so they can get profits holding illiquid stocks. This raises a question about interaction between liquidity and investment horizon, which is a main focus of this paper. To be more specific, I will study a direct relationship between liquidity and investment horizon, and pricing abilities of liquidity once controlled for investment horizon for both cases: liquidity characteristic and liquidity pricing factor.

To better understand this research questions I will start from the link between liquidity and investment horizon. There are few previous papers investigating horizon-liquidity correspondence. In their paper Amihud/Mendelson (1986) prove that assets with higher spreads are held by long term investors in equilibrium[1], but they do not test this proposition empirically. An empirical test of the proposition was done by Atkins/Dyl (1997), who find strong evidence in favor of the statement. They find that a longer investment horizon is associated with higher transaction cost. The result is much stronger for NASDAQ stocks. Here I am going to test the same hypothesis for broader spectrum of liquidity measures over a longer time span. The results of this exercise will enable a better understanding of the relation between liquidity and holding horizon.

Kamara et al. (2012) test a different kind of commonly used systematic factors across different investment horizons. They show that some factors behave differently in long and short holding periods. They also document that factors may vary in behavior at one month and one year investment horizons. For example, value and return on equity behave like characteristics in short horizons, while with long horizons, they behave like systematic risk factors. Taking into consideration all of the above, it is interesting to test the pricing ability of liquidity, controlling for investment horizons. Amihud (2002) shows that market illiquidity (measured by the ratio of the absolute value of returns to dollar traded volume) positively affects ex ante stock excess returns.

[1] Amihud/Mendelson (1986) in fact state that "assets with higher spreads are allocated in equilibrium to portfolios with longer expected holding horizons", but one can equate long term investors and portfolio with longer horizon.

Beber et al. (2011) investigate an impact of investment horizon on the liquidity adjusted asset pricing model. The authors find that allowing for heterogeneity of investment horizon reduces liquidity risk relative to standard CAPM market risk. That is similar to the question in the previous paragraph and is going to be considered in this paper as well. The objective of this research is a broad investigation of the relationships between the investment horizon and liquidity.

This research finds the following: concerning the relationship between liquidity and holding horizon I find that illiquidity (relative spread, Amihud measure) is positively related to the investment holding horizon. More liquid stocks are traded more often, by the short term investors. Secondly, liquidity risk is negatively related to the holding horizon. Specifically, I find that stocks that are held by the short term investors are, in general, more disposed to aggregate liquidity shocks. Short term investors are willing to take on liquidity risk, which might be in line with liquidity provision by hedge funds.

The rest of the paper is organized as follows. Section 2.2 provides relevant literature review and aims to establish connections between liquidity and the investment holding horizon literature. In section 2.3, objectives, methodology and variables are highlighted. Data is discussed briefly in section 2.4. Results of estimation are presented in section 2.5. I provide results for liquidity characteristics of the stock (relative spread, price impact measure, turnover etc.) and liquidity risk (we propose three definitions of liquidity risk). In section 2.6, robustness checks of my findings are proposed. Section 2.7 concludes with a summary of findings.

2.2 Literature review

There is a large amount of literature on liquidity in financial markets. It covers both idiosyncratic and systematic aspects of liquidity. One of the attributes of liquid asset is how often this asset is traded. This property is partially determined by the investment policy of the holder of a given asset. If an asset holder is a short term investor, it is the case that stock is traded more often. If the investor is a long term investor, the stock is traded more rarely. Literature on investment horizon is not as abundant as liquidity literature, which might be attributed to data scarcity and the difficulty of measuring investment horizon.

It is widely acknowledged that less liquid stocks have higher returns (Amihud, 2002; Amihud/Mendelson, 1986; Brennan et al., 1998; Jones, 2002). Chordia et al. (2001a) make a broad study of liquidity and trading activity. Other papers (Acharya/Pedersen, 2005; Pástor/Stambaugh, 2003; Sadka, 2006) based on a different measure of aggregate liquidity, show that liquidity is priced in a cross-section of stocks. Watanabe/Watanabe (2008) show that liquidity premium is time varying and in general correlates with market cycles. The liquidity risk in the cross-section of hedge funds is investigated in two more recent papers. Sadka (2010) shows that liquidity risk based on his liquidity factor is an important determinant in the cross-section of hedge fund returns. Teo (2011) demonstrates that in the group of liquid hedge funds, those loading on liquidity risk earn almost 5% higher annual returns than those shunning liquidity risk. He also argues that, taking this into account, hedge funds often have much higher exposures to liquidity risk than is necessary. Further, a strong relation between stock market liquidity and business cycles is reported by Næs et al. (2011). There are also papers showing importance of liquidity risk in foreign exchange markets (see Mancini et al. (2013)) Some of these and other results concerning liquidity are discussed in a survey by Amihud et al. (2005).

Considering institutional investor results, Sias/Starks (1997) report that institutional ownership and returns autocorrelation are positively related, implying that institutiona ownership has an impact on returns behavior. Boehmer/Kelley (2009) argue that stocks with greater institutional ownership are priced more efficiently. Another branch of investment horizon literature considers the impact of short- versus long-term investors on the financial market and individual stocks. Yan/Zhang (2009) show that short term investors are better informed and thus, they trade more often to exploit this informational advantage. A corporate finance paper by Gaspar et al. (2005) concludes that firms held by short term investors have weaker bargaining position in acquisitions. In addition, Attig et al. (2012) argue that institutional investors with longer investment horizon have greater incentives to establish effective monitoring. This mitigates the principal agent problem and improves the funding ability of a firm (Lei, 2009). On the other hand, if a stock is held by short-term investors, its price reflects fundamental value better than the price of a stock held by long-term investors. Thereby one may distinguish two different functions of the length of investment horizon of prevailing investors. First, for corporate governance reasons, long-term investors are preferred, and second, for more efficient pricing short-term investors do a better job.

Further, there are papers analyzing connections between transaction costs and investment horizon. The first paper in this stream of literature is by Amihud/Mendelson (1986). The authors provide formal proof of the statement "Asset with higher spreads are allocated in equilibrium to portfolios with longer horizon". Empirical confirmation of this proposition is made by Atkins/Dyl (1997), who also find that this relation is stronger for NASDAQ stocks than for NYSE stocks. Importance of this type of "clientele effect" for the financial policy is discussed in Amihud/Mendelson (1991).

Beber et al. (2011) propose an equilibrium model with short and long term investors. They show that liquidity risk effects decrease if heterogeneous investment horizons are allowed. Kamara et al. (2012) analyze different pricing factor behavior for short and long horizon. The authors analyze the nature of the factor at monthly and yearly horizon. They find that liquidity has a systematic nature at the short horizon. Other factors (market, value and return to equity) demonstrate systematic patterns at the long horizon. Book-to-market behaves as a characteristic at the short term scale.

Cella et al. (2013) study the trading behavior of short and long term investors around the 2008 financial crisis. Their intuition is that, during market declines, institutional investors with short trading horizon are forced to sell their holdings much more often than long term institutional investors. The outcome of such behavior translates into poor performance of stocks held by short term investors. Such stocks experience more severe price drops and larger reversals than stocks held by long term investors. Cella et al. (2013) summarize by stating that short term investment holding horizons amplify negative market-wide effects. A result of this paper is in line with the result of Lou/Sadka (2011), who report that during the 2008 financial crisis, liquid stocks experienced stronger drop in price than less liquid stocks. Given that liquid stocks are held by short term investors, both papers point in the same direction and support the idea of tight relationship between the holding horizon and liquidity.

2.3 Methodology and Variables

This section introduces methodology and variables used in the paper, starting from the liquidity-investment horizon relationship.

Relating liquidity and investment horizon. The theory underlying the relation of liquidity with the holding horizon is Proposition 1 from Amihud/Mendelson (1986), which states the following:

Proposition 1. Assets with higher spreads are allocated in equilibrium to portfolios with (the same or) longer expected holding periods.

I use a methodology that is similar to Atkins/Dyl (1997), in order to investigate question of this Proposition. The authors use two stage instrumental variables regression of holding period on lagged quoted spread. I test reverse relationship and regress different liquidity measures on investment horizon. To avoid a two step procedure I run liquidity on lagged average investment horizon of the stock. Estimation of my panel is done by two-way clustering, as suggested in Petersen (2009). I chose this approach because data on liquidity has two sources of potential dependencies: firm effects (large firms may be more liquid and thus, have shorter holding horizons) and time effects (liquidity and holding policies may change over time). Using usual panel approaches such as Fama/MacBeth or one dimensional clustering produces too high t statistics, which may signal some dependencies in the data and biases in coefficient estimates. Double clustering is an effective way to make estimated standard errors close to true ones[2]. To estimate the relation between liquidity and holding horizon, I estimate coefficients of the following equation:

[2]For more on the firm and time effects and best estimation procedures see in Petersen, 2009. I use *cluster2* Stata procedure proposed by Petersen on his web page: http://www.kellogg.northwestern.edu/faculty/petersen/htm/papers/se/se_programming.htm.

$$Liq_{i\,t+1} = \alpha + \beta_1 \cdot Horizon_{i\,t} + \sum_j \gamma_j \cdot Control^j_{i\,t} \tag{2.1}$$

where $Liq_{i\,t+1}$ is one of five liquidity measures, $Horizon_{i\,t}$ is a proxy of the investment holding horizon presented in Cella et al. (2013) or in Yan/Zhang (2009), and is discussed in details below. One of the control variables is a percentage of the stock which is held by institutional investors filing their holdings to SEC. Another two variables represented $Control^j_{i\,t}$ are: stock returns volatility and the market cap of the stock.

I consider five liquidity measures: Amihud liquidity ratio, quoted spread, relative spread (these are measures of illiquidity), volume traded and turnover (these variables measure liquidity). Amihud liquidity measure is an averaged over a month ratio of the absolute return of a stock to its dollar volume traded. This measure is widely used because of its simplicity and its ability to explain stock returns adequately as a liquidity measure (for more information on this measure see Amihud, 2002). Liquidity is constructed as follows:

$$Illiq_{i,m} = \frac{1}{D_{i,m}} \sum_{t=1}^{D_{i,m}} \frac{|r_{i,t,m}|}{DVol_{i,t,m}} \tag{2.2}$$

where $r_{i,t,m}$ and $DVol_{i,t,m}$ are return and dollar volume traded of the stock i on day t of month m. Because illiquidity changes considerably over time, I scale *Illiq* by the average market illiquidity in a given month; i.e. as the liquidity of an individual stock, I use

$$Illiq^S_{i,m} = \frac{Illiq_{i,m}}{Illiq^M_m} \tag{2.3}$$

where $Illiq_{i,m}$ is from equation (2.2) and average market illiquidity is

$$Illiq_m^M = \frac{1}{N_m}\sum_{i=1}^{N_m} Illiq_{i,m},$$

where N_m – is the number of stocks in month m.

Amihud (2002) argues that this variable is positively correlated to other liquidity variables, such as Kyle's price impact measure λ, and fixed-cost measure, ψ, connected to bid-ask spread[3]. Further, he shows that his variable does a good job in explaining stock returns both cross-sectionally and over time.

The quoted spread is a difference between ask and bid prices, both are reported in CRSP. The quoted spread captures the trading costs investor must incur while trading the stock. The relative or percentage spread is the quoted spread divided by price of the stock, or average of bid and ask prices if price is unavailable. The relative spread is less noisy than the quoted spread and shows the percentage cost for trading particular stock. Volume traded is the sum of the trading volumes during that month. It is a measure of trading activity for a given stock. The turnover is a ratio of the traded volume and total shares outstanding. This measure indicates the activity of a given stock as a fraction of total shares outstanding.

Liquidity and investment horizon of the stock are determined to a great extent by the breadth of ownership and its structure. Chen et al. (2002) argue that if institutional investors have low share of a given stock and only few investors have long positions, then short-sales constraint is binding. This has an impact on the liquidity of the stock and on its ownership structure. Thus, the percentage held by institutional investors is an important indicator for both liquidity and holding horizon, and must be taken into account when analyzing liquidity-horizon interplay.

Atkins/Dyl (1997) used two other controls which I utilize as well; standard deviation of returns and the stock size. Standard deviation of returns is a consequence of frequent trading, thus, one would expect to observe a negative (positive) relationship between illiquidity (liquidity) variables and variability of the stock returns.

[3]Both measures were used in Brennan/Subrahmanyam (1996) to investigate relations between return premium and different components of cost.

Market value is included because the stock size is related to liquidity; larger stocks are more liquid. Hence, one would expect size to reduce the impact of investment horizon on liquidity.

Liquidity risk. Liquidity risk is an important issue in financial market, together with other systematic risk factors. As mentioned, liquidity risk is priced at both the stock and institutional levels. Teo (2011) shows that hedge funds often load relatively heavy on liquidity risk. I extend this result by looking at the relation between liquidity risk, as measured by the liquidity beta, and investment holding horizon. Because I use all institutional investors reporting to SEC, my analysis also provides results for long term investors, i.e. mutual and pension funds.

I estimate equation (2.1) with the dependent variable being the liquidity beta. In this case, control variables are added to control for some non-horizon related factors. They include size, book-to-market, momentum, short interest and liquidity. The estimation is carried out similarly to the to analysis for the liquidity level.

I use three types of the liquidity beta, depending on the aggregate market liquidity measure; the aggregate liquidity measure proposed by Pástor/Stambaugh (2003), Sadka's (2006) permanent-variable component and the aggregate liquidity measure used in Acharya/Pedersen (2005). Liquidity risk is estimated as beta from the time-series regression[4]:

$$r_{it}^{e} = \beta_i^0 + \beta_i^M \cdot mkt_t + \beta_i^S \cdot smb_t + \beta_i^{BM} \cdot hml_t + \beta_i^L \cdot Mliq_t + \varepsilon_{it}, \tag{2.4}$$

where r_{it}^{e} is an excess return of stock i (stock return minus the risk-free rate), mkt is the excess return of the market index and smb and hmb are returns of long-short spreads obtained by sorting stocks according to their market value and book-to-market ratio[5]. $Mliq$ is one of the aggregate market liquidity mentioned above.

[4]The equation (2.4) is estimated on monthly data with three years moving window.

[5]More details on *smb* and *hmb* portfolios can be found in Fama/French (1993).

Liquidity betas are estimated with a 36 month rolling window, which causes them to be persistent. Given this and the fact that the holding horizon is highly persistent, one has to take into account biases in parameters estimation. To tackle this problem I am going to use a fixed effects model with standard errors clustered by time. For more robustness, double clustering is conducted as well.

Investment holding horizon variable. To define the holding horizon of an institutional investor, I utilize the methodology proposed in Gaspar et al. (2005). Their measure is based on the frequency with which an institutional investor changes its portfolio, the so called "churn ratio". It is based on the idea that short-term investors buy and sell stocks more often than long-term investors, who keep their positions unchanged for long time periods. This is captured by the investor's portfolio turnover. More specifically, we start by calculating the churn ratio of institutional investor i holding a set of stocks Q_i at quarter t as:

$$CR_{i,t} = \frac{\sum\limits_{j \in Q_i} |S_{i,j,t}P_{j,t} - S_{i,j,t-1}P_{j,t-1} - S_{i,j,t-1}\Delta P_{i,j}|}{\sum\limits_{j \in Q_i} \frac{S_{i,j,t}P_{j,t} + S_{i,j,t-1}P_{j,t-1}}{2}} \tag{2.5}$$

where $S_{i,j,t}$ is the number of firm j's shares held by institutional investor i at quarter t, and $P_{j,t}$ is firm j share price at quarter t.

One should think of the investment horizon as a permanent characteristic of an investor's strategy, reflecting its preferences, objectives or funding structure. To account for this, I further compute the average churn ratio for institutional investor i over the past four quarters:

$$aCR_{i,t} = \frac{1}{4}\sum_{l=1}^{4} CR_{i,t-l+1}.$$

The measure obtained above characterizes the investor's investment policy with respect to the time an investor holds the average stock in her portfolio. A value of the churn ratio ranges from 0 to 2. An institutional investor who has low churn ratio has a low portfolio turnover, and is a long-term investor, while an investor with high churn ratio is a short term investor. Now I am able to calculate the churn ratio of the stock, given the churn ratios of the investors holding this particular stock.

$$fCR_{j,t} = Horizon_{j,t} = \sum_{i \in I} w_{j,i,t} \cdot aCR_{i,t} \tag{2.6}$$

where I is the set of all institutional investors, $w_{j,i,t}$ is the share of the stock j held by the investor i at quarter t. This variable measures how often on average the stock is traded. High *Horizon* means that stock is mostly held by short-term investors, low *Horizon* indicates that stock is preferred by long term investors.

For robustness, I also use two alternative proxies for the investment horizon. These alternative measures are taken from Ben-David et al. (2012) and Yan/Zhang (2009).

2.4 Data

There are three main data sources for this study. The quarterly institutional investors holdings data for common stocks traded on NYSE, AMEX and NASDAQ come from Thompson Reuters. Thompson Reuters Institutional Holdings (13f) Database provides institutional common stock holdings and transactions as reported on the Form 13f filed with Securities and Exchanges Commission (SEC). This database contains ownership information by institutional managers with assets under management exceeding \$100 million and reports all equity positions greater than 10,000 shares or \$200,000. Information is reported at the end of each quarter to the SEC. The data contained in the 13f datafile is used to calculate the churn ratio for each institutional investor, associate it with a particular stock as a proxy for the holding horizon of an average stock, and to calculate institutional investors ownership for each stock.

The stock characteristics are retrieved from two main sources: The Center for Research in Security Prices (CRSP) and COMPUSTAT. We obtain stock returns, prices, bid and ask prices, spreads, traded volumes, shares outstanding and dividend to price ratio from the CRSP monthly and daily (to estimate Amihud liquidity ratio) tapes for all NYSE, AMEX and NASDAQ common stocks. Book values used to obtain book-to-market ratio is obtained from COMPUSTAT.

The aggregate liquidity measures (Pástor/Stambaugh and Sadka) and the Fama-French portfolios (*mkt*, *smb* and *hmb*) are downloaded from the Wharton Research Data Services (WRDS) web page.

All the data sources provide data for time period from the beginning of 1990 through the end of 2011. Stocks with price lower than 1$ are excluded from the analysis, since they are generally considered to be highly speculative. Table 2.1 displays summary statistics of all the variables. Investment horizon variables as required ranges from 0 to 2, and have averages 0.105, 0.0292 and 0.107. These variables do not have high variation and are slightly positively skewed (means are higher than medians). Table 2.2 provides correlation of dependent and independent variables. Correlation between holding horizon variables is quite high; for example, my measure has correlation of 0.93 with measure from Ben-David et al. (2012). From Panel B one can see that indeed liquidity risk variables have small correlation with liquidity variables.

2.5 Main results

In this section I present my empirical findings. The order is the same as in section 2.3. I start from the results on holding-horizon-liquidity relation and further move to the relationship between liquidity risk and investment horizon.

2.5.1 Liquidity and investment horizon

Results of the double clustered regressions (2.1) are presented in Tables 2.3-2.7. The tables present results of the regression of liquidity characteristics on lagged holding horizon, lagged size, and lagged standard deviation of the returns. The percentage held by institutional investors is added to control for possible effects of institutional ownership. Since lower values of the horizon variable corresponds to long term investment horizon, one should expect a negative sign of *Horizon* when regressing illiquidity measures, and a positive sign when regressing volume and turnover – measures of liquidity.

The results of regressing the Amuhud liquidity measure onto the holding horizon are presented in the Table 2.3. When the holding horizon is the only explanatory variable, it is significant at the 1% confidence level. The coefficient is negative, confirming that indeed stocks held by short (long) term investors have lower (higher) price impact of trading volume. In other words, short term investors prefer more liquid stocks. The result holds after controlling for different variables. Controlling for size (column 2 of Table 2.3) does not impair the significance of the *Horizon* coefficient, though it decreases in absolute value from -0.64 to -0.15. The reduction of the coefficient reflects the relationship between holding horizon and the stock size. Controlling for standard deviation of the returns (column 3) has mild impact on the *Horizon* coefficient. It remains highly significant and is equal to -0.44, implying that the holding horizon does not reflect a variability of the stock returns in its liquidity. Column 4 presents the result of the regression with size and volatility as control variables. The t-statistics of the $Horizon$ coefficient drops to 2.76 (which is still significant at the 1% confidence level) and the coefficient is compatible with the one from the regression in the column 2. Further, I control for the percentage held by institutional investors (column 5). The $Horizon$ coefficient is highly significant and is equal to -0.183. Column 6 reports results of the regression with the volatility and institutional ownership as controls. The coefficient for the holding horizon is highly significant, with the t-statistics -3.55, and is equal to -0.16. The control variables in all regressions are of expected sign and are highly significant.

Summarizing findings on the relation between the Amihud liquidity measure and holding horizon, I can confirm that liquidity of the stock and the investment strategy of the investors holding this stock are connected. Effectively, the $Horizon$ coefficient is close to -0.15. This means that an increase in the holding horizon by one standard deviation leads to 0.15 times the standard deviation of the Amihud ratio decrease in the Amihud liquidity measure for the stock.

Table 2.4 reports results for the quoted spread. The general conclusion one can draw from the table is that the holding horizon does not influence the quoted spread. In addition, the sign of the horizon coefficient alters. For regressions in columns 1 to 4, it is negative as one should expect, but once the percentage held by the institutional investors is included in the regression, the sign changes to positive. Such ambiguity might be a consequence of the noise in the quoted spread.

In Table 2.5, results for the relative spread and the holding horizon are presented. There the holding horizon is the only explanatory variable (column 1), it is significant, and is equal to -0.02. Controlling for the size and standard deviation of the returns in general confirms Atkins/Dyl's (1997) results. The significance of the $Horizon$ coefficient decreases. Then, controlled for the volatility, the coefficient (see column 3) decreases slightly but remains significant at the 1% confidence level. Controlling for the stock size takes some explanatory power from holding horizon. In the regression reported in column 2, t-statistics of the $Horizon$ coefficient drops to -2.01 and its value shrinks to -0.002. In the regression from column 4, the $Horizon$ coefficient is significant at the 10% confidence level. Columns 5 and 6 add the percentage held by institutions as a control. This is not done in Atkins/Dyl (1997). Results indicate that the holding horizon does not have explanatory power to explain variation in the relative spread once controlled for the percentage held. The sign of the coefficient is still negative but very low. Control variables are highly significant and have the expected signs.

In summary, the results in Table 2.5 confirm the results of Atkins/Dyl (1997), and expand them in a sense that I demonstrate that relation between the relative spread and the holding horizon streams from the institutional ownership.

A relationship between the volume traded and investment horizon is reported in Table 2.6. It is evident from the table that the impact of the holding horizon on the traded volume is, to some extent, due to the size. In the regressions 2 and 4, the size is a control variable, the $Horizon$ coefficient is insignificant and has a negative sign. The holding horizon is highly significant, when used as the only regressor, and then regressed together with the volatility. Controlling for the percentage held by the institutions makes the $Horizon$ coefficient only marginally significant and decrease it by a factor of 10. Overall, the holding horizon has an impact on the trading volume, but it is not robust. In fact, including the size into the regression wipes out all explanatory power of the holding horizon.

Table 2.7 reports the results of the regression for the turnover rate. Holding horizon has an impact on turnover. The $Horizon$ coefficient is significant at the 1% confidence level in all regressions. Then, regressed as the only explanatory variable, the holding horizon is highly significant and is positive as expected. Controlling for the size and volatility of the returns alters the coefficient, but significance and the sign of the coefficient remains unchanged. Then, when the percentage held by institutional investors is included, the $Horizon$ coefficient decreases to 0.4, but its t-statistics even goes up. Overall, the holding horizon has an impact on the turnover rate. An average value of the $Horizon$ coefficient is 0.63, and in every regression it is significant at the 1% confidence level. However, this result might be attributed to the way we constructed our horizon proxy. The point is that the $Horizon$ variable is constructed as average investor's turnover and investor's turnover is the part of a stock turnover. Thus, results for the turnover rate may reflect a pure mechanical relation. It is worth mentioning that the correlation between the $Horizon$ variable and turnover in our panel is only 0.14.

In general, this subsection confirms earlier results on the relationship between liquidity level and the investment holding horizon, and expands them for other measures of the liquidity. My study confirms the findings of Atkins/Dyl (1997) that holding horizon is positively related to the relative spread. In addition, I find that the relation is true for the Amihud liquidity measure, turnover rate, and partially, for the trading volume, although I cannot exclude that results for the turnover might be purely the mechanic effect of variables construction.

2.5.2 The holding horizon and liquidity risk

To analyze the impact of the holding horizon on the liquidity risk of the stock, I estimate the following regression:

$$LiqBeta_t = \alpha + \beta \cdot Horizon_{t-1} + \gamma' \cdot Controls_{t-1}, \tag{2.7}$$

where *LiqBeta* is one of three possible liquidity betas measuring liquidity risk, *Controls* is a vector of control variables composed of systematic characteristics (book-to-market, size, momentum) to control for possible indirect impacts of the holding horizon onto liquidity betas through these systematic channels. Another control variable is *short interest*, which is an important indicator of the sentiments of investors toward a given stock, and influences investment policy with respect to a given stock (see Asquith et al., 2005). Liquidity measures are also included, since they are tightly related to both indicators analyzed here.

Results of the regression (2.7) are presented in Table 2.8. Overall the coefficient on the holding horizon is positive and significant. Columns denoted P&S report regressions for the Pástor/Stambaugh liquidity risk. One can see that a coefficient at holding horizon is highly significant and positive, ranging from 1.98 to 2.93. For the Sadka liquidity risk coefficients are much higher in magnitude (from 10.65 to 13.94) but they are less significant. Nevertheless, they are significant at the 5% significance level. Acharya/Pedersen's liquidity betas are positively related to holding horizon as well. Thought the relationship is not very strong statistically; only in the column (3) the coefficient on *Horizon* is significant at the 10% significance level.

A positive coefficient on the holding horizon implies that a stock with higher average churn ratio (and thus, shorter investment horizons of the majority of its investors) has higher exposure to the liquidity risk. In other words, a stock held by short term investors is exposed to the aggregate market liquidity more than one held by long term investors. This is consistent with Teo (2011), who argues that hedge funds, which happen to be mostly short term investors, take on a greater amount of liquidity risk than it is optimal for them. Our finding is an extension of this result, indicating that, in general, short term investors prefer more liquidity risk.

Control variables have the following patterns. The book-to-market coefficient has a positive sign for the Pástor/Stambaugh and the Acharya/Pedersen liquidity betas. It varies in significance from insignificant for the model in column (3) to strongly significant, for the model (7). The sign of the coefficients indicates that value stocks carry more liquidity risk. This finding is consistent with the fact that value stocks are in general more risky. Their price is below its book value, and a shock to the aggregate liquidity puts them into an even worse conditions. In the regression of the Sadka liquidity beta, the coefficient on the book-to-market is negative and strongly significant. This is most probably attributed to the nature of the variable component of liquidity risk. Sadka (2006) relates this variable component of liquidity to private information and information asymmetry. Given the value of the stock, private information allows one to disentangle good and bad stocks, thus, making value stocks an attractive targets for investors exploiting informational asymmetries.

The effect of the size is negative for the Pastor and Stambaugh and the Sadka liquidity risk. The latter relationship is much stronger. The negative coefficient for the size agrees with common wisdom that bigger stocks are safer, here in a sense of the liquidity risk. In the Acharya and Pedersen regression, the size coefficient is positive and this is in conflict with findings of Acharya/Pedersen (2005).

Momentum does not have an impact on the Acharya and Pedersen liquidity beta. Coefficients are low in their magnitude and insignificant. For Sadka and Pastor and Stambaugh liquidity risk measures, momentum has a pronounced positive impact, implying that stocks with stronger tendency to persist are more likely to have higher liquidity risk. Liquidity level variables take the part of size effects and attempts to compensate each other.

2.6 Robustness

In this section I present robustness checks of my results. I start from the results for liquidity measures and then move to liquidity risk variables.

2.6.1 Alternative investment horizon measures

To check results of the regression (2.1), reported in tables 2.3-2.7, I repeat estimation with other proxies of the investment horizon. As mentioned in section 2.3, I use alternative churn ratios from Yan/Zhang (2009) and Ben-David et al. (2012).

The churn ratio in Yan/Zhang differs from the one used above in a way that the authors use minimum among the aggregate purchase and the aggregate sale for an institution's churn ratio, while here I use a sum of both. This variable minimizes the impact of investors' cash flow on portfolio turnover. A summary of the results is presented in Table 2.9, and results are in line with findings reported in Section 2.5.

Another proxy for investment horizon is the one used in Ben-David et al. (2012). The difference between their variable and the one used here is that the churn ratio of institutional investors are not averaged over the previous year. This might be justified by the fact that churn ratios are stable over time, thus, there is no need to smooth them even more. This measure reflects the most recent churn ratio, but not the previous year average. A summary of the regressions using this variable as proxy for the investment horizon is provided in Table 2.10. Results are consistent with the results reported for other proxies of the investment horizon.

2.6.2 Results by year

In this subsection I present results by year, to see how the relationship reported above changes through time.

For the liquidity level variables, results are identical to the results reported in Section 2.5. Coefficients vary in their values, but are of the same sign and are significant. I do not report these results here due to the lack of space; they are available upon request. The analysis of the relationship between liquidity risk and investment horizon is more interesting. Table 2.11 reports results of the estimation of the regression (2.7) implemented year by year for the Pastor and Stambaugh liquidity betas as a dependent variable[6]. One can see that most of the significant coefficients are positive, as was for the whole sample. There are, though, years in the middle of the sample (1999, 2000, 2001) where the sign is negative. For these years, stocks held by short term investors also have low liquidity risk. This means that during this period, short term investors abstained from carrying liquidity risk in their portfolios. Stocks with returns tightly connected with the marked liquidity were too risky to hold because of movements in the market liquidity, possibly caused by market turmoils of these years (LTCM, Russian and Argentine debt crises and dot-com bubble burst). This may be one of the reasons for adopting a decimal trading format.

2.6.3 Instrumenting churn ratio

So far have I assumed implicitly that churn ratio variables, which are a proxy for the institutional investors holding horizon, are exogenous with respect to the liquidity variables. It is natural to question whether the churn ratio, measured as investor's flows, may determine liquidity characteristics of the stock. It is hard to argue what comes first: is it that long term investors choose illiquid stocks, or stocks become illiquid because they are held by institutions with long term investment horizon and thus, these stocks are traded infrequently? The results I reported earlier are for lagged churn ratios, but since churn ratio is very persistent it might not resolve the problem. In this section, I introduce an instrument for the stock's investment holding horizon and report results obtained utilizing this instrument.

Firstly, I try to answer the question what is a main clause of the profit of long- and short term investors. Obviously a main source of short term investors' profit is a capital gain from trading, while for the long term investors profits mainly come

[6] Results for other liquidity betas are similar.

from dividend payments. This difference is important, but it does not help in disentangling stocks preferred by long term investors and those held by short term investors. What is crucial here is taxation, or so called tax clienteles. Poterba/Summers (1985) and Pérez-González (2003) analyze the impact of tax changes, and argue that a change in taxation affects dividend payouts. Pérez-González concludes that dividend payouts increased in years when dividends were more tax-advantaged relative to capital gains[7].

More systematic description of the theory of tax clienteles is described in Allen et al. (2000). In this paper, the authors aim to explain why some firms prefer to pay dividends rather than repurchase shares. They find that the decision to pay out dividends is based on the preferences of main stockholders, on their taxation[8]. Together with stated above difference in the preferences of long and short term investors, it is reasonable to propose dividends as a good instrument for measuring the holding horizon. The simplest measure, which I am going to use is the dividend-price ratio.

The dividend-price ratio is calculated from the CRSP monthly datafile. We must check if the dividend-price ratio satisfies the conditions of a good instrumental variable. Changes in the instrument should be associated with changes in the instrumented variable, but must not cause changes in the dependent variable. This means, firstly, dividend-price ratio must be correlated with the instrumented variable; holding horizon in my case. Table 2.2 reports correlations of the dividend-price ratio with three holding horizon variables. One can see that correlation varies from -3.6% to -3.9%. This correlation is significant at the 0.1% confidence level. The correlation between dividend-price ratio and Pastor and Stambaugh liquidity beta is 0.01% and insignificant, which makes dividend-price ratio an admissible instrument. Correlations of our instrument with Sadka and Acharya and Pedersen liquidity betas are -0.45% and -0.61% respectively.

[7]He also gives a brief analysis of tax reforms and how they have influenced a relative position of dividends with respect to capital gains.

[8]A similar idea of catering firm's dividend policy towards its main investors is proposed by Baker/Wurgler (2004), although they do not discuss determinants of investors demand for stocks paying dividends. They do not mention tax as a determinant for the purchase of paying or nonpaying stocks, arguing that for either psychological or institutional reasons, some investors have an uninformed and perhaps time-varying demand for dividend-paying stocks.

Second, dividend-price ratio must be conditionally uncorrelated with dependent variable; liquidity beta in this case. This assumption cannot be tested. The numbers above allow us to consider dividend-price ratio as a valid instrument. In addition, I will report statistics for verification of the instrument quality.

Results. Now, after finding and justifying the instrument, I discuss an estimation procedure and results. I reestimate equation (2.7), utilizing dividend-price ratio as an instrument for the holding horizon. Variables for controlling other effects are the same as in section 2.5.2. I control for systematic characteristic (BM, size, momentum), short interest and liquidity. As before, a variable of interest is the holding horizon, which is instrumented by the dividend-price ratio.

Equation (2.7) is estimated by means of the two-step feasible GMM estimation with time fixed effects and Eicker-Huber-White sandwich estimator of variance-covariance matrix. This variance-covariance matrix is robust to arbitrary heteroscedasticity.

Table 2.12 reports the results of the estimation. Before analyzing each model separately, a few general comments should be noted. The coefficients in the regression models are much higher in magnitude than the coefficients in Table 2.8, which is probably an outcome of low average values of dividend-price ratios. Next, results for Sadka liquidity betas are insignificant and have negative signs. Further, results for Pastor and Stambaugh and Acharya and Pedersen confirm findings from Section 2.5.2. Weak identification F-statistics are higher than 20 and t-statistics of the instrument in the first stage is higher than 2.5 in absolute value. These statistics support dividend-price ratio as a valid instrumental variable for the holding horizon.

In Table 2.12, results for Pastor and Stambaugh are denoted P&S. In column 1, the horizon coefficient is positive and highly significant with t-statistic 3.06. Book-to-market impact is insignificant. Size, as in section 2.5.2, has a significant negative sign, meaning that high cap stocks are safer, in the sense that they are less sensitive to aggregate liquidity shocks. Momentum is also consistent with the previous results: a positive sign implies that stocks with performance persistence are more (liquidity) risky. In column (4) model, short interest is added. Its coefficient is negative and significant, which means stocks about which investors are bearish have lower liquidity risk. In the model from column (7), the Amihud liquidity measure is added. It is negatively related to liquidity risk, implying that illiquid stock carries less liquidity risk. For Sadka liquidity beta the *Horizon* coefficient is insignificant with low t-statistics. Results for Acharya and Pedersen liquidity risk are essentially the same as results for Pastor and Stambaugh liquidity risk.

In this section, I tested the robustness of my findings. I used alternative measures of the holding horizon, and show that the results do not change depending on the churn ratio used. Further, I demonstrated results by year. This analysis confirms our findings for separate time spans, with the exception of the late 1990s. Such a breakdown may be caused by macroeconomic and liquidity problems the financial market experienced during this period. Both robustness checks were implemented for the liquidity level and liquidity risk results. Finally, I propose an admissible instrument for my holding horizon measure, and show that my results are robust to possible endogeneity issues.

2.7 Concluding remarks

This paper investigates the relationship between investment holding horizon and liquidity. I show that the theoretical statement of Amihud/Mendelson (1986) holds for the US stock market data. The proposition is true for a broad range of liquidity measures. The strongest relation is observed for Amihud's price impact measure. I find that stocks with higher (lower) values of Amihud ratio have shorter (longer) holding horizon. Similar results are observed for the relative spread, confirming that stocks with lower liquidity (higher spreads and price impact of trading) are allocated to portfolios with longer expected holding horizon. The same is true for trading frequency measures of liquidity. For stocks with higher turnover, I find that investors are mainly short term investors. Similarly, although statistically weaker, the result is reported for the trading volume. My conclusions remain valid after conducting a number of robustness checks.

Another interesting result is obtained for liquidity risk. I find that stocks with higher (lower) liquidity risk are held by investors with shorter (longer) investment horizon. Given that the market is driven mostly by short term investors, liquidity risk must be overseen and controlled thoroughly. The reason is, that short term investors seem to take more liquidity risk than its optimal for them (Teo, 2011), which together with the fact that shocks to short term investors' factors tend to propagate. This may put financial markets into big problems, one example the 2008 financial crisis (Cella et al., 2013).

Findings of this paper enhance understanding of liquidity and its relation to investment policies. Investment horizon is shown to have an impact on liquidity and riskiness of stocks.

Table 2.1: **Summary Statistics**

The table displays descriptive statistics of variables. Most of the variables cover the time period from 1990 through 2011. *Obs.* denotes the number of stock-month observation. Liquidity betas are winsorized on both sides at 1%.

	Obs.	Mean	Std. Dev.	Min	Median	Max
Investment horizon:						
Holding horizon	977750	0.105	0.100	0	0.0827	2
Investment horizon (Yan Zhang)	977750	0.0292	0.0278	0	0.0242	2
Investment horizon (Ben-David et al.)	977750	0.107	0.108	0	0.0835	2
Liquidity measures:						
Amihud measure	975733	0.333	3.386	0	0.00668	861.5
Quoted Spread	905326	0.269	0.540	0	0.125	172.5
Relative spread	905326	0.0247	0.0374	0	0.0123	1.980
Volume	975766	0.124	0.905	0	0.0133	201.2
Turnover	975766	1.287	2.452	0	0.687	368.7
Liquidity betas:						
Pastor and Stambaugh	976426	0.963	55.19	-213.3	0.357	218.1
Sadka	976112	5.895	732.8	-3025.9	1.209	2886.5
Acharya and Pedersen	976112	0.0296	13.10	-49.52	0.136	50.96
Control variables:						
Book-to-Market	882024	0.727	0.850	0	0.554	40.58
Log(size)	974595	12.44	1.997	4.820	12.26	20.22
Momentum	971079	0.142	0.695	-0.998	0.0526	81.80
Standard Deviation of Returns	975670	0.0335	0.0265	0	0.0267	2.533
Short Interest ratio	872596	0.0265	0.0458	0	0.00855	0.973
Institutional Ownership	977750	0.435	0.294	0	0.408	1
Instrument:						
Dividends-price ratio	974233	0.00104	0.0176	0	0	8.333

TABLE 2.2: **Correlation Matrix**

Panel A of the table provides pairwise correlation of all explanatory variables. Panel B reports pairwise correlation of all dependent variables. The correlations are reported over the 1990-2011 sample period for each pair of variables, except *ShortInterest* which is available only up to June, 2010.

Panel A. Correlation matrix of explanatory variables

	Holding horizon	Inv. horizon, Yan Zhang	Inv. horizon, Ben-David et al.	Book-to-Market	Log(size)	Momentum	Std. Dev of Returns	Short Interest	Institutional Ownership
Inv. horizon (Yan Zhang)	0.685								
Inv. horizon (Ben-David et al.)	0.930	0.651							
Book-to-Market	-0.128	-0.113	-0.119						
Log(size)	0.474	0.453	0.448	-0.251					
Momentum	0.0435	0.0579	0.0526	-0.0161	0.0701				
St. Dev. of Returns	-0.177	-0.128	-0.173	0.0999	-0.393	-0.0594			
Short Interest	0.439	0.327	0.406	-0.0849	0.217	-0.0125	0.0184		
Inst. Ownership	0.784	0.723	0.743	-0.120	0.650	0.00296	-0.256	0.406	
DP	-0.036	-0.0385	-0.0392	0.00609	0.0113	-0.00471	-0.0189	-0.00972	-0.00524

Panel B. Correlation matrix of dependent variables

	Amihud measure	Quoted Spread	Relative spread	Volume	Turnover	P&S beta	Sadka beta
Quoted Spread	0.00819						
Relative Spread	0.135	0.363					
Volume	-0.0132	-0.0458	-0.0752				
Turnover	-0.0388	-0.109	-0.157	0.158			
P&S beta	-0.00383	-0.00192	0.0315	-0.00842	0.0118		
Sadka beta	0.0173	0.00628	0.0361	-0.0180	-0.0108	0.118	
A&P beta	0.00154	0.00434	0.0211	-0.000472	0.00275	0.0235	0.0938

Table 2.3: **Cross-Sectional Regressions of the Amihud Liquidity Measure**

Cross-section regressions of the Amihud liquidity measure on the lagged holding horizon and the other lagged stock characteristics. The table reports results of the regression analysis of the equation

$$Amih_{i\ t+1} = \alpha + \beta_1 \cdot Horizon_{i\ t} + \sum_j \gamma_j \cdot Control^j_{i\ t},$$

where $Amih$ is taken to be the Amihud liquidity measure (2.3). Controls are logarithm of the stock size ($Ln(size)$), standard deviation of the stock returns ($RetStdev$), and institutional investor ownership in a given stock ($\%hold$). Estimation is made using two dimensional clustering procedure, which takes into account between firm and year effects and produces unbiased standard errors. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)
Horizon	-0.637***	-0.150***	-0.437***	-0.138***	-0.183***	-0.160***
	(-3.68)	(-3.58)	(-3.25)	(-2.76)	(-4.24)	(-3.55)
Ln(size)		-0.394***		-0.282***		
		(-16.02)		(-8.68)		
RetStdev			23.72***	16.93***		21.01***
			(10.76)	(6.99)		(8.84)
% hold					-1.818***	-1.156***
					(-24.55)	(-8.06)
Constant	0.654***	5.339***	-0.273***	3.347***	1.299***	0.242
	(15.37)	(16.81)	(-2.70)	(6.95)	(16.21)	(1.54)

TABLE 2.4: **Cross-Sectional Regressions of the Quoted Spread**

Cross-section regressions of the quoted spread on the lagged holding horizon and the other lagged stock characteristics. The table reports results of the regression analysis of the equation

$$spread_{i\ t+1} = \alpha + \beta_1 \cdot Horizon_{i\ t} + \sum_j \gamma_j \cdot Control^j_{i\ t},$$

where *spread* is taken to be the Quoted Spread. Controls are logarithm of the stock size ($Ln(size)$), standard deviation of the stock returns ($RetStdev$), and institutional investor ownership in a given stock ($\%hold$). Estimation is made using two dimensional clustering procedure, which takes into account between firm and year effects and produces unbiased standard errors. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)
Horizon	-0.0533	-0.0299	-0.0649*	-0.0339	0.00818	0.00580
	(-1.49)	(-1.19)	(-1.74)	(-1.30)	(0.62)	(0.41)
Ln(size)		-0.0197**		-0.0302***		
		(-1.98)		(-2.73)		
RetStdev			-0.923***	-1.651***		-1.660***
			(-2.88)	(-4.01)		(-5.45)
% hold					-0.256***	-0.307***
					(-2.86)	(-3.59)
Constant	0.290***	0.524***	0.326***	0.714***	0.381***	0.464***
	(5.85)	(3.56)	(5.64)	(4.08)	(5.55)	(6.21)

TABLE 2.5: **Cross-Sectional Regressions of the Relative Spread**

Cross-section regressions of the relative spread on the lagged holding horizon and the other lagged stock characteristics. The table reports results of the regression analysis of the equation

$$rspread_{i\ t+1} = \alpha + \beta_1 \cdot Horizon_{i\ t} + \sum_j \gamma_j \cdot Control^j_{i\ t},$$

where $rspread$ is taken to be the Relative Spread. Controls are logarithm of the stock size ($Ln(size)$), standard deviation of the stock returns ($RetStdev$), and institutional investor ownership in a given stock ($\%hold$). Estimation is made using two dimensional clustering procedure, which takes into account between firm and year effects and produces unbiased standard errors. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)
Horizon	-0.0208***	-0.00237**	-0.0154***	-0.00204*	-0.00150	-0.000665
	(-3.80)	(-2.01)	(-3.55)	(-1.85)	(-1.09)	(-0.53)
Ln(size)		-0.0155***		-0.0130***		
		(-8.07)		(-7.22)		
RetStdev			0.697***	0.383***		0.543***
			(6.78)	(4.90)		(5.46)
% hold					-0.0805***	-0.0640***
					(-8.23)	(-7.92)
Constant	0.0379***	0.222***	0.0106***	0.178***	0.0666***	0.0394***
	(8.59)	(8.72)	(3.82)	(7.46)	(9.10)	(6.96)

Table 2.6: **Cross-Sectional Regressions of the Volume Traded**

Cross-section regressions of the volume traded on lagged holding horizon and other lagged stock characteristics. The table reports results of the regression analysis of the equation

$$VolTr_{i\ t+1} = \alpha + \beta_1 \cdot Horizon_{i\ t} + \sum_j \gamma_j \cdot Control^j_{i\ t},$$

where $VolTr$ is taken to be the Relative Spread. Controls are logarithm of the stock size ($Ln(size)$), standard deviation of the stock returns ($RetStdev$), and institutional investor ownership in a given stock ($\%hold$). Estimation is made using two dimensional clustering procedure, which takes into account between firm and year effects and produces unbiased standard errors. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)
Horizon	0.0722***	-0.00340	0.0732***	-0.00197	0.00729*	0.00780*
	(2.65)	(-0.33)	(2.67)	(-0.20)	(1.69)	(1.72)
Ln(size)		0.0612***		0.0707***		
		(6.38)		(6.69)		
RetStdev			-0.250	1.439***		0.382***
			(-1.48)	(6.27)		(4.09)
% hold					0.260***	0.272***
					(8.06)	(8.34)
Constant	0.0772***	-0.651***	0.0867***	-0.820***	-0.0150**	-0.0343***
	(5.64)	(-6.20)	(4.76)	(-6.55)	(-2.49)	(-4.23)

TABLE 2.7: **Cross-Sectional Regressions of the Turnover**

Cross-section regressions of the turnover on lagged holding horizon and other lagged stock characteristics. The table reports results of the regression analysis of the equation

$$Turnov_{i\ t+1} = \alpha + \beta_1 \cdot Horizon_{i\ t} + \sum_j \gamma_j \cdot Control^j_{i\ t},$$

where $Turnov$ is taken to be the stock Turnover. Controls are logarithm of the stock size ($Ln(size)$), standard deviation of the stock returns ($RetStdev$), and institutional investor ownership in a given stock ($\%hold$). Estimation is made using two dimensional clustering procedure, which takes into account between firm and year effects and produces unbiased standard errors. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)
Horizon	0.831***	0.597***	0.905***	0.623***	0.397***	0.412***
	(3.53)	(3.92)	(3.48)	(3.92)	(6.92)	(6.92)
Ln(size)		0.190***		0.266***		
		(6.05)		(7.88)		
RetStdev			5.187***	11.56***		9.952***
			(3.58)	(8.35)		(7.69)
% hold					1.741***	2.056***
					(9.44)	(12.94)
Constant	1.113***	-1.146***	0.906***	-2.504***	0.496***	-0.00713
	(8.23)	(-3.80)	(6.15)	(-7.13)	(3.77)	(-0.08)

TABLE 2.8: **Cross-Sectional Regressions of Liquidity Risk on the Lagged Holding Horizon**

Cross-section regressions of liquidity risk on the lagged holding horizon and other lagged stock characteristics. The table reports results of the regression analysis of the equation

$$LiqBeta_t = \alpha + \beta \cdot Horizon_{t-1} + \gamma' \cdot Controls_{t-1},$$

where *Amih* is taken to be the Amihud liquidity measure (2.3). Controls are logarithm of the stock size ($Ln(size)$), ratio of book value of the stock to its market value ($Book-to-Market$), cumulated previous 9 month returns excluding month $t-1$ (*Momentum*), investors short interest toward a given stock (*Short Int*), and liquidity measures (*Amihud measure* and *Relative spread*). Columns denoted *P&S* report regressions conducted for Pastor and Stambaugh liquidity betas, columns denoted *Sadka* report results for Sadka liquidity betas, and *A&P* – for Acharya and Pedersen liquidity betas. Estimation is made using fixed effects with time clustered variance covariance matrix estimated using Huber-White sandwich VCE estimator. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	P&S (1)	Sadka (2)	A&P (3)	P&S (4)	Sadka (5)	A&P (6)	P&S (7)	Sadka (8)	A&P (9)
Horizon	2.090***	10.646**	0.174*	2.929***	13.937***	0.034	1.981***	11.636***	0.137
	(4.566)	(2.544)	(1.773)	(4.500)	(2.979)	(0.308)	(4.496)	(2.748)	(1.401)
Book-to-Market	0.798**	-28.655***	0.062	0.629	-24.669***	0.228**	1.002**	-23.948***	0.196**
	(1.986)	(-4.899)	(0.634)	(1.519)	(-3.751)	(2.207)	(2.432)	(-3.834)	(2.009)
Log(size)	-0.370*	-12.078***	0.211***	-0.419**	-11.152***	0.233***	0.248	-11.000***	0.336***
	(-1.785)	(-3.399)	(4.054)	(-1.985)	(-2.851)	(4.018)	(1.118)	(-2.752)	(5.960)
Momentum	1.432**	32.608***	-0.063	1.188*	37.106***	-0.037	1.440**	37.140***	-0.014
	(2.107)	(3.567)	(-0.433)	(1.705)	(3.947)	(-0.246)	(2.083)	(3.994)	(-0.094)
Short Int				-12.453***	-53.857	1.192*			
				(-4.312)	(-1.453)	(1.921)			
Amihud measure							-0.063***	1.493***	-0.010**
							(-2.645)	(6.036)	(-2.510)
Relative spread							32.620***	-61.214	6.207***
							(4.409)	(-0.712)	(5.523)
Constant	4.809*	168.440***	-2.705***	5.974**	153.383***	-3.221***	-3.772	153.008***	-4.495***
	(1.718)	(3.561)	(-3.733)	(2.126)	(2.943)	(-4.014)	(-1.268)	(2.866)	(-5.721)
# of periods	249	249	249	243	243	243	249	249	249

TABLE 2.9: **Cross-sectional Regressions of Liquidity on the Alternative Holding Horizon**

regressions of liquidity on the alternative lagged holding horizon and other lagged stock characteristics. Holding horizon is estimated as in Yan/Zhang (2009). The table reports results of the regression analysis of the equation

$$(Il)Liq_{i\ t+1} = \alpha + \beta_1 \cdot Horizon_{i\ t} + \sum_j \gamma_j \cdot Control^j_{i\ t},$$

where $(Il)Liq$ is one of the five (il)liquidity measures: Amihud liquidity ratio ($Amihud$), quoted spread ($Spread$), relative spread ($\%spread$), volume traded ($Volume$) and $turnover$. Controls are logarithm of the stock size ($Ln(size)$) and standard deviation of the stock returns ($RetStdev$). Holding horizon is estimated as in Yan/Zhang (2009). Estimation is made using two dimensional clustering procedure, which takes into account between firm and year effects and produces unbiased standard errors. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	Amihud	Spread	% spread	Volume	Turnover
Horizon	-0.666***	0.0318	-0.00350	-0.0404	1.767***
	(-3.43)	(0.62)	(-1.11)	(-1.21)	(3.42)
Ln(size)	-0.281***	-0.0311***	-0.0131***	0.0709***	0.269***
	(-8.76)	(-2.82)	(-7.26)	(6.70)	(7.82)
RetStdev	16.95***	-1.648***	0.383***	1.439***	11.51***
	(7.01)	(-3.98)	(4.91)	(6.26)	(8.41)
Constant	3.341***	0.720***	0.178***	-0.822***	-2.525***
	(6.98)	(4.12)	(7.48)	(-6.56)	(-7.09)

Table 2.10: **Cross-sectional Regressions of Liquidity on the Alternative Holding Horizon**

Cross-section regressions of liquidity on lagged holding horizon and other lagged stock characteristics. Holding horizon is estimated as in Ben-David et al. (2012). The table reports results of the regression analysis of the equation

$$(Il)Liq_{i\ t+1} = \alpha + \beta_1 \cdot Horizon_{i\ t} + \sum_j \gamma_j \cdot Control^j_{i\ t},$$

where $(Il)Liq$ is one of the five (il)liquidity measures: Amihud liquidity ratio ($Amihud$), quoted spread ($Spread$), relative spread ($\%spread$), volume traded ($Volume$) and $turnover$. Controls are logarithm of the stock size ($Ln(size)$) and standard deviation of the stock returns ($RetStdev$). Holding horizon is estimated as in Ben-David et al. (2012). Estimation is made using two dimensional clustering procedure, which takes into account between firm and year effects and produces unbiased standard errors. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	Amihud	Spread	% spread	Volume	Turnover
Horizon	-0.137***	-0.0315	-0.00204*	-0.00397	0.591***
	(-3.14)	(-1.17)	(-1.80)	(-0.44)	(3.73)
Ln(size)	-0.282***	-0.0303***	-0.0130***	0.0707***	0.266***
	(-8.70)	(-2.73)	(-7.22)	(6.69)	(7.86)
RetStdev	16.93***	-1.652***	0.383***	1.438***	11.57***
	(6.99)	(-4.01)	(4.90)	(6.27)	(8.35)
Constant	3.347***	0.715***	0.178***	-0.821***	-2.511***
	(6.95)	(4.08)	(7.46)	(-6.55)	(-7.10)

TABLE 2.11: **Cross-Sectional Regressions of Pastor and Stambaugh Liquidity Risk by Years**

Cross-section regressions of Pastor and Stambaugh liquidity risk (beta) on lagged holding horizon and other lagged stock characteristics by years. The table reports results of the regression analysis of the equation

$$LiqBeta_t = \alpha + \beta \cdot Horizon_{t-1} + \gamma' \cdot Controls_{t-1},$$

where *Amih* is taken to be the Amihud liquidity measure (2.3). Controls are logarithm of the stock size ($Ln(size)$), ratio of book value of the stock to its market value ($Book - to - Market$), cumulated previous 9 month returns excluding month $t-1$ ($Momentum$), investors short interest toward a given stock ($Short\ Int$), and liquidity measures ($Amihud\ measure$ and $Relative\ spread$). In each raw there are presented coefficient estimates for a given year (1990,1991, etc.). Estimation is made using fixed effects with time clustered variance covariance matrix estimated using Huber-White sandwich VCE estimator. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	Horizon	Ln(size)	Momentum	Amihud	% spread	Const
1990	-87.020	-3.230	27.085	.951	17.106	30.125
	(-5.059)***	(-3.265)***	(5.748)***	(.571)	(1.331)	(2.749)***
1991	28.148	-3.636	4.681	-.085	-16.777	37.293
	(1.151)	(-2.913)***	(1.348)	(-.085)	(-.474)	(2.499)**
1992	20.384	-1.790	-15.995	-.679	-12.997	24.094
	(3.325)***	(-1.687)*	(-4.307)***	(-.332)	(-.404)	(1.802)*
1993	12.074	.451	12.504	-1.386	54.782	-9.274
	(.501)	(.451)	(2.168)**	(-.745)	(1.617)	(-.667)
1994	.341	-1.867	19.051	-1.317	66.382	20.293
	(.017)	(-2.066)**	(2.864)***	(-.770)	(1.921)*	(1.453)
1995	5.144	-5.880	-1.255	.211	58.851	80.634
	(2.368)**	(-5.297)***	(-.217)	(.151)	(1.147)	(4.448)***
1996	9.443	-2.496	-9.851	-.225	4.860	39.579
	(.687)	(-4.102)***	(-2.462)**	(-.293)	(.116)	(4.193)***
1997	-5.652	-.981	-3.333	.066	-12.089	16.119
	(-.328)	(-1.935)*	(-.889)	(.091)	(-.619)	(2.340)**
1998	10.517	.876	-10.362	-.419	-40.645	-10.490
	(.791)	(1.538)	(-2.702)***	(-2.110)**	(-1.753)*	(-1.424)
1999	-18.460	-2.073	13.600	-.041	8.992	26.629
	(-.827)	(-4.700)***	(6.767)***	(-.173)	(.309)	(3.820)***
2000	-34.062	-1.479	1.482	.034	12.667	20.553
	(-1.334)	(-3.118)***	(.739)	(.235)	(.875)	(3.192)***
2001	-31.425	-.944	3.771	-.523	12.254	13.138
	(-1.891)*	(-2.040)**	(1.132)	(-1.290)	(8.192)***	(2.278)**
2002	-.275	.492	9.169	.716	51.591	-3.317
	(-.024)	(.843)	(3.396)***	(2.214)**	(2.095)**	(-.426)
2003	33.877	.762	-16.684	-1.209	.355	-5.104
	(3.031)***	(1.997)**	(-6.899)***	(-2.852)***	(.014)	(-.998)
2004	38.467	.236	4.948	-.001	-52.914	-2.388
	(2.224)**	(.423)	(1.196)	(-.011)	(-.936)	(-.336)
2005	29.187	1.852	16.425	-.024	-93.021	-29.228
	(2.317)**	(2.705)***	(4.147)***	(-.764)	(-1.892)*	(-2.929)***
2006	16.451	3.046	.589	.080	-171.065	-45.472
	(1.274)	(4.744)***	(.142)	(2.703)***	(-4.102)***	(-4.422)***
2007	3.033	1.062	6.196	.017	3.286	-16.983
	(3.385)***	(2.370)**	(1.886)*	(.472)	(.077)	(-2.375)**
2008	34.935	-.951	-4.883	.112	30.593	11.298
	(2.802)***	(-2.823)***	(-1.762)*	(1.811)*	(5.539)***	(2.798)***
2009	6.515	.453	-1.550	.089	13.843	-9.317
	(.501)	(1.384)	(-1.073)	(2.550)**	(.654)	(-2.026)**
2010	-7.317	-.154	5.393	.042	14.174	-2.591
	(-.538)	(-.391)	(3.339)***	(.741)	(.803)	(-.504)

TABLE 2.12: **Cross-Sectional Instrumental Variables (IV) Regressions of Liquidity Betas**

Cross-section instrumental variables (IV) regressions of liquidity betas on lagged holding horizon and other lagged stock characteristics. The table reports results of the regression analysis of the equation

$$LiqBeta_t = \alpha + \beta \cdot Horizon_{t-1} + \gamma' \cdot Controls_{t-1}, \tag{2.8}$$

where $Amih$ is taken to be the Amihud liquidity measure (2.3). Controls are logarithm of the stock size ($Ln(size)$), ratio of book value of the stock to its market value ($Book-to-Market$), cumulated previous 9 month returns excluding month $t-1$ ($Momentum$), investors short interest toward a given stock ($Short\ Int$), and liquidity measures ($Amihud\ measure$). Columns denoted $P\&S$ report regressions conducted for Pastor and Stambaugh liquidity betas, columns denoted $Sadka$ report results for Sadka liquidity betas, and $A\&P$ – for Acharya and Pedersen liquidity betas. Holding horizon is instrumented by the dividend-price ratio.

Equation (2.8) is estimated by means of the two-step feasible GMM estimation with time fixed effects and Eicker-Huber-White sandwich estimator of variance-covariance matrix. This variance-covariance matrix is robust to arbitrary heteroscedasticity. t statistics are provided in parentheses, significance levels are marked as follows: * coefficient is significant at the 10% level, ** coefficient is significant at the 5% level, and *** coefficient is significant at the 1% level.

	P&S (1)	Sadka (2)	A&P (3)	P&S (4)	Sadka (5)	A&P (6)	P&S (7)	Sadka (8)	A&P (9)
Horizon	343.1***	-779.3	58.01**	358.1***	-1620.4	49.62**	346.6***	-811.6	58.55**
	(3.06)	(-0.72)	(2.21)	(3.51)	(-1.57)	(2.23)	(3.06)	(-0.74)	(2.21)
Book-to-Market	-0.141	-22.67***	-0.0759	0.568***	-20.80***	0.247***	-0.131	-22.78***	-0.0744
	(-0.38)	(-6.04)	(-0.87)	(3.27)	(-9.40)	(5.83)	(-0.35)	(-6.09)	(-0.86)
Ln(size)	-15.31***	24.35	-2.310**	-13.63***	51.22	-1.584*	-15.49***	26.03	-2.338**
	(-3.11)	(0.51)	(-2.01)	(-3.57)	(1.33)	(-1.90)	(-3.11)	(0.54)	(-2.01)
Momentum	3.171***	27.65***	0.181	1.455***	35.05***	-0.0454	3.170***	27.67***	0.181
	(5.35)	(4.72)	(1.31)	(7.57)	(15.17)	(-1.03)	(5.33)	(4.72)	(1.30)
Short Int				-225.5***	931.0	-28.68**			
				(-3.69)	(1.51)	(-2.15)			
Amihud measure							-0.164***	1.576***	-0.0252***
							(-3.36)	(3.33)	(-2.67)
weak ident. F-stat	22.85	22.61	22.61	27.69	27.53	27.53	22.53	22.29	22.29
1-stage instrument t-stat	-4.780	-5.262	-4.746	-4.755	-5.247	-4.721	-4.755	-5.247	-4.721

Chapter 3.

Is the Price Kernel Monotone?

—by Giovanni BARONE-ADESI, Hakim DALL'O and Volodymyr VOVCHAK*—*

3.1 Introduction

According to economic theory, the shape of the state price density (SPD) per unit probability (also known as the asset pricing kernel, Rosenberg/Engle (2002) or stochastic discount factor (SDF), Campbell et al. (1997)) is a decreasing function in wealth.

Jackwerth (2000) finds a kernel price before the crash of 1987 in agreement with economic theory, but a discordant result for the post-crash period. After his work, a number of papers have been written on this topic trying to explain the reason for this puzzle. Rosenberg/Engle (2002), Detlefsen et al. (2007) and Jackwerth (2004) are among the most interesting papers on this subject. Unfortunately, none of them found an answer to this puzzle. In all of these papers the authors found problems in the methodology employed by previous papers and tried to improve them, but the result was the same: the puzzle remained.

An answer to this puzzle has been given in Chabi-Yo et al. (2005), where they argue that the main problem is the regime shifts in fundamentals: when volatility changes, the kernel price is no longer monotonically decreasing. In each regime they prove that the kernel price is consistent with economic theory, but when there is a shift in regime the kernel price changes in its shape and it is no longer consistent with economic theory.

In a recent paper, Barone-Adesi et al. (2008) compute again the kernel price and find kernel prices consistent with economic theory. In particular they find kernel price consistency for fixed maturities. They do not pool different maturities as Aït-Sahalia/Lo (1998) and therefore they avoid the problem that arises when maturities are different, but they do not consider the change in fundamentals as a relevant aspect of their computation. Their result can be explained by the fact that the sample they use is very short (3 years) and that throughout this period (2002 - 2004) the volatility does not change much.

In this paper we compute the kernel price both in a single day and as an average of kernel prices over a period of time, holding maturity constant. We want to understand the implication of the changing regime using two measures of moneyness: in the first case we consider the kernel price as a function of two parameters, the underlying and the interest rate (we do not take into consideration the changing regime) and then we add third parameter – the volatility of the underlying. As argued in Brown/Gibbons (1985), under some general assumptions one may substitute (in estimation) consumption with the market index while working with asset pricing models[1]. Thats why in order to evaluate the kernel price we need to take a broad index which attempts to cover the entire economy. As it is common in this kind of literature to use S&P500 index, we also use data on S&P500 index prices and options on the S&P500 index over a period of 12 years (from the 2nd of January 1996 to the 31st December 2007).

Evaluating the kernel price in a period of time, without taking into consideration the change in volatility, should lead to a kernel not consistent with economic theory. Surprising, when we compute the kernel price considering only two parameters (the underlying and the interest rate), the average kernel price is consistent with economic theory, with the exceptions of a few dates.

To check our result we also do kernel smoothing which is similar to averaging, but it has the advantage of producing smooth price kernel without the spikes one might get from simple averaging. Another robustness check for our results is the testing of monotonicity of the obtained kernel price. We take our estimated average price

[1] The aggregated consumption is inconvenient in two ways: (1) it is hard to measure, and (2) no options on aggregate consumption are traded.

kernel, consider its monotone version and then compare the monotone version with the estimated version by means of Kolmogorov-Smirnov test.

In order to estimate the risk neutral distribution, we use the well-known result in Breeden/Litzenberger (1978). The difference with previous works is in the options we use. Instead of creating option prices through nonparametric or parametric models (all the previous research use artificial price of options and this could introduce a bias in the methodology), we use only the options available on the market. We then construct the historical density using the GJR GARCH model with Filtered Historical Simulation already presented in Barone-Adesi et al. (2008).

As discussed in Rosenberg/Engle (2002), among the several GARCH models, the GJR GARCH with FHS has the flexibility to capture the leverage effect and the ability to fit daily S&P500 index returns. Then, the set of innovations estimated from historical returns and scaled by their volatility gives an empirical density function that incorporates excess skewness, kurtosis, and other extreme return behavior that is not captured in a normal density function. These features avoid several problems in the estimation of the kernel price. For example, using a simple GARCH model where the innovations are standard normal (0; 1) leads to a misspecification of the return distribution of the underlying index.

Once we have the two probabilities, under the pricing and the objective measures, we take the ratio between the two densities, discounted by the risk-free rate, in a particular day, to compute the kernel price for a fixed maturity. We repeat the same procedure for all the days in the time series which have options with the same maturity and then we take the average of the kernel price through the sample. At the same time we apply kernel smoothing on the estimated values of the price kernel to confirm our result.

We also evaluate how the shape of the price kernel changes before and during a crisis (the 2008 crisis). We notice that the three periods before the crisis (2005, 2006 and 2007) exhibit fairly monotonically decreasing paths, while during the crisis, the kernel price remains monotonically decreasing, but has higher values. this is consistent with the idea that during a crisis investors increase the risk aversion.

In order to evaluate the impact of the shifting regime, we repeat the computation of the different kernel prices considering the volatility as a parameter of the kernel function. As expected, results improve, but they are still quite similar, supporting our first intuition that the changing regime is relevant, but our methodological choices have a strong impact on the final result.

The remainder of this paper is organized as follows. In section 2, we present a review of the literature and we define the "pricing kernel puzzle". In section 3, we define our method to estimate the kernel price. We explain our application of the result of Breeden/Litzenberger (1978) and we derive the risk neutral distribution. We then estimate the historical density using a GJR GARCH method with FHS and we take the kernel price from a particular day as well as the kernel price over the time series of our sample. In the last part of the section we conduct two statistical tests. Namely, we use two Kolmogorov type tests of the monotonicity of the estimated pricing kernels. In section 4, we provide further evidence of our results. First we plot kernel price with different maturities to prove the robustness of our methodology, then we take the average of these different kernel prices and we show that the average of SPD per unit probabilities with close maturities have a monotonically decreasing path. In section 5, we present the change in the kernel price shape before and during the recent crisis. In section 6, we extend our model, using a kernel price with three parameters (underlying, volatility and risk-free), and in section 7 we offer conclusions.

3.2 Review of the Literature

In this section we derive the price kernel as in macroeconomic theory and also as in probability theory. We then present some methods, parametric and non parametric, to derive the kernel price.

3.2.1 Price kernel and investor preference

The ratio between the risk neutral density and the historical density is known as the price kernel or state price density per unit probability. In order to explain the relationship between the risk-neutral distribution and the historical distribution we need to introduce some basic concepts from macroeconomic theory. In particular, we use a representative agent with a utility function $U(\cdot)$. According to economic theory (the classical von Neumann and Morgenstern economic theory), we have three types of investors: risk averse, risk neutral and risk lover. The utility function $U(\cdot)$ of these investors is a twice differentiable function of consumption c: $U(c)$. The common property for the three investors is the non-satiation property: the utility increase with consumption, e.g. more consumption is preferred to less consumption, and the investor is never satisfied - he never has so much wealth that getting more would not be at least a little bit desirable. This condition means that the first derivative of the utility function is always positive. On the other hand, the second derivative changes according to the attitude the investor has toward risk.

If the investor is risk averse, his utility function is an increasing concave utility function. The risk neutral investor has a second derivative equal to zero, while the risk seeker - a convex utility function.

Defining $u(\cdot)$ as the single period utility function and β as the subjective discount factor, we can write the intertemporal two-period utility function as

$$U(c_t; c_{t+1}) = u(c_t) + \beta u(c_{t+1}).$$

We introduce ξ as the amount of an asset the agent chooses to buy at time t, e as the original endowment of the agent, P_t as the price of the asset at time t and x_{t+1} as the future payoff of the asset. The optimization problem is:

$$\max_{\xi} \left\{ u(c_t) + \beta E_t[u(c_{t+1})] \right\};$$

$$\text{subject to}$$

$$c_t = e_t - P_t \xi;$$

$$c_{t+1} = e_{t+1} + x_{t+1} \xi.$$

The first constraint is the budget constraint at time 1, while the second constraint is the Walrasian property, e.g. the agent will consume all of his endowment and asset's payoff at the last period. Substituting the constrains into the objective and setting the derivative with respect to ξ equal to zero we get:

$$P_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}x_{t+1}\right].$$

We define

$$\beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\right] = m_{t,t+1} = MRS, \tag{3.1}$$

as the *marginal rate of substitution* at time t. The MRS is also known as the Stochastic Discount Factor (SDF) or the price kernel. Therefore the price of any asset can be expressed as

$$P_t = E_t[m_{t,t+1}x_{t+1}].$$

In a continuous case, the price of any asset can be written as

$$P_t^p = \int_{\mathbb{R}} m_{t,T}(S_T)x_T(S_T)p_{t,T}(S_T)dS_T \tag{3.2}$$

where $p_{t,T}(S_T)$ is the physical probability of state S_T (for the rest of the paper we refer to this probability as the historical probability) and $x_T(S_T)$ is the payoff of an asset.

To define the price of an asset at time t, under the risk neutral measure, we can write equation (3.2) as:

$$P_t^q = e^{-rt}\int_{\mathbb{R}} x_T(S_T)q_{t,T}(S_T)dS_T \tag{3.3}$$

where $q_{t,T}(S_T)$ is the state price density (for the rest of the paper we refer to this probability as the risk neutral probability). At this point, combining equation

(3.2) and (3.3) we can derive the SDF as:

$$m_{t,T}(S_T) = e^{-rt}\frac{q_t(S_T)}{p_t(S_T)} \tag{3.4}$$

In this case we consider a two period model where the price kernel is a function only of the underlying, S_T, and the risk free rate, r. In the following part we will see how to have a kernel price with more parameters.

In their papers Arrow (1964) and Pratt (1964) find a connection between the kernel price and the measure of risk aversion of a representative agent. Arrow-Pratt measure of absolute risk-aversion (ARA) is defined as:

$$A_t(S_T) = -\frac{u''(S_T)}{u'(S_t)}$$

The absolute risk aversion is an indicator of willingness to expose some amount of wealth to risk as a function of wealth. An agent's utility function demonstrating decreasing (constant or increasing) absolute risk aversion implies that her willingness to take risk increases (does not change or decreases) as the agent becomes wealthier.

Classic economic theory assumes risk averse economy agents, i.e. the utility function of the economy is concave (mathematically $u''(S_T) \leq 0$). The following argument should unveil an impact of this basic property of pricing kernel behavior.

From (3.1), the pricing kernel can be written as function of the marginal utility as:

$$m_{t,T}(S_T) = \beta\frac{u'(S_T)}{u'(S_t)},$$

and its first derivative is:

$$m'_{t,T}(S_T) = \beta\frac{u''(S_T)}{u'(S_t)} = -\beta A_t(S_T),$$

which (remember $u''(S_T) \leq 0$ and $u'(S_t) > 0\ \forall t$) implies $m'_{t,T}(S_T) \leq 0$, or in words, the pricing kernel is decreasing as a function of the wealth. We are aiming to check if the pricing kernel is decreasing and, as a consequence, if agents in the economy are risk averse.

3.2.2 Nonparametric and parametric estimation

There are several methods to derive the kernel price. There are both parametric models and nonparametric models. In this section we give a review of the most well-known methods used in literature. We focus particularly on the nonparametric models because they do not assume any particular form for the risk neutral and historical density and also for the kernel price.

One of the first papers to recover the price kernel in a nonparametric way is Aït-Sahalia/Lo (1998). In their work they derive the option price function by nonparametric kernel regression and then, applying the result in Breeden/Litzenberger (1978), they compute the risk neutral distribution. Their findings are not consistent with economic theory. Because they look at the time continuity of $m_{t,T}$ across time, one may understand their results as estimates of the average kernel price over the sample period, rather than as conditional estimates.

Other problems in their article are discussed in Rosenberg/Engle (2002). In particular they suggest that the non specification of the investors beliefs about future return probabilities could be a problem in the evaluation of the kernel price. Also they use of a very short period of time, 4 years, to estimate the state probabilities. Moreover, they depart from the literature on stochastic volatility, which suggests that future state probabilities depend more on recent events than past events. In fact, past events remain useful for prediction of future state probabilities. In order to take this into account we use a dataset of 12 years of option prices.

A work close in spirit to Aït-Sahalia/Lo (1998) is Jackwerth (2000). His article is one of the most interesting pertaining to this literature. Beyond the estimation technique used, his paper is noteworthy because it also opened up the well-known "pricing-kernel puzzle". In his nonparametric estimation of the kernel price, Jackwerth finds that the shape of this function is in accordance with economic theory before the crash of 1987, but not after the crash. He concludes that the reason is the mispricing of options after the crash.

Both articles could incur some problems that cause the kernel price and the relative risk aversion function (RRA) to be not consistent with economic theory. In Aït-Sahalia/Lo (1998), we see that, if the bandwidth changes, the RRA changes as well and this means that the bandwidth chosen influences the shape of the RRA; on the other hand, in Jackwerth (2000), the use of option prices after the crisis period could influence the shape of the kernel price if volatility is misspecified.

Another nonparametric estimation model for the kernel price is given by Barone-Adesi et al. (2008), where they use a procedure similar to the one used by Rosenberg/Engle (2002), but with a nonparametric estimation of the ratio $q_{t,t+\tau}/p_{t,t+\tau}$. While in the papers by Aït-Sahalia/Lo (2000) and Jackwerth (2000) results are in contrast with the economic theory, Barone-Adesi et al. (2008) find a kernel price which exhibits a fairly monotonically decreasing shape.

Parametric methods to estimate the kernel price are often used in literature. Jackwerth (2004) provides a general review on this topic, but for the purpose of our work we do not go into much detail on parametric estimation. As pointed out by Birke/Pilz (2009) there are no generally accepted parametric forms for asset price dynamics, for volatility surfaces or for call and put functions and therefore the use of parametric method may introduce systematic errors.

Our goal is to test whether a different nonparametric method, starting from option pricing observed in the market, respects the conditions of no-arbitrage present in Birke/Pilz (2009). In particular, we test if the first derivative of the call price function is decreasing in the strike and the second derivative is positive. These conditions should guarantee a kernel price monotonically decreasing in wealth.

It is important to stress that our kernel price is a function of three variables: the underlying price, the risk-free rate and volatility. In the first part, we use only two factors: the underlying and the risk-free rate. In last sections we introduce also volatility.

3.3 Empirical kernel price

In this section we compute the kernel price as the ratio of the risk-neutral and the historical density, discounted by the risk-free interest rate. First we describe how we compute the risk-neutral density. Then, we explain our computation of the historical density. In each part we describe the dataset we use and our filter for cleaning it.

3.3.1 Theoretical backgrounds of risk-neutral density

Breeden/Litzenberger (1978) shows how to derive the risk-neutral density from a set of call options with fixed maturity. The formula for risk-neutral density is (see Appendix A for derivation):

$$f(K) = e^{rT} \frac{\partial^2 C(S_t, K, T)}{\partial K^2}\bigg|_{S_T = K} \tag{3.5}$$

We can approximate this result for the discrete case as:

$$f(K_i) \approx e^{rT} \frac{C_{i+1}(S_t, K, T) - 2C_i(S_t, K, T) + C_{i-1}(S_t, K, T)}{(K_{i+1} - K_i)(K_i - K_{i-1})}\bigg|_{S_T = K} \tag{3.6}$$

and for puts

$$f(K_i) \approx e^{rT} \frac{P_{i+1}(S_t, K, T) - 2P_i(S_t, K, T) + P_{i-1}(S_t, K, T)}{(K_{i+1} - K_i)(K_i - K_{i-1})}\bigg|_{S_T = K} \tag{3.7}$$

Note that in equations (3.6) and (3.7) we wrote numerical derivatives for values of K_{i-1} and K_{i+1} which are not symmetric around K_i. Breeden/Litzenberger (1978) used symmetric strikes while deriving (3.5). But having non symmetric strikes does not hurt our estimation in any sense, to the contrary it gives us more observations and as a result may improve our estimation.

Now we discuss some other methods of deriving the risk neutral density and compare it to the one we use.

A recent paper by Figlewski (2008) is very close in spirit to our work. In his paper he derives the risk neutral distribution using the same result in Breeden/ Litzenberger (1978). We differ from him in some aspects. First, we use the bid and ask prices that are given on the market to construct butterfly spreads. e.g. for the long position the ask price is used and for the short position the bid price. Our choice removes negative values in the risk-neutral distribution and we therefore find that the no-arbitrage condition described in Birke/Pilz (2009) holds. Second, we do not need to convert the bid, ask, or mid-prices into implied volatility to smooth the transition from call to put because we take the average of butterfly prices from several days with equal maturities and this improves the precisionof our result. Other similar works are discussed in Bahra (1997), Pirkner et al. (1999) and Jackwerth (2004).

In Bahra (1997), the author proposes several techniques to estimate the risk neutral density. For every method he explains the pros and the cons. He then assumes that the options prices can be derived either using a parametric method, by solving a least squares problem, or nonparametric only, using kernel regression. In our work, using a time series of options over a sample of 12 years and taking averages we avoid the parametric or nonparametric pricing step and therefore we rely only on pricing available on the market.

In Pirkner et al. (1999), they use a combination approach to derive the risk neutral distribution. They combine the implied binomial tree and the mixture distributions to get the approach called "Mixture Binomial Tree". Our work differs from their work due to our use of European options. In their work, they use American options and therefore they could have the problem of the early exercise. In our sample, we consider only European options to be sure to have the risk neutral density for that expiration time only.

Jackwerth (2004) may be considered as a general review of different methods and problems. He concentrates in particular on nonparametric estimation, but he gives a general overview also on parametric works, sorting parametric works into classes and explaining the positive and negative aspects of each one.

3.3.2 Historical density

To obtain historical probability density we need to account for features of the empirical returns of S&P500 index. There are lots of evidence suggesting that return innovations are (i) not normal, (ii) volatility is stochastic and, (iii) that positive and negative shocks to return have diverse effect on returns' volatility (see for example Ghysels et al. (1996)). That's why we are going to use a GARCH model together with the filtered historical simulation (FHS) approach used by Barone-Adesi et al. (1998). FHS approach allows to model volatility of returns without specifying any assumption on return innovations.

Among variety of GARCH models we are going to use Glosten et al. (1993) (GJR GARCH) model. The choice of the GJR model relies on two properties: 1) its ability to capture an asymmetry of positive and negative returns effect on return volatility and, 2) its fitting ability (Rosenberg/Engle (2002) document that GJR GARCH model fits S&P500 returns data better than other GARCH models).

Under the historical measure, the asymmetric GJR GARCH model is

$$log\frac{S_t}{S_{t-1}} = \mu + \epsilon_t,$$

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2 + \gamma I_{t-1}\epsilon_{t-1}^2,$$

where S_t – the underlying price, $\epsilon_t = \sigma_t z_t$, and $z_t \sim f(0,1)$, and $I_{t-1} = 1$ when $\epsilon_{t-1} < 0$, and $I_{t-1} = 0$ otherwise. The scaled return innovation, z_t, is drawn from the empirical density function $f(\cdot)$, which is obtained by dividing each estimated return innovation, $\hat{\epsilon}_t$, by its estimated conditional volatility $\hat{\sigma}_t$. This set of estimated scaled innovations gives an empirical density function that incorporates excess skewness, kurtosis, and other extreme return behavior that is not captured in a normal density function.

The methodology we use to estimate the historical density is as follows. We have a set of risk neutral densities, $f(K_i)$, for each day over 12 years. They are calculated from S&P500 index options with constant maturities. Our $f(K_i)$ are prices of hypothetical butterfly strategies constructed from two (call or put) options with strike K_i and two long options of the same type with strikes K_{i-1} and K_{i+1}. Our triplets K_{i-1}, K_i and K_{i+1} are not necessary symmetric. For each day, we estimate the parameters of the GJR GARCH using a time series of 3500 returns from the S&P500. Once we have the estimated parameters for each day, we simulate 35000 paths of S&P500 index using as a distribution of z_t the empirical distribution of the normalized innovation (FHS). We estimate the probability that at maturity we exercise the butterfly, e.g. we count the fractions of paths that at maturity are in the range $[K_{i-1},\, K_{i+1}]$[2]:

$$p(K_i) = \frac{\frac{\#of\ paths\ in\ the\ interval\ [K_{i-1}, K_{i+1}]}{total\ number\ of\ paths}}{K_{i+1} - K_{i-1}}. \tag{3.8}$$

Once we have computed the probability for each day, we can apply the same methodology we use for the risk-neutral distribution. We round the butterfly moneyness to the second digit after the decimal point and we take the average over the sample period.

We use the mid-strike for the butterfly and we round the moneyness to two decimal places. We take the average throughout the time series and we plot the resulting distribution as a function of moneyness. The historical density is drawn for one day at figure 3.1, and averaged at figure 3.2.

In subsection 3.3.3 we explain the estimation method we use for the risk neutral distribution.

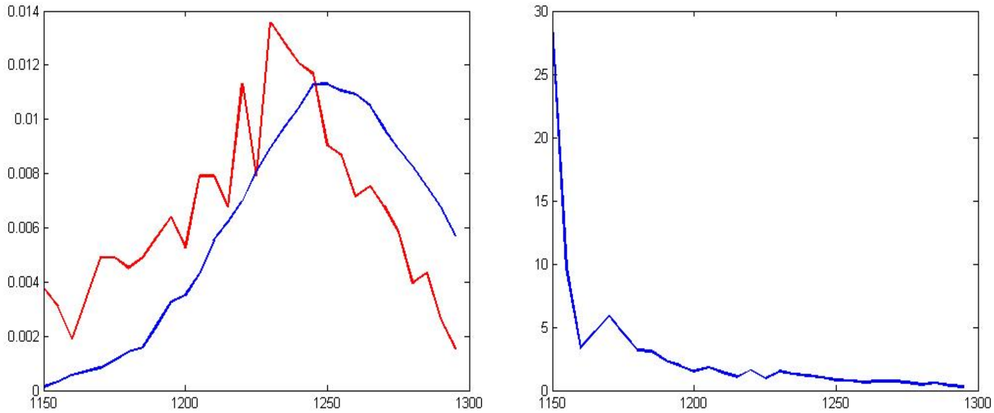
3.3.3 Risk-neutral estimation

We use European options on the S&P 500 index (symbol: SPX) to implement our model. We consider the closing prices of the out-of-the-money (OTM) put and call SPX options from 2^{nd} January 1996 to 29^{th} December 2007. It is known that OTM options are more actively traded than in-the-money options and by using

[2] In our sample we use intervals with different lengths: most of them are intervals with a length of 10 index points, but we also have some intervals with 25 or 50 points, and these intervals are in some cases overlapping.

Figure 3.1: **Risk-Neutral and Historical Distributions**

Left: Risk-neutral distribution (red line) and the historical distribution (blue line). We take one day at random from our sample (11 August 2005) with maturity equal to 37 days. Right: Price kernel for this particular day (11 August 2005).



only OTM options one can avoid the potential issues associated with liquidity problems.

Option data and all the other necessary data are downloaded from OptionMetrics. We compute the risk-neutral density at two different maturities: 37, 46, 57 and 72 days[3]. The choice of maturities is random and the same procedure can be applied for all other maturities. We download all the options from our dataset with the same maturities (we provide analysis and graphs for four maturities: 37, 46, 57 and 72 days; for other maturities results are similar) and we discard the options with an implied volatility larger than 75%, an average price lower than 0.05 or a volume equal to 0. In table 3.1 we summarized the number of options available for each maturity.

We construct then butterfly spreads using the bid-ask prices of the options. The butterfly spread is formed by two short call options with strike K_i and long two call with strikes K_{i+1} and K_{i-1}, the same for puts. We divide the dataset and we construct a butterfly spread for every day. We try to use the smallest distance possible in the strikes to construct the butterfly spread. Following the quotation for the SPX we use a difference of 5 basis points. However, for the deep-out-of-the-money options we need to take into consideration a larger distance because

[3]We work with this four maturities through the paper, although we provide sometimes graphs and p-values for more maturities.

TABLE 3.1: **The Options Number for All Maturities**

Table reports number of options used for estimations. Panel A reports number of call and put options used for main calculation. Panel B reports number of call and put options used for subsamples around crisis.

Panel A. Main sample (1996-2007)

Maturity	36	37	38	39	43	44	45	46
Calls	2753	2789	2667	2537	2424	2321	2318	2171
Puts	3406	3421	3308	3217	3112	3057	3026	2870
Maturity	54	57	58	59	71	72	73	74
Calls	1879	1873	1853	1934	1460	1466	1414	1281
Puts	2469	2488	2497	2569	1976	1985	1966	1837

Panel B. Around a crisis samples

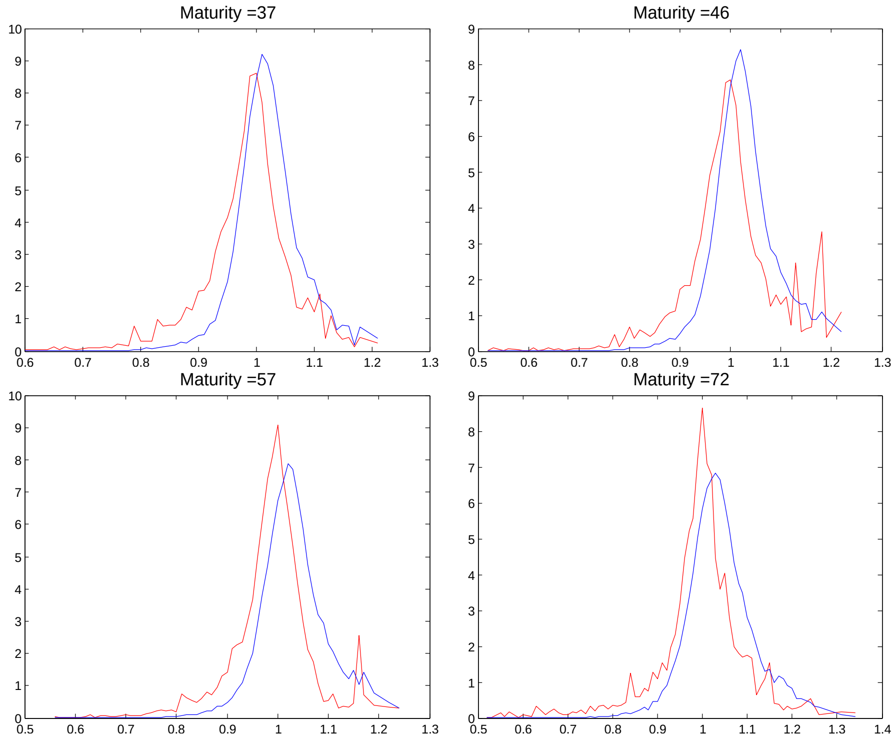
Maturity	37	46	57	72
	Aug 12, 2004 to Sep 15, 2005			
Calls	340	250	207	147
Puts	385	297	217	179
	Nov 10, 2005 to Oct 10, 2006			
Calls	364	280	256	172
Puts	424	329	303	199
	Jun 14, 2006 to Jun 14, 2007			
Calls	486	355	325	187
Puts	608	441	386	250
	Oct 11, 2007 to Aug 14, 2008			
Calls	607	505	403	271
Puts	727	609	478	301

there are less options traded. In that case, we arrive to have spreads of 10 to 50 points. We download option prices, order by strike, from smallest to largest. We take the second difference of option prices using formulas (3.6) and (3.7), for calls and puts separately, and then combine them. We do it for each day available in our dataset.

In figure 3.1 we take a day at random from our sample and we show the risk-neutral distribution, the historical distribution and their ratio as the price kernel. As an example, we take 11 August 2005, and we look at options with a maturity equal to 37 days. We see that for this choice, the kernel price shows a monotonically decreasing path in S_T, with some jumps because we do not smooth the curve.

FIGURE 3.2: **Average Risk Neutral and Historical Distribution**

Risk neutral and historical distribution as the 12 year average of risk neutral and historical distributions for a fixed number of days to maturity.



At this point, we take into consideration the moneyness of each butterfly. As reference moneyness of the butterfly spread, we use the moneyness of the middle strike. We round all the butterfly moneyness to the second decimal digit and we take the average of all the butterfly prices with equal moneyness[4]. We can now plot the risk-neutral distribution as an average of the butterfly prices for a fixed maturity over a twelve year period. Figure 3.2 draws the risk-neutral and historical (physical) distributions of the underlying index. As expected, the risk neutral distribution is shifted to the left with respect to the historical distribution.

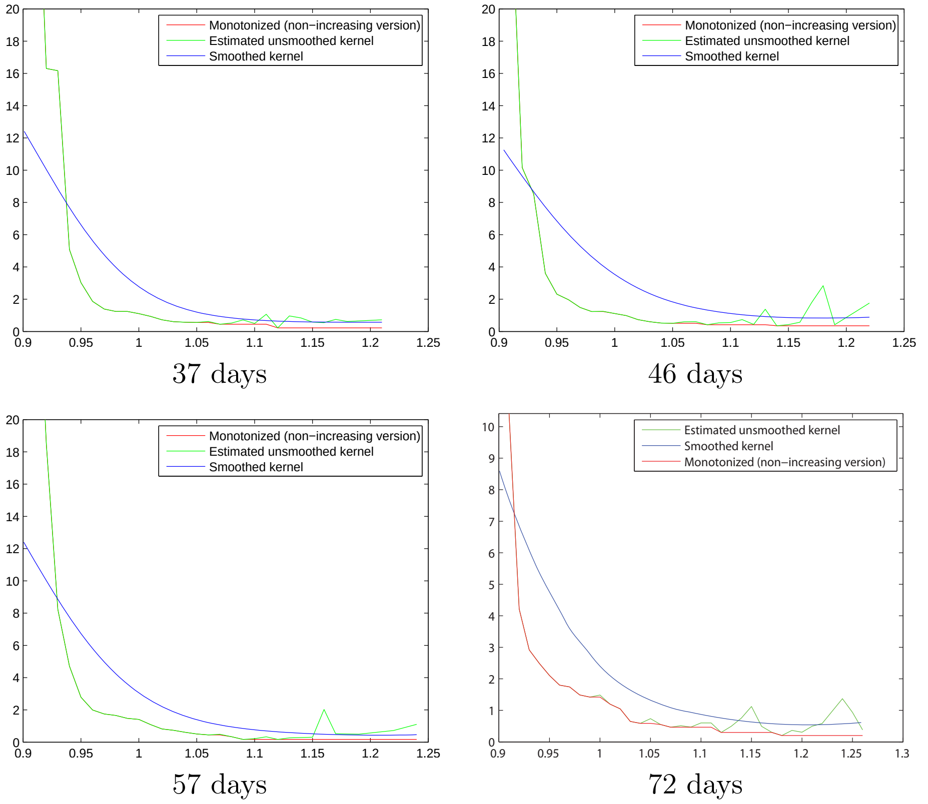
3.3.4 Kernel price

We apply the definition given in equation (3.4) in order to get the kernel price. From previous calculations we obtain the average risk-neutral distribution for the fixed maturity and also the average historical density. In order to get the average

[4]In order to find an equal moneyness it is necessary to round the moneyness values to the second decimal digit. Otherwise we cannot average and we are left with a lower number of points.

Figure 3.3: **SPD Per Unit Probability Averaged Over 12 Years**

SPD per unit probability as the average of the SPD per unit probability throughout the time series of 12 years and with equal maturity. It is important to keep in mind that this SPD per unit probability is not derived from the two distributions given in figure 3.2.



kernel price we take the kernel price of each day and then we compute the average from all the days in our time series. Averaging across time allows us to increase the otherwise small number of data points.

It is important to recall that our kernel price is the average of the kernel prices estimated each day. In another words, we estimate the risk neutral and the historical densities for each day, calculate the price kernel and then average these daily kernels, rather than calculate average densities over the entire sample.

Generally, for all different maturities we get a monotonically decreasing path for the kernel price and all of these are in accordance with economic theory.

We also obtain a kernel-smoothed version of the price kernel by applying to our unaveraged pricing kernel the kernel smoothing. Our smoothed pricing kernels are also monotonic decreasing (see figure 3.3).

TABLE 3.2: **Test of Monotonicity of the Pricing Kernel**

Maturity,	Intuitive test		Durot test	
days	$H_0{}^a$	P-value	$H_0{}^b$	P-value
36	Not rej.	0.3213*	Not rej.	0.2398
37	Not rej.	0.0221**	Not rej.	0.2138
38	Not rej.	0.0259**	Not rej.	0.4809
39	Not rej.	0.3420*	Not rej.	0.3338
43	Not rej.	0.1088*	Not rej.	0.3535
44	Not rej.	0.6976*	Not rej.	0.1824
45	Not rej.	0.1088*	Not rej.	0.4186
46	Not rej.	0.0259*	Not rej.	0.1046
54	Not rej.	0.1844*	Not rej.	0.2855
57	Not rej.	0.0259**	Not rej.	0.2079
58	Not rej.	0.1088*	Not rej.	0.4088
59	Not rej.	0.0343**	Not rej.	0.1097
71	Not rej.	0.1315*	Not rej.	0.1021
72	Not rej.	0.0244**	Not rej.	0.1034
73	Rej.	0.0082	Rej.	0.0493
74	Rej.	0.00013	Not rej.	0.5114

[a] In the intuitive test we use 1% confidence interval. Starred numbers denote that in this case H_0 is not rejected for the 5% confidence level, double starred – for the 1%.
[b] While using the Durot testing procedure we use 5% confidence level.

3.3.5 Monotonicity testing

In this section we introduce some monotonicity testing. We do two kinds of monotonicity tests. Both of them, as many nonparametric tests, involve the notion of Kolmogorov distance. The first one is very simple, may be not completely justified by theory but it is very intuitive. The second one is more thorough and is more sound statistically proves and results.

Simple test Our first test (call it *simple*), considers a monotonized version of the price kernel obtained earlier. We test that the estimated and monotonized versions are equal.

We create our monotonized version, $\hat{m}(x)$, as follows. From estimation of the pricing kernel, $x_i \to m(x_i)$, we inspect each point $m(x_i)$ for monotonicity. If it is between its adjacent points, the monotonized version is defined to be equal to estimated one, otherwise – the monotonized version is defined to be constant and equal to previous value. More precisely, the monotone version is given as:

$$\hat{m}(x_1) = m(x_1)$$
$$\hat{m}(x_{i+1}) = \begin{cases} m(x_{i+1}), & \text{if } m(x_{i+1}) \leq m(x_i) \\ \hat{m}(x_i), & \text{if } m(x_{i+1}) \geq m(x_i) \end{cases}$$

where $m(x)$ – the estimated price kernel. After getting the monotone version we compare it with the estimated price kernel, $m(x)$, by means of Kolmogorov-Smirnov test[5].

Results Results of testing are given in the table 3.2, test result in "Not rej." if $H_0 : \hat{m} \equiv m$ is not rejected and in "Rej." – if it was rejected at the 1% significance level. Also one can observe p-values of H_0 and that comes from the table is that we are not able to reject null hypothesis of monotonicity of the price kernel at the confidence level 1% (for some maturities it is 10%). Only for maturities of 73 and 74 days we reject monotonicity, possibly because of discretization errors. In any case this test is rather weak. Thus we are going to introduce a more powerful test.

Sophisticated test To test monotonicity of the pricing kernel more thoroughly we use a Kolmogorov-type test for monotonicity of a regression function described in Durot (2003). Hypothesis testing is performed within the following regression model

$$y_i = f(x_i) + \varepsilon_i,$$

[5] One can do it using MatLab standard function `kstest2`.

where, in our case x_i is the moneyness of the option, y_i is the price kernel, and ϵ_i – random errors with mean 0. Our second test is based on the fact that f is non-increasing (decreasing) if and only if $\hat{F} \equiv F$, here $F(t) = \int\limits_0^t f(s)ds, \quad t \in [0,1]$, and $\hat{F}$ is the least concave majorant (lcm) of F. One should reject H_0 about monotonic decrease of pricing kernel in case the difference between F and $\hat{F}$ corresponding to our price kernel is too large.

Test construction From sections 3.3.2 - 3.3.4 we can obtain the pricing kernel, so we have function f given as

$$\textit{moneyness } (x_i) \xrightarrow{f} \textit{pricing kernel } (y_i)$$

As mentioned above f is non-increasing on $[0,1]$ if and only if F is concave on $[0,1]$. Denote i_t integer part of nt and define

$$F_n(t) = \frac{1}{n}\sum_{j \le i_t} y_i + (t - x_{i_t})y_{i_t}, \quad t \in [0,1]$$

F_n is approximation of F, thus we consider Kolmogorov-type test statistic

$$S_n = \frac{\sqrt{n}}{\hat{\sigma}_n} \sup_{t \in [0,1]} \left| \hat{F}_n(t) - F_n(t) \right| \tag{3.9}$$

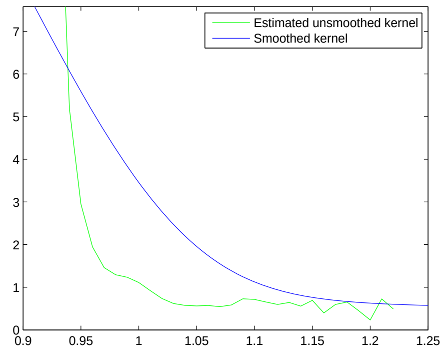
where $\hat{F}_n$ is lcm of F_n and $\hat{\sigma}_n$ consistent estimator of σ_n[6]. Durot (2003) proves that under H_0 S_n converges in distribution to $Z = \left\| \hat{W} - W \right\|$, where W is standard Brownian motion, $\hat{W}$ its lcm and $\|\cdot\|$ – supremum distance.

Results In Table 3.2 results of described testing procedure are presented on the right. We can see that all p-values support our hypothesis, namely we can not reject H_0 for 5% confidence level (except for the price kernel obtained from 73 days maturity options where p-value is 0.0493. Even for the 10% confidence level most of the samples would not contradict monotonicity of pricing kernels.

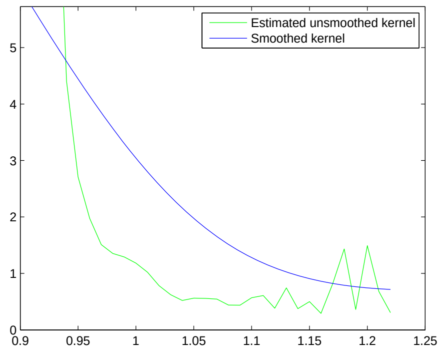
[6]We use the one provided in Durot paper: $\hat{\sigma}_n^2 = \frac{1}{2(n-1)} \sum\limits_{i=1}^{n-1} (y_i - y_{i+1})^2$.

FIGURE 3.4: **Averaged SPD per Unit Probability**

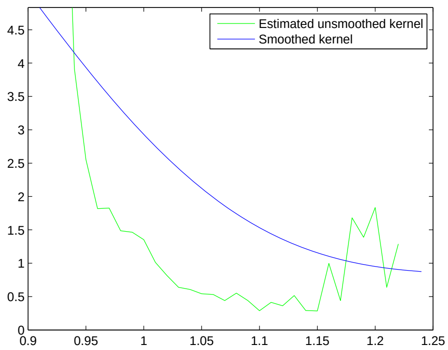
SPD per unit probability over time. It is averaged over close maturities.



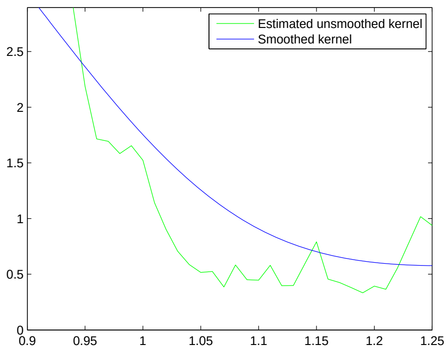
The first sample: 36 - 39 days



The second sample: 43 - 46 days



The third sample: 54, 57 - 59 days



The fourth sample: 71 - 74 days

3.4 Averaging price kernel over time

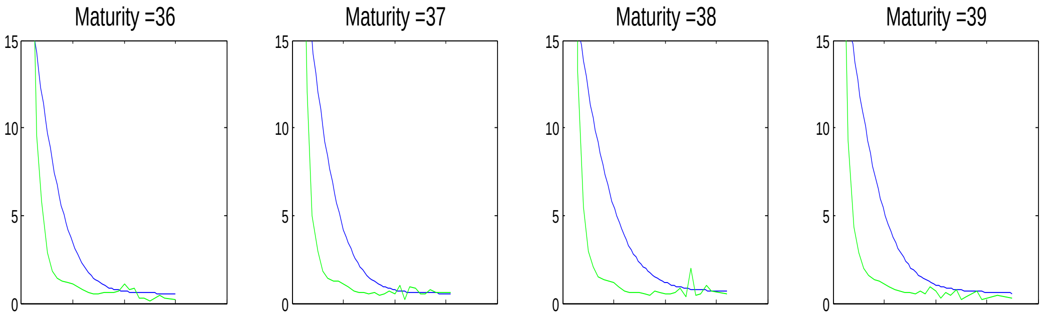
In this section we check the robustness of our methodology and we try to find a smoothness criteria for smoothing our price kernels. First of all, we show different price kernels with maturities close to those we showed before. According to economic theory, price kernels with close maturities should have similar shape. Different price kernels with close maturity should be similar to each other. In order to check this, we create four samples: the first one has maturities ranging 36 - 39 days, the second – 43 - 46 days, the third – 54, 57-59 days[7], and the last one consists of 71 - 74 days maturities.

We use the approach explained in previous section. By this method we derive the price kernels for the maturities in all four samples and in Figure 3.5 we plot the results of our estimations. The unsmoothed kernel prices show a clearly monoton-

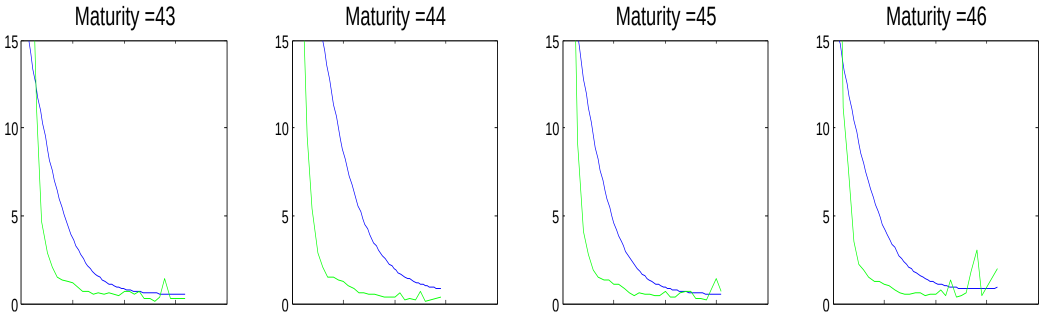
[7]There are no options for 55 and 56 days to maturity in OptionMetrix.

FIGURE 3.5: **SPD per Unit Probability for Different Maturities**

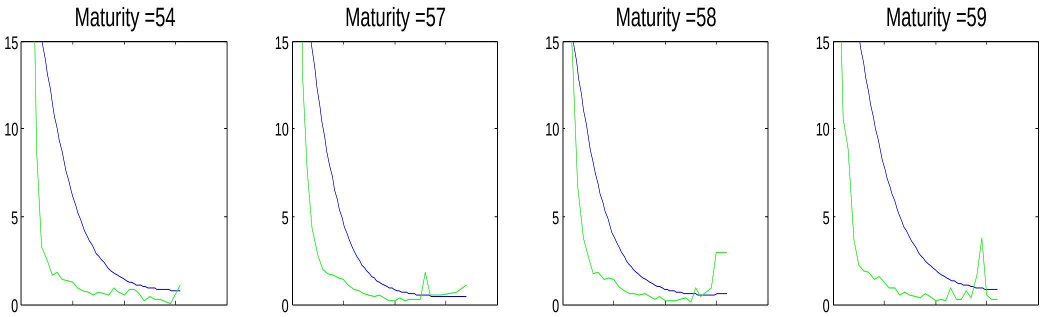
Figure shows SPD per unit probability for different maturities. Maturities are written above each figure.



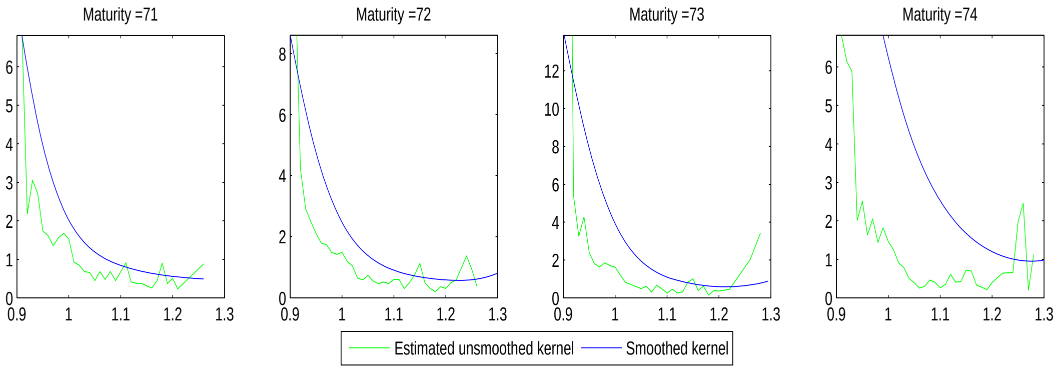
$\sim$ 30 days maturity



$\sim$ 40 days maturity



$\sim$ 50 days maturity



$\sim$ 70 days maturity

ically decreasing path, except in some points that may be due to the discretization of the data. Our smoothed pricing kernels are all smoothly decreasing. In order to verify that the price kernels are monotonous over time, we plot the kernel price as the average of different maturities (see figure 3.4). In particular, referring to our four samples (the first one is for maturities 36 - 39 days, the second – 43 - 46 days, the third – 54, 57 - 59, and the last – 71 - 74 days), we take the average over the 4 different maturities. We expect to find a kernel price that is monotonically decreasing in wealth, because of the fact that we average over close maturities in our sample.

As we see in figure 3.5, the kernel prices close in maturity, have similar path, supporting the robustness of our methodology. Test statistics for monotonicity of these price kernels are presented in table 3.2.

In figure 3.4, we plot the average for each sample. We find decreasing kernel prices for smoothed estimation, and mainly decreasing kernel prices for unsmoothed estimation, although 50th and 70th samples have some jumps. In this way we were able to have some sort of smoothing criteria without using a method which biases our findings.

3.5 Price Kernel around a crisis

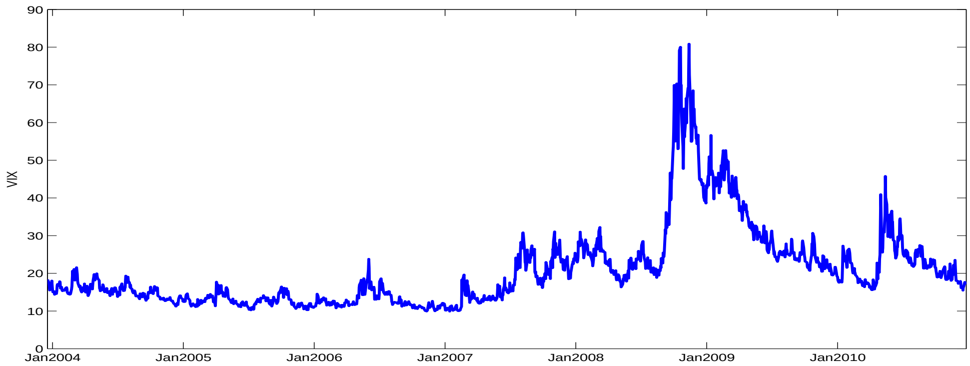
In this section we evaluate the change of kernel price during the crisis. In particular, we look at kernel prices before and during the recent financial crisis. We divide our sample in 4 periods. Every period is from 9 to 12 months and we take periods which show a similar range in volatility according to the VIX index (Figure 3.6).

3.5.1 Estimates of pricing kernels in different periods

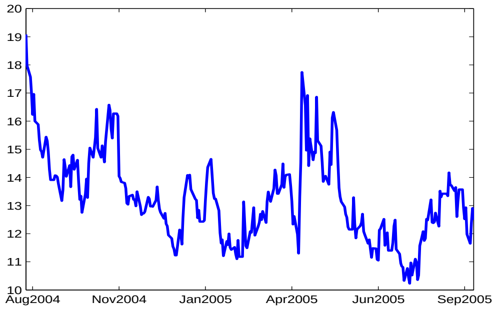
We identify four different periods between August 2004 and August 2008. The first period goes from of August 2004 to the 15^{th} of September 2005. In this period volatility is between 10 and 20 points. The second period is between 10 November 2005 to 10 October 2006. In this second period the volatility is again in a fixed range between 10 to 20 points. The third period, which is before the crisis period, is between 14 June 2006 and 14 June 2007. Even here the volatility is in a range of 10 to 20 points. The last period, the period of the beginning of the crisis is between 11 October 2007 and 14 August 2008. In this period the volatility is much higher and it is in a range between 10 and 30 points.

FIGURE 3.6: **Four Subsamples of The VIX index**

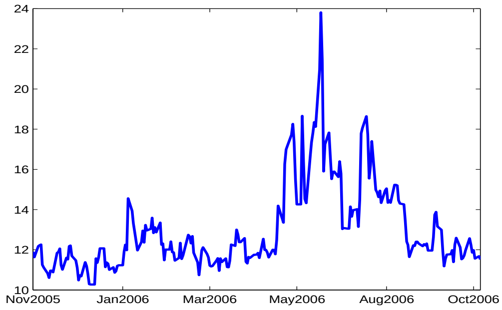
The four samples we use to compute the different SPD per unit probabilities over different years.



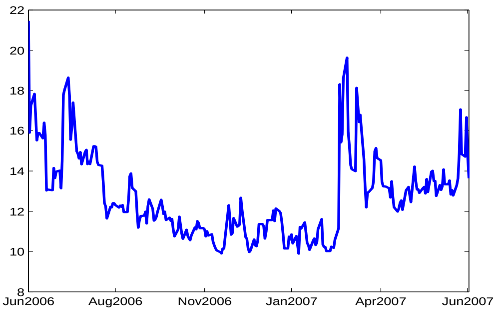
The VIX index between 2^{nd} of January 2004 and 29^{st} of December 2010



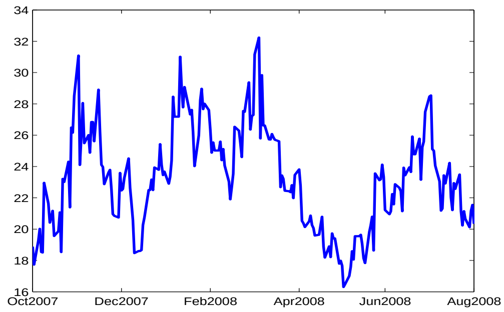
The sample: Aug 12, 2004 to Sep 15, 2005



The sample: Nov 10, 2005 to Oct 10, 2006



The sample: Jun 14, 2006 to Jun 14, 2007



The sample: Oct 11, 2007 to Aug 14, 2008

For each period, we compute the price kernel by the methodology presented in section 3.3. We fix a maturity (in this case we look at maturities of 37, 46, 57 and 72 days) and we plot the kernel price of each period.

As expected, for the three periods before the crisis we get price kernels monotonically decreasing and very similar in shape one each other. For the kernel price of the crisis period, we have a different shape. It is higher for moneyness smaller than 1 and constant for moneyness larger than 1. For the value smaller then 1, this is exactly what we expected to obtain. The probability of negative outcome is higher therefore we give more weight of negative outcomes. On the other hand, we do not expect to have a constant kernel price for moneyness values larger than 1.

In the subsection 3.5.2, we focus only on the kernel price of the crisis period and we try to solve it.

3.5.2 Kernel Price in crisis time

In the previous subsection we show kernel prices for different periods (see Fig 3.7). We see in figure 3.7 that the kernel prices in period where the volatility is stable (it remains in a determinate range of 10 to 20 points) the SPD per unit probabilities exhibit a monotonically decreasing shape.

When we enter in a period of crisis volatility changes dramatically. In this case, we observe a kernel price that is no more monotonically decreasing, but decreasing on the left, with constant value after the moneyness equal to 1.

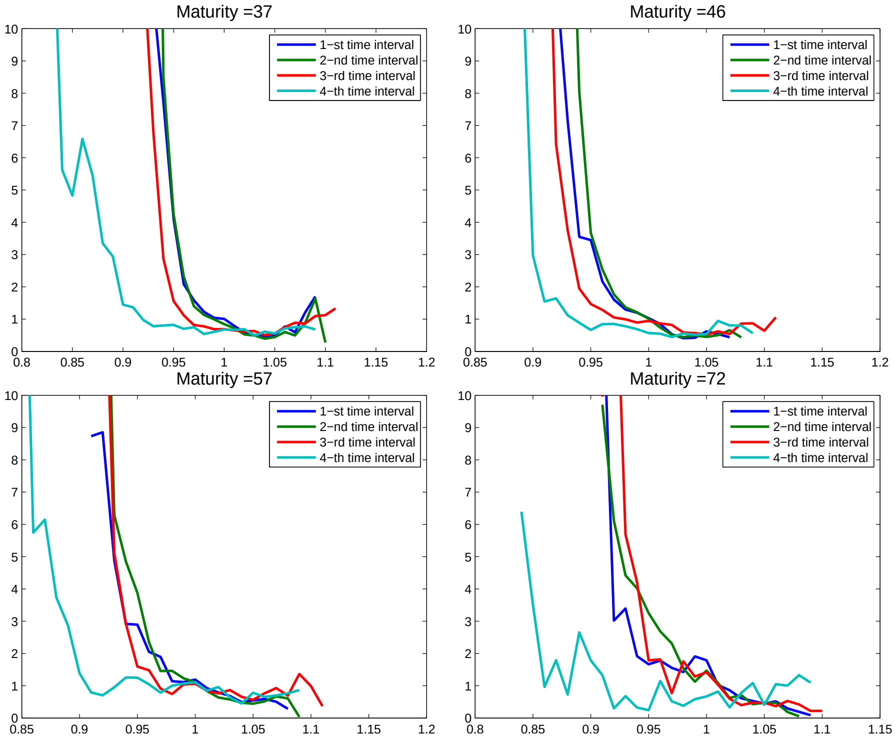
However, it is interesting to notice that the method we use to derive kernel prices is sufficiently robust to guarantee that even in a crisis we get kernel prices in agreement (in part of the graph) with economic theory.

3.6 Kernel price as a function of volatility

In this section we would like to extend our model and consider the kernel price as a function of more variables. In fact, as explained in Chabi-Yo et al. (2005), one possible explanation for the non-monotonicity of the price kernel is volatility. In a previous section we compute the price kernel as a function of one variable: the underlying, $m_{t,T}(S_T)$. We know from Pliska (1986), Karatzas et al. (1987),

FIGURE 3.7: **The Kernel Prices for Different Levels of VIX**

The kernel prices for four samples we create looking at different levels of volatility index.



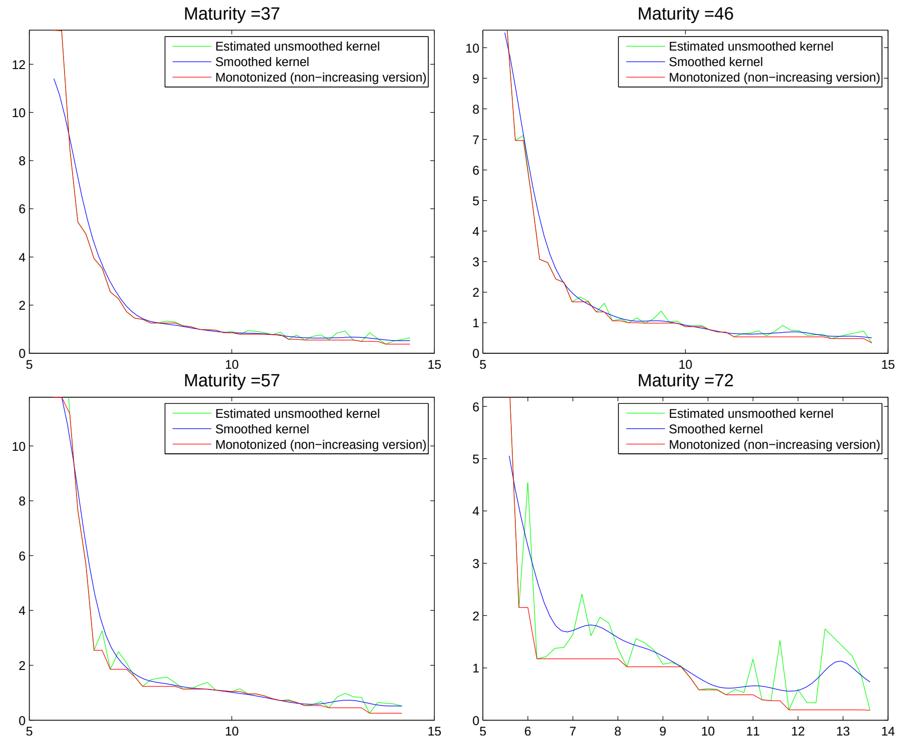
and Cox/fu Huang (1989) that the kernel price is characterized by at least two factors: the risk-free rate and the market price of risk. In our analysis we would like to consider the kernel price as a function of three different factors: the risk-free rate, the underlying price and the volatility. We have already introduced the underlying price and the risk free-rate. Now we want to introduce also the volatility, so $m_{t,T}(S_T, r_f, \sigma)$.

We saw that the risk-free rate is a parameter that does not enter in our analysis for as a decisive factor. Both probabilities are forward looking. In fact, the historical probability is seen as the probability to exercise a butterfly spread at maturity, while the risk-neutral is seen as the probability of a particular state. The moneyness is nothing else than the K/S_t. In order to introduce the volatility we take as a reference the idea by Carr/Wu (2003). They use a moneyness defined as:

$$moneyness = \frac{log(K/F)}{\sigma\sqrt{T}},$$

FIGURE 3.8: **The Kernel Prices with a Moneyness Corrected for Volatility**

The Kernel prices when we use a moneyness factor that take into consideration underlying price and volatility. See text, in particular equation 3.10.



where F is the futures contract price, T is the maturity time and σ is the average volatility of the index.

For our propose, we can change this formula to look:

$$moneyness = \frac{K}{S_t * \sigma}, \tag{3.10}$$

In fact the futures price is already considered for the above explanation of the forward looking probabilities, while the time to maturity is constant over the sample we consider. In our case the volatility is not anymore the average volatility, but the implied volatility of each option.

The procedure to derive the kernel price is again the same we have seen in the previous sections[8] and therefore our result for maturities equal to 37, 46, 57 and 72 are as at the figure 3.8.

[8]There is only a small difference when we round the new moneyness in order to average different periods. We do not take the second digit after the point, but we arrive only at the first one.

Table 3.3: **Test of Monotonicity of the Pricing Kernel with Modified Moneyness**

The pricing kernel monotonicity testing in case of volatility being additional parameter for moneyness. Table provides p-values for monotonicity tests.

Maturity	Intuitive	Durot	Maturity	Intuitive	Durot
36	0.7651	1.0000	54	0.1489	1.0000
37	0.2951	1.0000	57	0.4075	0.5313
38	0.2823	0.9401	58	0.5480	0.9037
39	0.0569	1.0000	59	0.2436	0.0058
43	0.1078	0.9715	71	0.0042	0.0000
44	0.0372	0.5398	72	0.0042	0.0000
45	0.2823	0.9363	73	0.6403	0.1268
46	0.0569	0.8069	74	0.0000	0.0446

At figure 3.8 we plotted pricing kernels with volatility accounted in the moneyness parameter. In this case the results are consistent with the economic theory. In table 3.3 we give p-values for all maturities we have seen above. One can see that these p-values confirm our graph, namely for smooth pricing kernels p-values are high in both test. Only for maturities equal to 71, 72 p-values suggest that pricing kernels are not monotone which one can also notice at the graph.

3.7 Conclusion

We propose a method to evaluate the kernel price in a specific day for a fixed maturity as well as the average of different kernel prices in a time series of 12 years for a fixed maturity. Using option prices on the S&P 500, we derive the risk-neutral distribution through the well-known result in Breeden/Litzenberger (1978).

We compute the risk neutral distribution in each day where we have options with a fixed maturity. Then, we compute the historical density, for the same maturity, in each day, using a GARCH method, based on the filter historical simulation technique. We then compute the ratio between the two probabilities in order to derive the kernel price for that given day. We show that in a fixed day (chosen at random in our sample) the risk-neutral distribution implied in the option prices satisfies the no-arbitrage condition.

We provide a smoothed version of the pricing kernel, to test its monotonicity. Our tests support the pricing kernel monotonicity. Therefore, we show that the ratio between the two probabilities, is monotonically decreasing in agreement with economic theory (see figure 3.1). We then show how the average of the different kernel prices across 12 years display the same monotonically decreasing path (see figure 3.3 and p-values in table 3.2).

We also prove that average price kernels over time, if we take close maturities, exhibit a monotonically decreasing path in agreement with economic theory.

Furthermore, we try to explain the reason why with different methods it could be possible to incur into the "pricing kernel puzzle" and have a different shapes for the kernel price. We can conclude that in most cases the model used to estimate the kernel price or the sample taken into consideration could introduce some errors in the estimation of the kernel price.

In the last part, we show the changing in shape of different price kernels before and during the recent crisis. We see that before the crisis the price kernels are monotonically decreasing while during the crisis it becomes decreasing in a part and then constant for moneyness value higher than 1. We understand this result in a very simple way: the risk neutral probability changes faster with respect to the historical one and therefore the ratio between the two remain constant.

Appendices

Chapter A.

Derivation of risk-neutral density

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We start from a portfolio with two short call options with strike K and long two call with strikes $K-\varepsilon$ and $K+\varepsilon$ and they consider $\frac{1}{2\varepsilon}$ shares of this portfolio. The result is a butterfly spread which pays nothing outside the interval $[K-\varepsilon,\ K+\varepsilon]$. Letting ε tend to zero, the payoff function of the butterfly tends to a Dirac delta function with mass at K[1], i.e. this is nothing else than an Arrow-Debreu security paying \$1 if $S_T = K$ and nothing otherwise (see Arrow, 1964). In this case, define K as the strike price, S_t the value of the underlying today, r as the interest rate, and T as the maturity time, the butterfly price is given by

$$P_{butterfly}(S_T) = \frac{1}{2\varepsilon}\left[2C(S_t, K, T, r) - C(S_t, K-\varepsilon, T, r) - C(S_t, K+\varepsilon, T, r)\right]$$

taking limit of this expression as $\epsilon \ \to 0$ we get

$$\lim_{\varepsilon \to 0} P_{butterfly}(S_T) = \frac{\partial^2 C(S_t, K, T)}{\partial K^2} \tag{A.1}$$

Now substitute the butterfly payoff, $x_{butterfly}(S_T) = \mathbb{I}_{[K-\varepsilon,\ K+\varepsilon]}(S_T)$, into equation (3.3) we get that the price of the butterfly is:

$$P_{butterfly} = e^{-rT} \int_{K+\varepsilon}^{K+\varepsilon} q_t(S_T) dS_T.$$

[1] More formally, payoff of the butterfly is $x_{butterfly}(S_T) = \mathbb{I}_{[K-\varepsilon,\ K+\varepsilon]}(S_T)$, or when $\varepsilon \to 0$ is $\lim\limits_{\varepsilon \to 0} x_{butterfly}(S_T) = \delta_K(S_T)$.

If we take limit as $\varepsilon \to 0$ and calculate this integral using properties of Dirac delta function, we get that

$$\lim_{\varepsilon \to 0} P_{butterfly} = e^{-rT} q_t(S_T)\Big|_{S_T=K}. \tag{A.2}$$

Rearranging equations (A.1) and (A.2) we can have that

$$\lim_{\varepsilon \to 0} P_{butterfly} = e^{-rT} q_t(S_T) = \frac{\partial^2 C(S_t, K, T)}{\partial K^2}\Bigg|_{S_T=K} \tag{A.3}$$

This result suggests that the second derivative of a call price (we will see that it is also true for a put price) with respect to the strike price gives the risk neutral distribution[2]

$$q_t(S_T) = e^{rT} \frac{\partial^2 C(S_t, K, T)}{\partial K^2}\Bigg|_{S_T=K}$$

In the next part of this section we see how to apply this result in the discrete case.

In the following, we consider three call options with strikes K_i, K_{i-1}, K_{i+1}, where $K_{i+1} > K_i > K_{i-1}$. We have seen that the price of a call option can be written as:

$$C(S_t, K, T) = \int_K^{\infty} e^{-rT}(S_T - K) f(S_T) dS_T.$$

We define $F(x)$ as the cumulative distribution function, $f(x)$ as the probability density, $C(S_t, K, T)$ as the price of a European call option, $P(S_t, K, T)$ as the price of a European put option, and K as the strike price of the reference option. According to the result in Breeden/Litzenberger, 1978 taking the first derivative

[2]This can also be obtained by differentiating $C(S_t, K, T) = \int_K^{\infty} e^{-rT}(S_T - K) f(S_T) dS_T$ w.r.t. S_T as in Birke/Pilz, 2009

with respect to the strike price, we get:

$$\begin{aligned}\frac{\partial C(S_t,K,T)}{\partial K} &= \frac{\partial}{\partial K}\left[\int_K^\infty e^{-rT}(S_T-K)f(S_T)dS_T\right] = \\ &= e^{-rT}\left[-(K-K)f(K)+\int_K^\infty -f(S_T)dS_T\right] = \\ &= e^{-rT}\int_K^\infty -f(S_T)dS_T = -e^{-rT}[1-F(K)]\end{aligned}$$

Solving for $F(K)$ one gets:

$$F(K) = e^{rT}\frac{\partial C(S_t,K,T)}{\partial K} + 1 \tag{A.4}$$

Now, taking the second derivative, we have equation (3.5).

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