

Essays in Macro-Finance

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Overview

Macro-finance is the subfield of economics at the border between macroeconomics and finance.

Finance must have macroeconomic underpinnings. The basic prediction of asset pricing theory is that asset prices equal expected discounted cashflows (Harrison and Kreps, 1979; Hansen and Richard, 1987). The discounting is done by means of a stochastic discount factor which, by DSGE reasoning, equals the growth rate in the marginal utility of wealth and is a function of macroeconomic fundamentals (Merton, 1973). The mimicking portfolio theorem shows how some linear combination of asset returns—a portfolio formed by projecting the discount factor on the payoff space—has the same pricing implications as the stochastic discount factor. However, a structural link to macroeconomic fundamentals remains ultimately necessary to motivate any asset pricing model and to limit the in-sample fishing of spurious pricing factors (Fama, 1991; Cochrane, 2005, 2008b). In fact, the sample mean-variance efficient portfolio always prices all asset returns perfectly in the sample and is typically not robust across samples (a warning that goes back at least to Roll, 1977). It follows that the only testable content of any asset pricing model comes from the discipline imposed by economic theory in motivating the discount factor as a particular function of the observables, so as to avoid coming up with some ad hoc pricing factor that really is just the sample mean-variance efficient portfolio.

Monetary macroeconomics, in turn, cannot abstract from its asset pricing implications. The asset pricing implications of a model—or, more specifically, its implications for the marginal utility of wealth—are crucial for welfare analysis and to understand the transmission mechanism from monetary policy to the real economy. First, the transmission mechanism is mediated by financial markets. In most monetary macromodels (e.g., Woodford, 2003; Galí, 2008) a single return, the real risk-free rate, mediates all decisions to save and invest, while risk premia are modeled as second-order terms. Yet, the business cycle seems to correlate much more with changes in risk premia than with changes in short-run interest rates (e.g., Cochrane, 2011). At the same time there is evidence that a policy shock has a first-order effect on risk premia (Rigobon and Sack, 2004; Bernanke and Kuttner, 2005), hence either on the price or on the quantity of systematic risk. A proper account of the policy transmission mechanism seems to require an explanation of the documented asset market's reaction to monetary policy, which requires the incorporation of risk premia variation into monetary macromodels.

Second, asset prices reveal information about the underlying states that drive the economy and about consumer preferences and/or firm behavior that can be used to test competing macromodels, on the one hand, and that have potentially important welfare implications, on the other. For example, a structural model able to account for observed equity and bond market data, say by modeling a high risk sensitivity of households, opens up the possibility that people value more a stable cashflow path than stable inflation.

Against this background, there is a major challenge that may question the usefulness of a macro-finance approach. Tallarini (2000) points out a potential dichotomy between quantity and price dynamics. He embeds Epstein and Zin (1989) preferences within an otherwise standard RBC model. He then calibrates the intertemporal elasticity of substitution to have a standard dynamic IS equation and shows how the quantity dynamics are nearly independent of the choice of the risk aversion parameter. The degree of freedom in the risk aversion coefficient then allows him to replicate the empirically high unconditional equity premium. People now fear much more the shocks that hit the economy but there is little they can do to avoid them; thus, the quantity predictions stay the same. If Tallarini's model correctly describes reality, then any macromodel designed to study quantity dynamics may disregard its asset pricing implications without harm.

However, there is an important reason why Tallarini's dichotomy does not condemn the macro-finance approach. Tallarini works in a RBC model in which monetary policy has no real effects. Outside monetary neutrality different policy rules can make a large difference in terms of welfare, as people are now highly sensitive to fluctuations.¹ Tallarini's work provides an important lesson nonetheless; as we adjust preferences to match more asset pricing facts, the equations driving quantities—including the dynamic IS equation—should likely remain unchanged in order to avoid counterfactual quantity dynamics and a risk-free rate puzzle (this is also among the main points of Campbell and Cochrane, 1999—see also Lettau and Uhlig, 2000 for a discussion). Thus, from a macroeconomic perspective, nearly all structural consequences of the macro-finance exercise should be on welfare and on our understanding of the transmission mechanism.

The important welfare implications of a macromodel that incorporates the properties of risk premia we see in the data is perhaps most easily seen in the model's predictions for the welfare cost of cashflow uncertainty. Lucas (1987) famously calculates a trivial cost of consumption fluctuations. However, within the representative-consumer power-utility model used by Lucas, the gains from stabilizing fluctuations are small for the same reason that the model predicts a small equity premium. Consumption is a fairly smooth time series, so that a model with a low and constant risk aversion can only predict that the representative consumer does not fear the typical empirical fluctuations much. This prediction is precisely why the model is controversial, for reasons that go back to Shiller (1981), Hansen and Singleton (1983) and Mehra and Prescott (1985). In this regard, the famous asset pricing puzzles forcefully suggest a departure from power utility (see, for example, Campbell and Cochrane, 1999; Tallarini, 2000; Alvarez and Jermann, 2004; Bansal and Yaron, 2004; Hansen and Sargent, 2005; Barillas et al., 2009; Croce et al., 2012; Gabaix, 2012; Wachter, 2013).

I show in part 1 how the welfare cost of uncertainty is closely related to the risk premium on equity. Namely, I link the notion of cost of uncertainty to a rich set of recent financial market evidence—in particular, the term structure of equity (Binsbergen et al., 2012, 2013). The negative slope of the estimated term structure cannot be captured by today's leading consumption-based

¹Tallarini's result is robust to moving from the highly risk-sensitive Epstein-Zin-Weil preferences to a power-utility model in which one introduces some fear of model misspecification, for example as formalized in the ambiguity averse multiplier preferences of Barillas et al. (2009). Note however that there are important ingredients missing in Tallarini's model, who is in particular unable to replicate the time-variation of asset prices. For example, Tallarini's model predicts counterfactually constant stock prices (Q) because he works without frictions to capital accumulation such as adjustment costs.

asset pricing models (Binsbergen et al., 2012; Ludvigson, 2013) and therefore represents a puzzling piece of evidence with seemingly crucial welfare consequences. The finding of large and volatile costs imposed by an increase in short-run uncertainty inscribes into a burgeoning literature that finds high and time-varying short-maturity risk premia as a pervasive phenomenon across different asset classes (Duffee, 2010; Binsbergen et al., 2012, 2013; Aït-Sahalia et al., 2012; Palhares, 2012).² The paper also contributes to the large literature that exploits asset market data to reveal information about the marginal utility of wealth (main examples of work in this agenda are Hansen and Jagannathan, 1991; Alvarez and Jermann, 2004, 2005; Hansen and Scheinkman, 2009; Backus et al., 2013).³

Once we depart from Tallarini's dichotomy, the asset pricing implications of a monetary model can even allow, if one accepts to take them seriously, for using a rich body of financial market data to test competing macromodels. For example, central ingredients of modern monetary models such as sticky prices may have peculiar predictions for asset prices (some examples are Gallmeyer et al., 2007; Bekaert et al., 2010; Berg, 2010; Palomino, 2012; Gorodnichenko and Weber, 2013). In part 2 I exploit this idea to test a New Keynesian Q theory of investment. The central ingredient of New Keynesian models (sticky prices and monopolistic competition) is able to break the equivalence between the average and marginal Q—which Hayashi (1982) showed to be very robust in a frictionless setting—and thereby to provide a rational explanation for a comovement between stock prices and expected inflation. I thus provide a test of the New Keynesian model versus the frictionless alternative based on asset market data.

Financial economics: stylized facts and theory

Why do asset prices vary? How do fundamentals move asset prices? What are those fundamentals? These are the crucial questions of asset pricing (e.g., Cochrane, 2005). A full-fledged model able to answer them would ideally work in general equilibrium, model both the consumption and the production sides of the economy, and reduce all motion to variation in some credibly exogenous states. This is the goal but we are not there yet: both small-scale (Jermann, 1998; Boldrin et al., 2001) and medium-scale (Christiano et al., 2005; Smets and Wouters, 2007) state-of-the-art general equilibrium models are not yet able to explain the variation of asset prices, neither through time nor across securities. For the time being, it may be useful to study consumption- and production-side implications separately, to better understand where refinements are necessary.

On the production side, the benchmark story is the Q theory of investment. By making the capital stock costly to adjust, the Q theory predicts that investments are high when market-book ratios are high (Hayashi, 1982). The Q theory performs surprisingly well at tracking investments in the last 20 years or so (Cochrane, 2011) but does not over, say, a 60-year sample. Simple versions of the Q theory find more empirical support along other dimensions (e.g., Cochrane, 1991, 1996;

²A related strand of literature extracts measures of equity duration of individual stocks and finds that short-duration stocks associate with higher average returns than long-duration stocks (Dechow et al., 2004; Da, 2009; Weber, 2012). Under the assumption that value stocks have shorter duration than growth stocks, part of the literature then uses the evidence to explain the value premium (see also Santons and Veronesi, 2010; Lettau and Wachter, 2007, 2011) and even a partially downward-sloping term structure of equity (Ai et al., 2012).

³See Hansen (2013) for a review of this literature.

Lamont, 2000; Lettau and Ludvigson, 2002; Kogan, 2004; Zhang, 2005; Liu et al., 2009), while a recent literature is refining the micro-foundations underlying firms' behavior and thereby the asset pricing implications of the production side (e.g., Belo, 2010; Jermann, 2010, 2013).

On the consumption side, the benchmark story is the power-utility model (Grossman and Shiller, 1981; Hansen and Singleton, 1983), which predicts that the price of a financial asset is low if its payoff covaries positively with consumption (Lucas, 1978). This prediction is correct qualitatively but is a disaster quantitatively: the model suffers from the equity premium puzzle (Mehra and Prescott, 1985), the risk-free rate puzzle (Weil, 1989) and the equity volatility puzzle (Shiller, 1981). The reason is that the quantity of risk, say in an equity index, is measured by the covariance of its return with consumption, which is empirically low and stable, and the price of risk is measured in the power-utility model by the constant risk aversion of the average consumer. A high relative risk aversion could make sense of the high and volatile empirical equity risk premium but only a low relative risk aversion (which in this model equals the inverse of the intertemporal elasticity of substitution) could make sense of the low and stable risk-free rate. The power-utility model has therefore two mutually inconsistent needs.

The equity volatility puzzle stems from a body of empirical evidence about return forecastability, i.e., the observation that expected (excess) returns are predictable and volatile (e.g., Shiller, 1981; Campbell and Shiller, 1988; Fama and French, 1988; Campbell, 1991; Cochrane, 1992, 2008a; Lettau and Nieuwerburgh, 2008; Bollerslev et al., 2009; Binsbergen and Koijen, 2010; Bakshi et al., 2011; Golez, 2013).⁴ The evidence in favor of time-varying risk premia seems a robust and pervasive phenomenon across several asset classes, in spite of the documented issues in the typical predictive regressions, such as parameter instability, poor out-of-sample forecasting power (Goyal and Welch, 2008) or statistical bias in finite samples (Stambaugh, 1999). Several variables appear to forecast returns but their statistical significance must be motivated theoretically to avoid the in-sample fishing of return predictors. The dividend-price ratio (and other stationary transformations of stock prices, such as the market-book ratio), the consumption-wealth ratio and the investment-capital ratio all appear to have return-forecasting abilities and have a special status as predictors, because they are robustly related to returns by an identity. The Campbell and Shiller (1988) identity tightly links the dividend-price ratio to market returns; the consumption-wealth ratio also is in an identity relation with returns on the wealth portfolio (Lettau and Ludvigson, 2001a); and the investment-capital ratio should be in an equilibrium relation with market-book ratios—by the basic Q story—so that it too should be related to market returns by the Campbell-Shiller identity (Cochrane, 1991). The return-forecasting regressions besides show that predictors comove and are highly correlated with the business cycle (Fama and French, 1989; Lettau and Ludvigson, 2010; Cochrane, 2011). In particular, when the state of the economy is bad, the average investor commands a higher expected return to get exposed to systematic risk.⁵

Asset pricing theory can rationalize the stylized asset pricing facts—which include the size, time-variation and term structure properties of stock and bond risk premia—either by changing

⁴See Cochrane (2011) and Koijen and Nieuwerburgh (2011) for a survey of the evidence.

⁵A richer set of evidence that modern asset pricing models are only recently trying to match is the one coming from equity term structure data (Binsbergen et al., 2012, 2013). At the state of the art no structural model seems able to explain the observed downward-sloping term structure of equity and an upward-sloping term structure of interest rates.

the price of risk (e.g., through additional factors affecting preferences) or the quantity of risk (e.g., through richer dynamics in consumption). Among the most common ingredients to model risk-premia variation are financial frictions introduced by the behavior of intermediaries and new preferences that introduce some time or state non-separability that affects the risk-bearing ability of investors (see Cochrane, 2011, for a survey). Examples of these preference specifications include the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the long-run risk model under limited information of Croce et al. (2012), the non-expected utility models of Tallarini (2000) and Barillas et al. (2009), and the rare disasters models of Gabaix (2012) and Wachter (2013).

For example, Campbell and Cochrane (1999) are able to replicate, at least partially, the stylized asset pricing facts by introducing a slow-moving habit in the utility function. The quantitative implications of the model are much better than the power-utility alternative. Risk aversion no longer equals the intertemporal elasticity of substitution; there no longer is a tradeoff between solving the equity premium and the risk-free rate puzzles. Moreover, the price of risk becomes time-varying, which helps accounting for the return-forecasting evidence about the cyclicity of risk premia. Formally, Campbell and Cochrane reverse-engineer a stochastic discount factor whose time-varying slope (the price of risk) and intercept have just enough degrees of freedom as to make by construction the model immune to the equity premium and the risk-free rate puzzles; and to have the dividend-price ratio reveal—up to first order—the unique state of the system, thus solving in part the equity volatility puzzle.⁶

Another example is provided by the long-run risk literature opened by Bansal and Yaron (2004). They build on Epstein-Zin-Weil utility, which are preferences able to break down the equality between the risk aversion parameter and the inverse of the intertemporal elasticity of substitution that holds in the power-utility model. Since two distinct parameters control the two quantities, the model is then able to avoid the tension between a large and volatile equity premium and a low and stable risk-free rate and thereby generate realistic risk premia by assuming rich dynamics in consumption.

Monetary economics: stylized facts and theory

What should be the goals of monetary policy? How should the central bank use its policy tools to react to systematic and unpredictable movements in the economy? What is the transmission mechanism from policy tools to macroeconomic variables? These are the fundamental questions of monetary policy (e.g., Galí, 2008). I focus here on a particular family of monetary macromodels—the New Keynesian framework—which is suitable to answer these questions.

The New Keynesian framework arises in the 1980s as a family of microfounded monetary macromodels describing a non-trivial relation between monetary policy and the business cycle

⁶A stochastic discount factor conditionally linear in consumption growth (such as Campbell and Cochrane's) seems also able to price the cross-section of returns remarkably well and thereby to make progress relative to one last empirical failure of the benchmark consumption-based model which, in its CCAPM form, is unable to line up its predicted average cross-sectional returns to the empirical mean returns. Lettau and Ludvigson (2001b) show how a particular specification of a discount factor with Campbell and Cochrane's shape can replicate (quasi-)structurally the positive results of the nonstructural 3-factor model of Fama and French (1993, 1996) on their 25 portfolios.

and capable of giving goals and tools for the conduct of monetary policy (Mankiw and Romer, 1991; Yun, 1996; King and Wolman, 1996; Goodfriend and King, 1997; Clarida et al., 1999). The microfoundations of the New Keynesian model make it immune to Lucas (1976) critique; predictable actions by the government generally affect the reduced form of modeled variables, so that a structural model is necessary when studying systematic monetary policy. The New Keynesian framework offers a structural model capable of studying the implementation of monetary policy through systematic policy rules such as targeting and interest-rate rules as well as to characterize the best monetary policy in an environment where people take their decisions also depending on their expectations about the way the central bank will act (Prescott, 1977; Kydland and Prescott, 1977, 1980). Moreover, the basic New Keynesian model offers a structure that has proved suitable to accommodate more complex frictions, such as labor market frictions (e.g., Erceg et al., 2000; Blanchard and Galí, 2010) or financial frictions (e.g., Bernanke et al., 1999; Cúrdia and Woodford, 2009; Gertler and Kiyotaki, 2011; Gertler and Karadi, 2011).⁷

The introduction of a parsimonious friction—monopolistic competition and Calvo (1983) price stickiness in the market for goods—on top of an otherwise efficient RBC model economy (Kydland and Prescott, 1982; Long and Plosser, 1983) breaks the neutrality of money that prevents monetary policy from having real effects in the baseline RBC structure (Lucas, 1972). The assumption of sticky prices is key to make the nominal dimension more than just a veil; since goods prices do not all move at the same time, whenever the state of the economy gives incentives to firms to change prices, there will be an inefficient movement in relative prices that in turn associates to an inefficient general equilibrium. The nonneutrality of monetary policy is then crucial because it implies that the policy rule adopted by the central bank makes a difference for overall welfare. The central bank should therefore design a policy rule in line with its goals and with its understanding of the way our decentralized market economies work. Formally, the goal monetary policy should achieve is the constrained maximization of overall welfare, a concept that arises naturally in a microfounded macromodel as the utility of the representative consumer (Lucas, 1987; Rotemberg and Woodford, 1997). Then, welfare maximization typically means that the central bank should stabilize inflation, real activity and the financial variables near their first-best levels. However, the precise definition of the policy goals, just as the way the central bank should achieve them with its policy tools, depends a lot on the structure of the model economy.

The basic New Keynesian model consists of three theoretical pillars (Galí, 2008). First, a New Keynesian Phillips curve describes the optimal price-setting decision of firms. The central New Keynesian friction is that only a fraction of firms is allowed to choose prices at each time period. Therefore, firms would set their prices by considering future economic developments; this behavior results, up to first order, in the forward-looking optimality condition linking inflation, π , and the aggregate markup, μ ,

$$\pi_t = -\lambda E_t \sum_{j=0}^{\infty} \beta^j (\mu_{t+j} - \mu)$$

⁷Galí (2008) and Galí (2011) survey many of the recent extensions. The examples that possibly epitomize the gain in complexity of New Keynesian models are the medium-scale models of Christiano et al. (2005) and Smets and Wouters (2007).

Inflation is high when firms expect long-run markups below the flexible-price level. In that case resetting firms choose a price above the average price level to realign their markup to the desired level. Deviations of aggregate markups from the desired level, μ , then associate to a gap in aggregate activity relative to the flexible-price equilibrium.

Second, an Euler equation describes the dynamic relation between consumption and saving—which is an asset pricing equation that holds unchanged in the baseline frictionless model—and controls the policy transmission mechanism. In a power-utility world characterized by certainty equivalence, the stochastic discount rate equals the risk-free rate and the dynamic IS equation is driven only by an intertemporal substitution motive as

$$i_t - E_t \pi_{t+1} - \rho = \gamma E_t \Delta c_{t+1}$$

where Δc is consumption growth, ρ is a subjective time-discount factor, and $1/\gamma$ is the intertemporal elasticity of substitution. A positive shock to the real rate—either following a policy shock or expected deflation—increases the incentives to save and thereby displaces some consumption forward in time. In a frictionless setting, all firms can adjust nominal prices to leave real output choices, and thereby overall welfare, unchanged. In a world with nominal rigidities, only some firms can reset prices; the remaining firms choose to inefficiently adjust quantities to meet the new consumption profiles. These effects on prices and/or real consumption restore the no-arbitrage condition by increasing ex-ante inflation and/or discount rates. This impact on the economy is dampened by a Taylor rule that responds positively to expected inflation or current consumption because people would expect some automatic decrease in the nominal rate to restore the no-arbitrage condition; the required adjustments in current consumption and prices would then be smaller. The costly distortions caused by price rigidities are therefore reduced.⁸ This property justifies inflation targeting as the optimal monetary policy.

Third, a policy rule describes the monetary policy and formalizes the presence of a predictable and of an unpredictable policy component. The policy tool that is simplest to consider in the New Keynesian framework is some reference interest rate. Through an interest-rate rule—for example of the Taylor type—you can devise an endless number of systematic policy reactions to developments in the economy. The dynamic IS equation then transmits changes in the interest rate to the real economy, thus affecting consumers' and firms' decisions.

These three theoretical pillars have reasonable empirical success. From a full-information perspective, the New Keynesian model beats the benchmark RBC model in the qualitative replication of the empirical impulse responses to monetary and technology shocks (Christiano et al., 1999, 2005; Galí, 1999; Galí and Rabanal, 2005). However, these impulse responses are extracted with structural vector autoregressions (SVAR), a technique that remains controversial (Fernández-Villaverde et al., 2007).

From a limited-information perspective, the New Keynesian Phillips curve has empirical support (Galí and Gertler, 1999; Galí et al., 2005), although there is still debate about specification and

⁸Note that, on the one hand, inflation volatility is equivalent, up to first order, to a cross-sectional dispersion in prices, which in turn associates to a cross-sectional dispersion in output and causes welfare losses because it associates to inefficient employment; on the other hand, people dislike output-gap volatility because they are risk averse.

weak-identification issues (Mavroeidis, 2005; Nason and Smith, 2008; Canova and Sala, 2009). In part 2 I present a new limited-information test of the New Keynesian story versus the frictionless alternative based on return forecastability—and therefore on reduced-form evidence.

Roadmap

My broad research agenda is that asset pricing models have implications for monetary models, and vice versa.

Part 1 is an example of the implications of finance for macroeconomics, both in terms of providing a measure of people's current preferences between growth, short-run and long-run stability and in terms of pointing out how a macromodel that fails to explain these asset market features is likely to miss crucial welfare implications.

Part 2 is an example of macro-financial linkages running in both directions. On the one hand, the same monetary nonneutralities that are routinely used in monetary models are able to explain the observed comovement between inflation and dividend yields; on the other, stock market data help estimating important structural parameters and testing some implications of nominal rigidities.

Part 3 studies an example in which the explicit integration of some time-variation in stock prices can have non-trivial normative implications.

Part 1. The term structure of the welfare cost of uncertainty

I study the macroeconomic priorities as revealed by the notion of cost of consumption fluctuations. I show how the marginal cost of fluctuations has a term structure that is a simple transformation of the term structures of equity and interest rates (e.g., studied by Lettau and Wachter, 2007, 2011; Binsbergen et al., 2012, 2013). I am therefore able to use current finance models and recent index option market evidence about the term structures (Binsbergen et al., 2012; Boguth et al., 2012) to infer a downward-sloping and volatile term structure of welfare costs.

On average, cashflow stability is a macroeconomic priority and short-run stability is a greater priority than long-run stability. I estimate that at the margin the elimination of one-year ahead cashflow volatility is worth around twelve percentage points of additional growth. This number compares to a marginal cost of all consumption fluctuations of about two percentage points.

Over time, the term structure of welfare costs, hence the priority between growth, short-run and long-run stability, varies substantially. The macroeconomic priorities vary across the business cycle. Against this background, I show how a policy-maker can assess the current position and evolution of the term structure of welfare costs and forecast its movements by looking at innovations in the information set made by excess return predictors.

Finally, the link between welfare costs and risk premia can make the case for risk premia targeting as a welfare-enhancing policy regime.

Part 2. A New Keynesian Q theory of investment and the link between inflation and the stock market

I study a New Keynesian version of the Q theory, which is non-standard if prices are sticky. The benchmark real business cycle (RBC) Q theory of investment links investment with stock prices. In this study, I demonstrate how a New Keynesian Q (NKQ) theory links investment, stock prices, and inflation, providing a rational explanation for a comovement between expected inflation and stock

prices. This result is an example of how some features typical of monetary models (in this case, sticky prices) have implications for finance. The NKQ theory offers a parsimonious story to make sense of the correlation between stock prices and inflation as a rational outcome which, within a frictionless setting, appears to have to be explained outside the rational expectations framework.

Like the standard Q theory, the NKQ equation contains a specification error that complicates estimation. Because the specification error can have any shape and property, I must restrict its distribution for the theory to be rejectable. Namely, I choose to reject the NKQ theory if the market-book ratio predicted by the theory does not have the same return-forecasting ability as the observed market-book ratio; the NKQ theory might be inaccurate along several dimensions but, at least, predicted and observed market-book ratios should reveal the same sources of priced movement in the economy, which is what really matters from an asset pricing perspective.

This strategy allows me to overidentify the two free parameters. With this overidentification, I am able to both estimate and test the NKQ theory. This test is a good test for the NKQ theory, because nothing implies that there is a linear combination of inflation and the investment-capital ratio that displays a return-forecasting power similar to that of the market-book ratio. The replication of the return-forecasting ability of the market-book ratio is not perfect but the simple NKQ theory already succeeds along several dimensions. The benchmark frictionless alternative has instead a more difficult time accounting for the return-forecasting ability of the market-book ratio, because the investment-capital ratio alone cannot forecast long-run returns over the whole sample; the NKQ theory captures a long-run component of the market-book ratio that is missing in the benchmark Q theory.

Finally, the fit of the NKQ theory is a dramatic improvement over the benchmark.

Part 3. Reassessing the role of stock prices in the conduct of monetary policy

I reassess the role of stock prices in the conduct of monetary policy. I build a New Keynesian model with capital adjustment costs and exogenous sources of policy tradeoffs in which central banks should not respond to stock price movements. Even though the policy exercise is not trivial (financial stability affects consumer welfare in the model in addition to price and production stability), in the model a policy that focuses on stabilizing inflation is close to optimal. I show this fact under exogenous cost-push disturbances and financial frictions to discuss the robustness of the result to different sources of policy tradeoffs.

I then solve numerically for the optimal Taylor-type rule that responds to stock prices by using the typical approach adopted by the extant literature, which consists in fixing the response coefficient to inflation to subsequently search numerically for desirable policy reactions to stock prices and output. The numerical approach can easily prescribe all possible qualitative reactions to stock prices; the model highlights some pitfalls in a numerical study of stock prices and monetary policy that can explain and reconcile the conflicting policy prescriptions found in the literature. For fixed values of the Taylor rule coefficient attached to inflation, the optimal coefficients attached to stock prices and output are non-monotonic and nonlinear in the deep-parameter space, which can explain the qualitative variation in the prescription found in the literature. However, the only prescription that survives after a closer look is a strong commitment to stabilizing inflation with no response to stock price movements, as the other prescriptions are just roundabout ways of increasing the anti-inflationary stance.

Part 1

The Term Structure of the Welfare Cost of Uncertainty

Abstract. The marginal cost of aggregate fluctuations has a term structure that is a simple transformation of the term structures of equity and interest rates. I extract evidence from index option markets to infer a downward-sloping, volatile and pro-cyclical term structure of welfare costs. On average, cashflow stability is a macroeconomic priority and short-run stability is a greater priority than long-run stability. I estimate that at the margin the elimination of one-year ahead cashflow volatility is worth around twelve percentage points of additional growth. This number compares to a marginal cost of all consumption fluctuations of about two percentage points. Over time, the term structure of welfare costs varies substantially, pro-cyclically and with a volatility that decreases with maturity. Return predictors reveal the states that drive the term structure of welfare costs and thereby signal its current position and future developments. Finally, the link between welfare costs and risk premia can make the case for risk premia targeting as a welfare-enhancing policy regime.

1.1. Introduction

How much growth are people willing to trade against a marginal stabilization of macroeconomic fluctuations? The marginal welfare cost of aggregate uncertainty (Alvarez and Jermann, 2004) answers this important question in economics, which goes back at least to Lucas (1987). I decompose the marginal cost of uncertainty into a term structure. This decomposition allows for studying how cashflow fluctuations at different horizons contribute to the total cost of uncertainty (proposition 1). The term structure of welfare costs allows for understanding the tradeoff between growth and macroeconomic stability as well as the tradeoff between cashflow stability at different periodicities (for example, between short-run and long-run stability).

Reviving the insight that asset market data reveal the marginal cost of fluctuations (Alvarez and Jermann, 2004), I then show how the components of the term structure of welfare costs are tightly linked to the risk premia on market dividend strips (proposition 2); the term structure of welfare costs is a simple transformation of the term structures of equity and interest rates (e.g., studied in Lettau and Wachter, 2007, 2011; Binsbergen et al., 2012, 2013). This link allows for a measure of the cost of fluctuations that is directly observable over the last two decades. Like Alvarez and Jermann (2004), my approach requires only the absence of arbitrage opportunities and does not require a parametric specification of consumer preferences.

In the empirical section, I extract evidence about the term structure of equity from index option markets to infer the cost of uncertainty as a function of its time horizon. I find costs that are large,

volatile, pro-cyclical and have non-trivial term structure features. The point estimates, reported in figure 1.1a, suggest a negatively sloped term structure of welfare costs; people command a larger premium to shoulder short-term cashflow uncertainty than to shoulder longer-term uncertainty. The premium at one-year frequency is 12-13 percent on average and its volatility has at least the same size. Since both are measures of the premium people command to shoulder aggregate risk, it is natural to compare this number with the equity premium, which averages 6-9 percent over the last two decades and also displays significant time-variation.

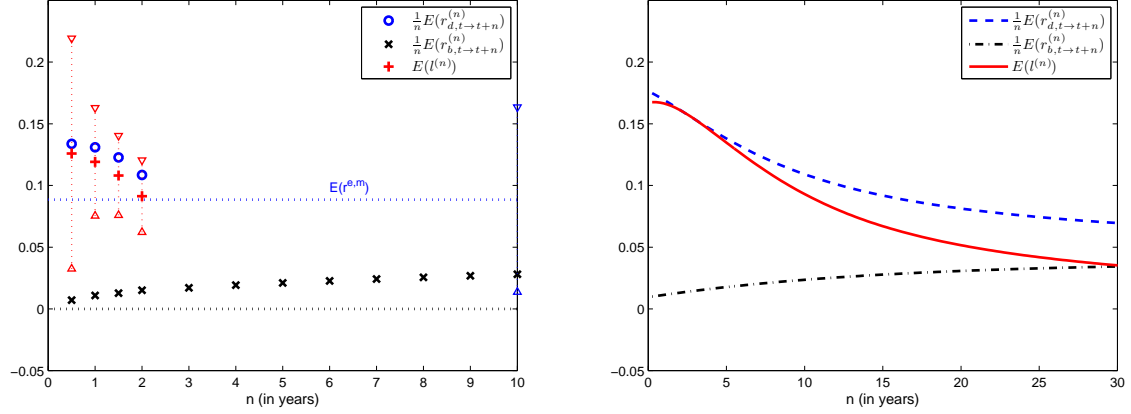
The evidence of a downward-sloping term structure of welfare costs helps identifying a model to capture and quantify the entire term structure. I study the implications of today's leading consumption-based asset pricing models for the three term structures.⁹ I consider the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the long-run risk model under limited information of Croce et al. (2012), the ambiguity averse multiplier preferences of Barillas et al. (2009), and the rare disasters model of Gabaix (2012). Although these models do not study the term structure of welfare costs directly, they have implications for it and are calibrated to match several other asset pricing facts. Unfortunately, from a structural perspective, replicating a downward-sloping term structure of equity and an upward-sloping term structure of interest rates is problematic (Lettau and Wachter, 2007; Binsbergen et al., 2012; Croce et al., 2012, make this point). I therefore have to discard a structural explanation and turn to a quasi-structural model. In this regard, Lettau and Wachter (2011) offer a parsimonious model designed to capture precisely a downward-sloping term structure of equity, an upward-sloping term structure of interest rates, and time-varying risk premia. The quantitative implications of the model, reported in figure 1.1b, are a marginal cost of lifetime fluctuations of about 3 percent—which compares with an equity premium of about 7 percent. Over a horizon of up to ten years, a marginal increase in uncertainty costs more than 10 percentage points of annual growth per unit of uncertainty, as measured by the conditional standard deviation of cashflows over the relevant horizon. These numbers compare with much smaller marginal benefits of long-run stabilization.

Thus, on average, cashflow stability is a macroeconomic priority, especially in the short run. Over time, the tradeoff among short-run, long-run stability and growth revealed by the term structure of welfare costs varies substantially because excess returns are predictable. Consequently, return predictors reveal the state that drives the time-variation in the term structure of welfare costs and thereby signal the current and future macroeconomic priorities. For example, in the model of Lettau and Wachter (2011), the cost of fluctuations at different periodicities is driven by one state, the price of risk, which is perfectly revealed by equity and bond yields. In the model, positive discount-rate news signal an increase in the cost of short-run fluctuations that slowly decays across maturities and over time.

Finally, from a welfare perspective, the link between risk premia and the cost of fluctuations could make the case for a risk premia targeting regime as a welfare-enhancing policy. I discuss in what sense a policy-maker could use the cost of fluctuations at different time horizons as a welfare criterion. A regime that targets the level of the term structure components, which I refer to as risk premia targeting, is unambiguously desirable provided it is neutral on the mean growth rate of

⁹See Cochrane (2011), Koijen and Nieuwerburgh (2011) and Ludvigson (2013) for a survey of the main asset pricing facts and of the progress state-of-the-art asset pricing models have made in explaining them.

cashflows and on the level of any additional factors that affect utility (proposition 3).



(a) Point estimates (with bootstrapped 95% confidence interval). $E(r^{e,m})$ is the equity premium.

(b) Model-based estimates in the model of Lettau and Wachter (2011).

Figure 1.1: Term structures of equity ('o' and dashed line), interest rates ('x' and dotted line) and welfare costs ('+' and solid line); annualized premia.

1.1.1. Marginal costs of uncertainty in the time domain

In his seminal work, Lucas (1987) uses a log utility representative consumer model and defines cashflow uncertainty as the deviations of consumption from a deterministic growth trend. He then calculates a cost of fluctuations of .01-.05% of aggregate consumption—a small amount that would make policies pursuing macroeconomic stability a low priority. However, within the representative-consumer power utility model, the gains from stabilizing fluctuations are small for the same reason that the model predicts a small equity premium (which is just the equity premium puzzle of Mehra and Prescott, 1985). Accordingly, a large literature studies the cost of fluctuations under different preferences or under consumer heterogeneity and possibly uninsurable idiosyncratic risk and finds highly dispersed model-based estimates, from virtually zero to more than 20 percent (for example, Atkeson and Phelan, 1994; Krusell and Smith, 1999; Tallarini, 2000; De Santis, 2007; Barillas et al., 2009; Croce, 2012; Ellison and Sargent, 2012).

In this context, Alvarez and Jermann (2004) represent a breakthrough, as they manage to move the game to a preference-free environment by showing how we can directly use asset market data to measure the cost of fluctuations *at the margin*. Alvarez and Jermann provide two main lessons about the estimate of the cost of fluctuations. A model that is consistent with the observed equity premium and takes the deterministic trend in consumption as the stable stream increases Lucas's estimates by two orders of magnitude. If you consider instead as the stable stream some stochastic trend in consumption the estimates can be much closer to Lucas's. They conclude that what people really dislike is the low-frequency volatility in consumption.

My approach builds on and complements the analysis of Alvarez and Jermann (2004). First, by focusing on marginal welfare costs, I stick to their preference-free setting while linking the cost of fluctuations to a richer set of financial market evidence.

Second, the term structure of the cost of uncertainty answers roughly to the question ‘How much compensation do people command to bear n -year ahead cashflow uncertainty?’ This question compares to ‘How much compensation do people command to bear uncertainty at business-cycle frequency in the entire cashflow process?’, which is the one studied by Alvarez and Jermann. Their answer depends a lot on the parametric assumptions about the filter that separates the trend and the business-cycle frequencies of the cashflow process. The question I am interested in is nonparametric and complements the exercise of Alvarez and Jermann by decomposing the marginal cost of uncertainty in the time domain rather than in the frequency domain.¹⁰

Finally, I take a slight departure from the original definition and express the marginal benefits of stability in terms of extra cashflow *growth*, as opposed to a uniform increase in the level of lifetime cashflows. Along with still measuring the cost of fluctuations, my notion of welfare cost measures the tradeoff between growth and consumption stability and is therefore of more direct interest to economic policy-makers facing tradeoffs between economic growth and macroeconomic stability.

1.2. The term structure of the cost of uncertainty

People live in a stochastic world, have finite resources and decide how to allocate them across time. Identical risk-averse consumers $i \in [0, 1]$ have time- t preferences $U_t = E_t U(C_i, X_i)$, where $C \equiv \{C_{t+n}\}_{n=1}^\infty$ is consumption and $X \equiv \{X_{t+n}\}_{n=1}^\infty$ is any other factor that influences utility.¹¹ Without loss of generality, I let factor X_i depend on aggregate consumption $C = \int_0^1 C_i di$ but not on individual consumption C_i . Since there is a continuum of agents each of which has zero mass, this modeling strategy allows me to ask an individual how much he would pay in exchange for less cashflow uncertainty without thereby having to affect all aggregate quantities, including factor X .

Financial markets are without arbitrage opportunities and people can trade in the financial market the full set of zero-coupon bonds and the full set of single market dividend payments, so-called dividend strips.

I am interested in measuring the cost of consumption uncertainty, i.e., how much consumption growth a consumer is willing to trade against a stable consumption stream. Let \bar{C}_{t+n} denote the consumption level that is hypothetically offered to the i th individual at time $t + n$, which I refer to as *stable* consumption. Then, I parametrize stable consumption as $\bar{C}_{t+n}(\theta) = \theta E_t C_{t+n} + (1 - \theta) C_{t+n}$, where the parameter $\theta \in [0, 1]$ indexes a convex combination of ex-ante and ex-post consumption and represents the fraction of ex-post uncertainty that is removed.

Definition (Marginal cost of uncertainty). In line with Alvarez and Jermann (2004), I define the cost of fluctuations as \mathcal{L}_t in

$$\begin{aligned} E_t U\left(\{(1 + \mathcal{L}_t^N(\theta))^n C_{t+n}\}_{n \in \mathbb{N}}, \{C_{t+n}\}_{n \in \bar{\mathbb{N}} \setminus \mathbb{N}}, \{X_{t+n}\}_{n=1}^\infty\right) = \\ = E_t U\left(\{\theta E_t C_{t+n} + (1 - \theta) C_{t+n}\}_{n \in \mathbb{N}}, \{C_{t+n}\}_{n \in \bar{\mathbb{N}} \setminus \mathbb{N}}, \{X_{t+n}\}_{n=1}^\infty\right) \end{aligned} \quad (1.1)$$

¹⁰It is natural to compare my finding of a downward-sloping term structure of welfare costs to the conclusion of Alvarez and Jermann that long-run fluctuations are the fluctuations that people fear the most. Our results are not necessarily inconsistent because they focus on the volatility of the entire consumption process with a given band of spectral frequencies, whereas I focus on the entire volatility in consumption at a given time horizon.

¹¹In section 1.4 I relax the assumption of identical preferences across agents.

where the index set $\mathcal{N} \subset \bar{\mathbb{N}} \equiv \{1, \dots, \infty\}$ indicates which coordinates of consumption are stabilized and allows for focusing on any window of interest.

Two particularly interesting quantities are the *total cost* $\mathcal{L}_t^{\bar{\mathbb{N}}}(1)$, which measures how much extra growth the elimination of all cashflow uncertainty is worth, and the *marginal cost* $L_t^{\mathcal{N}} \equiv \frac{\partial \mathcal{L}_t^{\mathcal{N}}(\theta)}{\partial \theta} \Big|_{\theta=0}$, which represents the current assessment of how much extra growth a marginal stabilization is worth.

The appendix discusses the relationship between definition (1.1) and the original definitions studied by Lucas and Alvarez and Jermann.

I assume enough smoothness in preferences to guarantee that $L_t^{\mathcal{N}}$ is a differentiable map on $\theta \in [0, 1]$. Thus, differentiating (1.1) with respect to θ ,

$$L_t^{\mathcal{N}} = \frac{\sum_{n \in \mathcal{N}} E_t(M_{t,t+n}) E_t(C_{t+n}) - E_t(M_{t,t+n} C_{t+n})}{\sum_{n \in \mathcal{N}} n E_t(M_{t,t+n} C_{t+n})} \quad (1.2)$$

where $M_{t,t+n} = (\partial U_t / \partial C_{t+n}) / (\partial U_t / \partial C_t)$ is the n -period stochastic discount factor. Note how $D_t^{(n)} = E_t M_{t,t+n} C_{t+n}$ is the no-arbitrage price of a n -period consumption strip and $E_t M_{t,t+n}$ is the no-arbitrage price of a n -period zero-coupon bond. Under no-arbitrage, equation (1.2) expresses the marginal cost of uncertainty around all coordinates $n \in \mathcal{N}$ as a function of the price of a claim to the trend in consumption and of the price and duration of a claim to future consumption at all coordinates $n \in \mathcal{N}$.

Definition (Term structure of the cost of uncertainty). Consider the singleton set $\mathcal{N} = \{n\}$, for $n = 1, 2, \dots$, and consider the marginal costs $l_t^{(n)} \equiv \frac{\partial \mathcal{L}_t^{(n)}(\theta)}{\partial \theta} \Big|_{\theta=0}$. The marginal cost $l_t^{(n)}$ is the cost of a marginal increase in uncertainty in the n th coordinate of consumption. Then, by equation (1.2), it follows that

$$l_t^{(n)} = \frac{1}{n} \left(\frac{E_t(M_{t,t+n}) E_t(C_{t+n})}{E_t(M_{t,t+n} C_{t+n})} - 1 \right)$$

The motivation for calling the map $l_t : n \mapsto l_t^{(n)}$ a term structure of the marginal cost of uncertainty is given by proposition 1. Given the prices of strips $\{D^{(n)}\}$ and the term structure components $\{l^{(n)}\}$ you can compute the marginal cost $L^{\mathcal{N}}$ for any coordinate set $\mathcal{N} \subset \bar{\mathbb{N}}$.

Proposition 1. *The marginal cost of uncertainty within any window of interest \mathcal{N} , $L_t^{\mathcal{N}}$, is the linear combination of the term structure components $\{l_t^{(n)}\}$ defined by*

$$L_t^{\mathcal{N}} = \sum_{n \in \mathcal{N}} \omega_{n,t} l_t^{(n)} \quad (1.3)$$

where the weights $\omega_{n,t} \equiv \frac{n D_t^{(n)}}{\sum_{n \in \mathcal{N}} n D_t^{(n)}}$ are positive and such that $\sum_{n \in \mathcal{N}} \omega_{n,t} = 1$.¹²

¹²No-arbitrage guarantees that the weights are positive. The weights $\omega_{n,t}$ depend on the coordinate set \mathcal{N} ; to keep formulas simple I omit such a dependence in the notation.

1.2.1. The term structures of equity, interest rates, and welfare costs

Proposition 2 shows how the term structure of welfare costs is a simple transformation of the term structures of equity and interest rates, by which I mean the maps $n \mapsto E_t R_{d,t \rightarrow t+n}^{(n)}$ and $n \mapsto E_t R_{b,t \rightarrow t+n}^{(n)}$ of ex-ante hold-to-maturity returns on dividend strips and zero-coupon real bonds, respectively, as a function of maturity.¹³

Proposition 2. *The n th component of the term structure of welfare costs is the risk premium for holding to maturity a portfolio long on a n -period dividend strip and short on a n -period zero-coupon bond. The term structure of welfare costs is the transformation of the term structures of equity and interest rates*

$$\begin{aligned} l_t^{(n)} &= \frac{1}{n} (E_t R_{t \rightarrow t+n}^{e,(n)} - 1) \\ &= \frac{1}{n} \left(\exp \left\{ E_t r_{d,t \rightarrow t+n}^{(n)} - E_t r_{b,t \rightarrow t+n}^{(n)} + \frac{1}{2} \mathcal{V}_t(R_{d,t \rightarrow t+n}^{(n)}) \right\} - 1 \right) \end{aligned} \quad (1.4)$$

where $\mathcal{V}_t(X) \equiv 2[\ln E_t X - E_t \ln(X)]$ denotes conditional entropy and $r \equiv \ln(R)$.

Equation (1.4) provides the link between the term structures of equity and interest rates and the term structure of welfare costs. In particular, since the entropy term is independent of the bond price, it shows clearly how the term structure of welfare costs would unambiguously slope downwards in the case of a downward-sloping term structure of equity and of an upward-sloping term structure of interest rates.

There is a powerful intuition behind these formulas. At the margin, people would trade $l_t^{(n)} \theta$ points of growth against the elimination of a fraction θ of the aggregate cashflow uncertainty around the n th consumption coordinate, C_{t+n} . Proposition 2 shows how this tradeoff is precisely the one offered by the financial market. In fact, by holding to maturity a portfolio short on the n -period zero-coupon bond and long an equal amount in the n -period dividend strip, people can experience an average growth rate of $\frac{1}{n} (E_t R_{t \rightarrow t+n}^{e,(n)} - 1)$ by shouldering a volatility of $\mathcal{V}_t(R_{t \rightarrow t+n}^{e,(n)}) = \mathcal{V}_t(C_{t+n})$. Therefore, the cost of n -year ahead uncertainty must be $l_t^{(n)} = \frac{1}{n} (E_t R_{t \rightarrow t+n}^{e,(n)} - 1)$.¹⁴

1.2.2. Relationship between the cost of uncertainty and the equity premium

A theoretically important relationship is the one between the cost of fluctuations and the equity premium, as both are measures of the premium people command to shoulder aggregate risk. A natural benchmark is the case of flat term structures of equity and interest rates. Indeed, when both term structures are flat the entire term structure of the cost of uncertainty equals the equity premium,

¹³For now, dividends and consumption are used interchangeably. I discuss in section 1.4 how dividends rather than consumption can be used to measure the welfare costs of uncertainty in a production economy.

¹⁴Note how the n th component of the term structure of uncertainty is the arithmetic version of the dividend risk premium, $\theta_t^{(n)}$, as defined by Binsbergen et al. (2013), namely $l_t^{(n)} = \frac{1}{n} (\exp\{n\theta_t^{(n)}\} - 1)$. Therefore, in their language, time-variation in forward equity yields must reveal a time-varying term structure of growth or a time-varying term structure of the welfare cost of fluctuations. Binsbergen et al. flesh out the former link; the present writing explains the latter and how the term structure components combine to measure the cost of uncertainty around arbitrary sets of cashflow components.

and L_t^N equals the equity premium, for all $N \in \overline{\mathbb{N}}$. In essence, the term structure of equity is flat if shocks to the cashflow opportunity set (e.g., shocks to expected cashflow growth and to cashflow volatility) are either absent or unpriced; the term structure of interest rates is flat in economies in which the state driving the risk-free rate is either absent or unpriced, so that the expectations hypothesis of bond valuation holds.¹⁵

While the benchmark case would allow for a quantification of the cost of uncertainty over a large sample, it is restrictive, both empirically and theoretically, and must therefore be taken with caution. On the one hand, recent evidence about the term structures of equity and interest rates challenges the trivial term structure properties required for the equality between the cost of fluctuations and the equity premium (Binsbergen et al., 2012, 2013; Boguth et al., 2012).

On the other hand, although it is not restrictive for some asset pricing models, the assumption of flat term structures is particularly severe for the long-run risk explanation (for example, of Bansal and Yaron, 2004). A flat term structure of equity in a long-run risk setting with Epstein-Zin-Weil type utility requires either a random walk in consumption or a unitary elasticity of intertemporal substitution. The random walk in consumption would imply that the main component of the long-run risk explanation is absent, while the unitary elasticity of substitution implies that the long-run risk component is not priced. Both features actually kill the mechanism that generates several interesting asset pricing facts within that framework. Therefore, the long-run risk literature shows how local alternatives, against which we have no statistical power, to the conditions granting flat term structures can have non-trivial asset pricing implications.

1.3. Empirics of the cost of uncertainty

Suppose that a full set of zero-coupon real bonds and a full set of put and call European options whose underlying is an aggregate equity index are traded on the market. In absence of arbitrage opportunities, put-call parity holds as

$$C_{t,t+n} - \mathcal{P}_{t,t+n} = P_t - \sum_{j=1}^n D_t^{(j)} - X P_{b,t}^{(n)} \quad (1.5)$$

where $C_{t,t+n}$ and $\mathcal{P}_{t,t+n}$ are the prices at time t of a call and a put European options on the market index with maturity n and strike price X , $P_{b,t}^{(n)} = E_t M_{t,t+n}$ is the price of a n -period zero-coupon bond, $P_t = E_t \sum_{j=1}^{\infty} M_{t,t+j} D_{t+j}$ is the price of the market portfolio, and $D_t^{(n)} = E_t M_{t,t+n} D_{t+n}$ is the price of the n th dividend strip. Since the only unknowns in equation (1.5) are the prices of the dividend strips, $\{D_t^{(n)}\}$, one can synthetically replicate them (Binsbergen et al., 2012).

1.3.1. Empirical results

I follow Binsbergen et al. (2012) and Golez (2013) in synthesizing the evidence on dividend claims from put and call European options on the S&P 500 index. Standard index option classes, with twelve monthly maturities of up to one year, and Long-Term Equity Anticipation Securities

¹⁵The technical appendix studies approximate analytical conditions that grant flat term structures in a general loglinear-lognormal environment.

(LEAPS), with ten maturities of up to three years, are exchange traded on the Chicago Board Options Exchange (CBOE) since 1990.¹⁶ The overall size of the index option market in the U.S. grows rapidly over the years. During the first year of the sample Options Clearing Corp reports an average open interest of \$60 billion for standard options and LEAPS with maturities of less than six months that gradually decreases across maturities to \$200 million for maturities larger than two years. The corresponding figures in the last year of the sample are an open interest of \$1,400 billion for maturities of less than six months and of \$40 billion for maturities larger than two years.

I use a dataset provided by Market Data Express containing end-of-day S&P 500 index option data for CBOE traded European-style options and running from January 1990 to December 2013. I obtain the daily S&P 500 price and one-day total return indices from Bloomberg and combine them to calculate daily index dividend payouts; I then aggregate the daily payouts to a monthly frequency without reinvestment. To measure real bond prices I use zero-coupon TIPS yields with maturities of up to ten years from Gürkaynak et al. (2010). Since TIPS yields are unavailable during the 1990s, I also use Treasury yields from Gürkaynak et al. as a proxy available over the same sample period as I have dividend strips.¹⁷ Both nominal and real government bonds are computed on the last trading day of the month.

There are three major difficulties when extracting options implied prices through the put-call parity relation. First, quotes may violate the law of one price for reasons that include measurement errors such as bid-ask bounce or other microstructural frictions. Second, the synthesized prices are extremely sensitive to the choice of the nominal risk-free rate, which multiplies the strikes in the put-call parity relation; since strike prices are large numbers, any error in the interest rate will magnify in the synthetic prices (see also Boguth et al., 2012). Third, end-of-day data quote the closing value of the index, whose components trade on the equity exchange, and the closing prices of derivatives that are exchange-traded on a market that continues to operate for 15 minutes after the equity exchange closes. An asynchronicity of up to 15 minutes may therefore drive a wedge between the reported quotes of the index value and the option prices and bias the synthetic prices.

To address these difficulties, I use the no-arbitrage relation to extract *both* the risk-free rate and the strip price in a unique step. This approach produces the appropriate interest rate for synthetic replication as well as it allows for spotting violations of the law of one price (LOOP) on any trade date and for any maturity.¹⁸ Moreover, I follow Golez (2013) in using ten days of data at the end of each month to compute options implied prices at monthly frequency. He shows how this

¹⁶Other data sources that allow for extracting the prices of index dividend claims include index futures and dividend futures (Binsbergen et al., 2013). CME S&P 500 futures have expiration dates only for eight months in a quarterly cycle over most of the available sample and thereby maturities of less than two years. Dividend futures have the clear advantage over index options and futures that they directly reveal strip prices without the need for synthetic replication and they do so for longer maturities. However, for S&P 500 dividend futures only proprietary datasets are available covering over-the-counter trades for a relatively short sample (for example, the dataset studied by Binsbergen et al., 2013, starts in October 2002). The exchange-traded nature of options makes it more likely that the preferences embedded in their pricing reflect those of the average investor.

¹⁷I adjust the means of the Treasuries to match the means of TIPS yields over the 1999-2013 period; this adjustment provides the correct average welfare costs up to a second-order term.

¹⁸The intra-daily approach of Binsbergen et al. (2012), who use tick-level prices on put and call European options on the S&P 500 index to synthesize the dividend claims, has the clear advantage of exploiting information from more data points and avoids the asynchronicity problem. However, Binsbergen et al. may be sacrificing accuracy as they choose a

strategy reduces substantially the distance between intra-day and end-of-day options implied prices and thereby the potential effect of asynchronicities and other microstructural frictions. I bring further support to his claim by showing large correlations with the intra-day options implied prices extracted by Binsbergen et al. (2012) over the 1996-2009 period; the correlation of the 6-, 12-, 18- and 24-month equity prices with Binsbergen-Brandt-Koijen data are of .91, .95, .95 and .94, respectively, with a mean-zero difference in levels.¹⁹

1.3.1.1. Data selection and synthetic replication

I drop weekly, quarterly, pm-settled and mini options, whose non-standard actual expiration dates are not tagged. Index mini options with three-year maturities are traded since the 1990s but standard classes appear only in the 2000s; for this reason I follow Binsbergen et al. (2012) and focus on options of up to two-year maturity. I eliminate all observations with missing values or zero prices and keep only paired call and put options. I use mid quotes between the bid and the ask prices on the last quote of the day and closing values for the S&P 500 index.

On any date t , consider all available put-call pairs that differ only in strike price. For the i th strike price, X_i , $i = 1, \dots, I$, define the auxiliary variable

$$\begin{aligned}\mathcal{A}_{it}^{(n)} &\equiv P_t - C_{t,t+n} + \mathcal{P}_{t,t+n} \\ &= P_{d,t}^{(n)} + X_i P_{b,t}^{(n)}\end{aligned}$$

where the last equality holds by put-call parity (1.5), with $P_{d,t}^{(n)} = \sum_{j=1}^n E_t M_{t,t+j} D_{t+j}$ the no-arbitrage price of the next n periods of dividends. Therefore, if there are no arbitrage opportunities and the LOOP holds then the map $\mathcal{A}_t^{(n)} : X_i \mapsto \mathcal{A}_{it}^{(n)}$ is strictly monotonic and linear. In practice, the relation does not always hold without error across all strike prices available; as long as more than two strike prices are available for a given maturity, I use the no-arbitrage relation to extract $P_{d,t}^{(n)}$ and $P_{b,t}^{(n)}$ as the least absolute deviations (LAD) estimators that minimize expression

$$\sum_{i=1}^I |\mathcal{A}_{it}^{(n)} - P_{d,t}^{(n)} - X_i P_{b,t}^{(n)}|$$

for a given trade date t and maturity n . The cross-sectional error term accounts for potential measurement error (e.g., because of bid-ask bounce, asynchronicities, or other microstructural frictions).

In many instances, non-monotonicities in the auxiliary variable are concentrated in deep in-

proxy for the interest rate and do not develop an approach to spot and drop observations in which the LOOP may be importantly violated. In this sense, I bring an additional robustness check to their synthesized prices by showing how my approach extracts very similar prices; the nearly white-noise deviation between their estimate and mine over the comparable sample are likely a mixture of asynchronicities and different proxies for the interest rate (I find options implied interest rates with nearly perfect correlations with the corresponding LIBOR and Treasury rates but with different levels that lie about halfway between the two proxies).

¹⁹End-of-month data using a one-day window have slightly higher volatilities and correlations between .80 and .95. The median or the mean over a three-day window centered on the end-of-month trading day increases correlations to .87-.95; the marginal increase in correlations for window widths of more than ten days is nearly imperceptible.

and out-of-the-money options. Whenever I spot non-monotonicities for low and high moneyness levels I restrict the sample to strikes with moneyness between 0.7 and 1.1 before I run the cross-sectional LAD regression. Over most of the sample the strikes and the auxiliary variables are in a nearly perfect linear relation except for a few points that violate the LOOP. The LAD estimator is particularly appropriate to attach little weight to those observations as long as their number is small relative to the sample size of the cross-sectional regression. Accordingly, I drop all trade dates and maturities that associate with a linear relation between X_i and $\mathcal{A}_{it}^{(n)}$ that fails to fit at least a tenth of the cross-sectional size (with a minimum of five points) up to an error that is less than 1% of the extracted dividend claim price.

Figure 1.2 box-plots the size of the LOOP violations present in the sample which, for the most part, concentrate around errors of less than 1%; larger violations associate with the first years of the sample—probably because of a relatively low liquidity—and to years of greater volatility such as 2001 or the last years of the sample. Data previous to 1994 are more problematic by this metric (see also Golez, 2013) and I therefore exclude them altogether from the sample.

The procedure results in a finite number of matches, which I combine to calculate the prices of options implied dividend claims and nominal bonds by using the put-call parity relation. The number of cross-sectional observations available to extract the options implied prices of bonds and dividend claims increases over time (from medians of around 25 observations per trading day up to more than 100) as the market grows in size and declines with the options maturity. Of the resulting extracted prices I finally discard all trading days that associate with prices that are non-increasing in maturity, as they would represent arbitrage opportunities. Overall, my selection method based on LOOP violations excludes almost a fifth of the available put-call pairs.

Finally, to obtain monthly implied dividend yields with constant maturities, I follow Binsbergen et al. (2012) and Golez (2013) and interpolate between the available maturities. As advocated by Golez (2013), I then construct monthly prices using ten days of data at the end of each month (namely, I consider the average price over a ten-day window centered on the end-of-month value). Figure 1.3a plots the prices of the synthetic dividend claims.

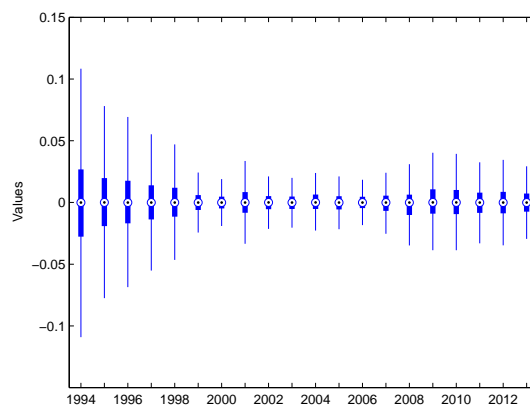
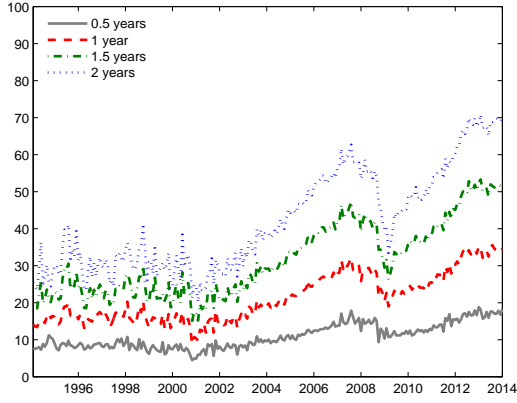
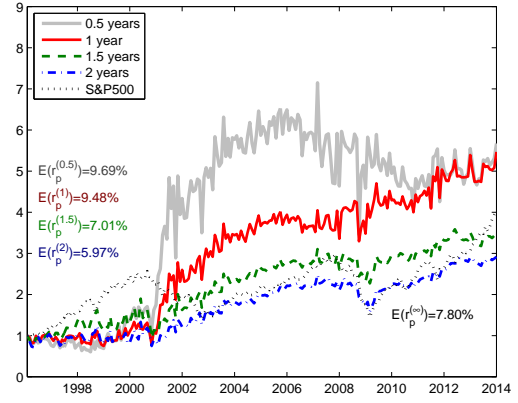


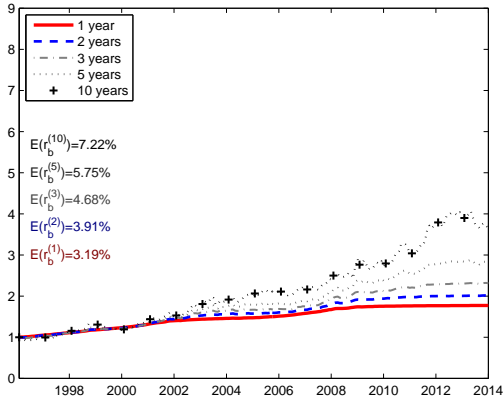
Figure 1.2: Law-of-one-price violations, in percent of the associated options implied dividend claim price; only observations not filtered out by my data selection approach are included.



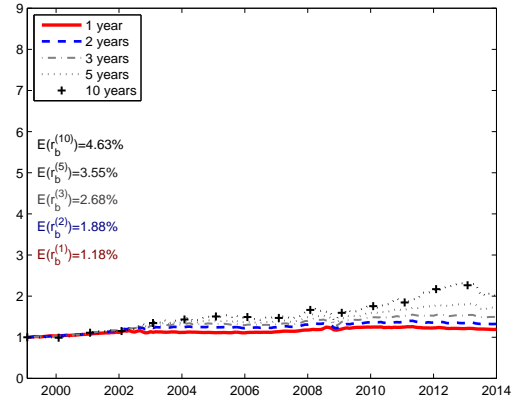
(a) Price of the next n years of dividends.



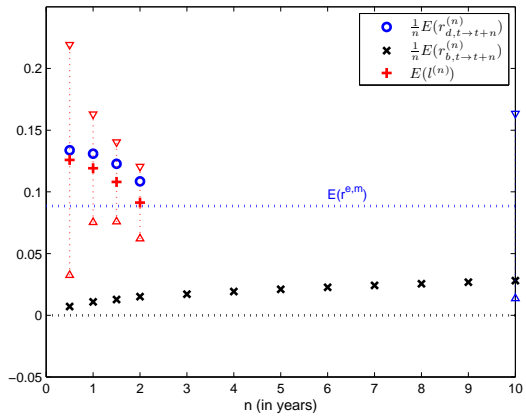
(b) Monthly cumulated returns on equities.



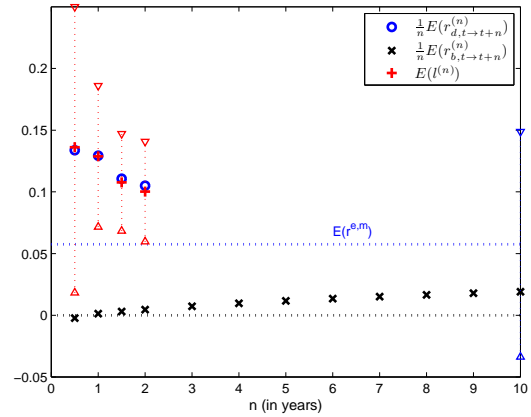
(c) Monthly cumulated returns on nominal bonds.



(d) Monthly cumulated returns on real bonds.



(e) Average term structures of equity, interest rates (mean-adjusted Treasuries) and welfare costs (with block bootstrapped 95% confidence intervals).



(f) Average term structures of equity, interest rates (TIPS) and welfare costs (with block bootstrapped 95% confidence intervals), 1999-2013.

Figure 1.3: Term structures of equity, interest rates and welfare costs over the last two decades. The term structure of equity is synthesized from index options; the term structure of interest rates use Gürkaynak-Sack-Wright data.

1.3.1.2. Mean

To measure the cost of uncertainty I consider dividend strip data and thereby disregard the difference between aggregate consumption and dividends. Consumption and dividends coincide in an endowment economy. Moreover, although definition (1.1) is in terms of consumption, I show in section 1.4 how in a production economy the definition is in terms of dividends. This result has the convenient consequence that the term structure of welfare costs links to the observable term structure of equity rather than to the unobservable term structure of consumption equity, even in a production economy.

I follow Binsbergen et al. (2012) and focus on a semestral periodicity; the first strip pays off the next six months of dividends, the second strip the dividends paid out six to twelve months out, and so on. I measure the hold-to-maturity return on the first semestral strip as the return on a six-month buy-and-hold strategy that pays off the next six months of dividends; namely, I roll over three times on a two-month buy-and-hold strategy that goes long in the six-month strip.²⁰ Accordingly, I measure the hold-to-maturity return on the n -semester strip as the return for holding for $n - 1$ semesters a n -semester strip times the semestral return on the first semestral strip.

Risk premia are large for short-duration equities and small for short-duration bonds. To address the potential concerns raised by Boguth et al. (2012) that microstructural frictions could cause spuriously large arithmetic high-frequency returns on synthetic dividend claims, I report log returns and hold-to-maturity returns on strategies with maturities between 0.5 and 2 years, which Boguth et al. argue are much less biased by microstructure effects related to highly-leveraged positions.²¹

Figure 1.3b illustrates the size of average annualized monthly log returns on six-month strategies over different subsamples by plotting the cumulated return on an investment strategy that goes long on January 31, 1996 by a dollar on a claim to the next n years of dividends, holds the investment for six months and then rolls over the position. Monthly average log returns are large and positive for short-duration equities (close to ten percent for claims to the next semester and year of dividends) and larger than the return on the index. The economic significance of the large returns on short-term equities becomes even stronger once we note that the initial and final years of the 1994-2013 period are years in which the index performed particularly well.

Figures 1.3c and 1.3d plot the analogous cumulated monthly returns on six-month bond strategies long a dollar on zero-coupon bonds with maturities between six months and ten years; average returns steadily increase in maturity across nominal as well as real government bonds, consistent with an upward-sloping average term structure of interest rates.

The annualized average hold-to-maturity return over the available dataset is of 13.4%, 13.1%, 12.3% and 10.9% for a strategy that goes long in the first to fourth semestral dividend strip,

²⁰This strategy, as opposed to holding to maturity a six-month strip, has the advantage of being robust to potential bias in prices resulting from preannouncement. Golez (2013) raises concerns that equity prices of up to three-month maturity may be biased as a result of firms routinely preannouncing part of their dividend payouts, which would lower their riskiness. Options implied data on the one-month and two-month strips display a negative risk premium (as opposed to a zero risk premium as would be expected of a riskless asset) while the three-month strip pays off an extremely large and positive average return over the sample. Along with the preannouncement of part of the dividend payouts, potential reasons for the likely biased properties of very short-term dividend claims include the microstructural issues described by Boguth et al. (2012) (the shorter the maturity the higher the leverage).

²¹The annual strategy on the six-month claims holds the claim for six months and then rolls over the same strategy.

respectively, and of 2.9%, 3.0%, 3.1% and 3.3% for a strategy that goes long in the first to fourth semestral Treasuries, respectively. Over the 1999-2013 sample period, in which TIPS data are available, the hold-to-maturity strategies pay off average returns of 17.0%, 14.0%, 11.5% and 10.5% for long positions in the short-term equities and 0.2%, 0.6%, 0.7% and 0.9% for long positions in the first to fourth semestral TIPS, respectively. These numbers compare with an average return on an annual buy-and-hold strategy on the market index of 10.2% over the 1994-2013 period and of 6.5% during 1999-2013.

As in Binsbergen et al. (2012), the economic significance of this evidence is that the average term structure of equity is downward-sloping owing to the fact that the equity premium is a linear combination of the entire sequence of dividend strip premia.

1994-2013					1999-2013			
	0.5 years ($n = 1$)	1 year ($n = 2$)	1.5 years ($n = 3$)	2 years ($n = 4$)	0.5 years ($n = 1$)	1 year ($n = 2$)	1.5 years ($n = 3$)	2 years ($n = 4$)
$l_t^{(n)}$								
Mean	0.1259 (0.0325) [0.0689]	0.1192 (0.0160) [0.0303]	0.1081 (0.0120) [0.0204]	0.0913 (0.0097) [0.0180]	0.1362 (0.0408) [0.0906]	0.1288 (0.0196) [0.0412]	0.1077 (0.0134) [0.0255]	0.1002 (0.0107) [0.0243]
$L_t^{\{1,...,n\}}$								
Mean	0.1259 (0.0325) [0.0689]	0.1184 (0.0190) [0.0398]	0.1113 (0.0139) [0.0276]	0.1036 (0.0107) [0.0210]	0.1362 (0.0408) [0.0906]	0.1299 (0.0239) [0.0535]	0.1177 (0.0170) [0.0375]	0.1110 (0.0129) [0.0295]
$l_t^{(1)}, 1 \text{ period} = n \text{ semesters}$								
Mean	0.1259 (0.0325) [0.0689]	0.0905 (0.0187) [0.0444]	0.0782 (0.0126) [0.0305]	0.0711 (0.0097) [0.0243]	0.1362 (0.0408) [0.0906]	0.1124 (0.0230) [0.0571]	0.1032 (0.0154) [0.0384]	0.0972 (0.0116) [0.0306]
Equity premium, semestral								
Mean		0.0885 (0.0169) [0.0398]				0.0575 (0.0207) [0.0471]		

Table 1.1: Options implied average term structure of the welfare cost of uncertainty. $l^{(n)}$ is the cost of a marginal increase in uncertainty in n -semester ahead cashflows. $L^{\{1,...,n\}}$ is the cost of a marginal increase in uncertainty in 1 to n -semester ahead cashflows. The third panel reports the cost of a marginal increase in one-period ahead uncertainty, $l^{(1)}$, for different period lengths. The short sample (1999-2013) uses TIPS yield data to measure the term structure of real interest rates. The full sample (1994-2013) uses Treasury yield data, whose means are adjusted to match the corresponding TIPS means over the 1999-2013 period, as a proxy for the term structure of real interest rates. The equity premium is the average six-month buy-and-hold return on the S&P 500 index in excess over the six-month risk-free rate. Bootstrapped standard errors use block sizes of one (in parentheses) and ten (in brackets) observations.

Table 1.1 reports the point estimates for the term structure of welfare costs. The welfare costs of aggregate uncertainty around semestral cashflows 0.5, 1, 1.5 and 2 years out are of 12.6%, 11.9%, 10.8% and 9.1%, respectively. I then use equation (1.3) to compute the costs of uncertainty around multi-period cashflows. Namely, I estimate average welfare costs of 11.8% associated with up to one-year ahead uncertainty, of 11.1% with up to 18-month uncertainty, and of 10.4% with up to

two-year ahead uncertainty, respectively. Restricting the attention to the 1999-2013 period, over which TIPS data are available to measure real interest rates, the estimated welfare costs around semestral cashflows 0.5, 1, 1.5 and 2 years out are of 13.6%, 12.9%, 10.8% and 10.0%, respectively; welfare costs associated with up to 1-, 1.5- and 2-year ahead cashflow uncertainty are of 13.0%, 11.8% and 11.1%, respectively. These figures compare to an average six-month buy-and-hold equity premium over the two sample periods of 8.8% (1994-2013) and of 5.7% (1999-2013).

Additionally, I compute the welfare cost of one-period ahead uncertainty for different periodicities, from semestral to biennial. These estimates complement the evidence about the term structure in a way that bypasses the somewhat arbitrary choice of the semestral periodicity of the strips. I find comparable results; the average cost of one-year ahead uncertainty over the two samples is of 9.0% (1994-2013) and 11.2% (1999-2013), whereas the cost of uncertainty over the next two years is of 7.1% and of 9.7%.

Figures 1.3e and 1.3f plot the point estimates for the term structures of equity, interest rates, and welfare costs. The term structures of equity and interest rates report the semestral zero-coupon equities and real bonds and the corresponding per-period hold-to-maturity returns for different maturities. The dotted lines represent the counterfactual case of flat term structures of equity and interest rates, expressed in excess over the first bond yield. To report estimates for the term structure of welfare costs, I use equation (1.4) to compute hold-to-maturity excess returns on the available semestral strips whose unconditional means are the first components of the term structure of welfare costs. The figures also show block-bootstrapped one-sided critical values based on bootstrap- t percentiles corresponding to a five-percent size for the means of $l_t^{(n)}$ and the six-month market return; the block size of ten observations is slightly larger than the number of lags after which the correlogram of the underlying returns becomes negligible. I can reject the hypothesis that the term structure components are zero over both samples.

The evidence suggests a *downward*-sloping term structure of welfare costs, driven *both* by a downward-sloping term structure of equity and by an upward-sloping term structure of interest rates.

1.3.1.3. Volatility

The components of the term structure of welfare costs are volatile and pro-cyclical.

Present-value logic with time-varying expected returns and expected dividend growth imply that dividend yields contain information about both state variables (Golez, 2013). Since the same states would drive the risk premia that constitute the term structure of welfare costs I can use the semestral equity yields to signal variation in the term structure of welfare costs. In line with Binsbergen et al. (2013) and motivated by theoretical models such as Lettau and Wachter (2011) I consider a one-factor specification for these risk premia. To capture the factor I extract the first principal component of the semestral equity yields, which captures 82% of their volatility, and use it to forecast the hold-to-maturity excess returns whose ex ante values measure the welfare costs. Motivated by theoretical considerations, I control for the presence of additional factors by including the second principal component of equity yields (these two principal components account for 95% of the variance in equity yields) and the first principal component of bond yields.²²

²²Section 1.5 and the appendix show how the model of Lettau and Wachter (2011) implies that these three variables

	$r_{t \rightarrow t+1}^{e,(1)}$		$\frac{1}{2}r_{t \rightarrow t+2}^{e,(2)}$		$\frac{1}{3}r_{t \rightarrow t+3}^{e,(3)}$		$\frac{1}{4}r_{t \rightarrow t+4}^{e,(4)}$		$r_{t+1}^{e,m}$	
Constant	0.083	0.083	0.094	0.094	0.084	0.084	0.069	0.069	0.072	0.072
	[0.031]	[0.022]	[0.015]	[0.015]	[0.014]	[0.013]	[0.012]	[0.012]	[0.034]	[0.033]
$pc_{d,t}^1$	0.369	0.366	0.150	0.150	0.077	0.079	0.064	0.066		-0.094
	[0.100]	[0.056]	[0.028]	[0.032]	[0.024]	[0.023]	[0.019]	[0.017]		[0.056]
$pc_{b,t}^1$		0.095		-0.017		-0.067		-0.111		0.104
		[0.145]		[0.088]		[0.072]		[0.057]		[0.118]
$pc_{d,t}^2$		0.703		-0.063		-0.220		-0.152		
		[0.144]		[0.065]		[0.043]		[0.037]		
dp_t^m									0.345	0.318
									[0.152]	[0.194]
R^2	0.353	0.502	0.254	0.259	0.106	0.229	0.105	0.204	0.098	0.147
$\frac{std(E_t r)}{E(r)}$	2.68	3.19	1.00	1.01	0.57	0.85	0.57	0.79	1.10	1.35
$E(r)$	0.0832		0.0941		0.0837		0.0694		0.0719	

Table 1.2: Predictive regressions on hold-to-maturity semestral strip returns and on the semestral buy-and-hold market return. Annualized log returns in excess over the riskless return over the holding period. Regressors are the first two principal components of the semestral equity yields, pc_d^1 and pc_d^2 , the first principal component of up to ten-year bond yields, pc_b^1 , and the market dividend yield, dp^m . Monthly data, 1996m1-2013m12. Newey-West standard errors to correct for overlapping.

Thus, since excess returns are forecastable, the cost of uncertainty varies over time and considerably so. Table 1.2 shows a standard deviation of expected returns about as large as the already large level. Note how the forecasting regressions may miss some important return predictors. Therefore, since the variance of expected returns is increasing in the number of predictors, the estimates in table 1.2 understate the actual volatility of the cost of uncertainty. The cost of short-run cashflow uncertainty is huge at some junctures of the business cycle.

Figure 1.4 plots the estimated time series of the term structure of the welfare cost of uncertainty over time. The cost of uncertainty rises dramatically during the dot-com crash and the period immediately preceding the early 2000s recession as well as during the most recent recession as declared by the National Bureau of Economic Research. Moreover, the premium to hedge uncertainty six months out is considerably larger than the premium to hedge longer-run uncertainty. The estimated term structure remains downward-sloping during the downturns whereas it appears considerably flatter and even upward-sloping in normal times.

The evidence is consistent with people being highly sensitive to cashflow stability in bad times, in particular to short-run stability. The evidence indicates that some systematic stabilization policy to smooth the cost of uncertainty likely is a macroeconomic priority, especially in the short run and in particular states of the economy such as downturns.

1.3.2. Term structures in some consumption-based asset pricing models

Since the available sample allows for estimating only the first few components of the term structure of welfare costs at semestral frequency, I now turn to a model-based approach to capture

are sufficient to reveal the factor that drives the term structure of welfare costs.

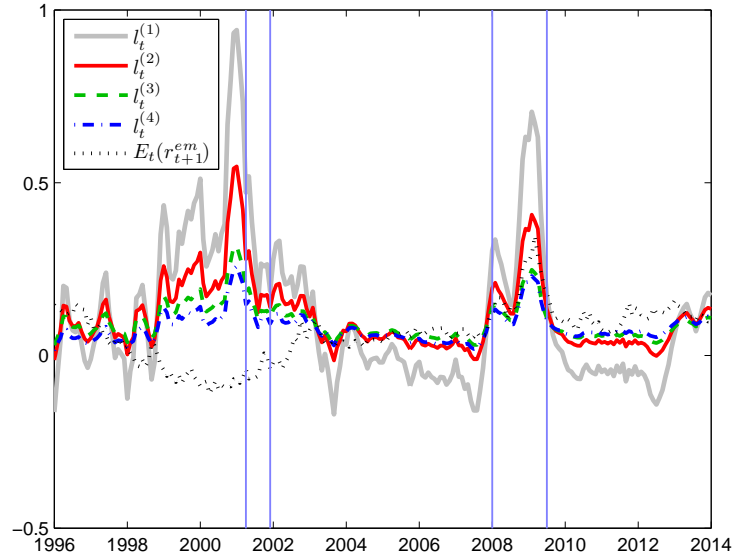


Figure 1.4: Semestral term structure of welfare costs and semestral equity premium, conditional on the information set spanned by equity and bond yields; annualized premia. Regressors are the first two principal components of semestral equity yields and the first principal component of up to ten-year bond yields. The semestral excess return on the index is additionally regressed on the market dividend yield. The vertical lines indicate business-cycle peaks and troughs as declared by the NBER (March–November 2001 and December 2007 to June 2009).

and quantify the rest of the term structure. The evidence in subsection 1.3.1 helps to identify a suitable model. The extant asset pricing literature offers several models, already calibrated to match several stylized asset pricing facts, that have implications for the term structure of welfare costs. Unfortunately, from a structural perspective, replicating a downward-sloping term structure of equity and an upward-sloping term structure of interest rates is problematic (Lettau and Wachter, 2007; Binsbergen et al., 2012; Croce et al., 2012). I therefore have to discard a structural explanation to answer the question I am interested in and be satisfied with a quasi-structural model. I thus turn to the model of Lettau and Wachter (2011), which is a parsimonious quasi-structural framework able to capture the term structures of equity and interest rates in line with the evidence in subsection 1.3.1.

1.3.2.1. Structural approach

Table 1.3 and figure 1.5 show the implications of some of today's leading consumption-based asset pricing models for the three term structures and for the welfare cost of uncertainty and the equity premium. I consider the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the long-run risk model under limited information of Croce et al. (2012), the recursive preferences of Tallarini (2000) and Barillas et al. (2009), the rare disasters model of Gabaix (2012), and the quasi-structural model of Lettau and Wachter (2011). In studying the term structures in the different asset pricing models, I consider the original calibrations, which the authors choose to match some asset pricing facts. Note that alternative calibrations and refinements of the models are possible and some of the model-based predictions about the term structures of equity and interest rates might change accordingly. The technical appendix works out

the details of each model. I refer to the original writings for a list of the stylized asset pricing facts each model is matched to.

The habit formation model of Campbell and Cochrane (1999) predicts a flat term structure of interest rates and an upward-sloping term structure of equity. The term structure of interest rates is driven by a particular calibration of the time-varying risk aversion that produces a constant risk-free rate. The term structure of equity is instead driven by the positive correlation between the pricing factor—shocks to consumption growth—and dividend growth, and by the perfectly negative correlation between the pricing factor and the shocks to the price of risk, which decreases as consumption grows away from the external habit. Since dividend strips load negatively on shocks to the price of risk, and the more so the longer the maturity, people command a greater risk premium to bear long-run dividend strip risk. Under the baseline calibration, the model of Campbell and Cochrane predicts a marginal cost of all fluctuations of 4.4% and an equity premium of 6.8%.

The long-run risk model of Bansal and Yaron (2004) generates an upward-sloping term structure of equity and a downward-sloping term structure of interest rates. Bansal and Yaron introduce rich dynamics in consumption growth, which is driven both by shocks to expected consumption growth and to consumption volatility. Epstein-Zin-Weil utility then makes all shocks to the consumption opportunity set show up as pricing factors. In the calibration of Bansal and Yaron, long-run dividend strips load more heavily on the shocks to the consumption opportunity set and therefore are more risky, as long as the elasticity of intertemporal substitution is larger than one. In the model the risk-free rate is driven by shocks to the predictable component of consumption, which is positively priced; since long-run zero-coupon bonds load less on this state than the risk-free rate, they provide long-run insurance. This property explains the downward-sloping term structure of interest rates. The quantitative implications of the long-run risk model is a marginal cost of all fluctuations of 7.7% and an equity premium of 4.9%.

Croce et al. (2012) consider the long-run risk model of Bansal and Yaron (2004) and change the information structure. Under limited information, not all shocks to the cashflow opportunity set are observable. The shocks that are priced are therefore a linear combination of both short-run and long-run cashflow shocks. Then, since long-run shocks have a relatively small volatility, long-run dividend strips load less on the shocks that are priced under limited information than short-run dividend strips. This strategy allows for generating a downward-sloping term structure of equity. Notably, at the time of the present writing Croce et al. (2012) is the only state-of-the-art structural model that generates a downward-sloping term structure of equity for both holding-period and hold-to-maturity returns; however, the curvature is not enough quantitatively, at least under the baseline calibration, it still predicts a downward-sloping term structure of interest rates, and it works in a world in which risk premia are not time-varying. The model predicts a marginal cost of all fluctuations of 8.8%, against a predicted equity premium of 6.6%.

The ambiguity averse multiplier preferences in Barillas et al. (2009) and the recursive preferences of Tallarini (2000) yield two flat term structures which imply the equality between the equity premium and the cost of fluctuations. The unitary elasticity of intertemporal substitution that characterizes the recursive preferences of Tallarini (2000) and the robust control literature implies constant dividend yields, as discount-rate effects exactly offset cashflow effects in pricing equity claims; the random walk in consumption in turn implies constant interest rates and thereby a flat bond term structure. The multiplier preferences of Barillas et al. (2009) and the observationally

equivalent model of Tallarini (2000) predict a marginal cost of all fluctuations of 2.0% and an equity premium of about the same size.

Finally, the rare disasters model of Gabaix (2012) produces two flat term structure of holding-period returns but a slightly downward-sloping term structure of hold-to-maturity equity returns. The intuition behind the flat term structure of holding period returns in the rare disasters model is that different dividend strips have the same exposure to the disaster event, whose probability is independent of the cashflow shocks that are priced. However, the mean-reversion in the state that drives equity prices makes long-duration equities load slightly less on it than short-duration equities, which produces a negative slope in the term structure of hold-to-maturity returns. The model implies a marginal cost of all fluctuations of 6.9% and a slightly larger equity premium.

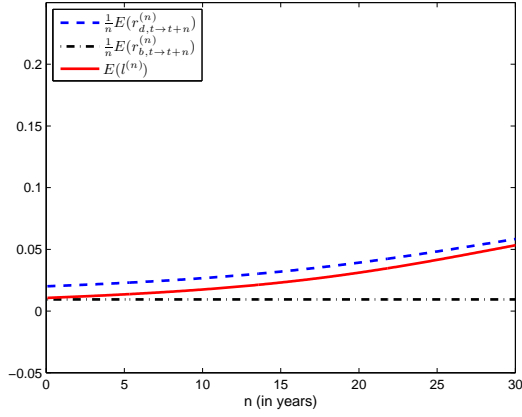
	$L^{\bar{N}}$	$\ln E(R^{e,m})$
Campbell and Cochrane (1999)	4.42	6.81
Bansal and Yaron (2004)	7.68	4.90
Croce et al. (2012)	8.82	6.56
Barillas et al. (2009)	2.00	1.92
Gabaix (2012)	6.87	7.89
Lettau and Wachter (2011)	2.79	7.18

Table 1.3: Mean marginal cost of uncertainty and equity premium (in percent per year) for different consumption-based asset pricing models. I consider the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the long-run risk model under limited information of Croce et al. (2012), the ambiguity averse multiplier preferences of Barillas et al. (2009), the rare disasters model of Gabaix (2012), and the quasi-structural model of Lettau and Wachter (2011).

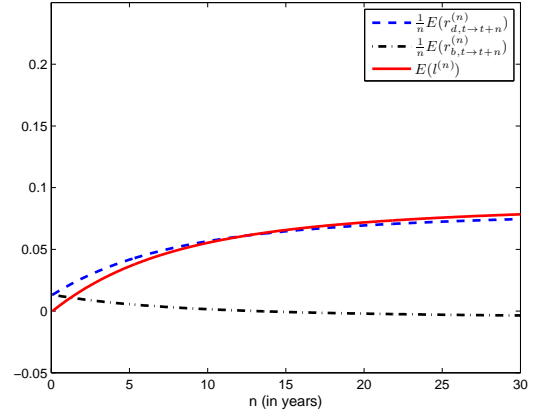
1.3.2.2. Quasi-structural approach

I next turn to the quasi-structural model of Lettau and Wachter (2011), which is designed to capture a downward-sloping term structure of equity and an upward-sloping term structure of interest rates. They combine lognormal pricing formulas and a loglinear state-space system with a price of risk, x_t , linear in the states of the economy. The exponential-Gaussian setting is particularly tractable to study term structures because their equilibrium values have closed-form solutions.

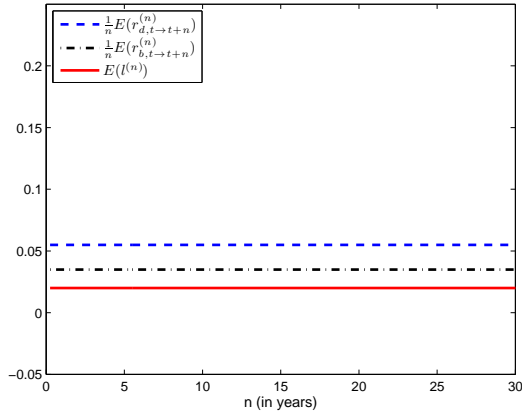
Without micro-founding it, Lettau and Wachter (2011) directly specify a stochastic discount factor, whose existence is guaranteed by the no-arbitrage theorems. To keep low the number of degrees of freedom in the model, they assume a single conditional pricing factor perfectly related to short-run cashflow shocks and a single state driving the price of risk. They then assume a zero correlation between cashflow and discount-rate shocks and show how that property seems crucial to generate a downward-sloping term structure of equity (see also Lettau and Wachter, 2007). To match the downward-sloping term structure of equity, Lettau and Wachter (2011) assume that the predictable component of cashflows is negatively related to priced shocks. Long-run dividend strips thus contain a component that provides long-run insurance. The independence of discount-rate shocks and cashflows shocks then avoids that the negative load of long-run dividend strips on the state that drives the price of risk offsets the long-run insurance effect.



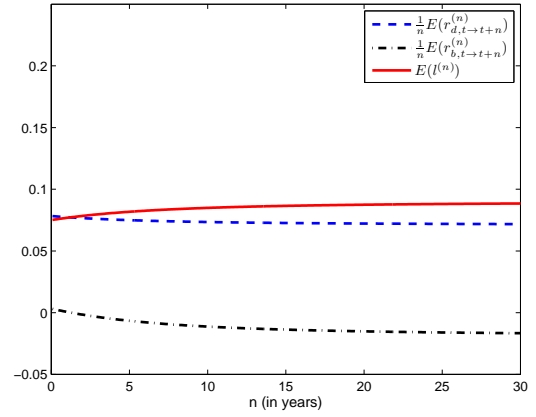
(a) *Habit formation (Campbell and Cochrane, 1999).*



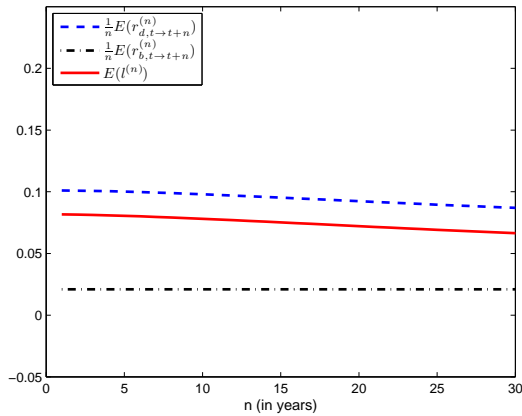
(b) *Long-run risk (Bansal and Yaron, 2004).*



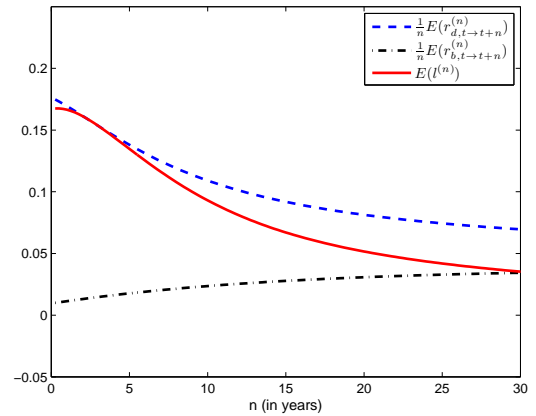
(c) *Ambiguity aversion (Barillas et al., 2009), Epstein-Zin-Weil log utility (Tallarini, 2000).*



(d) *Long-run risk (limited information) (Croce et al., 2012).*



(e) *Rare disasters (Gabaix, 2012).*



(f) *Quasi-structural model (Lettau and Wachter, 2011).*

Figure 1.5: The term structures of equity, interest rates and welfare costs of uncertainty in some consumption-based asset pricing models.

Finally, since only short-run cashflow shocks are priced, Lettau and Wachter manage to replicate the upward-sloping term structure of interest rates by assuming that shocks to the state driving the risk-free rate are negatively correlated with priced shocks. Since long-run zero-coupon bonds are less exposed to this state than short-run bonds, the assumption generates a positive bond risk premium as the maturity increases.

The model of Lettau and Wachter predicts a marginal cost of total uncertainty of 2.8% and an equity premium of 7.2%.

1.4. Robustness

This section examines the robustness of the results of section 1.3 along two dimensions. First, even though only the model of Lettau and Wachter (2011) captures the term structure features I am interested in, propositions 1' and 1'' show how all of today's leading consumption-based asset pricing models are unanimous in producing a high marginal cost of total uncertainty.

Second, I relax some important assumptions made in subsection 1.3.1. Namely, I show how the evidence of subsection 1.3.1 still measures the cost of uncertainty as we move, on the one hand, to a production economy in which the notions of consumption and market equity no longer coincide, and on the other hand, to a setting with consumers characterized by heterogeneous preferences and possibly uninsurable idiosyncratic risk.

1.4.1. Robustness across models

Even though I can only turn to the quasi-structural model of Lettau and Wachter (2011) to capture the term structure features I am interested in, I can draw some lessons that are robust across all asset pricing models considered.

Table 1.3 shows how the marginal cost of uncertainty in the entire consumption process is above two percent in each of today's leading asset pricing models. This robustness across models is in line with the result by Alvarez and Jermann (2004) that a model that is consistent with the main stylized asset pricing facts must increase the original estimates by Lucas (1987) by two orders of magnitude.

1.4.2. Market equity or consumption equity?

The definition of cost of fluctuations is in terms of consumption equity. The evidence I consider is in terms of market equity. These two notions of equity may not perfectly substitute. In fact, along with the choice of the moving-average filter, the other main empirical challenge of Alvarez and Jermann (2004) is to find a proxy for the price of a claim to the entire consumption stream.

The critique does not bite in an endowment economy, in which $C_t = D_t$, and the theoretical definition of cost of uncertainty can be stated in terms of dividends in the first place. However, in a production economy the equality breaks down. Fortunately, under mild general equilibrium assumptions one can save the link between the cost of fluctuations and market equity.

A natural approach in a production economy is to consider preferences that depend also on labor effort, N_t . Moreover, to model theoretically a difference between consumption and dividends I rely on assumption 1. The definition of dividends as current aggregate profits is standard in the Q literature (Hayashi, 1982). The optimality condition implies that there are no distortions in the

consumption-side of the economy that generate so-called labor wedges (such as, for example, the aggregate wage markup considered by Galí et al., 2007).

Assumption 1. Assume the first-best optimality condition $W_t = -(\partial U_t / \partial N_t) / (\partial U_t / \partial C_t)$, i.e., the marginal rate of substitution between labor and consumption equals the relative price—the real wage rate, W_t . Assume that the aggregate firm pays off all profits as dividends $D_t = Y_t - W_t N_t - I_t$, where Y_t is total output and I_t are gross investments made by the firm. Assume the market-clearing condition $Y_t = C_t + I_t$.

Definition (Marginal cost of uncertainty, production economy). I define the cost of fluctuations as

$$\begin{aligned} E_t U\left(\{(1 + \mathcal{L}_t^N(\theta))^n C_{t+n}\}, \{N_{t+n}\}, \{X_{t+n}\}\right) = \\ = E_t U\left(\{\theta E_t C_{t+n} + (1 - \theta) C_{t+n}\}, \{\theta \bar{N}_{t+n} + (1 - \theta) N_{t+n}\}, \{X_{t+n}\}\right) \end{aligned} \quad (1.6)$$

where stable hours worked \bar{N}_t are defined as $W_{t+n} \bar{N}_{t+n} \equiv E_t(W_{t+n} N_{t+n})$, i.e., stable hours provide a stable labor income. In a production economy, \mathcal{L}_t measures the cost of aggregate uncertainty around consumption and labor income, i.e., how much people would pay to hedge against uncertainty in consumption and in labor income.

Proposition 1'. Under assumption 1, the marginal cost of uncertainty around all coordinates $n \in \mathcal{N}$ is

$$L_t^N = \sum_{n \in \mathcal{N}} \omega_{n,t} l_t^{(n)}$$

with weights $\{\omega_{n,t}\}$ and term structure components $\{l_t^{(n)}\}$ defined by

$$\begin{aligned} \omega_{n,t} &= \frac{n E_t(M_{t,t+n} D_{t+n})}{\sum_{n \in \mathcal{N}} n E_t(M_{t,t+n} C_{t+n})} \\ l_t^{(n)} &= \frac{1}{n} \left(\frac{E_t(M_{t,t+n}) E_t(D_{t+n})}{E_t(M_{t,t+n} D_{t+n})} - 1 \right) \end{aligned}$$

In this production economy, as I distinguish between market equity and consumption equity, only the weights $\{\omega_{n,t}\}$ depend on consumption equity. Most importantly, the term structure components $\{l_t^{(n)}\}$ remain functions of market equities, which therefore exclusively control the slope of the term structure of welfare costs. In this context, the robust empirical finding about the term structure of the welfare cost of uncertainty is its slope. As I depart from the equality between market and consumption equity, an unobservable component appears (via weights $\omega_{n,t}$) in the level of the cost of uncertainty L_t^N . This issue adds to the fact that available direct data on both dividend strips and inflation-indexed bonds cover only a short period of time and thereby leave fairly large confidence intervals around the estimated level of the term structure of welfare costs. However, the downward slope in the term structure is robust to the departure from the equality between market and consumption equity.

1.4.3. Do we need identical consumers?

The term structure of marginal costs of uncertainty remains well-defined even in a heterogeneous-agent incomplete-market setting.

Consider agents with heterogeneous preferences, U_i , and an idiosyncratic consumption component, $\varepsilon_{i,t}$, driving their consumption stream, $C_{i,t} = C_t + \varepsilon_{i,t}$. Agents can only trade the entire term structures of dividend strips and zero-coupon bonds and therefore face possibly uninsurable idiosyncratic risk.

Definition (Marginal cost of uncertainty, heterogeneous-agent incomplete-market economy). In line with Alvarez and Jermann (2004), I define the cost of fluctuations as

$$E_t U_i \left(\{ (1 + \mathcal{L}_{i,t}^N(\theta))^n C_{t+n} + \varepsilon_{i,t+n} \} \right) = E_t U_i \left(\{ \theta (E_t C_{t+n} + \varepsilon_{i,t+n}) + (1 - \theta) C_{i,t+n} \} \right)$$

In this incomplete-market setting, \mathcal{L}_t measures the cost of uncertainty around the systematic part of the i th agent's consumption, i.e., how much people would pay to hedge against uncertainty in their systematic consumption component.

By the absence of arbitrage opportunities, the projection of their marginal rates of substitution on the payoff space is the same across people, so that their valuations of available asset prices are equal. Since all agents have access to the entire term structures of dividend strips and zero-coupon bonds, they end up equalizing their valuation of strips. Therefore,

Proposition 1''. *The marginal cost of uncertainty around all coordinates $n \in N$ for agent i is*

$$L_{i,t}^N = \sum_{n \in N} \frac{n E_t(M_{t,t+n}^i C_{t+n})}{\sum_{n \in N} n E_t(M_{t,t+n}^i C_{t+n})} \times \frac{1}{n} \left(\frac{E_t(M_{t,t+n}^i) E_t(C_{t+n})}{E_t(M_{t,t+n}^i C_{t+n})} - 1 \right) = L_t^N$$

and is constant across agents.

1.5. Policy implications

Lucas (1987, 2003) introduced the notion of cost of aggregate uncertainty as an indicator of the macroeconomic priorities. Similarly, the term structure of the marginal cost of uncertainty requires little structure to reveal a tradeoff between growth and macroeconomic stability as well as a tradeoff between macroeconomic stability at different time horizons.

1.5.1. Macroeconomic priorities

The negative slope of the term structure of the cost of fluctuations is a robust feature over the available sample, driven both by the upward-sloping term structure of interest rates and by the downward-sloping term structure of equity. The immediate consequence for policy-makers is that, on average, short-run economic stability should be a primary focus of attention. On the one hand, short-run aggregate consumption stability is a greater priority than long-run stability. On the other, the high level of the cost of fluctuations reveals that people are willing to trade a large amount of growth against short-run consumption stability.

Table 1.4 reports the cost of short- and long-run fluctuations over different coordinate sets as captured by the quasi-structural model of Lettau and Wachter (2011). An increase in consumption uncertainty by a fraction θ over a ten-year period has a marginal cost of more than 12θ percentage points of growth per year during the decade. These numbers compare to smaller yet non-trivial marginal benefits of long-run stability, which tend to zero as the stabilization becomes asymptotic, and are in line with the options implied estimates in table 1.1.

\mathcal{N}	$L^{\mathcal{N}}$	$L^{\overline{\mathcal{N}} \setminus \mathcal{N}}$
up to 1 year	16.70	2.77
up to 2 years	16.44	2.75
up to 3 years	16.04	2.71
up to 5 years	15.00	2.61
up to 10 years	12.31	2.35
up to 20 years	8.67	1.93
$\overline{\mathcal{N}}$	2.79	0

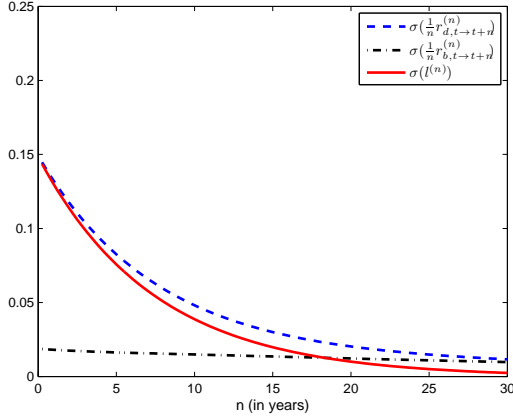
Table 1.4: Marginal cost of fluctuations at all periodicities $n \in \mathcal{N}$. Lettau and Wachter (2011) model-based estimates

Furthermore, the volatility of the cost of fluctuations at different frequencies is large so that this measure of the macroeconomic priorities varies substantially across the business cycle. The evidence in table 1.2 and figure 1.4 shows how the volatility of the first two term structure components is at least as large as the level. The cost of fluctuations is huge at some junctures of the business cycle, particularly during downturns.

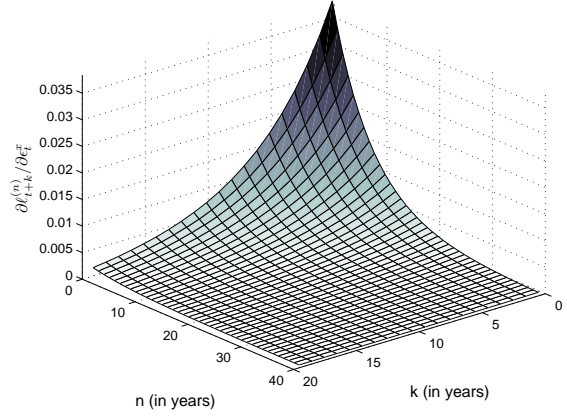
The volatility of the term structure as captured by the model of Lettau and Wachter, reported in figure 1.6a, confirms the direct evidence. The standard deviation of the cost of fluctuations at short periodicities is large and decays over long horizons. In their model, the term structure of welfare costs follows a one-factor structure driven entirely by movements in the time-varying market price of risk, as shown in the appendix. Figure 1.6b shows the impulse-response of the term structure of the cost of uncertainty after a one standard deviation discount-rate shock. News to the price of risk forecast a transitory change in the state of the economy that makes consumers temporarily increase their aversion to uncertainty, hence the cost of uncertainty and the risk premium they require to hold equities. The positive initial effect then decays across maturities and over time.

A policy-maker interested in following the developments of the macroeconomic priorities over the business cycle would therefore find useful some information about the state that drives the term structure of welfare costs. Since the term structure components are *expected* excess returns, present-value logic tells us that return predictors must reveal the state. Therefore, news to return predictors signal the tradeoff between growth and macroeconomic stability at each juncture of the business cycle and reveal what periodicities are the priority as well as they forecast future developments of the macroeconomic priorities.

I turn once more to the model of Lettau and Wachter (2011) to put the point formally. The model suggests that a sufficient information set to reveal news to the market price of risk is made by two equity yields and a bond yields. This theoretical consideration motivates the predictive regressions of section 1.3 that extract the time series of the term structure of welfare costs from the



(a) Model-based estimates of volatilities in the model of Lettau and Wachter (2011).



(b) Impulse response $\partial \ell_{t+k}^{(n)} / \partial \varepsilon_t^x$ to a one-standard deviation discount-rate shock.

Figure 1.6: Time-variation in the term structure of welfare costs in the model of Lettau and Wachter (2011).

hold-to-maturity excess returns on the dividend strips. Outside the world of Lettau and Wachter the intuition remains the same, provided the policy-maker has some belief about a sufficient set of variables to predict the hold-to-maturity excess returns that form the term structure of welfare costs.

1.5.2. Risk premia targeting

The result that the aggregate stability is a macroeconomic priority suggests that policy-makers should pay special attention to the quantity of aggregate cashflow uncertainty. A natural subsequent question is whether policy-makers have traction on the aggregate quantity of uncertainty. Unfortunately, even though it is generally accepted that greater policy forecastability reduces aggregate uncertainty (e.g., Clarida et al., 1999), the quantitative implications on the economy of a change in policy uncertainty are not yet well-understood (see for example Baker et al., 2011; Fernández-Villaverde et al., 2011).

Alternatively, we can think of directly targeting the cost of uncertainty. Such a policy regime would not just target the amount of uncertainty but also the market price of uncertainty. Therefore, the question is whether it is desirable, from a welfare viewpoint, to target the risk premia that form the term structure of welfare costs. The advantage of this prescription is that the effect of policy on equity premia is more documented than the effect of policy on the amount of uncertainty. For example, Bernanke and Kuttner (2005) report evidence about the effect of monetary policy shocks on a broad array of asset market data, although the theoretical mechanism explaining the documented effects is still an open question.

Proposition 3 provides a basis for interpreting the cost of uncertainty as a welfare criterion. It shows how people prefer a smaller cost of fluctuations for a given expected path of consumption growth and of factor X . The argument is one of second-order stochastic dominance.

Proposition 3. *For two lognormally distributed lotteries $C^{(1)}$ and $C^{(2)}$ that have the same expectation $E(C)$, and for a given level of the factor X , one has that*

$$L^{(1)} < L^{(2)} \quad \Rightarrow \quad E[U(C^{(1)}, X)] > E[U(C^{(2)}, X)]$$

There is an important caveat in proposition 3 and thereby a qualifier for the risk premia targeting regime that is desirable. An intervention that reduces the mean cost of fluctuations and that affects the trend of cashflows or the aggregate level of the other variables in the utility function is not necessarily welfare-enhancing. Therefore, the desirability of a regime that aims at reducing the cost of fluctuations at any frequency is not to be taken for granted.

The careful design of desirable policy rules requires a full-fledged structural model able to account for, on the one hand, the shape of the term structures of equity and interest rates and, on the other, the stock market's reaction to a policy shock documented in the literature.

1.6. Conclusion

The term structure of the marginal cost of uncertainty requires little structure to reveal both a tradeoff between growth and macroeconomic stability and a tradeoff between macroeconomic stability at different time horizons. I link the notion of cost of uncertainty to a rich set of recent financial market evidence—in particular, the term structure of equity. I use equity prices extracted from index option markets to show how recent evidence about the term structures of equity and interest rates is able to provide new insight into an old question (the first tradeoff) and allows for studying the new question (the second tradeoff).

Asset markets suggest that cashflow stability is on average a macroeconomic priority and that marginal increases in aggregate uncertainty are particularly costly, especially in the short run and during downturns such as in the early 2000s and during the recent financial crisis. However, macroeconomic priorities can vary substantially across the business cycle. In this context, a policy-maker can assess the current position of the term structure of welfare costs and forecast its movements by looking at innovations in the information set made by excess return predictors.

The negative slope of the estimated term structure cannot be captured by today's leading consumption-based asset pricing models (Binsbergen et al., 2012; Ludvigson, 2013) and therefore represents a puzzling piece of evidence with seemingly crucial welfare consequences. The finding of large and volatile costs imposed by an increase in short-run uncertainty inscribes into a burgeoning literature that finds high and time-varying short-maturity risk premia as a pervasive phenomenon across different asset classes (Binsbergen et al., 2012, 2013; Aït-Sahalia et al., 2012; Palhares, 2012). The paper also contributes to the large literature that exploits asset market data to reveal information about the marginal utility of wealth (for example, Hansen and Jagannathan, 1991; Alvarez and Jermann, 2004, 2005; Hansen and Scheinkman, 2009; Backus et al., 2013; Hansen, 2013).

The result that the welfare cost of uncertainty is a linear combination of risk premia makes one of the main tasks of macroeconomics—that of assessing the macroeconomic priorities (Lucas, 2003)—inextricably linked to finance. A major implication of this finding is that a structural model able to explain the stylized evidence about the term structures of equity and interest rates, and thereby of welfare costs, should be high on the research agenda. Such a project promises to deliver important new insights in terms of welfare analysis and in the policy transmission mechanism.

Appendix

1.A. Relationship with the definitions of Lucas (1987) and Alvarez and Jermann (2004)

Definition (1.1) is more general and slightly different from the one studied by Lucas (1987) and Alvarez and Jermann (2004). First, they measure the cost of fluctuations by the uniform compensation Ω_t in

$$E_t U(\{(1 + \Omega_t(\theta))C_{t+n}\}_{n=1}^{\infty}, \{X_{t+n}\}_{n=1}^{\infty}) = E_t U(\{\theta E_t C_{t+n} + (1 - \theta)C_{t+n}\}_{n=1}^{\infty}, \{X_{t+n}\}_{n=1}^{\infty})$$

whereas the definition I consider measures the cost of fluctuations by a compounded compensation. The alternative definition I study can be interpreted as the tradeoff between growth and macroeconomic stability and thereby has an arguably more intuitive appeal.

Second, I allow for considering the stabilization of only some coordinates of consumption—the set N in definition (1.1)—rather than of the whole stochastic process. This flexibility allows for a direct focus on the relevant periodicity of economic fluctuations.

1.B. Proof of proposition 1

I can rewrite equation (1.2) as

$$\begin{aligned} L_t^N &= \sum_{n \in N} \frac{n E_t(M_{t,t+n} C_{t+n})}{\sum_{n \in N} n E_t(M_{t,t+n} C_{t+n})} \times \frac{1}{n} \left(\frac{E_t(M_{t,t+n}) E_t(C_{t+n})}{E_t(M_{t,t+n} C_{t+n})} - 1 \right) \\ &= \sum_{n \in N} \omega_{n,t} l_t^{(n)} \end{aligned}$$

1.C. Proof of proposition 1'

Differentiating definition (1.6) with respect to θ , it follows that

$$L_t^N = \frac{\sum E_t(U_{1,t+n}) E_t(C_{t+n}) + E_t(U_{2,t+n} \frac{P_{t+n}}{W_{t+n}}) E_t(\frac{W_{t+n}}{P_{t+n}} N_{t+n}) - E_t(U_{1,t+n} C_{t+n} + U_{2,t+n} N_{t+n})}{\sum n E_t(U_{1,t+n} C_{t+n})} \quad (1.C.1)$$

Then, under assumption 1, expression (1.C.1) becomes

$$\begin{aligned} L_t^N &= \frac{\sum E_t(M_{t,t+n}) E_t(D_{t+n}) - E_t(M_{t,t+n} D_{t+n})}{\sum n E_t(M_{t,t+n} C_{t+n})} \\ &= \sum \omega_{n,t} l_t^{(n)} \end{aligned}$$

1.D. The term structure of welfare costs in the model of Lettau and Wachter

The variation of the term structure of welfare costs over the business cycle is driven by the time-varying components of $\{\ell_t^{(n)}\}$, where $\ell_t^{(n)} \equiv \ln(1 + n l_t^{(n)})^{1/n}$ is the continuously-compounded marginal cost of uncertainty associated with $l_t^{(n)}$. In the model of Lettau and Wachter (2011), the technical appendix shows how this component is driven entirely by movements in the market price

of risk x_t as, up to a constant term,

$$\ell_t^{(n)} = \frac{1}{n} \left\{ \frac{1 - \phi_x^n}{1 - \phi_x} \sigma_d + \left(\frac{1 - \phi_x^n}{1 - \phi_x} - \frac{\phi_x^n - \phi_z^n}{\phi_x - \phi_z} \right) \frac{\sigma_z}{1 - \phi_z} \right\} \frac{\sigma'_d}{\|\sigma_d\|} x_t$$

where the price of risk follows the autoregressive process $x_{t+1} = (1 - \phi_x)x + \phi_x x_t + \sigma_x \varepsilon_{t+1}$, with x the average discount rate. Vectors σ'_d , σ'_z and σ'_x are respectively the loadings of short-run and long-run cashflows and of the price of risk on the reduced-form shocks that drive the system, and ϕ_z is the persistence of the predictable component of cashflows.

The term structure of the costs of uncertainty starts from a level of $\|\sigma_d\|x$, which is about 17 percent per year in the calibration of Lettau and Wachter, to then decay with a slope determined by the persistence coefficients ϕ_z and ϕ_x .

The model suggests that a sufficient information set to reveal news to the market price of risk is made by the first two dividend yields, $e_t^{(1)}$ and $e_t^{(2)}$, and the first bond yield, $y_t^{(1)}$. In fact,

$$\begin{aligned} e_t^{(1)} &= -z_t + \|\sigma_d\|x_t + r_t \\ e_t^{(2)} &= -\frac{1 + \phi_z}{2} z_t + \frac{1 + \phi_r}{2} r_t + \frac{1 + \phi_x}{2} \|\sigma_d\|x_t + \frac{\sigma_z - \sigma_r}{2} \frac{\sigma'_d}{\|\sigma_d\|} x_t \\ y_t^{(1)} &= r_t \end{aligned}$$

where the variables are in log-deviations from the mean.

Therefore, the projection of hold-to-maturity excess strip returns on the information set induced by the history of the observable vector $s_t = [e_t^{(1)}; e_t^{(2)}; y_t^{(1)}]$ is the true cost of uncertainty $\ell_t^{(1)} = E[r_{d,t+1}^{e,(1)} | s^t] = \|\sigma_d\|x_t$. The residuals of the Wold representation of the information set s^t then reveal discount-rate news.

1.E. Proof of proposition 3

First note that

$$\begin{aligned} E[U((1 + L^{(1)})^n C_{t+n}^{(1)}, X)] &= E[U((1 + L^{(2)})^n C_{t+n}^{(2)}, X)] \\ &> E[U((1 + L^{(1)})^n C_{t+n}^{(2)}, X)] \end{aligned} \tag{1.E.2}$$

because utility is strictly increasing in consumption and the scalars $L^{(1)}, L^{(2)}$ are such that $L^{(1)} < L^{(2)}$. Since the two lotteries have the same mean and are conditionally lognormal, they must differ in variance.

Suppose $C^{(1)}$ has a larger variance than $C^{(2)}$, i.e., by the projection theorem in Hilbert spaces, $C^{(1)} = C^{(2)} + \varepsilon$ with $E[\varepsilon | C^{(2)}] = 0$. This implies $E[U(C^{(1)}, X)] = E[E[U(C^{(1)}, X) | C^{(2)}]] \leq E[U(E[C^{(2)} + \varepsilon | C^{(2)}], X)] = E[U(C^{(2)}, X)]$ by the projection theorem and Jensen's inequality.

However, the assumption also implies that the variance of $\{(1 + L^{(1)})^n C_{t+n}^{(1)}\} = \{(1 + L^{(1)})^n C_{t+n}^{(2)} + (1 + L^{(1)})^n \varepsilon_{t+n}\}$ is larger than the variance of $\{(1 + L^{(1)})^n C_{t+n}^{(2)}\}$. Therefore, $E[U((1 + L^{(1)})^n C_{t+n}^{(1)}, X)] \leq E[U((1 + L^{(1)})^n C_{t+n}^{(2)}, X)]$, which contradicts expression (1.E.2).

Therefore, it must be that $C^{(2)}$ has a larger variance than $C^{(1)}$ and $E[U(C^{(2)}, X)] < E[U(C^{(1)}, X)]$.

Part 2

A New Keynesian Q theory of investment and the link between inflation and the stock market

Abstract. The benchmark Q theory links investment with stock prices. In this study, I demonstrate how a New Keynesian Q (NKQ) theory links investment, stock prices, and inflation, providing a rational explanation for a comovement between expected inflation and stock prices. The NKQ equation contains a specification error that complicates estimation. I estimate and test the NKQ theory by matching the return-forecasting ability of predicted and actual stock prices; this strategy provides orthogonality conditions that bypass the specification error. Investment accounts for a portion of the component of stock prices that forecasts excess returns; inflation accounts for the component of stock prices that forecasts the risk-free rate. The benchmark Q theory fails to capture the second component, and it therefore presents difficulties in accounting for long-horizon return forecastability. Finally, the fit of the NKQ theory is a dramatic improvement over the benchmark.

2.1. Introduction

How does the Q theory of investment change if prices are sticky? Since the work of Tobin (1969), the Q theory of investment has been one of the main off-the-shelf production-based explanations of stock prices. In this study, I focus on the asset pricing implications of the production side of a widely used monetary macromodel—the New Keynesian model.²³

I show how sticky prices add inflation, π , to the standard Q theory relationship between the market-book ratio, s , and the investment-capital ratio, ik . Namely, the New Keynesian Q (NKQ) theory is the equilibrium condition²⁴

$$s_t = \eta ik_t - \frac{\vartheta}{\lambda} E_t \pi_{t+1} \quad (2.1)$$

where η is the inverse of the elasticity of investment to Q, λ is the slope of the New Keynesian Phillips curve, and ϑ is a function of other deep parameters. Therefore, monopolistic competition

²³A substantial literature attempts to replicate both time-series and cross-sectional asset pricing facts by studying asset pricing from the first-order conditions that describe an optimal firm's behavior (for example, Cochrane, 1991, 1996; Lamont, 2000; Lettau and Ludvigson, 2002; Kogan, 2004; Zhang, 2005; Liu, Whited and Zhang, 2009; Belo, 2010; Jermann, 2010, 2013).

²⁴The variables are log-deviations from the steady state.

and sticky prices in the goods market break down the equality between the marginal and average Qs that holds in the benchmark frictionless real business cycle model (Hayashi, 1982). In so doing, the New Keynesian framework provides a rational explanation for a comovement between expected inflation and stock prices, which is both strongly supported by the data and a long-standing puzzle in the literature (e.g., Campbell and Vuolteenaho, 2004; Bekaert and Engstrom, 2010). (“Stock prices,” “average Q,” and “market-book ratio” are all used interchangeably here.²⁵)

Like the standard Q theory, however, the NKQ equation is misspecified, i.e., it contains a specification error, e , defined by

$$\begin{aligned} s_t &= \eta i k_t - \frac{\vartheta}{\lambda} E_t \pi_{t+1} + e_t \\ &\equiv \widehat{s}_t + e_t \end{aligned} \quad (2.2)$$

The misspecification arises because expected inflation is restricted by the New Keynesian Phillips curve in a way that makes the NKQ equation (2.1) stochastically singular. Because the specification error is a stochastic process that can have any shape and property, I must restrict its space for the theory to be rejectable. Namely, I choose to reject the NKQ theory if the predicted market-book ratio, \widehat{s} , does not have the same return-forecasting ability as the observed market-book ratio, s . This strategy allows me to overidentify the two free parameters, η and λ . With this overidentification, I am able to both estimate and test the NKQ theory.

The NKQ theory captures a component of the market-book ratio that is missing in the benchmark Q theory. The investment-capital ratio forecasts returns in the short run. Inflation forecasts returns in the long run. Stock prices forecast returns both in the short and in the long run. As predicted by the NKQ theory, I can therefore replicate, at least in part, the return-forecasting ability of the market-book ratio by combining inflation with the investment-capital ratio. The benchmark frictionless alternative has instead a more difficult time accounting for the return-forecasting ability of the market-book ratio, because the investment-capital ratio alone cannot forecast long-run returns over the whole sample. Also the fit of the NKQ theory is dramatically better than that of the benchmark Q theory. The predicted and observed investment-capital ratios have a 61% correlation (a 120% improvement over the benchmark), while the predicted and observed market-book ratios have a 72% correlation (a 160% improvement over benchmark).

Stock price information helps therefore recover two important structural parameters that describe the monetary transmission mechanism. In particular, the restrictions on the joint distribution of observables imposed by the NKQ theory allow for estimates of the slope of the New Keynesian Phillips curve, λ , that are more accurate than is standard in the New Keynesian literature.

2.1.1. Long-horizon forecasting regressions

My approach to estimation most closely relates to Cochrane (1991) and Lettau and Ludvigson (2002), who study the successes and failures of the benchmark Q story in terms of long-horizon

²⁵There is a conceptual difference between average Q and the market-book ratio, however, in my setting the replacement cost of capital coincides with its book value and thereby average Q equals the market-book ratio.

forecasting regressions. Like these authors, I use return forecastability as a crucial factor to evaluate the Q theory.²⁶

In a dynamic stochastic general equilibrium (DSGE) setting, the benchmark Q theory describes the optimal investment choice for a firm in a RBC model with adjustment costs to capital accumulation (Hayashi, 1982) and consists of an exact linear relationship between the market-book ratio and the investment-capital ratio,

$$s_t = \eta i k_t \quad (2.3)$$

As shown by Cochrane (1991) and Restoy and Rockinger (1994), the benchmark Q theory can also be expressed in (quasi-)first differences as the equality

$$r_{t+1}^m = r_{t+1}^I \quad (2.4)$$

between the return on the market portfolio, r^m —a claim to the stream of the firms' profits—and the return to investment, r^I —a claim to the marginal product of capital, net of its depreciation, generated by the investment of a unit of value.

Against this background, there are two main reasons to focus on long-run forecasting regressions. First, the literature documents a failure of the benchmark Q theory in explaining the short-run correlation between investment and stock returns. In fact, some authors document a short-run correlation not statistically different from zero between investment and future stock returns and even a contemporaneous covariation with the wrong sign (Lamont, 2000; Lettau and Ludvigson, 2002). The problem disappears, however, over longer horizons, as investment expenditures seem to react with a lag to stock price developments.

Second, Cochrane (1991) uses forecasting regressions to test equation (2.4). Although, with no error, equation (2.4) is false, he shows how its weaker implication $E(r_{t+1}^m - r_{t+1}^I | X) = 0$, for any information set X known at time t , fares surprisingly well. Cochrane does not reject the prediction that forecasts of investment and stock returns are equal in an information set that includes the term spread (on both government and corporate bonds), the market return, the dividend-price ratio and the investment-capital ratio. He also finds that, depending on the frequency considered, the unconditional correlation between r^m and r^I is substantial, with a value ranging from 25-40%. Cochrane also finds, however, that dividend-price ratios can forecast stock returns but cannot forecast long-horizon investment returns; investment and stock returns appear to have a different low-frequency component.

The NKQ theory remains consistent with the hits of the benchmark Q theory; because the NKQ theory results from a first-order approximation, the NKQ theory still predicts $E(r_{t+1}^m - r_{t+1}^I | X) = 0$.²⁷ In addition, the NKQ theory appears successful where the benchmark Q theory fails. Inflation in the NKQ theory can account for a low-frequency component in stock prices.

²⁶There is extensive literature that provides empirical evidence of predictable and volatile expected returns (see Cochrane, 2011; Kojen and Nieuwerburgh, 2011, for a survey). In this regard, the equity dividend yields (and other stationary transformations of stock prices, such as the market-book ratio) have a special status as return predictors, because they robustly relate to returns by the Campbell and Shiller (1988) identity.

²⁷The loglinearization ignores all second-order risk-premia. The appendix shows in detail this fact by proving how, up to a first-order approximation, the return on investment equals the market return plus an unforecastable term.

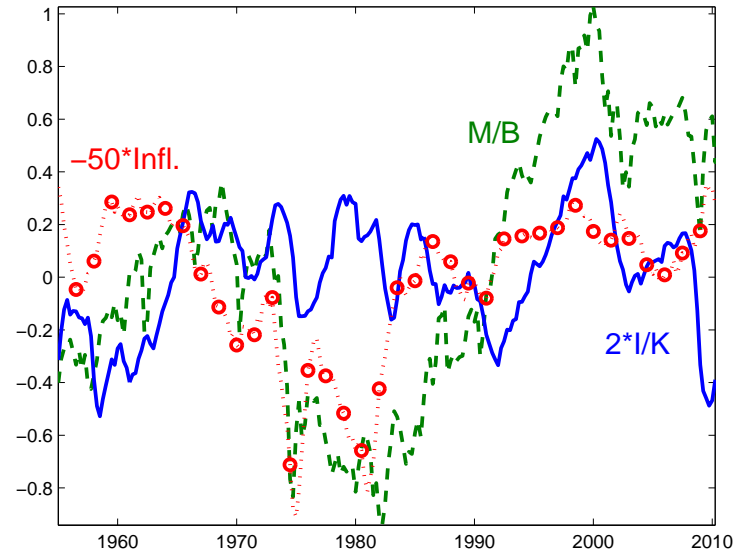


Figure 2.1: Market-book ratio, investment-capital ratio and inflation rate (demeaned logs).

2.1.2. Interpreting the relation between inflation and stock prices

Figure 2.1 offers a first measure to inspect the fit of the Q theory. The market-book ratio closely tracks the investment-capital ratio in the last 20 years of the sample (where the correlation is 73%), but it fails to do so over the whole 60-year sample (where the correlation is a mere 25%). Because there is room for improvement along this first dimension, some of the extant literature is skeptical of the standard Q theory (Caballero, 1999). A further difficulty for our study is that our theory should explain the dramatic increase in the correlation from the first two-thirds to the last third of the sample. Against this background, the inflation rate also displays a high unconditional correlation with both stock prices (-55%) and investment (32%). We can also observe how inflation loses its volatility in the last 20 years or so, which is when the investment-capital ratio starts to effectively track the market-book ratio. This is exactly what the NKQ theory would predict. The addition of inflation to the standard Q relationship helps explain the time-varying correlation between the market-book ratio and the investment-capital ratio.

The strong negative correlation between a stationary transformations of stock prices (such as market-book and price-dividend ratios) and inflation (or expected inflation) is both well-documented and puzzling, at least from a rational expectations perspective (e.g., Modigliani and Cohn, 1979; Ritter and Warr, 2002; Sharpe, 2002; Asness, 2003; Campbell and Vuolteenaho, 2004; Cohen, Polk and Vuolteenaho, 2005; Bekaert and Engstrom, 2010). In a context of price rigidities, expected inflation rationally signals something about the real cashflow component of market-book ratios;²⁸ high inflation associates to low expected markups and thereby to low expected aggregate dividends. Therefore, the New Keynesian framework offers a parsimonious story to make sense of the correlation between stock prices and inflation as a rational outcome.

²⁸The NKQ theory provides micro-foundations to a hypothesis that goes back at least to Fama (1981).

2.1.3. Identification of important structural parameters

Another contribution of this paper is the use of stock market data to estimate macroeconomically important structural parameters and test the importance of nominal rigidities.²⁹

On the one hand, by only adding adjustment costs of capital accumulation to the basic New Keynesian model (Galí, 2008), one obtains the NKQ theory, which restricts the joint distribution of stock prices, investment and inflation, and thus offers more structure to identify the slope of the New Keynesian Phillips curve, λ . This structure significantly increases the accuracy of the estimated λ , as shown in table 2.2. Notice in fact how the parameter λ is quite controversial quantitatively in the extant literature. Galí and Gertler (1999), Galí, Gertler and López-Salido (2005) and Kleibergen and Mavroeidis (2009) provide point estimates between 0.01 and 0.03, with standard deviations of a similar magnitude, and there is still debate about the strength of the identification (e.g., Mavroeidis, 2005; Nason and Smith, 2008; Canova and Sala, 2009).

On the other hand, the NKQ theory allows for identifying the elasticity of Q to investment, η , whose range of values is also subject to considerable disagreement in the literature.³⁰

2.2. A New Keynesian Q theory

A continuum of monopolistically competitive firms $i \in [0, 1]$ choose nominal prices $\{P_{t+h}^*(i)\}$, real output $\{Y_{t+h}(i)\}$, labor demand $\{N_{t+h}(i)\}$ and real investment $\{I_{t+h}(i)\}$ to maximize the expected discounted value of profits

$$E_t \sum_{h=0}^{\infty} M_{t,t+h} \left[\frac{P_{t+h}(i)}{P_{t+h}} Y_{t+h}(i) - (1 - \tau) \frac{W_{t+h}}{P_{t+h}} N_{t+h}(i) - (1 - \tau) I_{t+h}(i) - T_{t+h} \right]$$

subject to

$$\begin{aligned} Y_{t+h}(i) &= \left(\frac{P_{t+h}(i)}{P_{t+h}} \right)^{-\varepsilon} Y_{t+h} \\ Y_{t+h}(i) &= A_{t+h} K_{t+h}(i)^\alpha N_{t+h}(i)^{1-\alpha} \\ K_{t+h+1}(i) &= (1 - \delta) K_{t+h}(i) + \Phi \left(\frac{I_{t+h}(i)}{K_{t+h}(i)} \right) K_{t+h}(i) \\ P_{t+h}(i) &= \begin{cases} P_{t+h}^*(i), & \text{with probability } (1 - \theta) \\ P_{t+h-1}(i), & \text{with probability } \theta \end{cases} \end{aligned}$$

In this formula, M is the stochastic discount factor for real payoffs, P is the nominal price of a unit of consumption (the numeraire), W is the nominal wage rate, A is technology and K is the real capital stock. Prices are sticky (Calvo, 1983), so that firms can reset their prices at any given time with probability $1 - \theta$ only. There are capital adjustment costs. The adjustment cost function $\Phi(\cdot)$ is such that $\Phi(I/K) = \delta$, $\Phi'(I/K) = 1$ and its curvature $\eta \equiv -[\Phi''(I/K)I/K]/\Phi'(I/K)$ are constant and

²⁹Bekaert, Cho and Moreno (2010) use bond term structure data to (just-)identify a New Keynesian macromodel.

³⁰One portion of the literature (e.g., King and Wolman, 1996; Bernanke, Gertler and Gilchrist, 1999) suggests values for η between 0 and 1, while another area (e.g., Abel, 1980; Jermann, 1998) prefers values between 2 and 5.

positive in the steady state. The government levies lump-sum taxes, $T_t = \tau W_t/P_t + \tau I_t$, to finance an employment and investment subsidy, τ , which is in place to offset any steady-state distortion caused by the monopolistic competition.³¹ The parameters δ and α are the average depreciation rate of capital and capital share in value added, respectively.

This problem is the one of the firm in a New Keynesian model with firm-specific capital accumulation (Woodford, 2005; Sveen and Weinke, 2005).

2.2.1. Exact model

The first-order conditions for an interior optimum are

$$\sum_{h=0}^{\infty} \theta^h E_t M_{t,t+h} P_{t+h}^{\varepsilon-1} Y_{t+h}(i) [P_t^*(i) - \widehat{MC}_{t+h}(i) P_{t+h}] = 0 \quad (2.5)$$

$$\begin{aligned} MC_t(i) MPN_t(i) &= (1 - \tau) \frac{W_t}{P_t} \\ Q_t(i) &= \left[\Phi' \left(\frac{I_t(i)}{K_t(i)} \right) \right]^{-1} \end{aligned} \quad (2.6)$$

$$E_t M_{t,t+1} R_{t+1}^I(i) = 1 \quad (2.7)$$

with

$$R_{t+1}^I(i) \equiv \frac{\widehat{MC}_{t+1}(i) MPK_{t+1}(i) + \left((1 - \delta) + \Phi \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) - \Phi' \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) \frac{I_{t+1}(i)}{K_{t+1}(i)} \right) Q_{t+1}(i)}{Q_t(i)}$$

where Q is marginal Q, MPN is the marginal productivity of labor, MC is the real marginal cost of output, R^I is the gross real rate of return to investment, MPK is the marginal productivity of capital and $\widehat{MC} \equiv MMC$, with $M \equiv \varepsilon/(\varepsilon - 1)$ the average price markup that firms charge over marginal costs because of monopolistic competition in the market for goods.

Firms then distribute profits as dividends $P_{t+h} D_{t+h}$ to consumers.

Next, I define average Q as

$$S_t(i) \equiv \frac{E_t \sum_{h=1}^{\infty} M_{t,t+h} D_{t+h}(i)}{K_{t+1}(i)}$$

the ratio of two different valuations of the capital stock of firm i : the market price (expected discounted profits) and the book value.

Average Q is related to marginal Q through

$$S_t(i) K_{t+1}(i) = Q_t(i) K_{t+1}(i) + \sum_{h=1}^{\infty} E_t M_{t,t+h} \left[\frac{P_{t+h}(i)}{P_{t+h}} - \widehat{MC}_{t+h}(i) \right] Y_{t+h}(i) \quad (2.8)$$

Equation (2.8), which I prove in the appendix, states that the total value of the i th firm is the sum of the marginal value of the capital stock and the expected discounted value of future profits

³¹ $\tau = 1/\varepsilon$ achieves this goal.

due to imperfect competition on the market for goods. This result extends the finding of Hayashi (1982) to a model with nominal rigidities and monopolistic competition in the market for goods.

Finally, by the degree-one homogeneity of the production function, the aggregate real marginal cost, $MC_t = W_t/P_t MPN_t$, relates to the aggregate labor share of income, $L_t \equiv W_t N_t/P_t Y_t$, through

$$(1 - \alpha)MC_t = L_t$$

hence, in deviations from the mean, $mc_t = \ell_t$ (Galí and Gertler, 1999).

2.2.2. Approximate model

A loglinearization of equation (2.5) around the zero-inflation steady state yields a standard New Keynesian Phillips curve (Woodford, 2005; Sveen and Weinke, 2005), which drives inflation as

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \lambda \ell_t \\ &= -\lambda E_t \sum_{j=0}^{\infty} \beta^j \mu_{t+j}\end{aligned}\tag{2.9}$$

where $\mu_t = -\ell_t$ is the average price markup. Thus, λ is the elasticity of inflation to expected long-run markups.³² Because of nominal rigidities, the wedge μ_t is responsible for all deviations from the first best equilibrium.³³

Equation

$$q_t = \eta i k_t\tag{2.10}$$

approximates the optimal investment choice (2.6) whereby in equilibrium the marginal value of a unit of new capital equals its marginal cost.

Finally, as indicated in the appendix, the linearized version of equation (2.8) around the undistorted steady state combined with equations (2.9) and (2.10) is

$$\begin{aligned}s_t &= \eta i k_t - \frac{\vartheta}{\lambda} E_t \pi_{t+1} \\ &= \eta i k_t - \frac{\vartheta}{\beta \lambda} \pi_t + \frac{\vartheta}{\beta} \ell_t\end{aligned}\tag{2.11}$$

where $\vartheta \equiv [1 - \beta(1 - \delta)]/\alpha$. This equation represents the NKQ theory, which describes an equilibrium condition that holds when prices adjust sluggishly in the goods market and are flexible in financial

³²Note that the relation between the elasticity, λ , and the Calvo frequency of price adjustment, θ , is nonstandard under firm-specific capital accumulation. This issue is however irrelevant to my purposes, because I focus on estimating the parameter λ . In this respect, one may ask why I estimate λ rather than θ . The answer is that the parameter θ has a nice microeconomic interpretation, but a fragile link to the structural equations of the New Keynesian model. The structural equations depend on θ only through λ . However, the link between θ and λ is not robust, for example, to the choice of modeling investment through a rental market or through firm-specific capital accumulation. The connection between the structural equations and λ is more robust.

³³There is inflation if the desired markup differs from the constant, natural markup. The natural markup is zero when the efficient unemployment subsidy is in place.

markets. Note in particular how the equation is robust to systematic changes in the policy rule in place; for example, the equation is robust to a switch in policy regime of the type studied by Clarida et al. (2000) or to entering a period in which the zero lower bound on the interest rate is binding.

The economic mechanism behind equation (2.11) is straightforward. The rewards to stock owners come from two sources: the marginal product of capital, which they own, and the monopolistic profits that firms realize by charging a markup on output. Through the marginal Q, the investment-capital ratio signals future changes in the first source; equation (2.10) predicts that periods of high investment associate to times in which the marginal value of a unit of new capital is high. Using the New Keynesian Phillips curve, inflation signals future changes in the second source; equation (2.9) suggests that inflation is high if long-run expected markups, hence profits and dividends, are low.

In this study, equation (2.11) is the theory tested. Note that I can rewrite equation (2.11) as the (quasi-)first differenced NKQ equation

$$r_{t+1}^m = r_{t+1}^I - \frac{\vartheta}{\lambda}(E_{t+1} - E_t)\pi_{t+1} \quad (2.12)$$

where $r_{t+1}^I = \alpha\vartheta(mpk_{t+1} - \mu_{t+1}) + \beta\eta ik_{t+1} - \eta ik_t$, which is available in the appendix. Shocks to inflation signal low future profits and thereby decrease the return on equity relative to the return on investment.

2.3. Identification

I choose standard values for the uncontroversial parameters. I estimate the key parameters η and λ . The values I consider are given in table 2.1.

α	capital share in value added	0.35
β	subjective discount factor	0.99
δ	depreciation rate	0.025
η	inverse of elasticity of investment-capital ratio to marginal Q	to be estimated
λ	elasticity of inflation to expected long-run markups	to be estimated

Table 2.1: Deep parameters

In estimation, I use U.S. time series at a quarterly frequency running from 1952 to 2011.³⁴ The market-book ratio is the market value of nonfarm nonfinancial corporate business net worth over the book value of corporate equities (Fed Flow of Funds Accounts). Investment is the real private nonresidential fixed investment (BEA-NIPA database). Following Cochrane (1991, 2011), capital is constructed from investment using the quarterly depreciation rate δ .³⁵ Inflation is the

³⁴The 1952-2011 period is likely characterized by non-linearities in policy, such as changes in the policy regime (Clarida et al., 2000) and prolonged periods at the zero lower bound. This non-linearity is inconsequential for the estimation of the NKQ equation because its derivation is independent of the policy rule in place.

³⁵Given the time series for investment $\{i_t\}$, I use the capital accumulation equation to compute $ik_{t+1} = \Delta i_{t+1} + ik_t - \delta(ik_t - \ln \delta)$; this construction is accurate up to a term of second-order importance.

change in GDP deflator (BEA-NIPA). The labor share is the nonfarm business sector labor share of income (BEA-NIPA). The market return is the return on the CRSP NYSE value-weighted stock index (CRSP database).

Using the calibrated values, the last term in the NKQ equation (2.11) becomes quantitatively irrelevant.³⁶ Thus, the NKQ theory really adds only one regressor—the inflation rate. Any increase in the fit of the NKQ theory with respect to the benchmark Q theory is entirely due to inflation.

2.3.1. Misspecification

The Q theory—both in the RBC version (2.3) and in the NK version (2.11)—is an exact relationship among the variables that should hold in every time period. Of course, the data reject the stochastic singularity predicted by equation (2.11) at any level of significance.³⁷ Therefore, equation (2.11) contains some specification error, which by definition can have any shape and property.³⁸

Formally, I assume that there is a DSGE model describing the relevant time series. The model's solution then has the state-space representation, which I assume to be linear,

$$\begin{aligned}\zeta_t &= A\zeta_{t-1} + Bu_t \\ X_t &= C\zeta_t + Du_t\end{aligned}\tag{2.13}$$

where $X_t = [s_t; ik_t; \pi_t; \ell_t]$, ζ_t is a vector of states and $u_t \sim Niid(0, I)$ is a vector of structural shocks. Thus, the specification error, e , defined in the NKQ equation (2.2) is

$$\begin{aligned}e_t &= \begin{bmatrix} 1 & -\eta & \frac{\partial}{\beta\lambda} & -\frac{\partial}{\beta} \end{bmatrix} X_t \\ &= \begin{bmatrix} 1 & -\eta & \frac{\partial}{\beta\lambda} & -\frac{\partial}{\beta} \end{bmatrix} (C[I - AL]^{-1}B + D)u_t\end{aligned}$$

where L is the lag operator. The NKQ theory is silent regarding the orthogonality of e_t and the elements of X_t . The specification error is likely autocorrelated and conditionally heteroskedastic. These facts forbid any estimation of the NKQ equation based on the orthogonality of its residual with some instrument (including, e.g., OLS).

³⁶The labor share of income has a relatively low variance, while the constant term $[1 - \beta(1 - \delta)]/\alpha\beta \approx 0.10$ drives the variance further down by two orders of magnitude.

³⁷Consider the New Keynesian Phillips curve (2.9) to write the NKQ equation as $E_t \varepsilon_{t+1} \equiv E_t[(\partial/\lambda)\pi_{t+1} - \eta ik_t + s_t] = 0$. Then, a possible approach is a GMM estimation on the orthogonality condition $E(x_t \varepsilon_{t+1}) = 0$, where $x_t = [s_t; ik_t; \pi_t; \ell_t]$. Although attractive in principle, this approach fails because of the prediction of stochastic singularity: (a) since the four moments $E(x_t \varepsilon_{t+1}) = 0$ are linearly dependent, the efficient weighting matrix diverges to infinity and therefore rejects the theory at any level of significance; (b) if we consider only a subset of three moments out of four, then estimates differ widely across subsets; moreover, $[s_t; ik_t; \pi_t]$ leads to strong rejection, since ℓ is quantitatively irrelevant, so that we have a near stochastic singularity problem.

³⁸The specification error includes approximation errors due to the Taylor expansion, likely measurement errors in the time series, variations in flexible-price desired markups, the possibility that the labor share of income is not an exact proxy for the real marginal cost, or the fact that the simple NKQ theory abstracts from elements that might affect the problem of firms (e.g., taxation).

2.3.2. Identification strategy

My identification strategy is that observed stock prices, s_t , and predicted stock prices, $\widehat{s}_t = \eta i k_t - (\vartheta/\beta\lambda)\pi_t + (\vartheta/\beta)\ell_t$, should have the same ability to forecast returns.³⁹ This requirement restricts the space of the stochastic process $\{e_t\}$ and defines what I require to not reject the NKQ theory.

Regardless of the characteristics of the specification error e , I only take the Q theory seriously if it explains some important component of stock prices. From an asset pricing perspective, the component of s_t that forecasts returns is of crucial importance because, by ICAPM logic, it is the component that is priced (Merton, 1973; Harrison and Kreps, 1979). The NKQ theory might be inaccurate along several dimensions (s_t and \widehat{s}_t might move on different structural shocks and also on different states, as shown by the general form of e_t) but, at least, s_t and \widehat{s}_t should reveal the same sources of priced movement in the economy.

To better grasp the identification strategy, it is important to recall that there is an identity linking the market return and the market-book ratio: a version of the approximate Campbell and Shiller (1988) identity, $r_{t+1}^m = \beta s_{t+1} - s_t + (1 - \beta)dk_{t+1}$, where $dk_{t+1} \equiv d_{t+1} - k_{t+1} + \frac{\beta}{1-\beta}\Delta k_{t+2}$. This identity allows us to recover from the state-space model (2.13) the reduced form of the market return as $r_{t+1}^m = \psi'_{r,\zeta}\zeta_t + \psi'_{r,u}u_t + \varepsilon'_{t+1}$, where ε'_t is some linear combination of the structural shock, u_t . Therefore, the predictable component of the cumulated return from t to $t+h$, $r_{t \rightarrow t+h}^m \equiv \sum_{j=1}^h r_{t+j}^m$, is

$$\begin{aligned} E_t(r_{t \rightarrow t+h}^m) &= E(r_{t \rightarrow t+h}^m | u^t) \\ &= \psi'_{r,\zeta}[I - A]^{-1}[I - A^h]\zeta_t + \psi'_{r,u}u_t \end{aligned}$$

where $u^t = [u_0, \dots, u_t]$ is the true information set at time t , and by the law of iterated projections

$$E(r_{t \rightarrow t+h}^m | s^t) = \psi'_{r,\zeta}[I - A]^{-1}[I - A^h]E(\zeta_t | s^t) + \psi'_{r,u}E(u_t | s^t)$$

Thus, the predictor s^t (and analogously for \widehat{s}^t) reveals a slice of the true information set, u^t , i.e., the slice of the systematic movement in ex-ante returns that is perfectly correlated with s^t .

2.3.3. Moments for GMM estimation

In the language of the generalized method of moments (GMM), I am assuming that the observed market-book ratio, s , is orthogonal to the forecast error in predicting the cumulative market return, $r_{t \rightarrow t+h}^m$, with the theoretical market-book ratio, \widehat{s} , at all yearly horizons, h .

Formally, I define the predicted market-book ratio, $\widehat{s}_t = \eta i k_t - (\vartheta/\beta\lambda)\pi_t + (\vartheta/\beta)\ell_t$, so that $s_t = \widehat{s}_t + e_t$, and I consider the forecasting regressions with \widehat{s}_t as a predictor,

$$r_{t \rightarrow t+h}^m = a^{(h)} + b^{(h)}\widehat{s}_t + \varepsilon_{t+h} \quad (2.14)$$

³⁹I focus in particular on replicating the ability of the market-book ratio to forecast nominal returns, rather than real returns. This choice is motivated by the fact that the market-book ratio forecasts nominal returns more strongly than real returns; nominal returns provide therefore more information to estimate and test the NKQ equation. The results are however similar if I consider real returns.

Since the regressors are log-deviations from the mean, the constant terms simply equal the sample mean cumulative returns. For a best mean squared error forecast, I then estimate (2.14) through the orthogonality condition $E(\widehat{s}_t \varepsilon_{t+h}) = 0$. These H moments estimate H parameters $b^{(h)}$. I then define \widehat{s}_t as a good approximation for s_t if \widehat{s}_t contains the part of s_t that forecasts returns, i.e., if $E(s_t \varepsilon_{t+h}) = 0$, so that adding s_t as a predictor in equation (2.14) does not improve the forecasts (in the appendix I demonstrate this implication).⁴⁰ These H moments estimate the two key parameters $[\eta, \lambda]$.

Because the moment

$$E\left(\begin{bmatrix} \widehat{s}_t \\ s_t \end{bmatrix} \varepsilon_{t+h}\right) = 0, \quad h \in \{1, \dots, H\} \quad (2.15)$$

pins down $[\eta, -\vartheta/\beta\lambda]$ only up to a multiple, I add the moment condition,

$$\text{var}(s_t) = \text{var}(\widehat{s}_t) \quad (2.16)$$

which is a weak implication of the NKQ equation (2.11), to pin down $[\eta, \lambda]$ uniquely.⁴¹ Thus, the identifying condition (2.15) restricts the stochastic process $\{e_t\}$ to the process that has the smallest ability to forecast returns. This restriction pins down the stochastic process up to a scale factor. Then, the identifying condition (2.16) pins down the scale factor by restricting the variance of the stochastic process.⁴²

Note how the estimation procedure allows for a test of overidentification with $H - 1$ degrees of freedom. This test is a good test for the NKQ theory, because nothing implies that there are parameters $[\eta, \lambda]$ that make \widehat{s} display a return-forecasting power similar to that of s .

The final object I need to specify is H , hence the set of h -year ahead forecasts relevant for estimation. Ideally, I would match the forecasts indefinitely into the future, i.e., $H = \infty$. However, because the sample is finite, I must pick some finite H . To determine which value of H to use, one must trade off, as H increases, more information due to more moments and less information due to a shorter remaining sample. To gain insight into this tradeoff, I initially pick different values for H .

2.4. Estimation

Table 2.2 reports one- and two-step GMM estimates of the key parameters and J tests of the theoretical orthogonality conditions. The estimated η is lower for longer forecast horizons, H . With a threshold value H as high as 15 years, I can still observe how the information gained with more estimating moments outweighs the information lost due to the fact that a higher H leaves fewer data points available for estimation. Thus, from this point on, I use $H = 15$.

⁴⁰ An equivalent way to state the identifying condition $E(s_t \varepsilon_{t+h}) = 0$ is as $E(e_t \varepsilon_{t+h}) = 0$, because $E(s_t \varepsilon_{t+h}) = E(\widehat{s}_t \varepsilon_{t+h}) + E(e_t \varepsilon_{t+h})$ and $E(\widehat{s}_t \varepsilon_{t+h}) = 0$.

⁴¹ I relax the identifying condition (2.16) in section 2.4.4 below.

⁴² Note how the $2H + 1$ moment conditions can all be true in population at the same time, if e_t does not account for the part of s_t that forecasts returns (by present-value logic it must forecast future dividend-capital ratios) and if it allows \widehat{s}_t to display fluctuations with an amplitude equal to s_t .

	η	λ	$\sigma(\eta)$	$\sigma(\lambda)$	$t(\eta)$	$t(\lambda)$	J	$P(\chi^2_{H-1} > J)$
$H = 10$								
1st step	3.54	0.001	(0.91)	(0.0003)	3.89	3.84	12.52	0.19
2nd step	2.98	0.001	(0.89)	(0.0001)	3.34	7.70	18.83	0.04
$H = 12$								
1st step	3.30	0.001	(0.84)	(0.0003)	3.95	4.57	13.52	0.26
2nd step	3.01	0.001	(0.81)	(0.0001)	3.73	8.42	16.89	0.11
$H = 15$								
1st step	2.91	0.001	(0.78)	(0.0002)	3.73	5.61	23.12	0.06
2nd step	2.62	0.001	(0.43)	(0.0001)	6.06	14.31	24.92	0.04

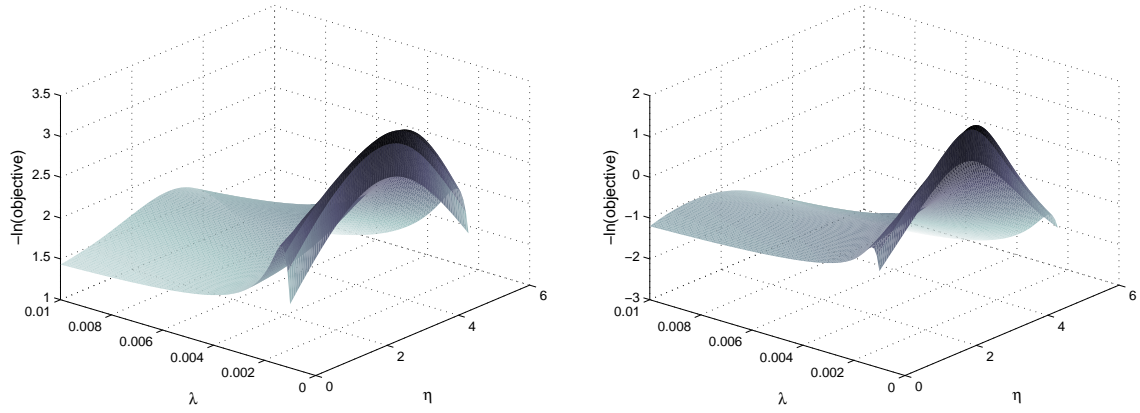
Table 2.2: GMM estimates, matching up to H -year ahead return forecasts and variance of the market-book ratio, observed and predicted. Quarterly data, 1952q1-2011q2 (238 points). J is the Hansen (1982) statistic for testing overidentifying moments. Newey and West (1987) HAC standard errors use eight leads and lags and a Bartlett kernel. Weighting matrices are the identity (first step) and the spectral density at frequency zero (second step).

2.4.1. Estimates

The precision in both the one- and two-step estimates is quite high, and the precision in the second step rises with a higher J statistic. Thus, at a 5% size level, I could reject that moments are zero in population (although not at a 1% level). However, I must use some judgment to place this rather low statistical support into perspective. After all, I am working with a very simple theory, which reduces all priced variation in stock prices to movements in investment and inflation. The high values of the J statistics point to a margin for improvement, but the simple NKQ theory already succeeds along several dimensions. By simply adding inflation to the benchmark Q relation, both the fit and the return-forecasting performance of the Q theory improve dramatically. Moreover, the evidence forcefully rejects $\vartheta/\lambda = 0$, which indicates that the benchmark Q theory with only the investment-capital ratio fails to pass the same J test at a much higher level of significance, because the investment-capital ratio cannot possibly mimic the forecasting power of the market-book ratio over long horizons.⁴³

To give a visual complement to the low volatility of the estimates, figure 2.2 shows the shape of the in-sample GMM objective in the $[\eta, \lambda]$ -plane. The high curvature along both dimensions at the estimated values can be observed in this figure. There appears to be no evidence of weak identification issues, especially for the New Keynesian Phillips curve parameter λ . This is a large gain in precision when compared to the estimates available in the literature (e.g., Galí and Gertler, 1999; Galí, Gertler and López-Salido, 2005; Kleibergen and Mavroeidis, 2009).

⁴³There is a long-standing debate about the statistical significance of long-horizon forecasting regressions. These regressions might not carry more statistical power than short-run regressions (Hodrick, 1992; Campbell, 2001; Boudoukh, Richardson and Whitelaw, 2008; Cochrane, 2008a). The debate concerns my identification strategy in that difficulties in estimating the distribution of long-run forecasts affect the second GMM step. Table 2.2 and figure 2.2, however, suggest a small difference between the first and the second GMM step.



(a) 1st step objective.

(b) 2nd step objective.

Figure 2.2: Graph of GMM objective as functions of $[\eta, \lambda]$, optimal $b^{(h)}$ fixed.

Horizon h (years)	$r_{t \rightarrow t+h}^m = a + bs_t + \varepsilon_{t+h}$		
	b	$t(b)$	R^2
1	-0.10	-2.62	0.08
3	-0.24	-3.12	0.23
5	-0.35	-4.26	0.33
8	-0.55	-5.17	0.53
10	-0.70	-5.37	0.61
12	-0.82	-5.00	0.62
15	-1.02	-8.86	0.63

Horizon h (years)	$r_{t \rightarrow t+h}^m = a + b\widehat{s}_t^{rbc} + \varepsilon_{t+h}$			$r_{t \rightarrow t+h}^m = a + b\widehat{s}_t^{nk} + \varepsilon_{t+h}$		
	b	$t(b)$	R^2	b	$t(b)$	R^2
1	-0.11	-1.98	0.04	-0.04	-1.23	0.01
3	-0.28	-2.54	0.11	-0.11	-1.74	0.05
5	-0.38	-2.29	0.15	-0.23	-3.52	0.14
8	-0.45	-1.82	0.14	-0.42	-6.78	0.34
10	-0.40	-1.42	0.08	-0.53	-7.36	0.39
12	-0.06	-0.21	0.00	-0.57	-5.44	0.36
15	0.48	2.34	0.08	-0.65	-5.63	0.35

Table 2.3: OLS regressions of h -year nominal returns on the market-book ratio, observed and predicted. Standard errors correct for overlapping, with a Bartlett kernel to ensure positive semidefiniteness of the asymptotic variance.

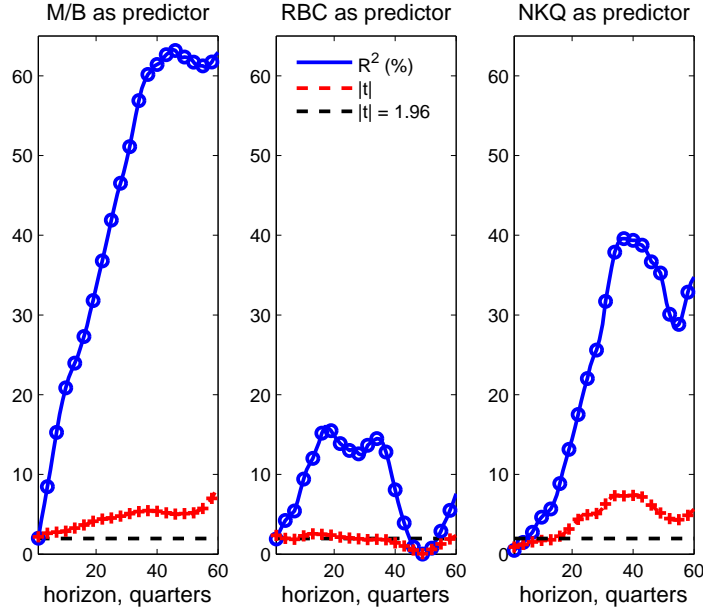


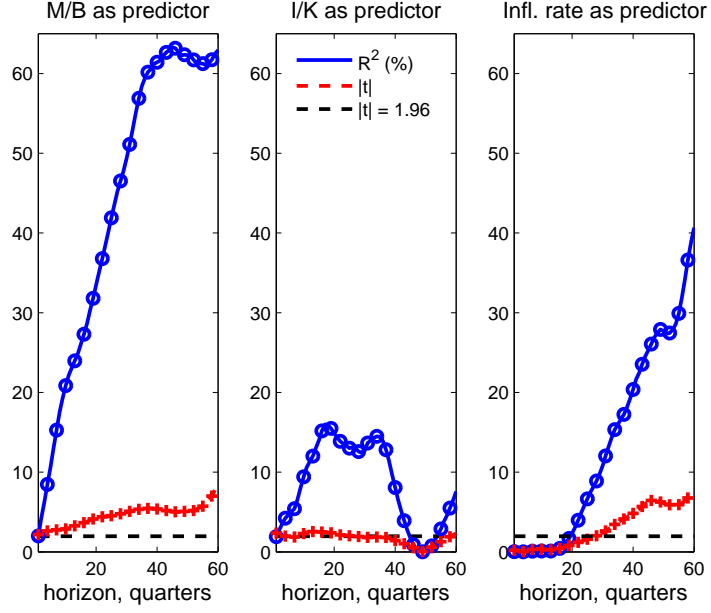
Figure 2.3: Stylized return-forecasting ability of NKQ theory vs. benchmark Q theory. R^2 is of the OLS regression $r_{t \rightarrow t+h}^m = a + bx_t + \varepsilon_{t+h}$, for different predictors x . The t statistic is for testing $b = 0$.

Table 2.3 compares the return-forecasting performance of the observed market-book ratio, of the benchmark Q prediction, $\widehat{s}^{bc} = \eta ik$, and of the NKQ prediction, $\widehat{s}^{nk} = \eta ik - (\vartheta/\beta\lambda)\pi + (\vartheta/\beta)\ell$. The benchmark prediction performs well in the short term, but it fails in the long term. The NKQ prediction instead replicates reasonably well the return-forecasting ability of the observed market-book ratio. Figure 2.3 offers a visual representation of these facts, by reporting R^2 in the return-forecasting regressions with the three predictors. We can observe the reason for the high values of the J statistics in table 2.2: there is still a margin for improvement in replicating the strong return-forecasting ability of the market-book ratio.

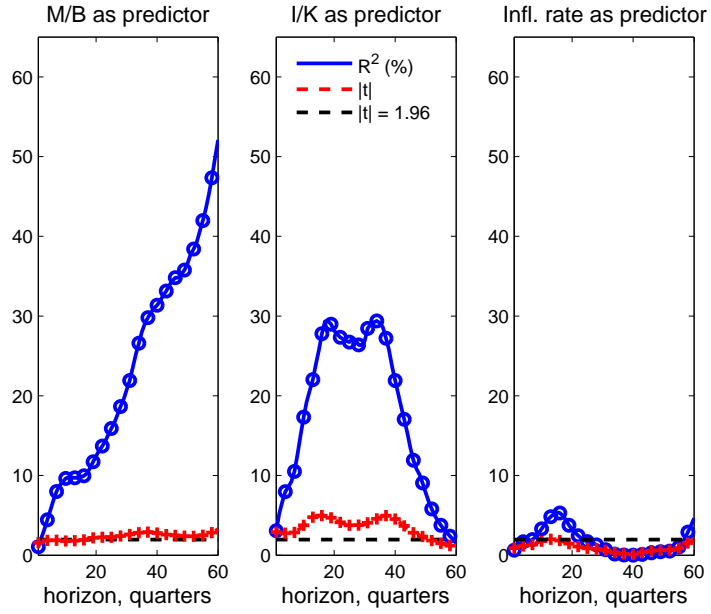
2.4.2. Stylized facts driving the results

The predicted market-book ratio in the NKQ theory is a linear combination of the investment-capital ratio and the inflation rate. Table 2.4 indicates how the investment-capital ratio and the inflation rate forecast market returns at different yearly horizons. The NKQ theory predicts that either investments are high or inflation is low when ex-ante returns are low. The regressions in table 2.4 verify these claims. Of course, the return-forecasting ability of the investment-capital ratio is equal to that of the benchmark Q prediction for the market-book ratio. The inability of the benchmark Q prediction to account for the component of stock prices that forecast long-run returns is therefore entirely due to the inability of investment to forecast long-run returns.

Figure 2.4 plots the results of the forecasting regressions for investment and inflation, which are predictors of both returns and excess returns. We can decompose any expected return into a risk premium and the risk-free rate. Figure 2.4 demonstrates how inflation forecasts risk-free rates but not excess market returns, except perhaps over short horizons. The investment-capital ratio forecasts excess returns, but not over long horizons. The market-book ratio forecasts excess returns at all horizons, and it forecasts returns even more strongly. Thus, a natural interpretation of the



(a) *Returns.*



(b) *Excess returns.*

Figure 2.4: Stylized return-forecasting ability of market-book ratio, investment-capital ratio and inflation rate. R^2 is of the OLS regressions $r_{t \rightarrow t+h}^m = a + bx_t + \varepsilon_{t+h}$ and $r_{t \rightarrow t+h}^m - r_{t \rightarrow t+h}^f = a + bx_t + \varepsilon_{t+h}$, for different predictors x . The t statistic is for testing $b = 0$.

Horizon h (years)	$r_{t \rightarrow t+h}^m = a + b i k_t + \varepsilon_{t+h}$			$r_{t \rightarrow t+h}^m = a + b \pi_t + \varepsilon_{t+h}$		
	b	$t(b)$	R^2	b	$t(b)$	R^2
1	-0.30	-1.98	0.04	-0.10	-0.03	0.00
3	-0.74	-2.54	0.11	1.32	0.25	0.00
5	-1.00	-2.29	0.15	7.99	0.94	0.02
8	-1.17	-1.82	0.14	23.01	2.87	0.13
10	-1.04	-1.42	0.08	32.59	4.84	0.20
12	-0.15	-0.21	0.00	39.49	6.40	0.27
15	1.26	2.34	0.08	51.30	7.67	0.41

Table 2.4: OLS regressions of h -year nominal returns on market-book ratio, investment-capital ratio and inflation rate. Standard errors correct for overlapping, with a Bartlett kernel to ensure positive semidefiniteness of the asymptotic variance.

results in table 2.3 is that inflation embeds the component of the market-book ratio that forecasts risk-free rates and the investment-capital ratio part of the component that predicts excess returns.

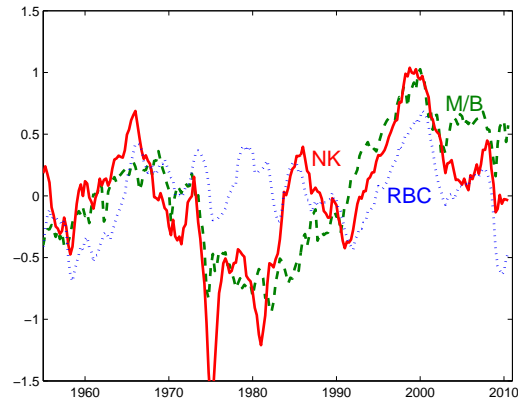
Thus, figure 2.4 shows the reason for the high values of the J statistics in table 2.2. The investment-capital ratio and inflation cannot account for the part of the market-book ratio that forecasts long-run excess returns. This missing component that forecasts excess returns would account for the remaining difference in forecasting power between observed and predicted market-book ratios that we see in figure 2.3. Against this background, note how the high correlation between market-book and investment-capital ratios in the last 20 years of the sample suggests that the two ratios have a similar forecasting power over this period. This observation suggests that the seemingly missing component in the predicted market-book ratio has stabilized in the last 20 years.

2.4.3. Fit

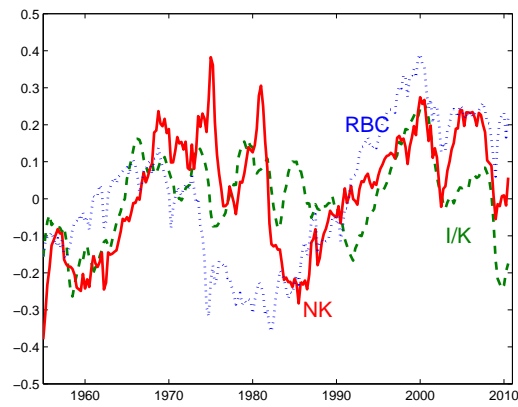
Equation (2.11) is an equilibrium condition that should hold between the market-book ratio, the investment-capital ratio and the inflation rate. From a limited-information approach, if any two of the three variables (market-book, investment-capital and inflation) are fixed, they should track the remaining variable relatively well—in spite of the unknown error term in the relation. In this section, I compare each one of the three time series to the respective NKQ prediction, which is obtained by rearranging equation (2.11) with the market-book ratio, the investment-capital ratio or the inflation rate on the left-hand side.

Figure 2.5 demonstrates how the predicted time series track their observable counterparts. To analytically evaluate the fit of the predicted market-book ratio to the observed time series, some R^2 measure is required. Because, for a generic linear regression $y_t = x_t' \beta + \varepsilon_t$, the equality $\text{var}(E[y_t|x_t])/\text{var}(y_t) = \text{corr}(y_t, E[y_t|x_t])^2$ breaks down outside standard OLS, I report both R^2 measures. $\text{corr}(y_t, E[y_t|x_t])$ is a meaningful measure because a correlation of ρ indicates that the qualitative prediction is correct $50(1 + \rho)\%$ of the times. $\text{var}(E[y_t|x_t])/\text{var}(y_t)$ quantitatively compares the predicted variance to the actual variance, which is also a useful measure.⁴⁴

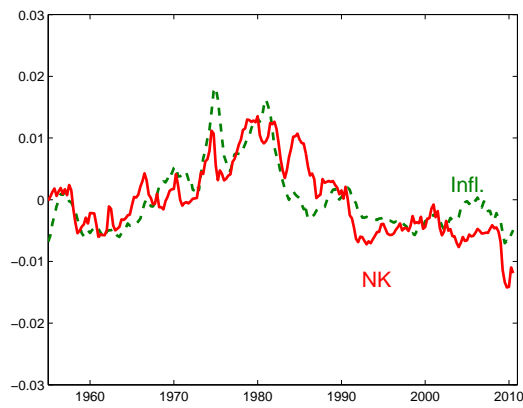
⁴⁴A third R^2 measure is $1 - [\text{var}(\varepsilon_t)/\text{var}(y_t)]$, however, outside OLS this measure is likely to take on values that are negative or greater than one, which are meaningless.



(a) *Market-book ratio.*



(b) *Investment-capital ratio.*



(c) *Inflation.*

Figure 2.5: U.S. time series, observed and predicted by benchmark and New Keynesian Q theories.

Market-book ratio				
RBC:	$corr(s, \widehat{s})$	0.28	$var(\widehat{s})/var(s)$	0.37
NK:	$corr(s, \widehat{s})$	0.72	$var(\widehat{s})/var(s)$	0.93
Investment-capital ratio				
RBC:	$corr(ik, \widehat{ik})$	0.28	$var(\widehat{ik})/var(ik)$	2.72
NK:	$corr(ik, \widehat{ik})$	0.61	$var(\widehat{ik})/var(ik)$	2.39
Inflation				
NK:	$corr(\pi, \widehat{\pi})$	0.72	$var(\widehat{\pi})/var(\pi)$	1.12

Table 2.5: R^2 measures of fit of predictions.

Table 2.5 reports the unconditional correlation between the observed and predicted time series. The simple addition of inflation dramatically improves the fit of the Q theory with respect to the benchmark frictionless case. Predicted and observed investment-capital ratios have a 61% correlation, which represents an improvement of 120% over the benchmark. The predicted and observed market-book ratios have a 72% correlation, which is a 160% improvement over benchmark. The predicted inflation rates track the observed rates with a 72% correlation.

2.4.4. Robustness

I next turn to relaxing the identifying condition (2.16), $var(s_t) = var(\widehat{s}_t)$, and use only the requirement that theoretical and actual stock prices have equal ability in forecasting returns. Note how identifying condition (2.15),

$$E\left(\begin{bmatrix} \widehat{s}_t \\ s_t \end{bmatrix} \varepsilon_{t+h}\right) = 0, \quad h \in \{1, \dots, H\}$$

is only able to set-identify the key parameters η and λ , because \widehat{s}_t and $\kappa\widehat{s}_t$ have the same forecasting ability, for any nonzero multiple κ . Indeed, this fact shows how the success of the NKQ story in explaining a relation between stock prices, investment and inflation is robust to relaxing assumption (2.16).

Figure 2.6 plots the first step GMM objective and shows the identified set for the key parameters. The GMM objective displays a ridge that describes all combinations of the key parameters $[\eta; \lambda]$ that associate to an equal return-forecasting ability, which remains as depicted in figure 2.3. For obvious reasons, this estimation procedure is no longer suitable to pin down the structural parameters uniquely or to discuss the fit of the NKQ theory, but it remains entirely appropriate to test the NKQ equation and to give empirical content to the predicted linkage between inflation and stock prices.

2.5. Conclusion

Along several dimensions, the New Keynesian Q theory fits well postwar U.S. data. The addition of inflation to the benchmark Q theory goes a long way in accounting for the states that drive both

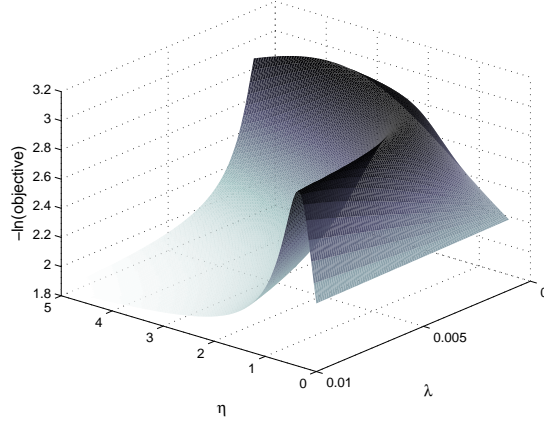


Figure 2.6: Graph of GMM objective as function of $[\eta, \lambda]$, optimal $b^{(h)}$ fixed, set-identified case. The weighting matrix is the identity matrix.

stock prices and investment. Coincidentally, the results contribute to the macro-finance literature by providing theoretical support to the finding of Bekaert and Engstrom (2010) that expected inflation is responsible for the high comovement between stock yields and nominal variables such as bond yields (which, for example, is the basis of the so-called ‘Fed model’).

There is still a relatively small low-frequency component in excess returns that is forecast by empirical stock prices and not by predicted stock prices. However, this discrepancy between predicted and observed stock prices seems to have nearly disappeared in the last 20 years. This observation suggests that such a missing component in the predicted market-book ratio has stabilized in the last 20 years.

Appendix

2.A. Average Q and marginal Q

Along the lines of Hayashi (1982), I derive the relation between the unobservable marginal Q and the observable average Q , which I denote by S_t .

By the degree-one homogeneity of the production function and the optimality conditions,

$$\begin{aligned} \bar{P}_t(i)Y_t(i) - (1 - \tau)\frac{W_t}{P_t}N_t(i) - (1 - \tau)I_t(i) - T_t &= \bar{P}_t(i)Y_t(i) - \frac{W_t}{P_t}N_t(i) - I_t(i) + \bar{T}_t(i) \\ &= \widehat{MC}_t(i)MPK_t(i)K_t(i) + [\bar{P}_t(i) - \widehat{MC}_t(i)]Y_t(i) + \bar{T}_t(i) \end{aligned} \quad (2.A.1)$$

where $\bar{P}_t(i) \equiv P_t(i)/P_t$ and $\bar{T}_t(i) \equiv \tau W_t[N_t(i) - N_t] + \tau[I_t(i) - I_t]$ is a term that aggregates to zero.

Next, using $M_{t+1} \equiv \beta \Lambda_{t+1} P_{t+1} / \Lambda_t P_t$, with Λ the marginal value of nominal income, rewrite the

optimality conditions (2.6) and (2.7) as

$$\begin{aligned} \beta E_t \Lambda_{t+1} P_{t+1} [\widehat{MC}_{t+1}(i) MPK_{t+1}(i) K_{t+1}(i) - I_{t+1}(i)] = \\ = \Lambda_t P_t Q_t(i) K_{t+1}(i) - \beta E_t \Lambda_{t+1} P_{t+1} Q_{t+1}(i) K_{t+2}(i) \end{aligned} \quad (2.A.2)$$

which I use, along with equation (2.A.1), to derive equation (2.8)

$$\begin{aligned} S_t(i) &= \frac{\sum_{h=1}^{\infty} \beta^h E_t \frac{\Lambda_{t+h} P_{t+h}}{\Lambda_t P_t} [\bar{P}_{t+h}(i) Y_{t+h}(i) - (1-\tau) \frac{W_{t+h}}{P_{t+h}} N_{t+h}(i) - (1-\tau) I_{t+h}(i) - T_{t+h}]}{K_{t+1}(i)} \\ &= \frac{\sum_{h=1}^{\infty} \beta^h E_t \frac{\Lambda_{t+h} P_{t+h}}{\Lambda_t P_t} [\widehat{MC}_{t+h}(i) MPK_{t+h}(i) K_{t+h}(i) - I_{t+h}(i) + [\bar{P}_t(i) - \widehat{MC}_{t+h}(i)] Y_{t+h}(i) + \bar{T}_{t+h}(i)]}{K_{t+1}(i)} \\ &= Q_t(i) + \frac{1}{K_{t+1}(i)} \sum_{h=1}^{\infty} E_t M_{t,t+h} \{ [\bar{P}_{t+h}(i) - \widehat{MC}_{t+h}(i)] Y_{t+h}(i) + \bar{T}_{t+h}(i) \} \end{aligned}$$

After aggregation⁴⁵ and a loglinearization around the undistorted zero-inflation steady state,⁴⁶

$$\begin{aligned} s_t - q_t &= -\vartheta \sum_{j=0}^{\infty} \beta^j E_t mc_{t+j+1} \\ &= -\frac{\vartheta}{\lambda} E_t \pi_{t+1} \end{aligned} \quad (2.A.3)$$

where the last equality uses the New Keynesian Phillips curve (2.9). (The technical appendix derives equation (2.A.3) in more detail and generalizes it to the case of a distorted steady state.) I once more use the New Keynesian Phillips curve (2.9) and the relation between real marginal costs and the labor share of income, $mc_t = \ell_t$, to derive equation (2.11).

Note how, in an efficient economy, $mc_t = 0$ at all t ; thus, I obtain $s_t^e = q_t^e$, which is the result obtained by Hayashi (1982) that the average and marginal Q are equal when there are no imperfections in the market for goods.

2.B. First-differenced NKQ theory

The gross return to investment, R_{t+1}^I , is such that

$$\begin{aligned} 1 &= E_t M_{t+1} \frac{\widehat{MC}_{t+1}(i) MPK_{t+1}(i) - \frac{I_{t+1}(i)}{K_{t+1}(i)} + \frac{K_{t+2}(i)}{K_{t+1}(i)} Q_{t+1}(i)}{Q_t(i)} \\ &\equiv E_t M_{t+1} R_{t+1}^I(i) \end{aligned} \quad (2.B.4)$$

⁴⁵Note that $\int_0^1 \bar{T}_t(i) di = 0$.

⁴⁶Note that $\int_0^1 \bar{p}_t(i) di = 0$, up to a first-order approximation.

The gross return on stock i is

$$R_{t+1}^m(i) = \frac{E_{t+1} \sum_{h=0}^{\infty} M_{t+1,t+h+1} D_{t+h+1}(i)}{E_t \sum_{h=0}^{\infty} M_{t,t+h} D_{t+h}(i) - D_t(i)}$$

Then, proceeding as in the derivation of (2.8),

$$R_{t+1}^m(i) = \frac{Q_t(i)K_{t+1}(i)R_{t+1}^I(i) + E_{t+1} \sum_{h=1}^{\infty} M_{t+1,t+h}[\bar{P}_{t+h}(i) - \widehat{MC}_{t+h}(i)]Y_{t+h}(i)}{Q_t(i)K_{t+1}(i) + E_t \sum_{h=1}^{\infty} M_{t,t+h}[\bar{P}_{t+h}(i) - \widehat{MC}_{t+h}(i)]Y_{t+h}(i)}$$

where the denominator is the current value of the i th firm and the numerator is tomorrow's payoff for owning the i th firm.

Note how in absence of imperfections in the market for goods, $\widehat{MC}_{t+h}(i) = \bar{P}_{t+h}(i)$, and therefore one has the benchmark Q prediction $R_{t+1}^m = R_{t+1}^I$ (Cochrane, 1991; Restoy and Rockinger, 1994).

When $\widehat{MC}_{t+h}(i) \neq \bar{P}_{t+h}(i)$, a loglinear approximation around the zero-inflation, undistorted steady state yields the first-differenced NKQ theory for the aggregate stock market return

$$r_{t+1}^m = r_{t+1}^I - \frac{\vartheta}{\lambda}(E_{t+1} - E_t)\pi_{t+1}$$

with $r_{t+1}^I = \alpha\vartheta(mpk_{t+1} + mc_{t+1}) + \eta\beta ik_{t+1} - \eta ik_t$.

2.C. Moments for estimation

Let $r_{t \rightarrow t+h} = E(r_{t \rightarrow t+h}|\bar{s}^t) + \varepsilon_{t+h}$. Suppose you increase the information set from \bar{s}^t to $[\bar{s}^t \ s^t]$, so that $r_{t \rightarrow t+h} = E(r_{t \rightarrow t+h}|\bar{s}^t, s^t) + \widetilde{\varepsilon}_{t+h}$. Suppose $v_t \equiv s_t - E(s_t|\bar{s}^t) \neq 0$, i.e. $[\bar{s}^t \ s^t]$ is an information set strictly bigger than \bar{s}^t . Assume all projections are linear. Then, $E(r_{t \rightarrow t+h}|\bar{s}^t, s^t) = E(r_{t \rightarrow t+h}|\bar{s}^t)$ if and only if $E(s_t \varepsilon_{t+h}) = 0$.

Proof. Let $X = \bar{s}^T$, $Z = s^T$, $Y = r^{T \rightarrow T+h}$, $\varepsilon = \varepsilon^T$ and $v = v^T$ be the sample vectors. Let $\widehat{\varepsilon}$ and \widehat{v} denote estimates of the unobservable vectors ε and v . Then, the projection matrix on the $[X \ Z]$ -space is

$$\begin{aligned} P_{[X \ Z]} &= [X \ Z]([X \ Z]'[X \ Z])^{-1}[X \ Z]' \\ &= [X \ Z] \begin{bmatrix} (X'X)^{-1} - (X'X)^{-1}X'ZV^{-1}Z'X(X'X)^{-1} & -(X'X)^{-1}X'ZV^{-1} \\ -V^{-1}Z'X(X'X)^{-1} & V^{-1} \end{bmatrix} [X \ Z]' \end{aligned}$$

where $V = Z'Z - Z'X(X'X)^{-1}X'Z = Z'M_XZ = Z'M'_X M_X Z = \widehat{v}\widehat{v}'$, with $M_X = I - P_X$ and $P_X = X(X'X)^{-1}X'$ the projection matrix on the X -space. Therefore, you can check that

$$\begin{aligned} P_{[X \ Z]}X\beta &= X\beta & \xrightarrow{a.s.} & E(E(r_{t \rightarrow t+h}|\bar{s}^t)|\bar{s}^t, s^t) = E(r_{t \rightarrow t+h}|\bar{s}^t) \\ P_{[X \ Z]}\widehat{\varepsilon} &= M_X Z V^{-1} Z' \widehat{\varepsilon} = \frac{Z' \widehat{\varepsilon}}{\widehat{v}' \widehat{v}} \widehat{v} & \xrightarrow{a.s.} & E(\varepsilon_{t+h}|\bar{s}^t, s^t) = \frac{E(s_t \varepsilon_{t+h})}{E(v_t^2)} v_t \end{aligned}$$

where the asymptotic property holds by the law of large numbers for ergodic, stationary stochastic processes.

Thus, $E(r_{t \rightarrow t+h}|\widehat{s}^t, s^t) = E(E(r_{t \rightarrow t+h}|\widehat{s}^t)|\widehat{s}^t, s^t) + E(\varepsilon_{t+h}|\widehat{s}^t, s^t) = E(r_{t \rightarrow t+h}|\widehat{s}^t) + [E(s_t \varepsilon_{t+h})/E(v_t^2)]v_t$, so that $E(r_{t \rightarrow t+h}|\widehat{s}^t, s^t) = E(r_{t \rightarrow t+h}|\widehat{s}^t)$ if and only if $E(s_t \varepsilon_{t+h}) = 0$ (since by assumption $v \neq 0$, hence $E(v_t^2) > 0$). \square

2.D. Asymptotics of the estimates

I can write moments (2.15) and (2.16) for the GMM estimation as

$$\begin{aligned} E(\varepsilon_{t+h}\widehat{s}_t) &= 0, & h \in \{1, \dots, H\} \\ E(\varepsilon_{t+h}s_t) &= 0, & h \in \{1, \dots, H\} \\ E(\widehat{s}_t^2 - s_t^2) &= 0 \end{aligned} \quad \Leftrightarrow \quad E(f_t(\theta)) = 0$$

where $\theta \equiv [b^{(h)}, \eta, \lambda]$, $\varepsilon_{t+h} = r_{t \rightarrow t+h}^m - b^{(h)}\widehat{s}_t$ and $\widehat{s}_t = \eta i k_t - (\vartheta/\beta\lambda)\pi_t + (\vartheta/\beta)\ell_t$.

The GMM estimator of θ minimizes $E_T(f_t(\theta))'W_T E_T(f_t(\theta))$, with notation $E_T(\cdot) = 1/T \sum_{t=1}^T \cdot$, where W_T is a weighting matrix. I use a two-step approach to efficient GMM estimation (Hansen and Singleton, 1982), where the first-step weighting matrix is the identity matrix. In the second step, therefore, W_T is a matrix that is asymptotically consistent for the efficient weighting matrix S^{-1} , where S is the spectral density at frequency zero

$$S = \sum_{j=-\infty}^{\infty} E(f_t(\theta)f_{t-j}(\theta)')$$

which I estimate nonparametrically (Newey and West, 1987), with a Bartlett kernel and eight leads and lags. Notice how, for any weighting matrix W_T , the interior solution satisfies the first-order condition

$$d_T' W_T E_T(f_t(\theta)) = 0$$

where $d_T \equiv \partial E_T(f_t(\theta))/\partial \theta'$, which fully defines the GMM estimate considered. Under standard regularity conditions, Hansen (1982) shows how to derive the sampling distribution of the estimate and the sample moments. The estimate distributes asymptotically as

$$\sqrt{T}(\widehat{\theta} - \theta) \xrightarrow{d} N(0, (d' W d)^{-1} d' W S W d (d' W d)^{-1})$$

where $d \equiv \partial E(f_t(\theta))/\partial \theta'$, while the sample moments—under the null hypothesis that they are zero in population—conform to

$$\sqrt{T} E_T(f_t(\theta)) \xrightarrow{d, H_0} N(0, (I - d(d' W d)^{-1} d' W) S (I - d(d' W d)^{-1} d' W)')$$

and thereby allow for a test of the $H - 1$ overidentifying restrictions

$$T E_T(f_t(\theta))' \left[(I - d(d' W d)^{-1} d' W) S (I - d(d' W d)^{-1} d' W)' \right]^{\dagger} E_T(f_t(\theta)) \xrightarrow{d, H_0} \chi_{H-1}^2$$

where ‘ \dagger ’ denotes the Moore-Penrose pseudo-inverse operator.

Part 3

Reassessing the Role of Stock Prices in the Conduct of Monetary Policy

Abstract. In a New Keynesian model with capital adjustment costs and exogenous sources of policy tradeoffs central banks should not respond to stock price movements; a policy that focuses on stabilizing inflation is close to optimal. I then solve numerically for the optimal Taylor-type rule that responds to stock prices by using the typical approach adopted by the extant literature, which consists in fixing the response coefficient to inflation to subsequently search numerically for desirable policy reactions to stock prices and output. The numerical approach can easily prescribe all possible qualitative reactions to stock prices. Therefore, the model highlights some pitfalls in a numerical study of stock prices and monetary policy that can explain and reconcile the conflicting policy prescriptions found in the literature.

3.1. Introduction

Should central banks respond to stock price movements? The extant literature provides conflicting answers to this question; we are still not sure how central banks should respond to stock prices. The discrepancy in the literature is qualitative, not merely quantitative. For example, Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002) find that central banks should not respond to stock price movements. Cecchetti et al. (2001, 2002) and Gilchrist and Saito (2008) find that they should, and they should do so by increasing interest rates during a stock price boom (a “leaning against the wind” prescription). Faia and Monacelli (2007) also prescribe a response, but they favor a rate decrease when stock prices increase.

The main contribution of this writing is to offer a potential explanation for the conflicting policy prescriptions found in the literature, and thereby a resolution. I adopt a simple New Keynesian setting with capital adjustment costs in which nominal rigidities are the only source of inefficiencies and policy tradeoffs are caused by some exogenous shocks. The model is sufficiently simple that I can derive, on the one hand, a clean analytical approximate welfare loss function and, on the other, the optimal monetary policy prescription, at least within the family of Taylor-type rules. This simplicity allows me to highlight and explain some potential pitfalls in the typical numerical approach adopted by the relevant literature, which consists in fixing the response coefficient to inflation to subsequently search numerically for desirable policy reactions to stock prices and output.

3.1.1. *Pitfalls in a numerical study of optimal Taylor-type rules*

A first problem is that, taken at face value, the prescribed coefficients can be misleading. For realistic and fixed values of the Taylor-rule coefficient attached to inflation the central bank can increase welfare by responding to movements in stock prices and output, but such a response increases welfare only because it strengthens the central bank's anti-inflationary stance. Therefore, an economist may report the desirability of some policy response to stock prices or output even though such a prescription is really just a roundabout way of increasing the response coefficient attached to inflation.

Second, when conducting the numerical exercise within my simple model I easily find all three different prescriptions given a change in stock prices: no response, a positive response and a negative response. Formally, the prescribed Taylor-rule coefficients are not robust, even qualitatively, to small changes in two key parameters—the elasticity of Q to investment, η , and the slope of the New Keynesian Phillips curve, λ . The fragility to changes across the (λ, η) -plane is particularly troubling in that η and λ are precisely two parameters that have a high dispersion in the literature. Moreover, numerical prescriptions differ substantially for different exogenous sources of policy tradeoffs. The dispersion in the policy prescriptions recommended by the literature appears therefore to be an inherent characteristic of the problem.

An analytical approach to welfare highlights the reason for the sensitivity of the prescription to changes in the key parameters. I show how, in addition to price and production stability, marginal Q stability (relative to its efficient level) directly affects consumer welfare in the model up to a second-order approximation; the welfare weights on Q stability and on inflation stability in turn depend on the key parameters η and λ , respectively.⁴⁷

Third, part of the reason for the fragility is that the prescribed responses to Q and output are often as close as possible to the indeterminacy border (those coefficients plus epsilon will send the system to indeterminacy), which moves as the inflation coefficient varies. This issue arises because a reaction to Q pushes capital towards a unit-root process. In this regard, the complementary response to output often found in the literature (e.g., Cecchetti et al., 2001) results to offset such a nonstationarity and is typically just enough to leave the system on the indeterminacy border.

The indeterminacy issue is even worse if capital is firm-specific. On the one hand, the firm-specific problem has stability requirements independent of the policy rule in place (a point made by Woodford, 2005).⁴⁸ On the other, sticky prices break the equality between marginal Q and its observable proxy, i.e., average Q (Hayashi, 1982). Namely, in the model average Q equals marginal Q less a constant multiple of expected inflation. Therefore, a central bank that proxies a positive prescribed reaction to the unobservable marginal Q by responding positively to average Q may push the economy to an indetermined equilibrium. It is customary in the literature to introduce a

⁴⁷Price rigidities are the only friction in the model. The priority of an aggressive inflation stabilization looks therefore rather intuitive. However, a low value of η increases the priority of Q stabilization relative to inflation stabilization. It is therefore not trivial that inflation stabilization remains priority across the (λ, η) -plane when there are exogenous sources of policy tradeoffs. Therefore, an analysis across different values of the two key parameters combined with the exogenous sources of policy tradeoffs allows for an analytically tractable framework to study non-trivial tradeoffs among inflation, output and financial stability.

⁴⁸Sveen and Weinke (2005) pursue a different point, which states that for low values of the reaction coefficient to inflation and of the elasticity λ the system is indetermined.

proxy problem by assuming an exogenous bubble component in stock prices; but under nominal rigidities the proxy problem is true even in a setting without bubbles.⁴⁹

3.1.2. Literature review

Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002) find that focusing on inflation stability is sufficient. Bernanke and Gertler compute second moments of inflation and the output gap under two rules: one based solely on inflation and one based on inflation and stock prices. These authors define stock prices as marginal Q plus an exogenous bubble component. They find that the rule that includes stock prices generally increases the volatility of inflation and output and that a strong commitment to stabilizing inflation minimizes those volatilities, at least within the family of policy rules they consider. Cecchetti et al. (2001) and Cecchetti et al. (2002) counter by saying that Bernanke and Gertler do not choose the response coefficients optimally. Bernanke and Gertler compute the variances of inflation and output, but do not attach weights to the two variances to form a welfare criterion. In contrast, Cecchetti et al. use a welfare criterion in which they specify weights for both inflation and output. They find that some policy response to stock prices is typically optimal, at least if complemented by some response to output. More recently, Faia and Monacelli (2007) and Gilchrist and Saito (2008) conclude that some reaction to asset prices (marginal Q) in a Taylor-type rule is desirable, although they differ in the sign of the prescribed response. However, Faia and Monacelli argue that under a strong anti-inflationary stance (a good approximation to the optimal policy), the benefits of reacting to stock prices nearly vanish, a result that I find also in my simpler setting.

Against this background, note that the extant literature does not attempt an analytical derivation of welfare because it typically deals with rather complex models (e.g., that include endogenous financial frictions). The inflation-output welfare metric typically used in the literature is arbitrary and a quantitatively bad approximation in regions of the deep-parameter space in which the welfare weight attached to financial stability is relatively high.⁵⁰ Furthermore, a correct welfare analysis allows for a clear statement of which stock price measures (among, for example, marginal Q , average Q , or their gaps) affect welfare and which are potential arguments of a policy rule. My welfare analysis shows how the gap in marginal Q is the financial quantity that directly affects welfare and discusses the additional problems arising from the fact that marginal Q is unobservable.⁵¹

3.2. A monetary model

I work with a New Keynesian model with capital accumulation and adjustment costs (King and Wolman, 1996; Galí and Gertler, 2007). I choose the New Keynesian model since it is a standard

⁴⁹Moreover, the addition of bubbles raises some additional theoretical issues, including their definition; a bubble can be defined in terms of exogenous noise around Q , in terms of some gap measure relative to a benchmark value for Q , or as the violation of a transversality condition (or lack thereof, as in the OLG model of Galí, 2013) as in the case of rational bubbles.

⁵⁰The notable exception is Faia and Monacelli (2007) who, in their simulations, implicitly work with the correct welfare function, although they do not derive the function analytically.

⁵¹Gilchrist and Saito (2008) correctly argue that the marginal Q gap is the welfare-relevant quantity, although they also use an ad hoc inflation-output criterion and thereby cannot motivate the importance of the marginal Q gap from first principles.

workhorse for monetary policy evaluation. The capital adjustment cost function generates stock price variation (Hayashi, 1982). I make investment a firm-specific decision variable (Woodford, 2005; Sveen and Weinke, 2005) to induce a link between the marginal Q and the market-book ratio (average Q), so that I can discuss the relationship between stock prices and marginal Q. (“Stock prices,” “average Q,” and “market-book ratio” are all used interchangeably here.) Note how the pitfalls I point out in this writing do not hinge on the firm-specific capital specification; to maximize comparability to the literature, in what follows I derive the structural equations and the welfare criterion and conduct policy evaluation both in the firm-specific and in the rental-market cases.

The aggregate shocks that hit the economy result from either a cost-push or a financial disturbance. Such exogenous shocks are a simple device to break the divine coincidence (Blanchard and Galí, 2007) and make the optimal policy problem non-trivial. Moreover, these shocks break the divine coincidence but allow for a simple characterization of a nearly optimal Taylor-type rule. I do not therefore follow the rest of the literature in introducing additional complexity, such as endogenous financial frictions,⁵² because it would complicate the model—in particular the derivation of a welfare criterion—without necessarily changing the policy prescription. In fact, models that endogenize the financial disturbance (for example, Faia and Monacelli, 2007; Carlstrom, Fuerst and Paustian, 2010) similarly characterize the optimal Taylor-type rule as a rule aggressive on inflation.

The next subsections describe the model. The technical appendix derives the model in more detail.

3.2.1. Consumers

Identical consumers choose consumption demand $\{C_t\}$, labor supply $\{N_t\}$ and demand for one-period bonds $\{B_t\}$ to maximize the utility function

$$E_t \sum_{h=0}^{\infty} \beta^h \left[\frac{1}{1-\sigma} C_{t+h}^{1-\sigma} - \frac{1}{1+\varphi} N_{t+h}^{1+\varphi} \right]$$

subject to the budget constraint

$$\int_0^1 P_{t+h}(i) C_{t+h}(i) di + \frac{1}{1+i_{t+h}^f} B_{t+h} \leq W_{t+h} N_{t+h} + B_{t+h-1} + P_{t+h} D_{t+h}$$

where C is real consumption, P is the nominal price of a unit of consumption, B are nominal risk-free bonds with rate of return i^f , N are hours worked, W is the nominal wage rate, and D is the real dividend that firms pay to consumers (who own the firms). Parameters σ and φ are the inverse of the (constant) intertemporal elasticity of substitution and Frisch labor supply elasticity, respectively.

Aggregate real consumption C_t is the Dixit and Stiglitz (1977) aggregate of the continuum of goods $i \in [0, 1]$

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

⁵²See also Iacoviello (2005), Ravenna and Walsh (2006), Monacelli (2008), Carlstrom, Fuerst and Paustian (2010) and Leduc and Natal (2011).

where ε is the (constant) elasticity of substitution between consumption of any two goods. Consumers form consumption units by minimizing costs. The first-order condition for that problem yields the demand curve for good i

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \quad (3.1)$$

where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$ is the price index. The solution implies that the optimal expenditure is $P_t C_t = \int_0^1 P_t(i) C_t(i) di$.

The first-order conditions for an interior solution of the consumers' problem are

$$MRS_t = C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \quad (3.2)$$

$$E_t M_{t,t+1} \frac{1 + i_t^f}{\Pi_{t,t+1}} = E_t M_{t,t+1} R_t^f = 1 \quad (3.3)$$

where MRS is the marginal rate of substitution between consumption and labor, $M_{t+1} = \beta(C_{t+1}/C_t)^{-\sigma}$ is the stochastic discount factor for real payoffs, $\Pi_{t,t+1} \equiv P_{t+1}/P_t$ is the gross inflation rate between times t and $t+1$, and the Fisher equation $r_t = i_t^f - E_t \pi_{t+1}$ describes the real risk-free rate $r_t \approx \ln(R_t^f)$.

3.2.2. Firms

A continuum of monopolistically competitive firms $i \in [0, 1]$ choose nominal prices $\{P_{t+h}^*(i)\}$, production $\{Y_{t+h}(i)\}$, labor demand $\{N_{t+h}(i)\}$ and investment $\{I_{t+h}(i)\}$ to maximize the expected discounted value of profits

$$E_t \sum_{h=0}^{\infty} M_{t,t+h} \left[\frac{P_{t+h}(i)}{P_{t+h}} Y_{t+h}(i) - (1-\tau) \frac{W_{t+h}}{P_{t+h}} N_{t+h}(i) - (1-\tau) \int_0^1 P_{t+h}(j) I_{t+h}(j, i) dj - T_{t+h} \right]$$

subject to⁵³

$$\begin{aligned} Y_{t+h}(i) &= \left(\frac{P_{t+h}(i)}{P_{t+h}} \right)^{-\varepsilon} Y_{t+h} \\ Y_{t+h}(i) &= A_{t+h} K_{t+h}(i)^\alpha N_{t+h}(i)^{1-\alpha} \\ K_{t+h+1}(i) &= (1-\delta) K_{t+h}(i) + \Phi \left(\frac{I_{t+h}(i)}{K_{t+h}(i)} \right) K_{t+h}(i) \end{aligned}$$

⁵³Firms minimize costs in forming the investment aggregate $I_t^d \equiv \left[\int_0^1 I_t^d(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$. The demand equation for investment goods is thus $I_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} I_t^d$. One can combine this result with (3.1) and (3.9) to get the demand equation for a given firm i , $Y_t(i) = \left(\frac{P_{t+h}(i)}{P_{t+h}} \right)^{-\varepsilon} Y_{t+h}$. Moreover, the optimal expenditure is such that $\int_0^1 P_{t+h}(j) I_{t+h}(j, i) dj = P_{t+h} I_{t+h}(i)$.

$$P_{t+h}(i) = \begin{cases} P_{t+h}^*(i), & \text{with probability } (1 - \theta) \\ P_{t+h-1}(i), & \text{with probability } \theta \end{cases}$$

$$P_{t+h}I_{t+h}(i) = \int_0^1 P_{t+h}(j)I_{t+h}(j, i) dj$$

where Y is output, A is technology, K is the real capital stock and I is real investment. Prices are sticky as in Calvo (1983), so that firms can reset prices at any given time only with probability $1 - \theta$. There are capital adjustment costs. The adjustment cost function $\Phi(\cdot)$ is such that $\Phi(I/K) = \delta$, $\Phi'(I/K) = 1$ and its curvature $\eta \equiv -[\Phi''(I/K)I/K]/\Phi'(I/K)$ is constant and positive in the steady state. The government levies lump-sum taxes, $T_t = \tau W_t/P_t + \tau I_t$, to finance an employment and investment subsidy, τ , which is in place to offset any steady-state distortion caused by the monopolistic competition.⁵⁴ Parameters δ and $1 - \alpha$ are the average depreciation rate of capital and labor share of value added, respectively.

The first-order conditions for an interior optimum are

$$\sum_{h=0}^{\infty} \theta^h E_t M_{t,t+h} P_{t+h}^{\varepsilon-1} Y_{t+h}(i) [P_t^*(i) - \widehat{MC}_{t+h}(i) P_{t+h}] = 0 \quad (3.4)$$

$$MC_t(i) MPN_t(i) = (1 - \tau) \frac{W_t}{P_t} \quad (3.5)$$

$$Q_t(i) = \left[\Phi' \left(\frac{I_t(i)}{K_t(i)} \right) \right]^{-1} \quad (3.6)$$

$$E_t M_{t,t+1} R_{t+1}^I(i) = 1 \quad (3.7)$$

with

$$R_{t+1}^I(i) \equiv \frac{\widehat{MC}_{t+1}(i) MPK_{t+1}(i) + \left((1 - \delta) + \Phi \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) - \Phi' \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) \frac{I_{t+1}(i)}{K_{t+1}(i)} \right) Q_{t+1}(i)}{Q_t(i)}$$

where Q is marginal Tobin's Q , MPN is the marginal productivity of labor, MC is the real marginal cost of output, R^I is the gross real rate of return to investment, MPK is the marginal productivity of capital and $\widehat{MC} \equiv MMC$, with $M \equiv \varepsilon/(\varepsilon - 1)$ the average price markup that firms charge over marginal costs under monopolistic competition in the goods market.

Firms then distribute profits as dividends $P_{t+h} D_{t+h}(i)$ to consumers.

Next, I define average Tobin's Q as

$$S_t(i) \equiv \frac{E_t \sum_{h=1}^{\infty} M_{t,t+h} D_{t+h}(i)}{K_{t+1}(i)}$$

the ratio of two different valuations of the capital stock of firm i : the market price (expected discounted dividends) and the book value.

⁵⁴ $\tau = 1/\varepsilon$ achieves this goal.

Average Q is related to marginal Q through

$$S_t(i) = Q_t(i) + \frac{1}{K_{t+1}(i)} \sum_{h=1}^{\infty} E_t M_{t,t+h} \left[\frac{P_{t+h}(i)}{P_{t+h}} - \widehat{MC}_{t+h}(i) \right] Y_{t+h}(i) \quad (3.8)$$

which states that the market-book ratio is the marginal value of capital plus the expected discounted profits per unit of capital. These profits are driven by imperfections in the goods market. I prove equation (3.8) in the appendix. This result extends the result in Hayashi (1982) to a model with nominal rigidities and monopolistic competition in the goods market.

3.2.3. Market clearing

The goods market clears; for each firm i , supply equals demand,

$$Y_t(i) = C_t(i) + I_t^d(i) \quad (3.9)$$

where $I_t^d(i)$ denotes the demand for good i coming from all firms.

The labor market also clears; labor supply equals average labor demand as

$$N_t = \int_0^1 N_t(i) di$$

3.2.4. Structural form of the model solution

I loglinearize and aggregate the first-order conditions around the deterministic zero-inflation undistorted steady state. The full loglinear model is available in the appendix. The approximation yields a set of six equations, which together form the structural model economy that I consider from this point on. I express everything in terms of gaps. As is customary in the literature, I define a gap as the percentage deviation of a variable from its first-best level. Namely, $\widetilde{y}_t \equiv y_t - y_t^e$ is the output gap, $\widetilde{q}_t \equiv q_t - q_t^e$ is the marginal Q gap, and $\widetilde{k}_t \equiv k_t - k_t^e$ is the capital gap.

Marginal Q moves according to the dynamic equation

$$\widetilde{q}_t = \beta E_t \widetilde{q}_{t+1} + \vartheta E_t (\widetilde{y}_{t+1} - \widetilde{k}_{t+1}) + \vartheta E_t \widehat{mc}_{t+1} - \widetilde{r}_t + \chi_t \quad (3.10)$$

where ϑ is the composite parameter $\vartheta \equiv 1 - \beta(1 - \delta)$ and $\widetilde{r}_t \equiv i_t^f - E_t \pi_{t+1} - r_t^e$ is the real risk-free rate gap. The exogenous process χ_t represents a financial disturbance and follows the stationary autoregression

$$\chi_{t+1} = \rho_\chi \chi_t + \varepsilon_{\chi,t+1} \quad (3.11)$$

where ε_χ is a white-noise process.

The output gap evolves by the dynamic IS equation augmented with marginal Q gaps

$$\Psi \widetilde{y}_t = \Psi E_t \widetilde{y}_{t+1} + (\Lambda - 1) \widetilde{q}_t - \Lambda E_t \widetilde{q}_{t+1} - \widetilde{r}_t \quad (3.12)$$

where Ψ and Λ are the composite parameters $\Psi \equiv \sigma/\gamma_c$ and $\Lambda \equiv 1 + (1 - \delta)\Xi_{yq}$. The approximate

link between the real economy and financial markets, $\tilde{i}_t - \tilde{k}_t = \frac{1}{\eta} \tilde{q}_t$, allows me to interpret the curvature parameter η as the inverse of the elasticity of the investment-capital ratio to marginal Q.

A standard New Keynesian Phillips curve drives inflation as

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t \quad (3.13)$$

The link between the elasticity λ and the frequency of price adjustment is nonstandard under firm-specific capital accumulation (Woodford, 2005; Sveen and Weinke, 2005). The appendix solves the firm-specific problem. The wedge \widehat{mc}_t is responsible for all deviations from the first best, which are entirely due to nominal rigidities. Real marginal costs obey the approximate equilibrium condition

$$\widehat{mc}_t = \Xi_y \tilde{y}_t - \Xi_{yq} \tilde{q}_t - \Xi_{yk} \tilde{k}_t + \frac{1}{\lambda} u_t \quad (3.14)$$

where $\Xi_y, \Xi_{yq}, \Xi_{yk}$ are the weights in the welfare function (3.18).

Under firm-specific capital accumulation, average Q and marginal Q relate via the equation

$$\tilde{s}_t = \tilde{q}_t - \frac{\vartheta}{\alpha \lambda} E_t \pi_{t+1} \quad (3.15)$$

which I prove in the appendix. Equation (3.15) endogenously models a proxy problem for a central bank that were to react to marginal Q through stock prices.

Finally, two dynamic equations drive the states of the economy

$$\tilde{k}_{t+1} = \tilde{k}_t + \frac{\delta}{\eta} \tilde{q}_t \quad (3.16)$$

$$u_{t+1} = \rho_u u_t + \varepsilon_{u,t+1} \quad (3.17)$$

where ε_u is a white-noise process.

3.2.5. Exogenous shocks

I consider versions of the model that include either a cost-push disturbance u_t or a financial disturbance χ_t . I add these shocks to break the divine coincidence (Blanchard and Galí, 2007) and thereby avoid a trivial optimal monetary policy problem.⁵⁵ One can endogenize both disturbances from first principles. For example, Erceg et al. (2000) endogenize the cost-push disturbance through wage rigidities, and Bernanke et al. (1999) endogenize the financial disturbance through a costly state-verification mechanism under asymmetric information.⁵⁶ Against this background, the cost-push shock can be interpreted as a labor wedge due to imperfections in the labor market and the financial disturbance as a risk premium due to the presence of financial frictions.

⁵⁵Without any disturbance, any policy rule is optimal as long as it provides determinacy to the structural model.

⁵⁶With the financial accelerator in place, $\chi_t = \nu E_t \ln(N_t/Q_t K_{t+1})$, for some scale factor ν , where N_t is the net worth of the aggregate firm; the financial disturbance is related to the average leverage ratio in the economy.

3.2.6. Calibration

I select standard values for uncontroversial parameters. For the key parameters η and λ , I consider the range of values listed in table 3.1.

α	capital share in value added	0.35
β	discount factor	0.99
σ	inverse of intertemporal elasticity of substitution	1
φ	inverse of Frisch labor supply elasticity	0.8
δ	depreciation rate	0.025
$\Phi'(\delta)^{-1}$	steady-state value of marginal Q	1
λ	slope of the New Keynesian Phillips curve	(0, 0.05)
η	inverse of elasticity of investment-capital ratio to marginal Q	(0, 5)

Table 3.1: Calibration of the deep parameters

I conduct a robustness analysis to changes in the elasticities λ and η .⁵⁷

I consider values of the elasticity η between 0 and 5. There is considerable disagreement in the literature about the most likely range of values for η . One strand of the literature (e.g., Baxter and Crucini, 1993; King and Wolman, 1996; Bernanke et al., 1999) suggests values between 0 and 1 while another strand (e.g., Abel, 1980; Jermann, 1998) prefers values between 2 and 5.

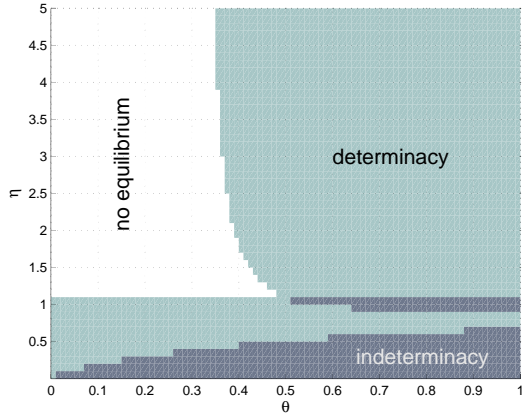
The parameter λ is also quite controversial quantitatively. Galí and Gertler (1999), Galí et al. (2005) and Kleibergen and Mavroeidis (2009) give point estimates between 0.01 and 0.03, but there is still debate about its identification (Mavroeidis, 2005; Nason and Smith, 2008; Canova and Sala, 2009). In line with the estimate in Galí and Gertler (1999), I consider a set for λ that ranges from 0 to 0.05.

3.2.7. Rental market

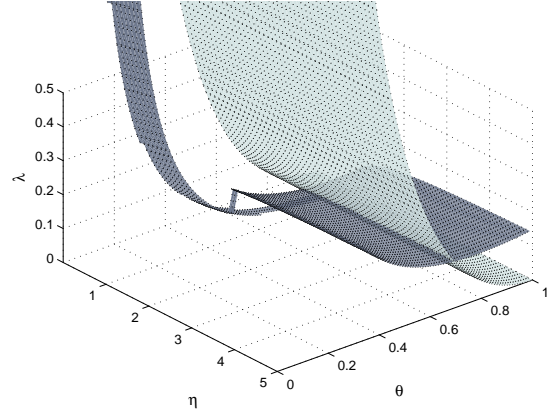
Under the baseline calibration, figure 3.1 analyzes quantitatively the firm-specific problem across the (θ, η) -plane and under the calibration of the remaining deep parameters reported in table 3.1. An important point, emphasized by Woodford (2005), is how an important region of the deep-parameter space yields no equilibria under the firm-specific capital specification. The appendix discusses the requirements for determinacy, which I used to construct figure 3.1. Moreover, another important region of the (θ, η) -plane yields multiple equilibria. These two facts characterize the first-order component of the solution to the firm-specific problem.

This determinacy issue vanishes if I model capital accumulation through a rental market. If firms purchase capital services on a competitive rental market at the common rental rate R_t^k , each firm repurchases from the consumers the required capital stock entirely at each period. Therefore,

⁵⁷I consider several calibrations for λ , rather than for the frequency of price re-optimization θ . The parameter θ has a nice microeconomic interpretation, but a fragile link to the structural equations of the New Keynesian model. The structural equations depend on θ only through λ . However, the link between θ and λ is not robust, for example, to the choice of modeling investment through a rental market or through firm-specific capital accumulation (Sveen and Weinke, 2005). The connection between the structural equations and λ is more robust.



(a) Determinacy regions, firm-specific capital.



(b) Slope of the New Keynesian Phillips curve under firm-specific capital (dark surface) and a rental market (light surface).

Figure 3.1: Determinacy in the firm-specific capital and rental-market specifications, as function of the key parameters θ (Calvo frequency of price adjustment) and η (curvature of capital adjustment costs).

all firms that reset at time t face an identical problem as they do not inherit different capital stocks. In particular, the firms' first-order conditions for optimal labor and capital purchases are

$$\begin{aligned} MC_t(i)MPN_t(i) &= (1 - \tau) \frac{W_t}{P_t} \\ MC_t(i)MPK_t(i) &= (1 - \tau)R_t^k \end{aligned}$$

Since $MPN_t(i) = X_t MPK_t(i)^{-\alpha/(1-\alpha)}$, where X_t is independent of i , then the distance between individual and aggregate average marginal costs is zero, and therefore $mpn_t(i) - mpn_t = mpk_t(i) - mpk_t = 0$. Therefore, $y_t(i) - y_t = k_t(i) - k_t = n_t(i) - n_t = -\varepsilon(p_t(i) - p_t)$; firms with higher demand for their goods purchase a proportionately higher quantity of capital and labor services.

The only first-order difference between the firm-specific and the rental-market specifications is the link between the slope of the New Keynesian Phillips curve, λ , and the frequency of price optimization, θ . In particular, in the rental-market case the slope of the New Keynesian Phillips curve is $\lambda^m = (1 - \theta)(1 - \beta\theta)/\theta$.

3.3. Welfare criterion

Following the seminal work of Rotemberg and Woodford (1997, 1999), the standard metric to evaluate alternative monetary policies is a welfare criterion, which arises naturally in the context of microfounded New Keynesian macromodels. A quadratic approximation to the unconditional consumers' objective function around the deterministic zero-inflation steady-state value provides a welfare criterion against which I can evaluate and rank different policies. I show in the appendix how the average welfare loss per period is directly proportional to (up to a term independent of

policy)

$$\begin{aligned}\mathbb{L} \equiv & \frac{1}{2}\Xi_y \text{var}(\tilde{y}) + \frac{1}{2}\Xi_q \text{var}(\tilde{q}) - \Xi_{yq} \text{cov}(\tilde{y}, \tilde{q}) + \frac{1}{2}\Xi_p \text{var}(\pi) \\ & + \frac{1}{2}\Xi_k \text{var}(\tilde{k}) - \Xi_{yk} \text{cov}(\tilde{y}, \tilde{k}) + \Xi_{qk} \text{cov}(\tilde{q}, \tilde{k})\end{aligned}\quad (3.18)$$

where the welfare weights are linked to the deep parameters as $\Xi_y \equiv \frac{\sigma}{\gamma_c} + \frac{\alpha+\varphi}{1-\alpha}$, $\Xi_q \equiv \frac{\gamma_c}{\sigma}\Xi_{yq}(1 + \Xi_{yq})$,

$\Xi_{yq} \equiv \frac{\sigma\gamma_i}{\gamma_c\eta}$, $\Xi_k \equiv \gamma_i + \frac{\sigma\gamma_i^2}{\gamma_c} + \frac{(1+\varphi)\alpha^2}{1-\alpha}$, $\Xi_{yk} \equiv \frac{\sigma\gamma_i}{\gamma_c} + \frac{(1+\varphi)\alpha}{1-\alpha}$ and $\Xi_{qk} \equiv \gamma_i\Xi_{yq}$.

In the firm-specific case, the welfare weight attached to inflation, Ξ_p , is a complex function of the deep parameters, which I derive in the appendix. Note how in the rental-market case average welfare losses maintain the form (3.18), with the only difference in the shape of the welfare weight attached to inflation volatility, which becomes $\Xi_p^{rm} = \varepsilon/\lambda^{rm}$.

The major difference between welfare criterion (3.18) and the inflation-output criterion of models without capital and adjustment costs

$$\frac{1}{2}\Xi_y \text{var}(\tilde{y}) + \frac{1}{2}\Xi_p \text{var}(\pi)$$

is the additional presence of two gap terms, one in marginal Q and one in capital.⁵⁸ The presence of a gap in marginal Q is particularly appealing in that it means that financial instability is per se a source of welfare losses. Namely, stock price movements affect welfare both through wealth effects to consumers and through investment effects to firms.

An important feature of the model is that, both in the firm-specific case and in the rental-market case, a relatively large slope of the New Keynesian Phillips curve associates to a relatively low welfare weight attached to inflation volatility. Parameter θ affects welfare weights almost entirely through λ . Therefore, I focus the policy evaluation exercise on different combinations of the deep parameters η and λ in the rental-market case. This exercise allows for showing the pitfalls in the numerical exercise both for the firm-specific and the rental-market cases and thereby to maximize the comparability with the literature. To correctly consider the firm-specific case you should then restrict your attention only to the regions of the $[\eta, \lambda]$ -plane that are available under the firm-specific problem (as shown in figure 3.1).

3.4. Policy evaluation

In the baseline model without exogenous shocks inflation targeting implements the first best and Q targeting introduces a nonstationarity in capital that causes no local equilibria to exist.

The presence of an exogenous cost-push or financial disturbance breaks the divine coincidence and introduces a policy tradeoff. Against this background, to compute the second-best policy (or

⁵⁸Sveen and Weinke (2009) derive welfare within a model similar to mine but do not explore the link between welfare and Q. Edge (2003) derives a welfare criterion within a New Keynesian model augmented by capital accumulation, and expresses it as function of capital and investment gaps. Edge however does not consider adjustment costs to investment, so that her model predicts constant stock prices. As a consequence, I cannot use Edge's welfare function to study the role of stock prices in the conduct of monetary policy.

the optimal monetary policy under commitment), the central bank chooses the stochastic processes $[\{\pi_{t+h}\}, \{\tilde{q}_{t+h}\}, \{\tilde{y}_{t+h}\}, \{\tilde{k}_{t+h+1}\}]_{h=0}^{\infty}$ that maximize welfare subject to the structural model made of equations (3.10) to (3.17).

The problem has a closed-form solution but the policy rule required to implement it is algebraically involved and I am not able to provide much intuition for it. Therefore I immediately discuss implementability and focus on Taylor-type interest-rate rules; this exercise directly compares to the literature and is more convenient to illustrate the point that inflation targeting is nearly optimal in the model. I focus on the family of feasible Taylor-type rules of the form

$$\widehat{i}_t^f = \phi_\pi \pi_t + \phi_s \widehat{s}_t + \phi_y \widehat{y}_t \quad (3.19)$$

and I numerically search for the parameters $[\phi_\pi, \phi_s, \phi_y]$ that minimize average welfare losses.⁵⁹

I carry out the exercise also under the unfeasible rule based on marginal Q,

$$\widehat{i}_t^f = \phi_\pi \pi_t + \phi_q \widehat{q}_t + \phi_y \widehat{y}_t \quad (3.20)$$

to discuss the potential problems of a central bank using average Q as a proxy for marginal Q in monetary policy design.

I present results as the two key parameters (η and λ) vary and under three calibrations of the response coefficient ϕ_π attached to inflation, $\phi_\pi = 1.5, 5, 20$. It turns out that the optimal coefficient ϕ_π is very large (practically infinite), both under cost-push shocks and under financial shocks.

3.4.1. Optimal Taylor-type rules under cost-push shocks

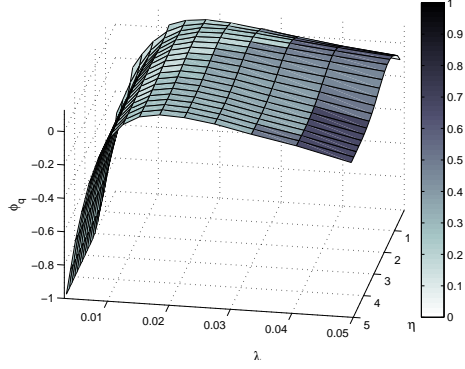
Figures 3.2 and 3.3 show the optimal response coefficients attached to output and Q in the unfeasible rules (3.19) and (3.20), respectively, when the economy is hit by cost-push shocks.⁶⁰

All graphs show the nonmonotonicity, nonlinearity and qualitative dispersion in the prescription. The difference in color between the graphs in the second and third rows shows how an aggressive anti-inflationary stance is the optimal reaction; the welfare gains from a reaction to anything other than inflation decrease as ϕ_π increases. The difference between figures 3.2 and 3.3 shows the proxy problem; the optimal responses to stock prices and marginal Q are similar under cost-push shocks only for large enough values of the reaction coefficient ϕ_π . Moreover, when there are cost-push shocks, the optimal reaction to marginal Q is zero in most of the key-parameter space and positive when η and λ are close to zero, although the welfare gains from such a response are small. In contrast, the optimal reaction to stock prices is usually negative and may yield large welfare gains.

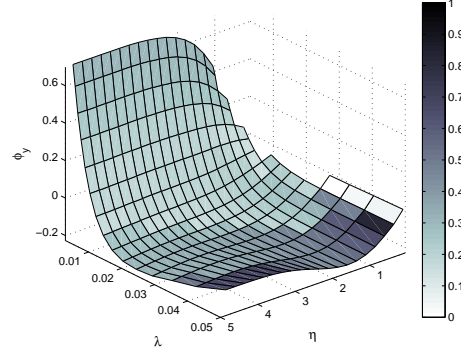
These results are driven by the property that in most of the key-parameter space, the top priority is inflation stabilization. Over the (λ, η) -plane, the welfare weights attached to everything but

⁵⁹The simplicity of the rule introduces an implementation disturbance in the system. For simplicity, I assume that no technology shocks hit the economy. This assumption disposes of the implementation disturbance, since gap terms become observable. A consideration of technology is crucial in the context of models that endogenize the cost-push or the financial disturbances, because technology drives those disturbances. With exogenous cost-push and financial disturbances, however, technology shocks only drive the implementation disturbance.

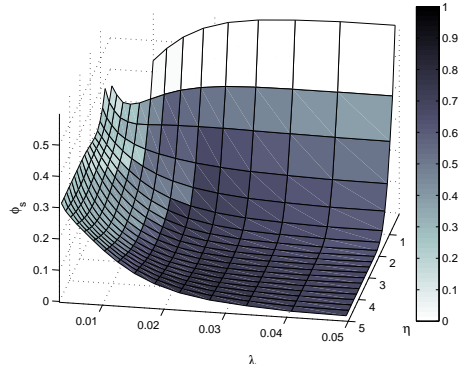
⁶⁰I set the persistence coefficient to a medium value, $\rho_u = 0.75$. None of the main results depends on the choice of this number.



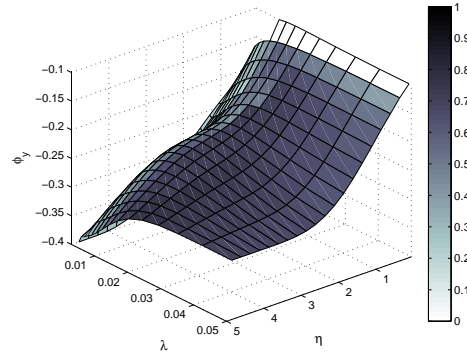
(a) Optimal ϕ_q under $\phi_\pi = 1.5$.



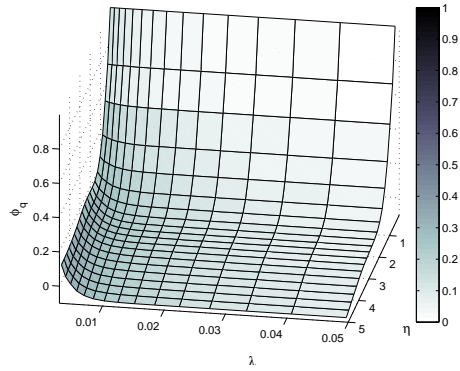
(b) Optimal ϕ_y under $\phi_\pi = 1.5$.



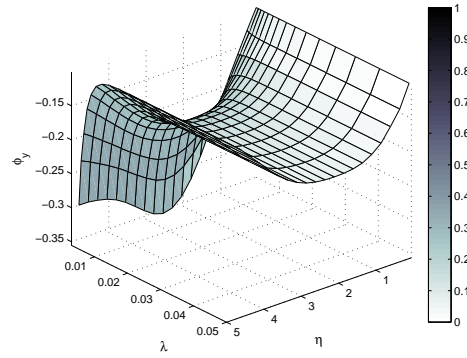
(c) Optimal ϕ_q under $\phi_\pi = 5$.



(d) Optimal ϕ_y under $\phi_\pi = 5$.

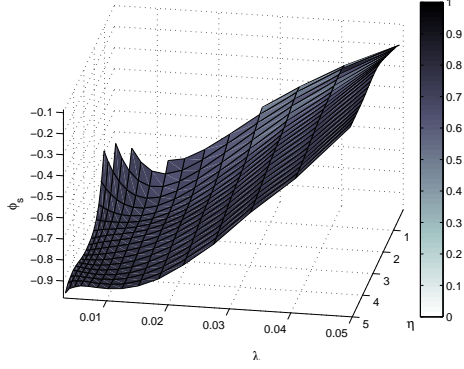


(e) Optimal ϕ_q under $\phi_\pi = 20$.

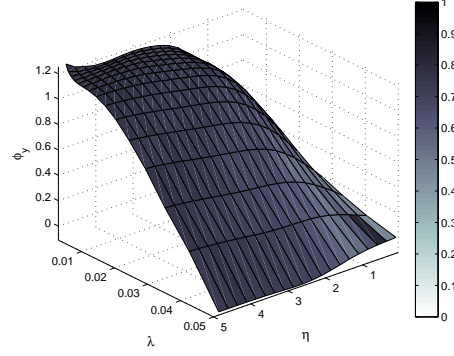


(f) Optimal ϕ_y under $\phi_\pi = 20$.

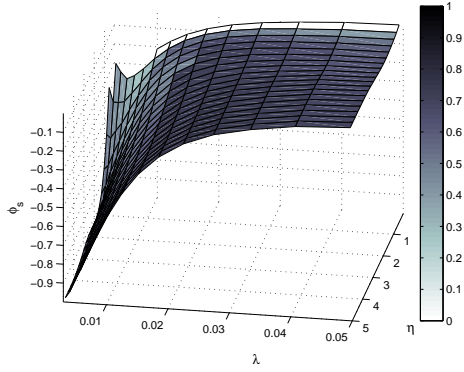
Figure 3.2: Interest-rate rule in inflation, output and marginal Q, when the economy is hit by cost-push shocks. The colors (on a scale from 0 to 1) indicate the percentage reduction in welfare losses from choosing rule $\widehat{i}_t^f = \phi_\pi \pi_t + \phi_q \widehat{q}_t + \phi_y \widehat{y}_t$ over rule $\widehat{i}_t^f = \phi_\pi \pi_t$.



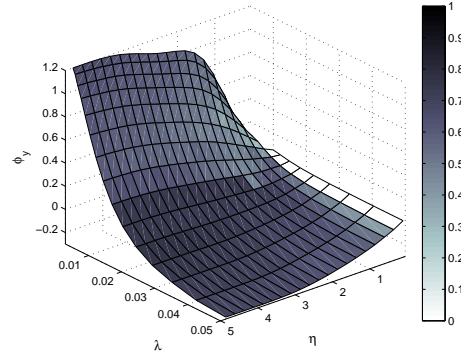
(a) Optimal ϕ_s under $\phi_\pi = 1.5$.



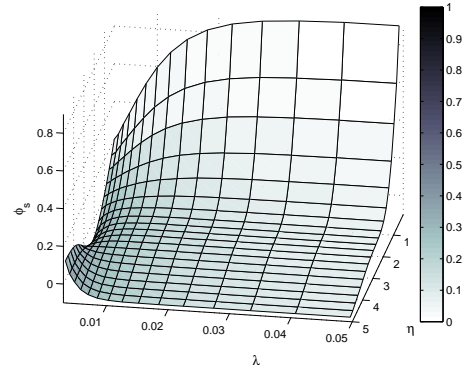
(b) Optimal ϕ_y under $\phi_\pi = 1.5$.



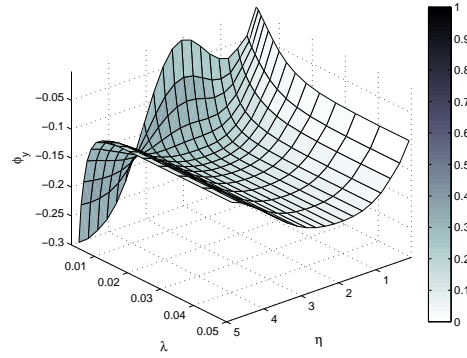
(c) Optimal ϕ_s under $\phi_\pi = 5$.



(d) Optimal ϕ_y under $\phi_\pi = 5$.



(e) Optimal ϕ_s under $\phi_\pi = 20$.



(f) Optimal ϕ_y under $\phi_\pi = 20$.

Figure 3.3: Interest-rate rule in inflation, output and average Q, when the economy is hit by cost-push shocks. The colors (on a scale from 0 to 1) indicate the percentage reduction in welfare losses from choosing rule $\widehat{i}_t^f = \phi_\pi \pi_t + \phi_s \widehat{s}_t + \phi_y \widehat{y}_t$ over rule $\widehat{i}_t^f = \phi_\pi \pi_t$.

inflation are small except when η is close to zero, where the welfare weight attached to financial stability becomes relatively big. This result explains why we see a positive reaction to stock price movements only when η is small and λ is relatively large; that is the only region of the key-parameter space in which financial stabilization is of higher priority than inflation stabilization. To avoid the case in which only inflation matters, we need either a very small η or very high risk aversion σ .

Therefore, in most of the key-parameter space, the central bank should not care about stabilizing output or asset prices per se; instead, it should react to them to indirectly increase the anti-inflationary stance. Since the exercise constrains ϕ_π to some finite value, there are two ways left to increase the aggressiveness in fighting inflation. First, the bank could react negatively to stock prices and, as a result, react positively to expected inflation (as seen in equation (3.15)). Then, when ϕ_π is high enough, the marginal increase in welfare from a positive reaction to marginal Q outweighs the marginal decrease in anti-inflationary stance from a positive reaction to average Q. Second, the bank could react non-negatively to marginal Q and negatively to output after a cost-push shock to cause a higher interest-rate increase than under a rule based on inflation only. In fact, a cost-push shock causes an increase in inflation and in the marginal Q gap. As a consequence, following the interest-rate rule (with $\phi_\pi > 0$ and $\phi_q \geq 0$) results in the central bank driving up interest rates. Such a restrictive policy depresses output. Finally, the drop in output causes a further rise in interest rates due to $\phi_y < 0$. Thus, $\phi_q \geq 0$ and $\phi_y < 0$ amplify the interest-rate increase, just as if the central bank had chosen a higher coefficient ϕ_π .

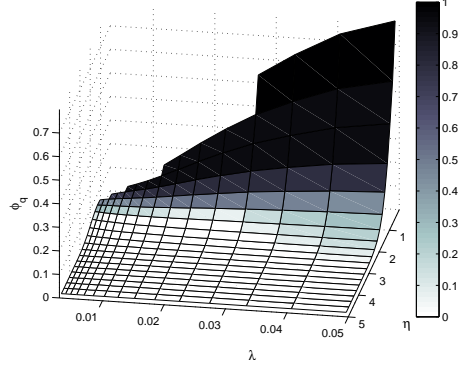
In a large region of the key-parameter space, the central bank wants to react to stock prices, output and marginal Q as an auxiliary way to strengthen the anti-inflationary stance. However, the boost to the reaction to inflation becomes irrelevant for sufficiently large values of ϕ_π ; hence, the marginal welfare gain of reacting to these ancillary signals vanishes as ϕ_π increases, as shown by the lighter color of the lower part of figures 3.2 to 3.5, which is associated to a high coefficient $\phi_\pi = 20$, relative to the upper part.

Finally, the difference in shape between the graphs in the first two rows shows the problem of indeterminacy. The positive reactions to marginal Q may increase welfare in some regions of the (λ, η) -plane because they increase the anti-inflationary stance. However, when ϕ_π is low such a reaction tends to cause a unit root in capital, thereby causing no local equilibria to exist. This is the reason we do not see positive reactions to Q when ϕ_π is low. In those regions of the key-parameter space the constraint that the system must be determined is binding for low values of ϕ_π ; in this regard, figure 3.6 shows how, for small values of ϕ_π and of either λ or η , nearly any reaction to stock prices causes indeterminacy. The system may regain determinacy provided a complementary positive reaction to output—which can avoid the unit root in capital—but it is dangerous to give any policy prescription at the indeterminacy border.

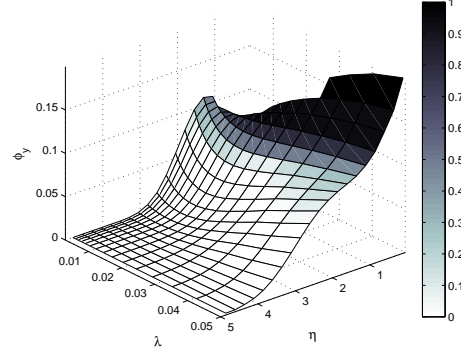
3.4.2. Optimal Taylor-type rules under financial shocks

Figures 3.4 and 3.5 show the optimal response coefficients attached to output and Q in the unfeasible rules (3.19) and (3.20), respectively, when the economy is hit by financial shocks.⁶¹

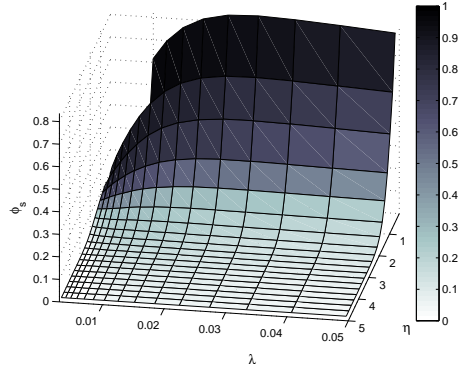
⁶¹I set the persistence coefficient to a medium value, $\rho_x = 0.75$. None of the main results depends on the choice of this number.



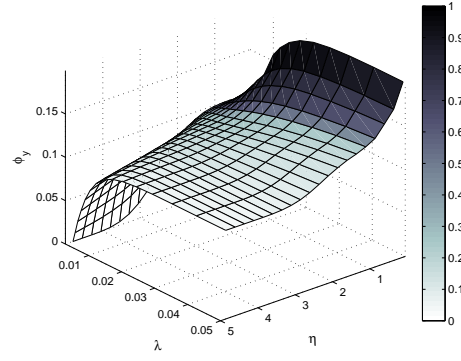
(a) Optimal ϕ_q under $\phi_\pi = 1.5$.



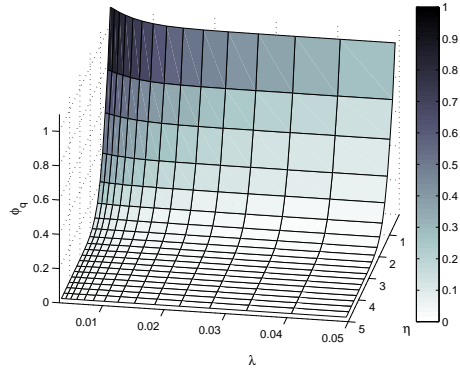
(b) Optimal ϕ_y under $\phi_\pi = 1.5$.



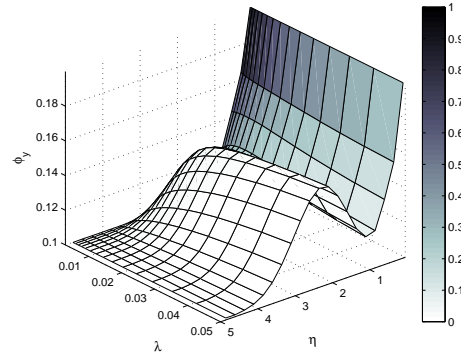
(c) Optimal ϕ_q under $\phi_\pi = 5$.



(d) Optimal ϕ_y under $\phi_\pi = 5$.

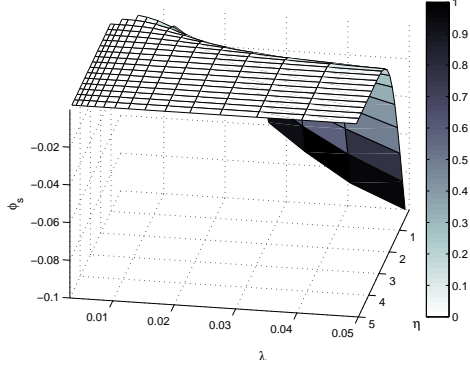


(e) Optimal ϕ_q under $\phi_\pi = 20$.

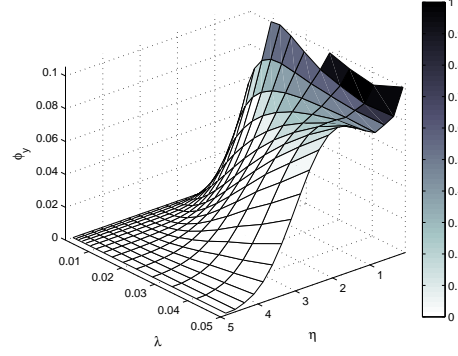


(f) Optimal ϕ_y under $\phi_\pi = 20$.

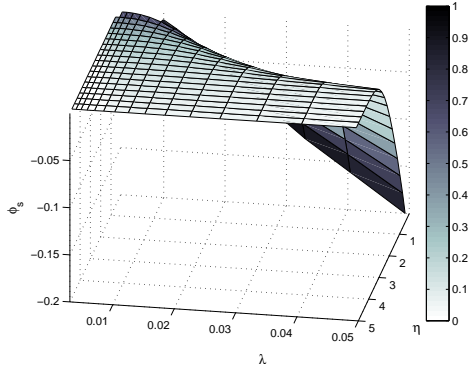
Figure 3.4: Interest-rate rule in inflation, output and marginal Q, when the economy is hit by financial shocks. The colors (on a scale from 0 to 1) indicate the percentage reduction in welfare losses from choosing rule $\widehat{i}_t^f = \phi_\pi \pi_t + \phi_q \widehat{q}_t + \phi_y \widehat{y}_t$ over rule $\widehat{i}_t^f = \phi_\pi \pi_t$.



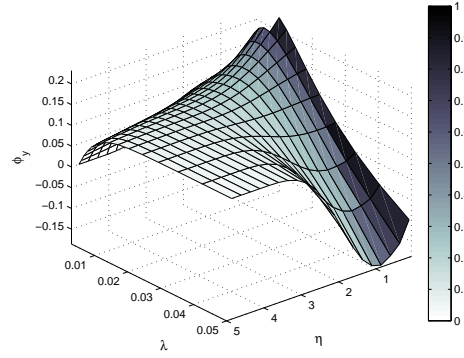
(a) Optimal ϕ_s under $\phi_\pi = 1.5$.



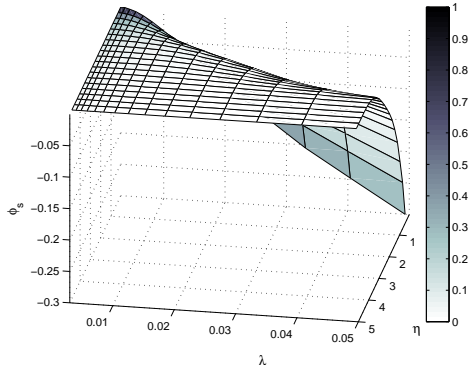
(b) Optimal ϕ_y under $\phi_\pi = 1.5$.



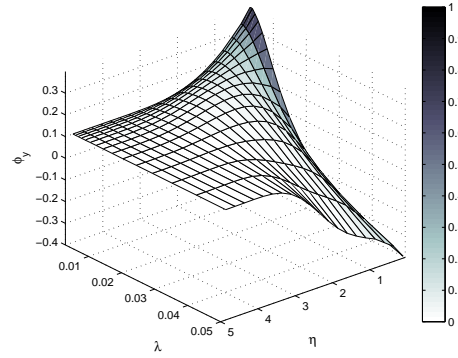
(c) Optimal ϕ_s under $\phi_\pi = 5$.



(d) Optimal ϕ_y under $\phi_\pi = 5$.



(e) Optimal ϕ_s under $\phi_\pi = 20$.



(f) Optimal ϕ_y under $\phi_\pi = 20$.

Figure 3.5: Interest-rate rule in inflation, output and average Q, when the economy is hit by financial shocks. The colors (on a scale from 0 to 1) indicate the percentage reduction in welfare losses from choosing rule $\widehat{i}_t^f = \phi_\pi \pi_t + \phi_s \widehat{s}_t + \phi_y \widehat{y}_t$ over rule $\widehat{i}_t^f = \phi_\pi \pi_t$.

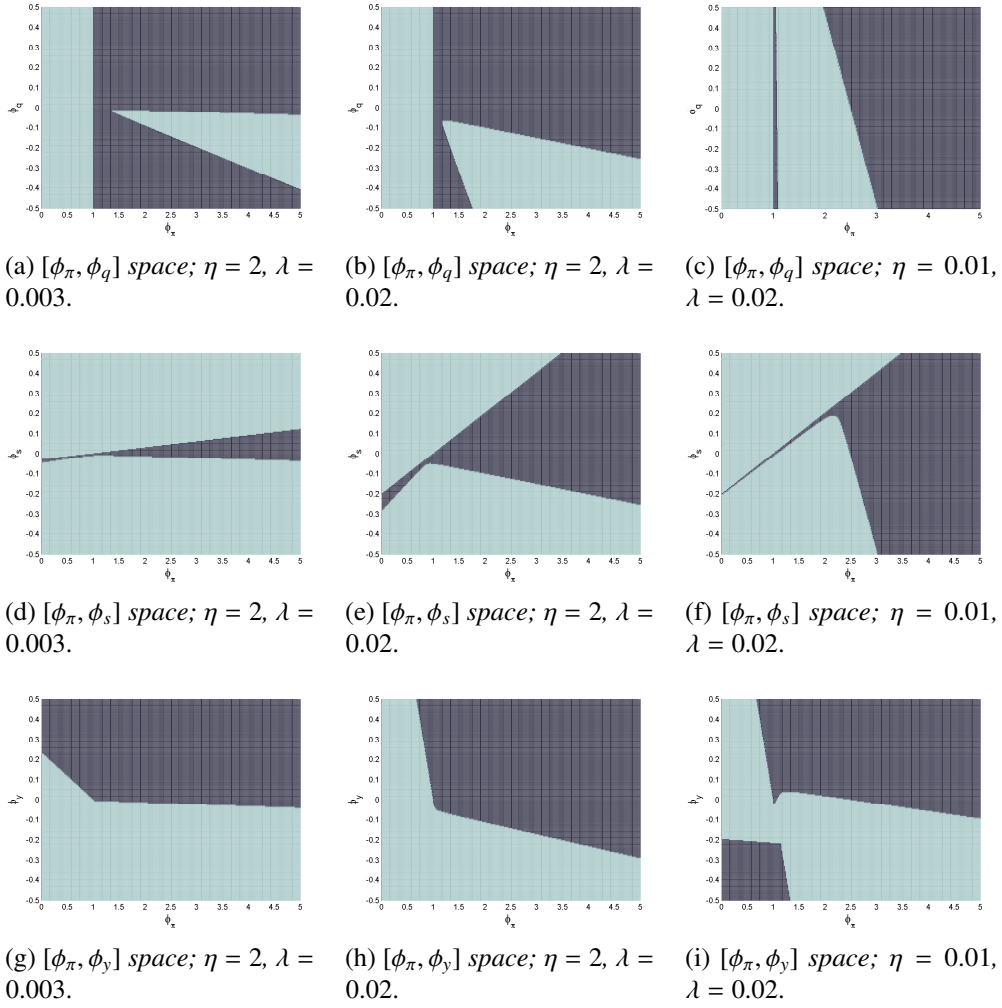


Figure 3.6: Determinacy regions (in black) under different calibrations of the key parameters.

When financial shocks hit the economy the optimal responses to stock prices and marginal Q differ, even qualitatively, from the cost-push case. When financial disturbances hit the economy, the optimal reaction to Q is positive, and truncated for low values of λ and ϕ_π , where determinacy is more of an issue. The optimal reaction to average Q is instead negative, which emphasizes that a stronger reaction to inflation is more a priority than a positive reaction to Q under financial frictions; this surface as well displays a truncation when λ is small.

Apart from the sign of the prescribed response and a greater proxy problem, however, the main lessons under the financial disturbance are similar to the case in which a cost-push disturbance hits the economy. In particular, the marginal welfare gain of reacting to stock prices and output vanishes as the reaction coefficient attached to inflation increases.

3.5. Conclusion

It seems imprudent to advise a central bank to react to stock prices in a New Keynesian model economy when such a policy prescription derives from simulations conducted under a fixed coefficient of reaction to inflation. The prescription is sensitive to the typical dispersion seen in the extant literature in some key parameters, such as the slope of the New Keynesian Phillips curve and the elasticity of investment to Q . The optimal reaction coefficients attached to average Q , marginal Q and output are non-monotonic and nonlinear in the arguments. In some cases, the prescription points to policy coefficients that, if slightly modified, may send the system to indeterminacy. These issues are robust across different sources of policy tradeoffs. Nonlinearity, non-monotonicity and the danger of indeterminacy are all reasons not to react to stock prices. Moreover, the difference between figures 3.2-3.3 and 3.4-3.5 shows how prescriptions differ for different sources of shocks. These problems are unfortunate, since the policy prescription seems intractable. However, in this simple model, all reactions to stock prices and output yield important welfare gains only when they increase the anti-inflationary stance. Therefore, a rule exclusively based on inflation seems to be the most robust prescription.

A final doubt concerning the robustness of the characterization of the optimal Taylor-type rule is that the sources of tradeoffs are exogenous in the model. The cost-push disturbance proxies for frictions in the labor market (e.g., sticky wages as in Erceg et al., 2000), the financial disturbance for frictions in the financial market (e.g., a financial accelerator mechanism as in Bernanke et al., 1999). Ultimately, a complete model with capital accumulation and adjustment costs endogenizes the disturbances and everything is then driven by technology shocks (see, for instance, Faia and Monacelli, 2007; Sveen and Weinke, 2009). In such a more complex setting, the conclusion that an aggressively anti-inflationary rule is optimal may break down.⁶²

However, the documented pitfalls are likely to remain with us in a more complex setting.

Appendix

3.A. Full loglinear model solution

Hat terms denote deviations from steady state.

Equations (3.2) and (3.5) combine as

$$\sigma \widehat{c}_t + \varphi \widehat{n}_t = \widehat{mc}_t + \widehat{y}_t - \widehat{n}_t$$

The approximate aggregate production function is

$$\widehat{y}_t = a_t + \alpha \widehat{k}_t + (1 - \alpha) \widehat{n}_t \quad (3.A.1)$$

The approximate market clearing conditions is

$$\widehat{y}_t = \gamma_c \widehat{c}_t + \gamma_i \widehat{i}_t$$

⁶²The result survives in the model of Faia and Monacelli (2007), who endogenize some financial frictions.

where $\gamma_i \equiv \alpha\beta\delta/[1 - \beta(1 - \delta)]$ is the investment share of output and $\gamma_c = 1 - \gamma_i$ is the consumption share of output. Equation (3.6) is approximately

$$\widehat{i}_t - \widehat{k}_t = \frac{1}{\eta} \widehat{q}_t \quad (3.A.2)$$

which links the real economy to financial markets.

Equation (3.3) holds approximately as the dynamic IS equation

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (\widehat{i}_t^f - E_t \pi_{t+1}) \quad (3.A.3)$$

The approximate no-arbitrage condition (3.7) is the forward-looking equation for marginal Q

$$\widehat{q}_t = \beta E_t \widehat{q}_{t+1} + \vartheta (\widehat{mc}_{t+1} + \widehat{y}_{t+1} - \widehat{k}_{t+1}) - (\widehat{i}_t^f - E_t \pi_{t+1}) \quad (3.A.4)$$

The New Keynesian Phillips curve relating inflation to marginal costs is

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t$$

which is the loglinearization of equation (3.4), and real marginal costs relate to the other variables as

$$\widehat{mc}_t = \Xi_y \widehat{y}_t - \Xi_{yq} \widehat{q}_t - \Xi_{yk} \widehat{k}_t - \frac{1 + \varphi}{1 - \alpha} a_t + \frac{1}{\lambda} u_t$$

with parameters $\Xi_y \equiv \sigma/\gamma_c + (\alpha + \varphi)/(1 - \alpha)$, $\Xi_{yq} \equiv \sigma\gamma_i/(\gamma_c\eta)$ and $\Xi_{yk} \equiv \sigma\gamma_i/\gamma_c + \alpha(1 + \varphi)/(1 - \alpha)$.

The approximate equation for capital accumulation is

$$\widehat{k}_{t+1} = (1 - \delta) \widehat{k}_t + \delta \widehat{i}_t \quad (3.A.5)$$

3.B. Firm-specific problem

The following derivation borrows heavily from Woodford (2005) and Sveen and Weinke (2005).

I use the notation $\bar{x}_t(i) \equiv x_t(i) - x_t$ to denote the difference between an individual and the corresponding aggregate variable.

Integrating individual prices, applying the law of large number and expanding individual prices around their aggregate level, one can show that the first-order dynamics of the aggregate price level is

$$p_t^* - p_t = \frac{\theta}{1 - \theta} \pi_t \quad (3.B.6)$$

Note also that the optimality conditions that characterize the choice of the i th firm—independently of when it last had the chance of resetting prices—imply

$$\begin{aligned} \overline{mpk}_t(i) &= -\varepsilon \bar{p}_t(i) - \bar{k}_t(i) \\ \overline{mc}_t(i) &= -\frac{\alpha\varepsilon}{1 - \alpha} \bar{p}_t(i) - \frac{\alpha}{(1 - \alpha)} \bar{k}_t(i) \end{aligned} \quad (3.B.7)$$

A distinguishing feature of the firm-specific problem is that not all firms that reset prices

at time t are equal, as they inherit a firm-specific capital stock that depends on when they last had the chance of resetting prices. Against this background, the optimal price chosen by the i th resetting firm satisfies the approximate first-order condition $\sum_{h=0}^{\infty} (\beta\theta)^h E_t^i [\bar{p}_{t+h}(i) - \widehat{mc}_{t+h}(i)] = 0$ with $E_t^i[p_{t+h}(i)] = p_t^*(i)$, where E^i denotes an expectation operator conditional on firm i not resetting in the future.⁶³ I can rewrite the last equation as $E_t^i \bar{p}_{t+h}(i) = \bar{p}_t^*(i) + (p_t^* - p_t) - \sum_{j=1}^h E_t \pi_{t+j}$ and therefore the optimal price setting equation as

$$(\bar{p}_t^*(i) + p_t^* - p_t) = \sum_{h=1}^{\infty} (\beta\theta)^h E_t \pi_{t+h} + (1 - \beta\theta)\Theta \sum_{h=0}^{\infty} (\beta\theta)^h E_t \widehat{mc}_{t+h} - \frac{\alpha(1 - \beta\theta)\Theta}{1 - \alpha} \sum_{h=0}^{\infty} (\beta\theta)^h E_t \bar{k}_{t+h}(i) \quad (3.B.8)$$

where $\Theta \equiv (1 - \alpha)/(1 - \alpha + \alpha\varepsilon)$, and where I used the fact that $\sum_{h=0}^{\infty} (\beta\theta)^h \sum_{j=1}^h \pi_{t+j} = \frac{1}{1 - \beta\theta} \sum_{h=1}^{\infty} (\beta\theta)^h \pi_{t+h}$.

I then guess that the equilibrium solution for the distance between individual and aggregate capital has the form

$$\bar{k}_{t+1}(i) = \psi_2 \bar{k}_t(i) + \psi_3 \bar{p}_t(i) \quad (3.B.9)$$

which must be true because $\bar{p}_t(i)$ and $\bar{k}_t(i)$ are the only states at time t idiosyncratic to firm i .

Equation (E.8) implies $E_t^i \bar{k}_{t+h}(i) = \psi_2 E_t^i \bar{k}_{t+h-1}(i) + \psi_3 [\bar{p}_t^*(i) + p_t^* - p_t - \sum_{j=1}^{h-1} E_t \pi_{t+j}]$. Plugging this expression into (E.7) we find

$$\kappa(\bar{p}_t^*(i) + p_t^* - p_t) = \kappa \sum_{h=1}^{\infty} (\beta\theta)^h E_t \pi_{t+h} + (1 - \beta\theta) \sum_{h=0}^{\infty} (\beta\theta)^h E_t \widehat{mc}_{t+h} - \frac{\alpha(1 - \beta\theta)}{(1 - \alpha)(1 - \beta\theta\psi_2)} \bar{k}_t(i)$$

where $\kappa \equiv 1/\Theta + \alpha\beta\theta\psi_3/[(1 - \alpha)(1 - \beta\theta\psi_2)]$. Then, using the approximate aggregate price dynamics (E.1),

$$\kappa(\bar{p}_t^*(i) + \frac{\theta}{1 - \theta} \pi_t) = \kappa \sum_{h=1}^{\infty} (\beta\theta)^h E_t \pi_{t+h} + (1 - \beta\theta) \sum_{h=0}^{\infty} (\beta\theta)^h E_t \widehat{mc}_{t+h} - \frac{\alpha(1 - \beta\theta)}{(1 - \alpha)(1 - \beta\theta\psi_2)} \bar{k}_t(i) \quad (3.B.10)$$

I can then integrate equation (E.9) and solve the resulting difference equation to find⁶⁴

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t$$

where $\lambda \equiv (1 - \theta)(1 - \beta\theta)/\kappa\theta$, which is the New Keynesian Phillips curve under firm-specific capital (Sveen and Weinke, 2005; Woodford, 2005). Moreover, subtracting the New Keynesian Phillips curve from (E.9), I find the solution for the newly reset prices, which obey the linear function of the individual capital shock

$$\bar{p}_t^*(i) = \psi_1 \bar{k}_t(i) \quad (3.B.11)$$

where

$$\psi_1 \equiv -\frac{\alpha(1 - \beta\theta)}{\kappa(1 - \alpha)(1 - \beta\theta\psi_2)}$$

⁶³Note that $E_t^i x_{t+j} = E_t x_{t+j}$ for an aggregate quantity x and $j \geq 0$.

⁶⁴Note that $\int_0^1 \bar{x}_t(i) di = O(\|\bar{x}\|^2)$ independent of the shape of the aggregator for variable X_t .

Next, the approximate optimality condition for the capital choice of firm i together with (E.6) imply

$$\Xi \bar{k}_{t+1}(i) = \beta E_t \bar{k}_{t+2}(i) + \bar{k}_t(i) - \varepsilon[\Xi - 1 - \beta] E_t \bar{p}_{t+1}(i) \quad (3.B.12)$$

where $\Xi \equiv [(1 + \beta)(1 - \alpha)\eta + \vartheta\delta]/\eta(1 - \alpha)$.

Note that, up to a first-order approximation,

$$\begin{aligned} E_t \bar{p}_{t+1}(i) &= \theta(\bar{p}_t(i) - E_t \pi_{t+1}) + (1 - \theta)E_t(\bar{p}_{t+1}^*(i) + p_{t+1}^* - p_{t+1}) \\ &= \theta \bar{p}_t(i) + (1 - \theta)E_t \bar{p}_{t+1}^*(i) \end{aligned} \quad (3.B.13)$$

where the last equality uses (E.1).

Then, I plug (E.8) into (E.13) and use (E.14) to identify ψ_2 and ψ_3 as

$$\Xi \psi_2 = 1 + \beta \psi_2^2 + \{\beta \psi_3 - \varepsilon[\Xi - 1 - \beta]\}(1 - \theta)\psi_1 \psi_2 \quad (3.B.14)$$

$$\Xi \psi_3 = \beta \psi_2 \psi_3 + \{\beta \psi_3 - \varepsilon[\Xi - 1 - \beta]\}[\theta + (1 - \theta)\psi_1 \psi_3] \quad (3.B.15)$$

which verifies the guess (E.8).

The problem of firms accumulating firm-specific capital under sticky prices adds therefore two idiosyncratic dynamic equations to the state-space system driving aggregate variables. Namely, equations (E.8) and (E.14) form the system

$$\begin{bmatrix} E_t \bar{p}_{t+1}(i) \\ \bar{k}_{t+1}(i) \end{bmatrix} = \begin{bmatrix} \theta + (1 - \theta)\psi_1 \psi_3 & (1 - \theta)\psi_1 \psi_2 \\ \psi_3 & \psi_2 \end{bmatrix} \begin{bmatrix} \bar{p}_t(i) \\ \bar{k}_t(i) \end{bmatrix} \quad (3.B.16)$$

To have a unique bounded solution, system (3.B.16) must have both eigenvalues within the unit circle. These eigenvalues are independent of the monetary policy rule in place. I can thus check over the (θ, η) -plane which combinations lead to indeterminacy, independently of the policy rule (Woodford, 2005). Note that this exercise is complementary to the exercise about the determinacy of the equilibrium for aggregate variables, which in general depends on the monetary policy rule in place (Sveen and Weinke, 2005).

3.C. Derivation of the welfare function

A second-order Taylor expansion of utility around the zero-inflation steady state reads

$$\begin{aligned} U(Ye^{\widehat{y}_t} - Ie^{\frac{1}{\eta}\widehat{q}_t + \widehat{k}_t}, Ne^{\widehat{n}_t}) &= \\ &= U + U_1 Y(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2) + \frac{1}{2}U_{11} Y^2 \widehat{y}_t^2 - U_1 \frac{1}{\eta}(\widehat{q}_t + \frac{1}{2}\widehat{q}_t^2) - U_1 I(\widehat{k}_t + \frac{1}{2}\widehat{k}_t^2) + \\ &\quad + \frac{1}{2}U_{11} I^2 (\frac{1}{\eta}\widehat{q}_t + \widehat{k}_t)^2 - U_{11} Y I \widehat{y}_t (\frac{1}{\eta}\widehat{q}_t + \widehat{k}_t) + U_2 N(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2) + \frac{1}{2}U_{22} N^2 \widehat{n}_t^2 \end{aligned}$$

where I used the market clearing condition $C_t = Y_t - I_t$ and the approximate optimality condition (3.A.2).

Noting that $U_{11}C/U_1 = -\sigma$, $U_{22}N/U_2 = \varphi$, $Y = C/\gamma_c$, and $I = C\gamma_i/\gamma_c$, then

$$\begin{aligned} U_t - U &= U_1 C \frac{1}{\gamma_c} \left[\widehat{y}_t + \frac{1}{2} \left(1 - \frac{\sigma}{\gamma_c} \right) \widehat{y}_t^2 \right] - U_1 C \frac{\gamma_i}{\gamma_c \eta} \left[\widehat{q}_t + \frac{1}{2} \left(1 + \frac{\sigma \gamma_i}{\gamma_c \eta} \right) \widehat{q}_t^2 \right] + U_1 C \frac{\sigma \gamma_i}{\gamma_c^2 \eta} \widehat{y}_t \widehat{q}_t + \\ &\quad - U_1 C \frac{\gamma_i}{\gamma_c} \left[\widehat{k}_t + \frac{1}{2} \left(1 + \frac{\sigma \gamma_i}{\gamma_c} \right) \widehat{k}_t^2 \right] + U_1 C \frac{\sigma \gamma_i}{\gamma_c^2} \widehat{y}_t \widehat{k}_t - U_1 C \frac{\sigma \gamma_i^2}{\gamma_c^2 \eta} \widehat{q}_t \widehat{k}_t + \\ &\quad + U_2 N \left[\widehat{n}_t + \frac{1 + \varphi}{2} \widehat{n}_t^2 \right] \end{aligned}$$

Recall that, in the undistorted steady state, $-U_2/U_1 = MRS = MPN = (1 - \alpha)Y/N$. Next, a second-order expansion of aggregate hours yields⁶⁵

$$\begin{aligned} N_t &\equiv \int N_t(i) di = \exp \left\{ \frac{1}{1 - \alpha} [y_t - a_t - \alpha k_t] \right\} \int \exp \left\{ \frac{1}{1 - \alpha} \bar{y}_t(i) - \frac{\alpha}{1 - \alpha} \bar{k}_t(i) \right\} di \\ &= \exp \left\{ \frac{1}{1 - \alpha} [y_t - a_t - \alpha k_t] \right\} \times \\ &\quad \times \left\{ 1 + \frac{1}{1 - \alpha} E_i \bar{y}_t(i) + \frac{1}{2} \frac{1}{(1 - \alpha)^2} \Delta_t^y - \frac{\alpha}{1 - \alpha} E_i \bar{k}_t(i) + \frac{1}{2} \frac{\alpha^2}{(1 - \alpha)^2} \Delta_t^k - \frac{\alpha}{(1 - \alpha)^2} \Delta_t^{yk} \right\} \end{aligned} \quad (3.C.17)$$

$$\begin{aligned} \Rightarrow (1 - \alpha)n_t &= y_t - a_t - \alpha k_t + \frac{1}{2} \frac{1 - \alpha + \alpha \varepsilon}{\varepsilon(1 - \alpha)} \Delta_t^y + \frac{1}{2} \frac{\alpha}{1 - \alpha} \Delta_t^k - \frac{\alpha}{1 - \alpha} \Delta_t^{yk} \\ &= y_t - a_t - \alpha k_t + \frac{1}{2} \frac{(1 - \alpha + \alpha \varepsilon) \varepsilon}{1 - \alpha} \Delta_t^p + \frac{1}{2} \frac{\alpha}{1 - \alpha} \Delta_t^k + \frac{\alpha \varepsilon}{1 - \alpha} \Delta_t^{pk} \end{aligned} \quad (3.C.18)$$

where I use the notation $\bar{x}_t(i) \equiv x_t(i) - x_t$, $\Delta_t^x \equiv \text{var}_i x_t(i)$ and $\Delta_t^{xy} \equiv \text{cov}_i(x_t(i), y_t(i))$, for any two random variables x and y , and where the transformation uses the fact that the demand curve implies $\bar{y}_t(i) = -\varepsilon \bar{p}_t(i)$.

Thus, I can use the quadratic approximation to the aggregate production function (3.C.18) to express welfare as

$$\begin{aligned} U_t - U &\propto \gamma_c \frac{U_t - U}{U_1 C} \\ &= \widehat{y}_t - \gamma_i \widehat{i}_t + \frac{1}{2} \left(1 - \frac{\sigma}{\gamma_c} \right) \widehat{y}_t^2 - \frac{1}{2} \left(\frac{\gamma_i}{\eta} + \frac{\sigma \gamma_i^2}{\gamma_c \eta^2} \right) \widehat{q}_t^2 - \frac{1}{2} \left(\gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c} \right) \widehat{k}_t^2 + \\ &\quad + \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t \widehat{q}_t + \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t - \frac{\sigma \gamma_i^2}{\gamma_c \eta} \widehat{q}_t \widehat{k}_t - \left[\widehat{y}_t - a_t - \alpha \widehat{k}_t \right] \\ &\quad - \left[\frac{1}{2} \frac{\varepsilon}{\Theta} \Delta_t^p + \frac{1}{2} \frac{\alpha}{1 - \alpha} \Delta_t^k + \frac{\alpha \varepsilon}{(1 - \alpha) \varepsilon} \Delta_t^{pk} + \frac{1 + \varphi}{2(1 - \alpha)} (\widehat{y}_t - a_t - \alpha \widehat{k}_t)^2 \right] \end{aligned}$$

⁶⁵By ‘ \Rightarrow ’ I mean that I disregard both terms independent of policy and terms of order higher than two.

where $\Theta \equiv (1 - \alpha)/(1 - \alpha + \alpha\varepsilon)$, and therefore, up to a term independent of policy,

$$\begin{aligned} \Rightarrow & -(\gamma_i \widehat{i}_t - \alpha \widehat{k}_t) + \frac{1}{2} \left(1 - \frac{\sigma}{\gamma_c}\right) \widehat{y}_t^2 - \frac{1}{2} \left(\frac{\gamma_i}{\eta} + \frac{\sigma \gamma_i^2}{\gamma_c \eta^2}\right) \widehat{q}_t^2 - \frac{1}{2} \left(\gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c}\right) \widehat{k}_t^2 + \\ & + \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t \widehat{q}_t + \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t - \frac{\sigma \gamma_i^2}{\gamma_c \eta} \widehat{q}_t \widehat{k}_t - \frac{1}{2} \frac{\varepsilon}{\Theta} \Delta_t^p - \frac{1}{2} \frac{\alpha}{1 - \alpha} \Delta_t^k - \frac{\alpha \varepsilon}{(1 - \alpha) \varepsilon} \Delta_t^{pk} \\ & - \left[\frac{1 + \varphi}{2(1 - \alpha)} \widehat{y}_t^2 + \frac{(1 + \varphi) \alpha^2}{2(1 - \alpha)} \widehat{k}_t^2 - \frac{(1 + \varphi) \alpha}{1 - \alpha} \widehat{y}_t \widehat{k}_t - \frac{1 + \varphi}{1 - \alpha} \widehat{y}_t a_t + \frac{(1 + \varphi) \alpha}{1 - \alpha} \widehat{k}_t a_t \right] \end{aligned}$$

Next, using (3.A.5) and recalling that $\gamma_i = \alpha \beta \delta / \vartheta$,

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t (\gamma_i \widehat{i}_t - \alpha \widehat{k}_t) &= \sum_{t=0}^{\infty} \beta^t \frac{\alpha}{\vartheta} (\beta \widehat{k}_{t+1} - \widehat{k}_t) \\ &= -\frac{\alpha}{\vartheta} \widehat{k}_0 \end{aligned}$$

which is a term independent of policy, so that, as I sum the period-by-period utilities, I can disregard the linear terms in $[\widehat{i}_t, \widehat{k}_t]$. Thus, rewrite the quadratic approximation as

$$\begin{aligned} \Rightarrow & -\frac{1}{2} w' \Delta_t - \frac{1}{2} \left\{ \left(\frac{\sigma}{\gamma_c} - 1\right) \widehat{y}_t^2 + \left(\frac{\gamma_i}{\eta} + \frac{\sigma \gamma_i^2}{\gamma_c \eta^2}\right) \widehat{q}_t^2 + \left(\gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c}\right) \widehat{k}_t^2 - 2 \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t \widehat{q}_t + \right. \\ & \left. - 2 \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t + 2 \frac{\sigma \gamma_i^2}{\gamma_c \eta} \widehat{q}_t \widehat{k}_t + \frac{1 + \varphi}{1 - \alpha} \widehat{y}_t^2 + \frac{(1 + \varphi) \alpha^2}{1 - \alpha} \widehat{k}_t^2 - 2 \frac{(1 + \varphi) \alpha}{1 - \alpha} \widehat{y}_t \widehat{k}_t - 2 \widehat{y}_t \frac{1 + \varphi}{1 - \alpha} a_t + 2 \alpha \widehat{k}_t \frac{1 + \varphi}{1 - \alpha} a_t \right\} \end{aligned}$$

where $w \equiv [\frac{1}{2} \frac{\varepsilon}{\Theta}; \frac{1}{2} \frac{\alpha}{1 - \alpha}; \frac{\alpha \varepsilon}{1 - \alpha}]$ and $\Delta_t \equiv [\Delta_t^p; \Delta_t^k; \Delta_t^{pk}]$.

I focus now on the term in curly brackets, which is what I want to express in terms of gaps. Add and subtract the quantities needed to make gaps appear and use a property of the efficient economy,

$$\frac{1 + \varphi}{1 - \alpha} a_t = \left(\frac{\sigma}{\gamma_c} + \frac{\alpha + \varphi}{1 - \alpha}\right) \widehat{y}_t^e - \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{q}_t^e - \left(\frac{\sigma \gamma_i}{\gamma_c} + \frac{(1 + \varphi) \alpha}{1 - \alpha}\right) \widehat{k}_t^e$$

to find

$$\begin{aligned} & \left[\Xi_y \widehat{y}_t^2 + \Xi_q \widehat{q}_t^2 + \Xi_k \widehat{k}_t^2 - 2 \Xi_{yq} \widehat{y}_t \widehat{q}_t - 2 \Xi_{yk} \widehat{y}_t \widehat{k}_t + 2 \Xi_{qk} \widehat{q}_t \widehat{k}_t \right] + \\ & + 2 \widehat{q}_t \left[\left(\frac{\gamma_i}{\eta} + \frac{\sigma \gamma_i^2}{\gamma_c \eta^2}\right) \widehat{q}_t^e - \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t^e + \frac{\sigma \gamma_i^2}{\gamma_c \eta} \widehat{k}_t^e \right] + 2 \widehat{k}_t \left[\left(\frac{\sigma \gamma_i^2}{\gamma_c \eta} - \frac{\sigma \gamma_i \alpha}{\gamma_c \eta}\right) \widehat{q}_t^e + \right. \\ & \left. - \left(\frac{\sigma \gamma_i}{\gamma_c} - \frac{\sigma \alpha}{\gamma_c} + \alpha\right) \widehat{y}_t^e + \left(\gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c} - \frac{\sigma \gamma_i \alpha}{\gamma_c}\right) \widehat{k}_t^e \right] \\ & = \left[\Xi_y \widehat{y}_t^2 + \Xi_q \widehat{q}_t^2 + \Xi_k \widehat{k}_t^2 - 2 \Xi_{yq} \widehat{y}_t \widehat{q}_t - 2 \Xi_{yk} \widehat{y}_t \widehat{k}_t + 2 \Xi_{qk} \widehat{q}_t \widehat{k}_t \right] + 2 \widehat{q}_t \zeta_{q,t} + 2 \widehat{k}_t \zeta_{k,t} \end{aligned}$$

where

$$\begin{aligned}
\zeta_{q,t} &\equiv \frac{\gamma_i}{\eta} \widehat{q}_t^e + \Xi_{yq} \left[\frac{\gamma_i}{\eta} \widehat{q}_t^e - \widehat{y}_t^e + \gamma_i \widehat{k}_t^e \right] \\
&= \frac{\gamma_i}{\eta} \widehat{q}_t^e - \Xi_{yq} \gamma_c \widehat{c}_t^e \\
&= \frac{\gamma_i}{\eta} (\widehat{q}_t^e - \sigma \widehat{c}_t^e) \\
\zeta_{k,t} &\equiv (\gamma_i - \alpha) \frac{\sigma}{\gamma_c} (\gamma_i \widehat{k}_t^e - \widehat{y}_t^e + \frac{\gamma_i}{\eta} \widehat{q}_t^e) + \gamma_i \widehat{k}_t^e - \alpha \widehat{y}_t^e \\
&= \gamma_i (\widehat{k}_t^e - \sigma \widehat{c}_t^e) - \alpha (\widehat{y}_t^e - \sigma \widehat{c}_t^e)
\end{aligned}$$

Note next that the intertemporal optimality conditions (3.A.3), (3.A.4) and (3.A.5) imply that

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t (\alpha \widehat{k}_t - \gamma_i \widehat{i}_t) \sigma \widehat{c}_t^e &= \sum_{t=0}^{\infty} \beta^t \frac{\alpha}{\vartheta} (\widehat{k}_t - \beta \widehat{k}_{t+1}) \sigma \widehat{c}_t^e \\
&\Rightarrow \frac{\alpha}{\vartheta} \sum_{t=1}^{\infty} \beta^t \widehat{k}_t \sigma \Delta \widehat{c}_t^e \\
&= \frac{\alpha}{\vartheta} \sum_{t=1}^{\infty} \beta^t \widehat{k}_t [\vartheta m \widehat{p} k_t^e + \beta \widehat{q}_t^e - \widehat{q}_{t-1}^e]
\end{aligned}$$

Thus, plugging this expression into the discounted sum $\sum_{t=0}^{\infty} 2 \widehat{q}_t \zeta_{q,t} + 2 \widehat{k}_t \zeta_{k,t}$, I get

$$\begin{aligned}
2 \sum_{t=0}^{\infty} \beta^t [\widehat{q}_t \zeta_{q,t} + \widehat{k}_t \zeta_{k,t}] &= \\
&= 2 \sum_{t=0}^{\infty} \beta^t \left[\frac{\alpha}{\vartheta} \widehat{k}_t (\vartheta \widehat{y}_t^e - \vartheta \widehat{k}_t^e + \beta \widehat{q}_t^e - \widehat{q}_{t-1}^e) + \frac{\gamma_i}{\eta} \widehat{q}_t \widehat{q}_t^e + \gamma_i \widehat{k}_t \widehat{k}_t^e - \alpha \widehat{k}_t \widehat{y}_t^e \right] \\
&\Rightarrow 2 \sum_{t=0}^{\infty} \beta^t [\alpha \widehat{k}_t \widehat{y}_t^e - \alpha \widehat{k}_t \widehat{k}_t^e + \frac{\alpha \beta}{\vartheta} \widehat{k}_t \widehat{q}_t^e - \frac{\alpha \beta}{\vartheta} \widehat{k}_{t+1} \widehat{q}_t^e + \frac{\gamma_i}{\eta} \widehat{q}_t \widehat{q}_t^e + \gamma_i \widehat{k}_t \widehat{k}_t^e - \alpha \widehat{k}_t \widehat{y}_t^e] \\
&= 2 \sum_{t=0}^{\infty} \beta^t [(\gamma_i - \alpha) \widehat{k}_t \widehat{k}_t^e + (\frac{\gamma_i}{\eta} \widehat{q}_t - \frac{\gamma_i}{\delta} \Delta \widehat{k}_{t+1}) \widehat{q}_t^e] = 2(\gamma_i - \alpha) \sum_{t=0}^{\infty} \beta^t \widehat{k}_t \widehat{k}_t^e \\
&\Rightarrow -2 \frac{\alpha \beta}{\vartheta} \sum_{t=0}^{\infty} \beta^t (\widehat{k}_{t+1} \widehat{k}_{t+1}^e - \widehat{k}_t \widehat{k}_t^e)
\end{aligned}$$

where I used $\gamma_i - \alpha = -\alpha(1 - \beta)/\vartheta$. By the stationarity of \widehat{k}_t and \widehat{k}_t^e , $E_t \sum_{t=0}^{\infty} \beta^t \widehat{k}_{t+1} \widehat{k}_{t+1}^e \approx E_t \sum_{t=0}^{\infty} \beta^t \widehat{k}_t \widehat{k}_t^e$, with equality when the expectation operator is unconditional, so that I can disregard the last term.

I next follow Sveen and Weinke (2009) in deriving the relationship between the dispersion term Δ_t and inflation, both under firm-specific capital accumulation and under a rental market for capital services.

3.C.1. Firm-specific capital

Up to a second-order approximation, Sveen and Weinke (2009) show how

$$\begin{aligned}\Delta_t^p &= \theta \Delta_{t-1}^p + (1 - \theta) \psi_1 \Delta_t^k + \frac{\theta}{1 - \theta} \pi_t^2 \\ \Delta_t^k &= \psi_2^2 \Delta_{t-1}^k + \psi_3^2 \Delta_{t-1}^p \\ \Delta_t^{pk} &= \theta \psi_2 \Delta_{t-1}^{pk} + \theta \psi_3 \Delta_{t-1}^p + (1 - \theta) \psi_1 \Delta_t^k\end{aligned}$$

I can represent this system of equations in equivalent vector notation,

$$\begin{bmatrix} 1 & -(1 - \theta) \psi_1^2 & 0 \\ 0 & 1 & 0 \\ 0 & -(1 - \theta) \psi_1 & 1 \end{bmatrix} \Delta_t = \begin{bmatrix} \theta & 0 & 0 \\ \psi_3^2 & \psi_2^2 & 0 \\ \theta \psi_3 & 0 & \theta \psi_2 \end{bmatrix} \Delta_{t-1} + \begin{bmatrix} \frac{\theta}{1 - \theta} \\ 0 \\ 0 \end{bmatrix} \pi_t^2 \quad \Leftrightarrow \quad B_\Delta \Delta_t = A_\Delta \Delta_{t-1} + C_\Delta \pi_t^2$$

which allows me to express the discounted sum of the dispersion term in welfare as

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t w' \Delta_t &= \sum_{t=0}^{\infty} \beta^t w' \sum_{h=0}^{t-1} (B_\Delta^{-1} A_\Delta)^h B_\Delta^{-1} C_\Delta \pi_{t-h}^2 + t.i.p. \\ &= \sum_{t=1}^{\infty} \beta^t w' \Delta_t = \frac{1}{2} \Xi_p \pi_t^2 + t.i.p.\end{aligned}$$

where $\Xi_p \equiv w' [B_\Delta - \beta A_\Delta]^{-1} C_\Delta$.

I can therefore compute the discounted sum of losses in the firm-specific capital case, ending up with expression (3.18).

3.C.2. Rental market

When firms purchase capital services on a competitive rental market, we have $\bar{y}_t(i) = \bar{k}_t(i)$ and therefore $\Delta_t^y = \Delta_t^k = \Delta_t^{yk} = \varepsilon^2 \Delta_t^p$. The recursive formulation for the cross-sectional dispersion of prices becomes (Woodford, 2003)

$$\Delta_t^p = \theta \Delta_{t-1}^p + \frac{\theta}{1 - \theta} \pi_t^2$$

because there is no dispersion in $p_t^*(i)$. Therefore, the dispersion component in the welfare loss measure is

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t w' \Delta_t &= \frac{\varepsilon}{2} \sum_{t=0}^{\infty} \beta^t \Delta_t^p \\ &= \frac{1}{2} \frac{\varepsilon \theta}{(1 - \theta)(1 - \beta \theta)} \sum_{t=1}^{\infty} \beta^t \pi_t^2 + t.i.p.\end{aligned}$$

I can therefore compute the discounted sum of losses in the rental-market case, ending up with expression (3.18) where $\Xi_p = \varepsilon / \lambda^{rm}$.

3.D. Average Q and marginal Q

Along the lines of Hayashi (1982), I derive the relation between the unobservable marginal Q and the observable average Q , which I denote by S_t .

By the degree-one homogeneity of the production function and the optimality conditions,

$$\begin{aligned}\bar{P}_t(i)Y_t(i) - (1 - \tau)\frac{W_t}{P_t}N_t(i) - (1 - \tau)I_t(i) - T_t &= \bar{P}_t(i)Y_t(i) - \frac{W_t}{P_t}N_t(i) - I_t(i) + \bar{T}_t(i) \\ &= \widehat{MC}_t(i)MPK_t(i)K_t(i) + [\bar{P}_t(i) - \widehat{MC}_t(i)]Y_t(i) + \bar{T}_t(i)\end{aligned}\quad (3.D.19)$$

where $\bar{P}_t(i) \equiv P_t(i)/P_t$ and $\bar{T}_t(i) \equiv \tau W_t[N_t(i) - N_t] + \tau[I_t(i) - I_t]$ is a term that aggregates to zero.

Next, using $M_{t+1} \equiv \Lambda_{t+1}P_{t+1}/\Lambda_tP_t$, with Λ the marginal value of nominal income, rewrite the optimality condition (3.7) as

$$\begin{aligned}\beta E_t \Lambda_{t+1} P_{t+1} [\widehat{MC}_{t+1}(i)MPK_{t+1}(i)K_{t+1}(i) - I_{t+1}(i)] &= \\ &= \Lambda_t P_t Q_t(i)K_{t+1}(i) - \beta E_t \Lambda_{t+1} P_{t+1} Q_{t+1}(i)K_{t+2}(i)\end{aligned}\quad (3.D.20)$$

which I use to derive equation (3.8)

$$\begin{aligned}S_t(i) &= \frac{\sum_{h=1}^{\infty} \beta^h E_t \frac{\Lambda_{t+h} P_{t+h}}{\Lambda_t P_t} [\bar{P}_{t+h}(i)Y_{t+h}(i) - (1 - \tau)\frac{W_{t+h}}{P_{t+h}}N_{t+h}(i) - (1 - \tau)I_{t+h}(i) - T_{t+h}]}{K_{t+1}(i)} \\ &= \frac{\sum_{h=1}^{\infty} \beta^h E_t \frac{\Lambda_{t+h} P_{t+h}}{\Lambda_t P_t} [\widehat{MC}_{t+h}(i)MPK_{t+h}(i)K_{t+h}(i) - I_{t+h}(i) + [\bar{P}_t(i) - \widehat{MC}_{t+h}(i)]Y_{t+h}(i) + \bar{T}_{t+h}(i)]}{K_{t+1}(i)} \\ &= Q_t(i) + \frac{1}{K_{t+1}(i)} \sum_{h=1}^{\infty} E_t M_{t,t+h} \{[\bar{P}_{t+h}(i) - \widehat{MC}_{t+h}(i)]Y_{t+h}(i) + \bar{T}_{t+h}(i)\}\end{aligned}$$

After aggregation⁶⁶ and a loglinearization around the undistorted zero-inflation steady state,⁶⁷

$$\begin{aligned}\widehat{s}_t - \widehat{q}_t &= -\frac{\vartheta}{\alpha} \sum_{j=0}^{\infty} \beta^j E_t \widehat{mc}_{t+j+1} \\ &= -\frac{\vartheta}{\alpha \lambda} E_t \pi_{t+1}\end{aligned}\quad (3.D.21)$$

where the last equality uses the New Keynesian Phillips curve (3.13). The technical appendix derives equation (3.D.21) in more detail and generalizes it to the case of a distorted steady state.

Note that, in the efficient economy, $\widehat{mc}_t = 0$ at all times t , so that $\widehat{s}_t^e = \widehat{q}_t^e$, which is the result in Hayashi (1982) that average and marginal Q are equal when there are no imperfections in the goods market.

⁶⁶Note that $\int_0^1 \bar{T}_t(i) di = 0$.

⁶⁷Recall that $\int \bar{p}_t(i) di = 0$, up to a first-order approximation.

To derive equation (3.15), combine (3.D.21) and $\widehat{s}_t^e = \widehat{q}_t^e$.

Equation (3.D.21) states that if the expected long-run markup, $-E_t \sum_{j=1}^{\infty} \beta^{j-1} \widehat{mc}_{t+j}$, is high, then firms' profits are high, hence dividends and stock prices are high. Note that there are two effects of markups on stock prices: one because markups affect q_t , through (3.A.4)—a high markup reduces the return to investment—one because markups affect profits.

Technical Appendix to Part 1

A. Relationship between the cost of uncertainty and the equity premium

In this section I use a version of the Campbell and Shiller (1988) approximate identity to study approximate loglinear analytical conditions that grant flat term structures.⁶⁸ Under assumptions 2 and 3, lemma 2 shows how the equality between the cost of uncertainty and the equity premium holds whenever $E_t r_{d,t+1}^{e,(1)} = E_t r_{t+1}^{em}$. Lemma 3 thus studies the relationship between the equity premium and the one-period welfare cost, $l_t^{(1)} = E_t r_{d,t+1}^{e,(1)}$, by showing how their difference is controlled by the systematic risk in the market dividend yield. Proposition 4, under the weak structure about the stochastic discount factor in assumption 4, studies conditions under which the two quantities are equal.

Assumption 2. Let either $E_t r_{d,t+1}^{e,(n)} \geq E_t r_{d,t+1}^{e,(1)}$ or $E_t r_{d,t+1}^{e,(n)} \leq E_t r_{d,t+1}^{e,(1)}$, for all $n > 1$.

Assumption 3. The expectations hypothesis of bond valuation holds, $E_t r_{b,t+1}^{e,(n)} = 0$, for all n .⁶⁹

Assumption 2, which rules out ample fluctuations across maturities in the term structure of holding-period equity risk premia, does not appear restrictive; it holds in every model among today's leading consumption-based asset pricing models. The expectations hypothesis in assumption 3 is restrictive (not to mention its empirical failures, that go back at least to Fama and Bliss, 1987), although it is still assumed in some benchmark calibrations of today's leading asset pricing models.

Lemma 1 (Lognormal no-arbitrage pricing). *The absence of arbitrage opportunities in the financial market implies the fundamental asset pricing representation $E_t M_{t+1} R_{t+1}^i = 1$ (Hansen and Richard, 1987). Under lognormality, the no-arbitrage pricing formula becomes*

$$E_t m_{t+1} + E_t r_{t+1}^i + \frac{1}{2} \text{var}_t(m_{t+1} + r_{t+1}^i) = 0$$

⁶⁸Recall the relationship between hold-to-maturity returns and holding period returns

$$\begin{aligned} r_{t \rightarrow t+n}^{(n)} &\equiv \ln \left(\frac{D_{t+n}}{E_t M_{t,t+n} D_{t+n}} \right) \\ &= \ln \left(\frac{D_{t+n}}{E_{t+n-1} M_{t+n} D_{t+n}} \right) + \dots + \ln \left(\frac{E_{t+1} M_{t+1,t+n} D_{t+n}}{E_t M_{t,t+n} D_{t+n}} \right) \\ &= r_{t+n}^{(1)} + \dots + r_{t+1}^{(n)} \end{aligned}$$

⁶⁹The expectations hypothesis is usually stated as $-f_{b,t}^{(n)} = \sum_{j=0}^{n-1} E_t r_{t+j}^f$, for all n , which implies $E_t r_{b,t+1}^{e,(n)} = 0$, for all n , because $E_t r_{b,t+1}^{e,(n)} = E_t f_{b,t+1}^{(n-1)} - f_{b,t}^{(n)} - r_t^f = -\sum_{j=0}^{n-2} E_t r_{t+j+1}^f + \sum_{j=0}^{n-1} E_t r_{t+j}^f - r_t^f = 0$.

This result also holds for the risk-free rate as

$$E_t m_{t+1} + r_t^f + \frac{1}{2} \text{var}_t(m_{t+1}) = 0$$

Combining the formulas for the ex-ante return of the i th security and of the risk-free rate we have the lognormal pricing formula for risk premia

$$\ln E_t R_{t+1}^{ei} = -\text{cov}_t(m_{t+1}, r_{t+1}^{ei})$$

Lemma 2. Let assumptions 2 and 3. Suppose that the first dividend strip premium, $E_t r_{d,t+1}^{e,(1)}$, equals the equity premium, $E_t r_{t+1}^{em}$. Then the cost of uncertainty L_t^N equals the equity premium, for any $N \subset \bar{\mathbb{N}}$.

Proof of lemma 2. By no-arbitrage, $E_t r_{t+1}^{em} = \sum_{n=1}^{\infty} w_{n,t} E_t r_{d,t+1}^{e,(n)}$, where $w_{n,t} \equiv D_t^{(n)} / \sum_{n=1}^{\infty} D_t^{(n)}$. Since $E_t r_{t+1}^{em} = E_t r_{d,t+1}^{e,(1)}$, then, under assumption 2, $E_t r_{d,t+1}^{e,(n)} \geq E_t r_{d,t+1}^{e,(n-1)}$, or viceversa. Suppose that the inequality is strict for some n . Then $E_t r_{d,t+1}^{e,(1)} = \sum_{n=1}^{\infty} w_{n,t} E_t r_{d,t+1}^{e,(n)} > E_t r_{d,t+1}^{e,(1)} \sum_{n=1}^{\infty} w_{n,t} = E_t r_{d,t+1}^{e,(1)}$, a contradiction. Therefore, the inequality must hold with equality, i.e., $E_t r_{d,t+1}^{e,(j)} = E_t r_{t+1}^{em}, \forall j \in \mathbb{N}$. Therefore, and by assumption 3, $l_t^{(n)} = \frac{1}{n} E_t r_{t \rightarrow t+n}^{e,(n)} = \frac{1}{n} \sum_{j=1}^n E_t (r_{d,t+n-j+1}^{e,(j)} - r_{b,t+n-j+1}^{e,(j)}) = \frac{1}{n} E_t r_{t+1}^{em}$. Thus, $L_t^N = \sum_{n \in N} \omega_{n,t} E_t r_{t+1}^{em} = E_t r_{t+1}^{em}, \forall N \subset \mathbb{N}$. \square

Lemma 3. Let a representative agent, lognormal environment without arbitrage opportunities in the financial market. By proposition 2, the cost at time t of fluctuations at time $t+1$, $l_t^{(1)}$, equals the premium on a strip that pays off aggregate consumption next period. Therefore, the distance between $l_t^{(1)}$ and the equity premium equals

$$\delta \text{cov}_t(m_{t+1}, dp_{t+1}) \tag{A.1}$$

where dp is the log dividend-price ratio of the market portfolio and $1/\delta$ is the unconditional market return.

Proof of lemma 3. I define the market portfolio as the portfolio that pays off the entire consumption stream. The approximate Campbell and Shiller (1988) identity for the market portfolio then is

$$r_{t+1}^m = k + dp_t - \delta dp_{t+1} + \Delta c_{t+1}$$

for some constant intercept k and $1/\delta$ the unconditional market return.

Then, by no-arbitrage pricing under lognormality (lemma 1) and by the definition of the one-period welfare cost $l_t^{(1)}$, the equity premium is

$$\begin{aligned} \ln E_t R_{t+1}^{em} &= -\text{cov}_t(m_{t+1}, r_{t+1}^m) \\ &= -\text{cov}_t(m_{t+1}, \Delta c_{t+1}) + \delta \text{cov}_t(m_{t+1}, dp_{t+1}) \\ &= l_t^{(1)} + \delta \text{cov}_t(m_{t+1}, dp_{t+1}) \end{aligned}$$

The residual $\delta cov_t(m_{t+1}, dp_{t+1})$, the systematic risk in the price-dividend ratio, measures the distance between the cost of uncertainty and the equity premium. \square

Assumption 4. Preferences U_t conform to the generic stochastic discount factor

$$m_{t+1} = -\rho_t - \gamma_t \sum_{j=0}^{\infty} \delta_j (E_{t+1} - E_t) \Delta c_{t+j+1} \quad (\text{A.2})$$

with $\delta_0 = 1$.

The stochastic discount factor in equation (A.2) is fairly general. For example, it embeds the preferences studied by Campbell and Cochrane (1999); Bansal and Yaron (2004); Hansen and Sargent (2005); Hansen, Heaton and Li (2008); Barillas, Hansen and Sargent (2009).

Assumption 5. Either the market price-dividend ratio is constant, or consumption is a random walk and news to consumption growth and to the price-dividend ratio are orthogonal.

Proposition 4. *Let a representative agent, lognormal environment without arbitrage opportunities in the financial market. Let assumptions 2, 3, and 4. Then, the distance between the one-period welfare cost and the equity premium is*

$$cov_t(m_{t+1}, dp_{t+1}) = -\gamma_t \sum_{j=0}^{\infty} \delta_j cov_t((E_{t+1} - E_t) \Delta c_{t+j+1}, dp_{t+1})$$

Moreover, let assumption 5. Then, the cost of uncertainty L_t^N equals the equity premium, for any coordinate set $N \subset \overline{N}$.

Proof of lemma 4. Under assumption 4, the distance between the cost of uncertainty and the equity premium characterized in lemma 3 becomes

$$\ln E_t R_{t+1}^{em} = l_t^{(1)} - \gamma_t \sum_{j=0}^{\infty} \delta_j cov_t((E_{t+1} - E_t) \Delta c_{t+j+1}, dp_{t+1}) \quad (\text{A.3})$$

which proves the first part of proposition 4.

Moreover, if consumption is a random walk, expression (A.3) collapses to

$$\ln E_t R_{t+1}^{em} = l_t^{(1)} - \gamma_t cov_t(\Delta c_{t+1}, \delta dp_{t+1})$$

If, additionally, news to consumption growth and to the price-dividend ratio are orthogonal,

$$\ln E_t R_{t+1}^{em} = l_t^{(1)}$$

i.e., the one-period welfare cost equals the equity premium. This result, combined with assumptions 2 and 3 and lemma 2, proves proposition 4. \square

To evaluate the restrictiveness of assuming trivial term structure properties, I use proposition 4 to study what conditions would grant them in some of today's leading consumption-based asset pricing models. Section B works out the details of each model.

Example A.1 (Log utility). Consider preferences captured by log utility, $U_t = \ln(C_t) + \beta E_t U_{t+1}$. Under log utility, the market portfolio has the convenient property that the price-dividend ratio of the market portfolio is constant. Therefore, by proposition 4, the welfare cost of uncertainty equals the equity premium.

Example A.2 (Campbell and Cochrane (1999) habits). In the habit formation model of Campbell and Cochrane (1999) the utility function $U(C_t, X_t) = (C_t - X_t)^{1-\gamma} + \beta E_t U_{t+1}$, where X_t represents an external habit that is a nonlinear function of past consumption such that the log stochastic discount factor has shape $m_{t+1} = -\rho_t - \gamma_t(E_{t+1} - E_t)\Delta c_{t+1}$. The nonlinearity is calibrated to ensure that the precautionary savings effect largely offsets the intertemporal substitution effect in the determination of the risk-free rate, thus avoiding the risk-free rate puzzle of Weil (1989). At the same time a large and time-varying risk-aversion coefficient, γ_t , matches a high and volatile equity premium. In their benchmark calibration, Campbell and Cochrane (1999) assume a constant risk-free rate and thereby a flat term structure of interest rates. Under these conditions, the distance between the one-period welfare cost and the equity premium is given by $\gamma_t \text{cov}_t(\Delta c_{t+1}, dp_{t+1})$.

Thus, under the further assumption that consumption growth and the price-dividend ratio are conditionally orthogonal, the distance (A.1) collapses to zero. Then, by proposition 4, the welfare cost of uncertainty equals the equity premium.

Under the assumed orthogonality, dp_{t+1} bears no systematic risk from time t to time $t + 1$. The baseline model of Campbell and Cochrane cannot however replicate the orthogonality assumption, because it has only one structural shock, a structure that imposes a perfect correlation between dividend growth and dividend yields.

Example A.3 (Bansal and Yaron (2004) long-run risk model). Consider preferences as described by non-expected utility $U_t = \{(1 - \beta)C_t^{1-\rho} + \beta[E_t U_{t+1}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}\}^{\frac{1}{1-\rho}}$, where $[E_t U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}$ is the certainty equivalent of utility at time $t + 1$ evaluated through the expected utility function $v(x) = x^{1-\gamma}$. Parameter ρ is the intertemporal elasticity of substitution and γ controls the risk aversion. An example are the recursive preferences of Epstein and Zin (1989).

I follow Campbell (1993) and Restoy and Weil (2011), who use a version of the loglinearized Campbell-Shiller identity and a lognormal no-arbitrage pricing framework with conditionally homoskedastic consumption growth to show that, up to first-order, the market price-dividend ratio and the stochastic discount factor are the functions of future consumption growth

$$\widehat{dp}_t = -(1 - \rho) \sum_{j=0}^{\infty} \delta^j E_t \widehat{\Delta c}_{t+j+1}$$

$$(E_{t+1} - E_t)m_{t+1} = -\gamma(E_{t+1} - E_t)\Delta c_{t+1} + (\rho - \gamma)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \delta^j \Delta c_{t+j+1}$$

Then, if consumption growth follows the generic process

$$\begin{aligned}\zeta_{t+1} &= A\zeta_t + Be_{t+1} \\ \Delta c_{t+1} &= \mu + C\zeta_t + De_{t+1}\end{aligned}$$

where ζ is a state vector and $e \sim WN(0, I)$, then the distance between the cost of uncertainty and the equity premium is

$$\begin{aligned}\delta cov_t(m_{t+1}, dp_{t+1}) &= cov_t(-\gamma De_{t+1} + (\rho - \gamma) \sum_{j=1}^{\infty} \delta^j CA^{j-1} Be_{t+1}, -(1 - \rho) \sum_{j=1}^{\infty} \delta^j CA^{j-1} Be_{t+1}) \\ &= (1 - \rho)(\gamma D + (\gamma - \rho)\delta C[I - \delta A]^{-1}B)(\delta C[I - \delta A]^{-1}B)'\end{aligned}\quad (A.4)$$

where $1/\delta$ is the steady-state value of the market return.

The price-dividend ratio is constant if the intertemporal elasticity of substitution, ρ , is unity or if consumption is a random walk ($C = 0$). In these two cases, the distance (A.1) collapses to zero. If consumption is a random walk, the term structure of interest rates is flat and assumption 3 holds. Then, by proposition 4, the welfare cost of uncertainty equals the equity premium.

Note how result (A.4) is true in the long-run risk model of Bansal and Yaron (2004) without stochastic volatility and the ambiguity averse preferences of the robust control literature (Barillas et al., 2009).

Example A.4 (Barillas, Hansen and Sargent (2009) ambiguity averse multiplier preferences). Consider agents who have a wish for robustness against some misspecification in the transition equation for the states of the economy. I follow Hansen and Sargent (2005) in modeling the misspecification through a non-negative martingale G_t that distorts the probability distribution $P(e'|\zeta_0)$ implied by the transition equation for the states of the economy. The stochastic process G_t in turn implies a factor g_{t+1} , defined recursively as $G_{t+1} = g_{t+1}G_t$ with $G_0 = 1$, that distorts the transition probability measure $P(e_{t+1}|e', \zeta_0)$. The ambiguity-averse agent then evaluates the objective function by drawing the worst-case scenario about the misspecification and penalizes the objective by a function of relative entropy, which is strictly greater than zero unless there are no distortions P -almost everywhere. I can write the objective function as

$$V_0 = \min_{\{g_{t+1}\}} E_0^Q \sum_{t=0}^{\infty} \beta^t \{ \ln(C_t) + \beta \theta E_t^P [g_{t+1} \ln g_{t+1}] \}$$

where parameter θ represents the agent's aversion to model misspecification and where Q represents the distorted probability measure, whose Radon-Nikodym derivative with respect to probability measure P is G_t . It follows that the optimized function g_{t+1} is the Esscher transform of probability measure P

$$g_{t+1} = \frac{\exp\{-V_{t+1}/\theta\}}{E_t^P \exp\{-V_{t+1}/\theta\}}$$

which implies the optimized value function

$$V_t = \ln(C_t) - \beta\theta \ln E_t^P \exp\left\{-\frac{V_{t+1}}{\theta}\right\} \quad (\text{A.5})$$

Value function (A.5) is observationally equivalent to Epstein and Zin (1989) preferences with unitary intertemporal elasticity of substitution, provided $V_t = \ln U_t/(1-\beta)$ and $\theta = -1/[(1-\beta)(1-\gamma)]$. Therefore, as in example A.3, the price-dividend ratio is constant and the cost of uncertainty equals the equity premium.

Example A.5 (Gabaix (2012) variable rare disaster model). Although these preferences are not a special case of (A.2), the relationship between the one-period cost of uncertainty and the equity premium can be easily studied. Gabaix assumes power utility and that log consumption growth falls by an amount b_{t+1} in the event of a disaster at time $t + 1$. These assumptions imply the log stochastic discount factor

$$m_{t+1} = \begin{cases} -\delta & \text{with probability } 1 - p_t \\ -\delta - \gamma b_{t+1} & \text{with probability } p_t \end{cases}$$

where p_t is the (potentially time-varying) probability of a disaster at time $t + 1$. Under the baseline calibration, the model implies flat term structures of holding-period risk premia on both zero-coupon equities and bonds and therefore the equality between the equity premium and the cost of uncertainty.

Example A.6 (Lettau and Wachter (2007, 2011) quasi-structural model). Lettau and Wachter (2007, 2011) assume an essentially affine exponential-Gaussian setting in which shocks to dividend growth $(E_{t+1} - E_t)\Delta d_{t+1}$ is the only pricing factor, the price of risk x_t is linear in the states of the economy, and shocks to the price of risk are uncorrelated to cashflow shocks, i.e., $cov_t(\Delta d_{t+1}, x_{t+1}) = 0$. Namely, consider a stochastic discount factor $m_{t+1} = -r_t^f - \frac{1}{2}\gamma_t^2 FF' - \gamma_t F \varepsilon_{t+1}$, where the price of risk $\gamma_t = \gamma + K\zeta_t$ is linear in the states $\zeta_t = A\zeta_{t-1} + B\varepsilon_{t+1}$, and the pricing factor is $F\varepsilon_{t+1} = \|D\|^{-1}(E_{t+1} - E_t)\Delta c_{t+1}$, where $\Delta c_{t+1} = C\zeta_t + D\varepsilon_{t+1}$, with $\varepsilon_t \sim \text{Niid}(0, I)$. Assume $KBF' = cov_t(\Delta d_{t+1}, \gamma_{t+1}) = 0$.

If on top of that consumption is a random walk ($C = 0$), we can easily verify that the term structure of welfare costs is flat:

$$\begin{aligned} E_t(r_{d,t+1}^{(n)} - r_{b,t+1}^{(n)}) - E_t(r_{d,t+1}^{(n-1)} - r_{b,t+1}^{(n-1)}) &= -(B^{d,(n-1)} - B^{b,(n-1)})(I - A)BF'\gamma_t \\ &= [I - \Gamma^{n-1}]DF'KBF'\gamma_t = 0 \end{aligned}$$

where the last equality is because $B^{d,(n)} - B^{b,(n)} = (B^{d,(n-1)} - B^{b,(n-1)})(A - BF'K) - DFK'$. Moreover, the term structure of interest rates is flat if $HBF' = cov_t(\Delta d_{t+1}, r_{t+1}) = 0$, i.e., if shocks to the state that drives the risk-free rate are either absent (as in Lettau and Wachter, 2007) or unpriced. Since both term structures are flat, the term structure of equity must also be flat, hence the welfare cost of uncertainty equals the equity premium.

B. Term structures in some consumption-based asset pricing models

Recall the definition of the components of the term structure of marginal costs of uncertainty,

$$l_t^{(n)} = \frac{1}{n} \left(\exp\{E_t r_{t \rightarrow t+n}^{e,(n)} + \frac{1}{2} \mathcal{V}_t(R_{t \rightarrow t+n}^{e,(n)})\} - 1 \right) \quad (\text{B.1})$$

$$\ell_t^{(n)} = \frac{1}{n} E_t r_{t \rightarrow t+n}^{e,(n)} + \frac{1}{2n} \mathcal{V}_t(R_{t \rightarrow t+n}^{e,(n)}) \quad (\text{B.2})$$

where $\mathcal{V}(X|\mathcal{F}) \equiv 2 \ln E[X|\mathcal{F}] - 2E[\ln(X)|\mathcal{F}]$ denotes entropy relative to the information set \mathcal{F} , and $\ell_t^{(n)} \equiv \ln(1 + nl_t^{(n)})^{1/n}$ is the continuously-compounded welfare cost of uncertainty associated with the discretely-compounded cost $l_t^{(n)}$.

Note that, unconditionally,

$$\begin{aligned} E(l_t^{(n)}) &= \frac{1}{n} (E(R_{t \rightarrow t+n}^{e,(n)}) - 1) \\ &= \frac{1}{n} \left(\exp\{E(E_t r_{t \rightarrow t+n}^{e,(n)}) + \frac{1}{2} \mathcal{V}(R_{t \rightarrow t+n}^{e,(n)})\} - 1 \right) \\ &= \frac{1}{n} \left(\exp\{E(\ln[1 + nl_t^{(n)}]) + \frac{1}{2} \mathcal{V}(E_t R_{t \rightarrow t+n}^{e,(n)})\} - 1 \right) \\ &= \frac{1}{n} \left(\exp\{E(n\ell_t^{(n)}) + \frac{1}{2} \mathcal{V}(\exp\{n\ell_t^{(n)}\})\} - 1 \right) \end{aligned} \quad (\text{B.3})$$

where the second equality uses equation (B.1) and the law of iterated expectations, and the third equality uses the law of total entropy.

B.1. Notation

Throughout the appendix, I use the notation for yields $F_{c,t}^{(n)} = D_{c,t}^{(n)}/C_t$, $F_{d,t}^{(n)} = D_{d,t}^{(n)}/D_t$ and $F_{b,t} = D_{b,t}$, where $D_{c,t}^{(n)} = E_t M_{t,t+n} C_{t+n}$ denotes the no-arbitrage price of a n -period consumption strip, $D_{d,t}^{(n)} = E_t M_{t,t+n} D_{t+n}$ denotes the no-arbitrage price of a n -period dividend strip, and $D_{b,t}^{(n)} = E_t M_{t,t+n}$ denotes the no-arbitrage price of a n -period zero-coupon real bond. Also, recall the definition $R_{t+1}^{(n)} = \frac{D_{t+1}^{(n-1)}}{D_t^{(n)}}$, which implies the recursive no-arbitrage pricing formula for the term structure components

$$\begin{aligned} F_{b,t}^{(n)} &= E_t M_{t+1} F_{b,t+1}^{(n-1)} \\ F_{c,t}^{(n)} &= E_t M_{t+1} \frac{C_{t+1}}{C_t} F_{c,t+1}^{(n-1)} \\ F_{d,t}^{(n)} &= E_t M_{t+1} \frac{D_{t+1}}{D_t} F_{d,t+1}^{(n-1)} \end{aligned}$$

with boundary condition $F_t^{(0)} = 1$ (Wachter, 2006; Lettau and Wachter, 2007, 2011). Finally, note the recursive structure of the term structure of growth $F_{g,t} = E_t(C_{t+n}/C_t)$:

$$F_{g,t}^{(n)} = E_t \frac{C_{t+1}}{C_t} F_{g,t+1}^{(n-1)}$$

B.2. Approximate Campbell-Shiller identities

A loglinearization of the identity $R_{t+1}^w = W_{t+1}/(W_t - C_t)$ for the wealth portfolio is

$$r_{t+1}^w = -\ln(\delta_c) + \widehat{\Delta c}_{t+1} - \widehat{c w}_{t+1} + \frac{1}{\delta_c} \widehat{c w}_t$$

where $\delta_c \equiv 1/E(R_t^w)$.

A loglinearization of the identity $R_{t+1}^m = (P_{t+1} + D_{t+1})/P_t$ for the market portfolio is

$$r_{t+1}^m = -\ln(\delta_d) + \widehat{\Delta d}_{t+1} - \delta_d \widehat{d p}_{t+1} + \widehat{d p}_t$$

where $\delta_d \equiv 1/E(R_t^m)$.

B.3. Log utility

Consider the simple log utility case $U_t = \ln C_t + \beta E_t U_{t+1} = \sum_{j=0}^{\infty} E_t \ln C_{t+j}$, which features the stochastic discount factor $M_{t+1} = \beta C_t / C_{t+1}$. Consumption claims and real bonds have therefore the respective no-arbitrage prices

$$E_t M_{t,t+n} C_{t+n} = \beta^n C_t \quad E_t M_{t,t+n} = \beta^n E_t (C_t / C_{t+n})$$

and therefore $R_{d,t+1}^{(n)} = C_{t+1} / \beta C_t = 1 / M_{t+1}, \forall n$. Under the additional assumption of lognormality,

$$\begin{aligned} E_t R_{t \rightarrow t+n}^{e,(n)} &= \frac{E_t M_{t,t+n} E_t (C_{t+n} / C_t)}{E_t (M_{t,t+n} C_{t+n} / C_t)} \\ &= e^{-E_t (c_{t+n} - c_t) + \frac{1}{2} \text{var}_t(c_{t+n}) + E_t (c_{t+n} - c_t) + \frac{1}{2} \text{var}_t(c_{t+n})} \\ &= e^{\text{var}_t(c_{t+n})} \end{aligned}$$

and therefore $\ell_t^{(n)} = \frac{1}{n} \text{var}_t(c_{t+n})$.

B.4. Habit formation

Let the factor X in the utility function $U(C, X)$ be a possibly nonlinear function of past consumption such that the log of the stochastic discount factor has form

$$m_{t+1} = -\rho_t - \gamma_t (E_{t+1} - E_t) \Delta c_{t+1}$$

An example are the habits of Campbell and Cochrane (1999), $U(C_t, X_t) = \frac{1}{1-\gamma} (C_t - X_t)^{1-\gamma} + \beta E_t U_{t+1}$, with surplus-consumption ratio $S_t \equiv (C_t - X_t) / C_t$ driven by $s_{t+1} = \phi s_t + \lambda(s_t)(E_{t+1} - E_t) c_{t+1}$.

As in Campbell and Cochrane (1999) I assume the structure

$$\begin{aligned}
m_{t+1} &= -\gamma\Delta c_{t+1} - \gamma\Delta s_{t+1} \\
\Delta c_{t+1} &= \mu + v_{t+1} \\
\Delta d_{t+1} &= \mu + w_{t+1} \\
s_{t+1} &= (1 - \phi)s + \phi s_t + \lambda(s_t)v_{t+1} \\
\lambda(s_t) &= e^{-s} \sqrt{1 - 2(s_t - s)} - 1 \quad \text{with} \quad e^s = \sigma_v \sqrt{\frac{\gamma}{1 - \phi - b/\gamma}} \\
r_t^f &= -\ln(\beta) + \gamma\mu - \frac{1}{2}\gamma(1 - \phi - b/\gamma) - b(s_t - s)
\end{aligned}$$

with $[v; w] \sim N(0, \Sigma)$ and $\Sigma = \begin{bmatrix} \sigma_v^2 & \rho_{cd}\sigma_v\sigma_w \\ \rho_{cd}\sigma_v\sigma_w & \sigma_w^2 \end{bmatrix}$.⁷⁰

Consumption and dividend yields are functions of the state s_t . Given the nonlinearity of the price of risk $\gamma_t = \gamma[1 + \lambda(s_t)]$ on the state s_t , the solution does not have a closed form (Wachter, 2005):

$$\begin{aligned}
F^{b,(n)}(s_t) &= E_t[\beta e^{-\gamma\Delta c_{t+1} - \gamma\Delta s_{t+1}} F_{t+1}^{b,(n-1)}(s_{t+1})] \\
&= \int_{-\infty}^{\infty} \beta e^{-\gamma\mu - \gamma v + \gamma(1-\phi)(s_t - s) - \gamma\lambda(s_t)v} F^{b,(n-1)}((1 - \phi)s + \phi s_t + \lambda(s_t)v) dP(v) \\
F^{c,(n)}(s_t) &= \int_{-\infty}^{\infty} \beta e^{(1-\gamma)\mu + (1-\gamma)v + \gamma(1-\phi)(s_t - s) - \gamma\lambda(s_t)v} F^{c,(n-1)}((1 - \phi)s + \phi s_t + \lambda(s_t)v) dP(v) \\
F^{d,(n)}(s_t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \beta e^{(1-\gamma)\mu + w - \gamma v + \gamma(1-\phi)(s_t - s) - \gamma\lambda(s_t)v} F^{d,(n-1)}((1 - \phi)s + \phi s_t + \lambda(s_t)v) dP(v, w) \\
&= \int_{-\infty}^{\infty} \beta E[e^w | v] e^{(1-\gamma)\mu - \gamma v + \gamma(1-\phi)(s_t - s) - \gamma\lambda(s_t)v} F^{d,(n-1)}((1 - \phi)s + \phi s_t + \lambda(s_t)v) dP(v) \\
&= \int_{-\infty}^{\infty} \beta e^{\rho_{cd} \frac{\sigma_w}{\sigma_v} v + \frac{1}{2}\sigma_w^2(1-\rho_{cd}^2) + (1-\gamma)\mu - \gamma v + \gamma(1-\phi)(s_t - s) - \gamma\lambda(s_t)v} F^{d,(n-1)}((1 - \phi)s + \phi s_t + \lambda(s_t)v) dP(v)
\end{aligned}$$

with the appropriate boundary conditions, and where the last equality uses the property of multi-variate normal variables $[v; w] \sim N(0, \Sigma) \Rightarrow w|v \sim N(\rho_{cd} \frac{\sigma_w}{\sigma_v} v, \sigma_w^2(1 - \rho_{cd}^2))$.

I solve the integrals numerically by a 40-point Gauss-Legendre quadrature, bounding the support of the shock by -8 and $+8$ standard deviations. I follow Campbell and Cochrane (1999) in first computing $F^{(n-1)}(s)$ over a grid of points for s , to then compute $F^{(n-1)}((1 - \phi)s + \phi s_t + \lambda(s_t)v)$ by loglinearly interpolating between grid points.

Next, compute the term structure of growth in this setting: guess $F_t^{g,(n)} \equiv E_t\Delta C_{t,t+n} = e^{A_n}$ and,

⁷⁰Note that this specification cannot guarantee that consumption and dividends are cointegrated.

under $\Delta c_{t+1} = \mu + v_{t+1}$, verify as

$$\begin{aligned} e^{A_n} &= E_t \Delta C_{t+1} e^{A_{n-1}} = E_t e^{\mu + v_{t+1} + A_{n-1}} \\ &= e^{\mu + A_{n-1} + \frac{1}{2}\sigma_v^2} \end{aligned}$$

hence $A_n = \mu + A_{n-1} + \frac{1}{2}\sigma_v^2 = n(\mu + \frac{1}{2}\sigma_v^2)$.

Then, I build $\ell_t^{(n)} = \frac{1}{n} \ln[F_t^{b,(n)} \frac{F_t^{g,(n)}}{F_t^{c,(n)}}] = \frac{1}{n} [f_t^{b,(n)} - f_t^{c,(n)} + f_t^{g,(n)}]$ and derive its mean and variance.

B.5. Long-run risk

Consider the preferences in Epstein and Zin (1989) and Weil (1989), $U_t = \{(1 - \beta)C_t^{1-\rho} + \beta X_t^{1-\rho}\}^{\frac{1}{1-\rho}}$, where $X_t \equiv [E_t U_{t+1}^{1-\gamma}]^{1/(1-\gamma)}$ is the certainty equivalent of utility at time $t + 1$ evaluated through the expected utility function $v(x) = x^{1-\gamma}$. Power utility is the special case $\gamma = \rho$. Tallarini (2000) is the special case $\rho = 1$, and is a case that can be linked to the robust control literature (Barillas et al., 2009).

Epstein-Zin-Weil preferences have the property $U_t = \sum_{j=0}^{\infty} C_{t+j} \partial U_t / \partial C_{t+j}$, by Euler's theorem, because U_t is homogeneous with $\partial U_t / \partial C_t = (1-\beta)U_t^\rho C_t^{-\rho}$ and $\partial U_t / \partial C_{t+1} = \beta U_t^\rho X_t^{\gamma-\rho} U_{t+1}^{-\gamma} \partial U_{t+1} / \partial C_{t+1}$. Thus,

$$M_{t+1} = \frac{\partial U_t / \partial C_{t+1}}{\partial U_{t+1} / \partial C_t} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{U_t}{X_t} \right)^{\rho-\gamma} \quad (\text{B.4})$$

$$\frac{W_t}{C_t} = \frac{U_t / C_t}{\partial U_t / \partial C_t} = \frac{1}{1-\beta} \left(\frac{U_t}{C_t} \right)^{1-\rho} \quad (\text{B.5})$$

The factor X in the utility function is

$$\begin{aligned} X_t &= [E_t U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}} \\ &= \beta^{-\frac{1}{1-\rho}} [U_t^{1-\rho} + (1-\beta)C_t^{1-\rho}]^{\frac{1}{1-\rho}} \\ &= \left(\frac{1-\beta}{\beta} \right)^{\frac{1}{1-\rho}} \left(\frac{W_t - C_t}{C_t} \right)^{\frac{1}{1-\rho}} C_t \end{aligned}$$

Result (B.5), combined with the definition of the return on the wealth portfolio $R_{t+1}^w = W_{t+1}/(W_t - C_t)$, allows me to rewrite equation (B.4) as

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left(\frac{1}{R_{t+1}^w} \right)^{\frac{\gamma-\rho}{1-\rho}} \quad (\text{B.6})$$

I follow Campbell (1993) and Restoy and Weil (2011), who use the loglinearized identity and a

lognormal no-arbitrage pricing framework to show that

$$cw_t = \text{const.} - \delta(1 - \rho) \sum_{j=0}^{\infty} \delta^j E_t \Delta c_{t+j+1} - \frac{1}{2} \delta \frac{1 - \gamma}{1 - \rho} E_t \sum_{j=0}^{\infty} \delta^j \text{var}_{t+j}(z_{t+j+1}) \quad (\text{B.7})$$

$$E_t m_{t+1} = \ln(\beta) - \rho E_t \Delta c_{t+1} + \frac{1}{2} \frac{(\gamma - \rho)(1 - \gamma)}{(1 - \rho)^2} \text{var}_t(z_{t+1})$$

where $z_t \equiv cw_t - (1 - \rho)\Delta c_t$.

Proof. Using (B.6), $m_{t+1} = \frac{1-\gamma}{1-\rho} \ln(\beta) - \rho \frac{1-\gamma}{1-\rho} \Delta c_{t+1} - \frac{\gamma-\rho}{1-\rho} r_{t+1}^w$, and the lognormal pricing formula, $0 = E_t m_{t+1} + E_t r_{t+1}^w + \frac{1}{2} \text{var}_t(m_{t+1} + r_{t+1}^w) = \frac{1-\gamma}{1-\rho} \ln(\beta) - \rho \frac{1-\gamma}{1-\rho} E_t \Delta c_{t+1} + \frac{1-\gamma}{1-\rho} E_t r_{t+1}^w + \frac{1}{2} \left(\frac{1-\gamma}{1-\rho} \right)^2 \text{var}_t(r_{t+1}^w - \rho \Delta c_{t+1})$, hence $E_t r_{t+1}^w = -\ln(\beta) + \rho E_t \Delta c_{t+1} - \frac{1}{2} \frac{1-\gamma}{1-\rho} \text{var}_t(r_{t+1}^w - \rho \Delta c_{t+1})$. The results then follow by using the loglinearized identity. \square

B.5.1. Approximate SDF

Let the nonlinear state-space model (Bansal and Yaron, 2004)

$$\begin{aligned} \zeta_{t+1} &= A\zeta_t + \sigma_t B \varepsilon_{t+1} \\ \Delta c_{t+1} &= \mu + C\zeta_t + \sigma_t D \varepsilon_{t+1} \\ \sigma_{t+1}^2 &= \sigma^2 + \nu(\sigma_t^2 - \sigma^2) + \Sigma \varepsilon_{t+1} \end{aligned}$$

where ζ is a state vector, σ_t^2 is a scalar random variable, $\varepsilon \sim WN(0, I)$ and $\Sigma \varepsilon_t \perp [B \varepsilon_t; D \varepsilon_t]$.

I guess that cw is a linear function of the states $[\zeta_t, \sigma_t^2]$, hence $(E_{t+1} - E_t)cw_{t+1} = (1 - \rho)\Psi_1 \sigma_t \varepsilon_{t+1} + (1 - \rho)\Psi_2 \Sigma \varepsilon_{t+1}$, and therefore

$$\begin{aligned} (E_{t+1} - E_t)cw_{t+1} &= -(1 - \rho) \sum_{j=1}^{\infty} \delta^j (E_{t+1} - E_t) \Delta c_{t+j+1} - \frac{1}{2} \frac{1 - \gamma}{1 - \rho} \sum_{j=1}^{\infty} \delta^j (E_{t+1} - E_t) \text{var}_{t+j}(cw_{t+j+1} - (1 - \rho)\Delta c_{t+j+1}) \\ &= -(1 - \rho) \sum_{j=1}^{\infty} \delta^j C A^{j-1} B \sigma_t \varepsilon_{t+1} - \frac{1}{2} (1 - \gamma)(1 - \rho) \|\Psi_1 - D\|^2 \sum_{j=1}^{\infty} \delta^j \nu^{j-1} \Sigma \varepsilon_{t+1} \\ &= -\delta(1 - \rho) C [I - \delta A]^{-1} B \sigma_t \varepsilon_{t+1} - \frac{1}{2} \delta \frac{(1 - \gamma)(1 - \rho)}{1 - \delta \nu} \|\Psi_1 - D\|^2 \Sigma \varepsilon_{t+1} \end{aligned} \quad (\text{B.8})$$

which verifies the guess for $\Psi_1 = -\delta C [I - \delta A]^{-1} B$ and $\Psi_2 = -\frac{1}{2} \delta \frac{1-\gamma}{1-\delta \nu} \|D - \Psi_1\|^2$. Therefore,

$$\begin{aligned} (E_{t+1} - E_t)m_{t+1} &= -\gamma(E_{t+1} - E_t)\Delta c_{t+1} + \frac{\gamma - \rho}{1 - \rho} (E_{t+1} - E_t)cw_{t+1} \\ &= -F \sigma_t \varepsilon_{t+1} - J \Sigma \varepsilon_{t+1} \\ (E_{t+1} - E_t)r_{t+1}^w &= [D - (1 - \rho)\Psi_1] \sigma_t \varepsilon_{t+1} - (1 - \rho)\Psi_2 \Sigma \varepsilon_{t+1} \end{aligned} \quad (\text{B.9})$$

where $F \equiv \gamma D + (\rho - \gamma)\Psi_1$ and $J \equiv (\rho - \gamma)\Psi_2$.

Thus,

$$\begin{aligned}\delta &= e^{-E(E_t r_{t+1}^w) - \frac{1}{2} \text{var}(E_t r_{t+1}^w) - \frac{1}{2} E \text{var}_t(r_{t+1}^w)} \\ &= e^{\ln(\beta) - \rho\mu + \frac{1}{2}(1-\gamma)(1-\rho)(\|D - \Psi_1(\delta)\|^2 \sigma^2 + \|\Psi_2(\delta)\Sigma\|^2) - \frac{1}{2}\rho^2 C \text{var}(\zeta_t) C' - \frac{1}{8}(1-\gamma)^2(1-\rho)^2 \|D - \Psi_1(\delta)\|^4 \text{var}(\sigma_t^2) - \frac{1}{2}(\|D - (1-\rho)\Psi_1(\delta)\|^2 \sigma^2 + \|(1-\rho)\Psi_2(\delta)\Sigma\|^2)}\end{aligned}$$

defines a map $\delta = f(\delta)$ whose fixed point yields a numerical value for δ and thereby for $[\Psi_1, \Psi_2, F, J]$.

Finally, note that

$$\begin{aligned}r_t^f &= -E_t m_{t+1} - \frac{1}{2} \text{var}_t(m_{t+1}) \\ &= -\ln(\beta) + \rho E_t \Delta c_{t+1} + \frac{1}{2}(\rho - \gamma)(1 - \gamma)(\|D - \Psi_1\|^2 \sigma_t^2 + \|\Psi_2 \Sigma\|^2) - \frac{1}{2}(\|F\|^2 \sigma_t^2 + \|J \Sigma\|^2) \\ &= r^f + G \sigma_t^2 + H \zeta_t\end{aligned}$$

where $r^f = -\ln(\beta) + \rho\mu + \frac{1}{2} \frac{1-\rho}{\rho-\gamma} \|\Psi_2 \Sigma\|^2$, $G = -\frac{1}{2}(\|F\|^2 - (\rho - \gamma)(1 - \gamma)\|D - \Psi_1\|^2)$, and $H = \rho C$.

B.5.2. Log utility and long-run risk ($EIS = 1$)

Consider Epstein-Zin-Weil non-expected utility as $\rho \rightarrow 1$:

$$\begin{aligned}\lim_{\rho \rightarrow 1} \frac{\ln U_t}{1 - \beta} &= \lim_{\rho \rightarrow 1} \frac{1}{(1 - \beta)(1 - \rho)} \ln((1 - \beta)C_t^{1-\rho} + \beta X_t^{1-\rho}) \\ &= \lim_{\rho \rightarrow 1} \frac{1}{1 - \beta} \frac{(1 - \beta)C_t^{1-\rho} \ln(C_t) + \beta X_t^{1-\rho} \ln(X_t)}{(1 - \beta)C_t^{1-\rho} + \beta X_t^{1-\rho}} \\ &= \ln(C_t) - \beta \theta \ln E_t e^{(1-\gamma) \ln U_{t+1}}\end{aligned}$$

Let $V_t \equiv \ln U_t / (1 - \beta)$, and rewrite utility when $\rho \rightarrow 1$ as

$$\begin{aligned}V_t &= \ln(C_t) - \beta \theta \ln E_t \exp\left\{-\frac{V_{t+1}}{\theta}\right\} \\ &= c_t + \beta E_t V_{t+1} - \frac{1}{2} \frac{\beta}{\theta} \text{var}_t(V_{t+1})\end{aligned}$$

where $\theta = -1/[(1 - \beta)(1 - \gamma)]$, and where the last equality uses the lognormality of the underlying states. These are the preferences in Tallarini (2000), which are isomorphic to the ambiguity averse preferences of the robust control literature (Barillas et al., 2009). When agents are highly risk averse, parameter θ is close to zero.

You can verify how these preferences imply the stochastic discount factor

$$M_{t+1} = \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} = \left(\beta \frac{C_t}{C_{t+1}}\right) \left(\frac{\exp\left\{-\frac{V_{t+1}}{\theta}\right\}}{E_t \exp\left\{-\frac{V_{t+1}}{\theta}\right\}}\right)$$

Next, under the previous state-space system, guess the solution for the value function $V_t =$

$V_0 + V_1\zeta_t + V_2\sigma_t^2 + V_3c_t$, and verify it as

$$V_0 + V_1\zeta_t + V_2\sigma_t^2 + V_3c_t = c_t + \beta V_0 + \beta V_1 A \zeta_t + \beta V_2 (1 - \nu) \sigma^2 + \beta V_2 \nu \sigma_t^2 + \beta V_3 c_t + \beta V_3 \mu + \beta V_3 C \zeta_t - \frac{1}{2} \frac{\beta}{\theta} [\text{var}_t(V_1\zeta_{t+1} + V_2\sigma_{t+1}^2 + V_3c_{t+1})]$$

which implies

$$\begin{aligned} V_0 &= \frac{\beta}{1 - \beta} \left(\frac{1}{1 - \beta} \mu + (1 - \nu) V_2 \sigma^2 - \frac{1}{2\theta} \|V_2 \Sigma\|^2 \right) \\ V_1 &= \frac{\beta}{1 - \beta} C [I - \beta A]^{-1} \\ V_2 &= -\frac{1}{2} \frac{\beta}{\theta (1 - \beta)^2 (1 - \beta \nu)} \|D + \beta C [I - \beta A]^{-1} B\|^2 \\ V_3 &= \frac{1}{1 - \beta} \end{aligned}$$

Therefore, since $-\frac{1}{2} \frac{\beta}{\theta} \text{var}_t(V_{t+1}) = V_t - c_t - \beta E_t V_{t+1}$,

$$\begin{aligned} m_{t+1} &= \ln(\beta) - \Delta c_{t+1} - \frac{1}{\theta} (E_{t+1} - E_t) V_{t+1} - \frac{1}{2\theta^2} \text{var}_t(V_{t+1}) \\ &= \ln(\beta) - \Delta c_{t+1} - \frac{1}{\theta} (E_{t+1} - E_t) V_{t+1} + \frac{1}{\beta\theta} V_t - \frac{1}{\beta\theta} c_t - \frac{1}{\theta} E_t V_{t+1} \\ &= \ln(\beta) - \Delta c_{t+1} - \frac{1}{\beta\theta} (\beta V_{t+1} - V_t + c_t) \\ &= \ln(\beta) - \mu - C \zeta_t - \frac{1}{2} \|(1 - \gamma)D + (1 - \gamma)\beta C [I - \beta A]^{-1} B\|^2 \sigma_t^2 - \frac{1}{2\theta^2} \|V_2 \Sigma\|^2 \\ &\quad - (\gamma D - (1 - \gamma)\beta C [I - \beta A]^{-1} B) \sigma_t \varepsilon_{t+1} - \frac{1}{\theta} V_2 \Sigma \varepsilon_{t+1} \\ &= \ln(\beta) - \mu - \frac{1}{2} (\|D - F\|^2 - \|F\|^2) \sigma_t^2 - C \zeta_t - \frac{1}{2} \|\sigma_t F + J \Sigma\|^2 - (\sigma_t F + J \Sigma) \varepsilon_{t+1} \end{aligned}$$

where $F = \gamma D - (1 - \gamma)\beta C [I - \beta A]^{-1} B$ and $J = -(1 - \beta)(1 - \gamma)V_2$. Therefore, $r^f = -\ln(\beta) + \mu$, $H = C$ and $G = -\frac{1}{2}(\|F\|^2 - \|D - F\|^2)$.

You can verify that these results are equivalent to the coefficient in the approximate $\rho = 1$ case found in the long-run risk case (except for parameter δ).

B.5.3. Exact solution under approximate SDF

Take as generic SDF

$$m_{t+1} = -\rho_t - \sum_{j=0}^{\infty} \sigma_t \delta_j (E_{t+1} - E_t) \Delta c_{t+j+1} - J \Sigma \varepsilon_{t+1}$$

which embeds Bansal and Yaron (2004), by (B.7), (B.8) and (B.9). Under the assumed state-space system that drives the economy, I can reduce it to

$$m_{t+1} = -\rho_t - F\sigma_t\varepsilon_{t+1} - J\Sigma\varepsilon_{t+1}$$

where it must hold that $-\rho_t = -r_t^f - \frac{1}{2}\|\sigma_t F + J\Sigma\|^2$ to guarantee that $r_t^f = -E_t m_{t+1} - \frac{1}{2}\text{var}_t(m_{t+1})$.

The assumed structure is

$$\begin{aligned}\zeta_{t+1} &= A\zeta_t + \sigma_t B\varepsilon_{t+1} \\ \Delta c_{t+1} &= \mu + C\zeta_t + \sigma_t D\varepsilon_{t+1} \\ \Delta d_{t+1} &= \mu + C_d\zeta_t + \sigma_t D_d\varepsilon_{t+1} \\ m_{t+1} &= -r_t^f - \frac{1}{2}\|\sigma_t F + J\Sigma\|^2 - (J\Sigma + \sigma_t F)\varepsilon_{t+1} \\ r_t^f &= r^f + G\sigma_t^2 + H\zeta_t \\ \sigma_{t+1}^2 &= (1-\nu)\sigma^2 + \nu\sigma_t^2 + \Sigma\varepsilon_{t+1} \\ \varepsilon_t &\sim \text{Niid}(0, I)\end{aligned}$$

where σ_t is a scalar random variable and for simplicity I assume $\Sigma\varepsilon_t \perp [B\varepsilon_t; D\varepsilon_t; F\varepsilon_t]$.

Now guess the solution

$$\begin{aligned}F_t^{b,(n)} &= \exp\{A^{b,(n)} + B_\zeta^{b,(n)}\zeta_t + B_\sigma^{b,(n)}\sigma_t^2\} \\ F_t^{c,(n)} &= \exp\{A^{c,(n)} + B_\zeta^{c,(n)}\zeta_t + B_\sigma^{c,(n)}\sigma_t^2\} \\ F_t^{d,(n)} &= \exp\{A^{d,(n)} + B_\zeta^{d,(n)}\zeta_t + B_\sigma^{d,(n)}\sigma_t^2\} \\ F_t^{g,(n)} &= \exp\{A^{g,(n)} + B_\zeta^{g,(n)}\zeta_t + B_\sigma^{g,(n)}\sigma_t^2\}\end{aligned}\tag{B.10}$$

and verify it for the coefficients $\{A^{(n)}, B^{(n)}\}$ with $A^{(0)} = 0$, $B_\zeta^{(0)} = 0$, $B_\sigma^{(0)} = 0$, and

$$\begin{aligned}e^{A^{c,(n)} + B_\zeta^{c,(n)}\zeta_t + B_\sigma^{c,(n)}\sigma_t^2} &= E_t M_{t+1} \frac{C_{t+1}}{C_t} e^{A^{c,(n-1)} + B_\zeta^{c,(n-1)}\zeta_{t+1} + B_\sigma^{c,(n-1)}\sigma_{t+1}^2} \\ &= e^{A^{c,(n-1)} + B_\sigma^{c,(n-1)}(1-\nu)\sigma^2 + B_\zeta^{c,(n-1)}A\zeta_t + B_\sigma^{c,(n-1)}\nu\sigma_t^2 + \mu + C\zeta_t} E_t M_{t+1} e^{[V_{c,\zeta,n-1}\sigma_t + V_{c,\sigma,n-1}]\varepsilon_{t+1}} \\ &= e^{A^{c,(n-1)} + B_\sigma^{c,(n-1)}(1-\nu)\sigma^2 + B_\zeta^{c,(n-1)}A\zeta_t + B_\sigma^{c,(n-1)}\nu\sigma_t^2 + \mu + C\zeta_t} \\ &\quad \times e^{-r^f - G\sigma_t^2 - H\zeta_t + \frac{1}{2}V_{c,\zeta,n-1}V'_{c,\zeta,n-1}\sigma_t^2 + \frac{1}{2}V_{c,\sigma,n-1}V'_{c,\sigma,n-1} - V_{c,\zeta,n-1}F'\sigma_t^2 - V_{c,\sigma,n-1}\Sigma'J'} \\ e^{A^{d,(n)} + B_\zeta^{d,(n)}\zeta_t + B_\sigma^{d,(n)}\sigma_t^2} &= E_t M_{t+1} \frac{D_{t+1}}{D_t} e^{A^{d,(n-1)} + B_\zeta^{d,(n-1)}\zeta_{t+1} + B_\sigma^{d,(n-1)}\sigma_{t+1}^2} \\ &= e^{A^{d,(n-1)} + B_\sigma^{d,(n-1)}(1-\nu)\sigma^2 + B_\zeta^{d,(n-1)}A\zeta_t + B_\sigma^{d,(n-1)}\nu\sigma_t^2 + \mu + C_d\zeta_t} \\ &\quad \times e^{-r^f - G\sigma_t^2 - H\zeta_t + \frac{1}{2}V_{d,\zeta,n-1}V'_{d,\zeta,n-1}\sigma_t^2 + \frac{1}{2}V_{d,\sigma,n-1}V'_{d,\sigma,n-1} - V_{d,\zeta,n-1}F'\sigma_t^2 - V_{d,\sigma,n-1}\Sigma'J'}\end{aligned}$$

$$\begin{aligned}
e^{A^{b,(n)} + B_{\zeta}^{b,(n)} \zeta_t + B_{\sigma}^{b,(n)} \sigma_t^2} &= e^{A^{b,(n-1)} + B_{\sigma}^{b,(n-1)} (1-\nu) \sigma^2 + B_{\zeta}^{b,(n-1)} A \zeta_t + B_{\sigma}^{b,(n-1)} \nu \sigma_t^2} \\
&\quad \times e^{-r^f - G \sigma_t^2 - H \zeta_t + \frac{1}{2} V_{b,\zeta,n-1} V'_{b,\zeta,n-1} \sigma_t^2 + \frac{1}{2} V_{b,\sigma,n-1} V'_{b,\sigma,n-1} - V_{b,\zeta,n-1} F' \sigma_t^2 - V_{b,\sigma,n-1} \Sigma' J'} \\
e^{A^{g,(n)} + B_{\zeta}^{g,(n)} \zeta_t + B_{\sigma}^{g,(n)} \sigma_t^2} &= e^{A^{g,(n-1)} + B_{\sigma}^{g,(n-1)} (1-\nu) \sigma^2 + B_{\zeta}^{g,(n-1)} A \zeta_t + B_{\sigma}^{g,(n-1)} \nu \sigma_t^2 + \mu + C \zeta_t} \\
&\quad \times e^{\frac{1}{2} V_{g,\zeta,n-1} V'_{g,\zeta,n-1} \sigma_t^2 + \frac{1}{2} V_{g,\sigma,n-1} V'_{g,\sigma,n-1}}
\end{aligned}$$

where $V_{c,\zeta,n-1} \equiv B_{\zeta}^{c,(n-1)} B + D$, $V_{c,\sigma,n-1} \equiv B_{\sigma}^{c,(n-1)} \Sigma$, $V_{d,\zeta,n-1} \equiv B_{\zeta}^{d,(n-1)} B + D_d$, $V_{d,\sigma,n-1} \equiv B_{\sigma}^{d,(n-1)} \Sigma$, $V_{b,\zeta,n-1} \equiv B_{\zeta}^{b,(n-1)} B$, $V_{b,\sigma,n-1} \equiv B_{\sigma}^{b,(n-1)} \Sigma$, $V_{g,\zeta,n-1} \equiv B_{\zeta}^{g,(n-1)} B + D$ and $V_{g,\sigma,n-1} \equiv B_{\sigma}^{g,(n-1)} \Sigma$, where the third equality is by the assumed conditional lognormality and where the fourth equality uses the assumed orthogonality of $\Sigma \varepsilon_t$ (Bansal and Yaron, 2004). The solution is correct for

$$\begin{aligned}
A^{c,(n)} &= \mu - r^f + A^{c,(n-1)} + B_{\sigma}^{c,(n-1)} (1-\nu) \sigma^2 + \frac{1}{2} V_{c,\sigma,n-1} V'_{c,\sigma,n-1} - V_{c,\sigma,n-1} \Sigma' J' \\
B_{\zeta}^{c,(n)} &= B_{\zeta}^{c,(n-1)} A + C - H \\
B_{\sigma}^{c,(n)} &= B_{\sigma}^{c,(n-1)} \nu + \frac{1}{2} V_{c,\zeta,n-1} V'_{c,\zeta,n-1} - V_{c,\zeta,n-1} F' - G \\
A^{d,(n)} &= \mu - r^f + A^{d,(n-1)} + B_{\sigma}^{d,(n-1)} (1-\nu) \sigma^2 + \frac{1}{2} V_{d,\sigma,n-1} V'_{d,\sigma,n-1} - V_{d,\sigma,n-1} \Sigma' J' \\
B_{\zeta}^{d,(n)} &= B_{\zeta}^{d,(n-1)} A + C_d - H \\
B_{\sigma}^{d,(n)} &= B_{\sigma}^{d,(n-1)} \nu + \frac{1}{2} V_{d,\zeta,n-1} V'_{d,\zeta,n-1} - V_{d,\zeta,n-1} F' - G \\
A^{b,(n)} &= -r^f + A^{b,(n-1)} + B_{\sigma}^{b,(n-1)} (1-\nu) \sigma^2 + \frac{1}{2} V_{b,\sigma,n-1} V'_{b,\sigma,n-1} - V_{b,\sigma,n-1} \Sigma' J' \\
B_{\zeta}^{b,(n)} &= B_{\zeta}^{b,(n-1)} A - H \\
B_{\sigma}^{b,(n)} &= B_{\sigma}^{b,(n-1)} \nu + \frac{1}{2} V_{b,\zeta,n-1} V'_{b,\zeta,n-1} - V_{b,\zeta,n-1} F' - G \\
A^{g,(n)} &= \mu + A^{g,(n-1)} + B_{\sigma}^{g,(n-1)} (1-\nu) \sigma^2 + \frac{1}{2} V_{g,\sigma,n-1} V'_{g,\sigma,n-1} \\
B_{\zeta}^{g,(n)} &= B_{\zeta}^{g,(n-1)} A + C \\
B_{\sigma}^{g,(n)} &= B_{\sigma}^{g,(n-1)} \nu + \frac{1}{2} V_{g,\zeta,n-1} V'_{g,\zeta,n-1}
\end{aligned}$$

which define a recursion linking the loadings of different dividend strip prices to the states of the economy.

When $\nu = 0$, $\Sigma = 0$ and $B_{\sigma}^{(n)} = 0$ we have the case without stochastic volatility.

Moreover, the risk premium on the n th consumption strip is

$$cov_t(-m_{t+1}, r_{c,t+1}^{(n)}) = (V_{c,\zeta,n-1} \sigma_t + V_{c,\sigma,n-1})(J \Sigma + \sigma_t F)' = V_{c,\sigma,n-1} \Sigma' J' + V_{c,\zeta,n-1} F' \sigma_t^2$$

which you can verify to be equal to $\ln E_t R_{c,t+1}^{e,(n)}$.

The analogous expression for the dividend strip is

$$\text{cov}_t(-m_{t+1}, r_{d,t+1}^{(n)}) = V_{d,\sigma,n-1} \Sigma' J' + V_{d,\zeta,n-1} F' \sigma_t^2$$

B.6. Reduced-form approach

I follow Lettau and Wachter (2007, 2011) in combining lognormal pricing formulas and a loglinear state-space system with a price of risk, x_t , linear in the states of the economy. Their exponential-Gaussian model is a standard and particularly tractable setting to study term structures with closed form equilibrium values. Lettau and Wachter assume the essentially affine stochastic discount factor

$$\begin{aligned} m_{t+1} &= -\rho_t - \gamma_t F \varepsilon_{t+1} \\ &= -r_t^f - \frac{1}{2} x_t^2 F F' - x_t F \varepsilon_{t+1} \end{aligned}$$

where matrix F selects which shocks of the economy are priced.

The assumed structure is

$$\begin{aligned} \zeta_{t+1} &= A \zeta_t + B \varepsilon_{t+1} \\ \Delta c_{t+1} &= \mu + C \zeta_t + D \varepsilon_{t+1} \\ \Delta d_{t+1} &= \mu + C_d \zeta_t + D_d \varepsilon_{t+1} \\ m_{t+1} &= -r_t^f - \frac{1}{2} \gamma_t^2 F F' - \gamma_t F \varepsilon_{t+1} \\ r_t^f &= r^f + H \zeta_t \\ \gamma_{t+1} &= \gamma + K \zeta_{t+1} \\ \varepsilon_t &\sim \text{Niid}(0, I) \end{aligned}$$

where γ_t is a scalar random variable.⁷¹

Now guess the solution

$$\begin{aligned} F_t^{b,(n)} &= e^{A^{b,(n)} + B^{b,(n)} \zeta_t} \\ F_t^{c,(n)} &= e^{A^{c,(n)} + B^{c,(n)} \zeta_t} \\ F_t^{d,(n)} &= e^{A^{d,(n)} + B^{d,(n)} \zeta_t} \\ F_t^{g,(n)} &= e^{A^{g,(n)} + B^{g,(n)} \zeta_t} \end{aligned}$$

⁷¹Lettau and Wachter (2007, 2011) assume $[d_t; c_t] \sim I(1)$ with cointegrating relation $c_t - d_t \sim I(0)$, hence $c_t - d_t = \lambda_0 + \lambda_1 \zeta_t$. Therefore, $\Delta c_{t+1} = \Delta d_{t+1} + \lambda_1 \Delta \zeta_{t+1} = \mu + (C_d - \lambda_1 [I - A]) \zeta_t + (D_d + \lambda_1 B) \varepsilon_{t+1} = \mu + C \zeta_t + D \varepsilon_{t+1}$, which imposes that $\lambda_1 = (C_d - C)[I - A]^{-1}$ and therefore $D = D_d + (C_d - C)[I - A]^{-1} B$.

and verify it for the coefficients $\{A^{(n)}, B^{(n)}\}$ with $A^{(0)} = 0$, $B^{(0)} = 0$, and

$$\begin{aligned}
e^{A^{c,(n)}+B^{c,(n)}\zeta_t} &= E_t M_{t+1} \frac{C_{t+1}}{C_t} e^{A^{c,(n-1)}+B^{c,(n-1)}\zeta_{t+1}} \\
&= e^{\mu+A^{c,(n-1)}+B^{c,(n-1)}A\zeta_t+C\zeta_t} E_t M_{t+1} e^{V_{c,n-1}\varepsilon_{t+1}} \\
&= e^{\mu+A^{c,(n-1)}+B^{c,(n-1)}A\zeta_t+C\zeta_t} e^{-r_t^f + \frac{1}{2}V_{c,n-1}V'_{c,n-1} - V_{c,n-1}F'\gamma_t} \\
&= e^{\mu-r^f-H\zeta_t+A^{c,(n-1)}-V_{c,n-1}F'\gamma+B^{c,(n-1)}A\zeta_t+C\zeta_t-V_{c,n-1}F'K\zeta_t+\frac{1}{2}V_{c,n-1}V'_{c,n-1}} \\
e^{A^{d,(n)}+B^{d,(n)}\zeta_t} &= e^{\mu-r^f-H\zeta_t+A^{d,(n-1)}-V_{d,n-1}F'\gamma+B^{d,(n-1)}A\zeta_t+C_d\zeta_t-V_{d,n-1}F'K\zeta_t+\frac{1}{2}V_{d,n-1}V'_{d,n-1}} \\
e^{A^{b,(n)}+B^{b,(n)}\zeta_t} &= e^{-r^f-H\zeta_t+A^{b,(n-1)}-V_{b,n-1}F'\gamma+B^{b,(n-1)}A\zeta_t-V_{b,n-1}F'K\zeta_t+\frac{1}{2}V_{b,n-1}V'_{b,n-1}} \\
e^{A^{g,(n)}+B^{g,(n)}\zeta_t} &= e^{\mu+C\zeta_t+A^{g,(n-1)}+B^{g,(n-1)}A\zeta_t+\frac{1}{2}V_{g,n-1}V'_{g,n-1}}
\end{aligned}$$

where $V_{c,n-1} \equiv B^{c,(n-1)}B + D$, $V_{d,n-1} \equiv B^{d,(n-1)}B + D_d$, $V_{b,n-1} \equiv B^{b,(n-1)}B$ and $V_{g,n-1} \equiv B^{g,(n-1)}B + D$, and where the third equality is by the assumed conditional lognormality. The solution is correct for

$$\begin{aligned}
A^{c,(n)} &= \mu - r^f + A^{c,(n-1)} + \frac{1}{2}V_{c,n-1}V'_{c,n-1} - V_{c,n-1}F'\gamma \\
B^{c,(n)} &= B^{c,(n-1)}A + C - H - V_{c,n-1}F'K \\
A^{d,(n)} &= \mu - r^f + A^{d,(n-1)} + \frac{1}{2}V_{d,n-1}V'_{d,n-1} - V_{d,n-1}F'\gamma \\
B^{d,(n)} &= B^{d,(n-1)}A + C_d - H - V_{d,n-1}F'K \\
A^{b,(n)} &= -r^f + A^{b,(n-1)} + \frac{1}{2}V_{b,n-1}V'_{b,n-1} - V_{b,n-1}F'\gamma \\
B^{b,(n)} &= B^{b,(n-1)}A - H - V_{b,n-1}F'K \\
A^{g,(n)} &= \mu + A^{g,(n-1)} + \frac{1}{2}V_{g,n-1}V'_{g,n-1} \\
B^{g,(n)} &= B^{g,(n-1)}A + C
\end{aligned}$$

which define a recursion linking the loadings of different dividend strip prices to the state of the economy.

Now,

$$\begin{aligned}
R_{c,t+1}^{(n)} &= \frac{C_{t+1}}{C_t} \frac{F_{t+1}^{(n-1)}}{F_t^{(n)}} \\
&= e^{r_t^f + V_{c,n-1}F'\gamma + V_{c,n-1}F'K\zeta_t + V_{c,n-1}\varepsilon_{t+1} - \frac{1}{2}V_{c,n-1}V'_{c,n-1}}
\end{aligned}$$

and therefore

$$\begin{aligned}
r_{c,t+1}^{(n)} + \frac{1}{2}V_{c,n-1}V'_{c,n-1} &= r_t^f + V_{c,n-1}F'\gamma + V_{c,n-1}F'K\zeta_t + V_{c,n-1}\varepsilon_{t+1} \\
&= \ln E_t R_{c,t+1}^{(n)} + V_{c,n-1}\varepsilon_{t+1}
\end{aligned} \tag{B.11}$$

where the last equality is because $V_{c,n-1}V'_{c,n-1} = \text{var}_t(r_{c,t+1}^{e,(n)})$ and $E_t\varepsilon_{t+1} = 0$. Moreover, the risk premium on the n th dividend strip is

$$\text{cov}_t(-m_{t+1}, r_{c,t+1}^{(n)}) = \gamma_t F V'_{c,n-1} = V_{c,n-1} F' \gamma + V_{c,n-1} F' K \zeta_t$$

which you can verify to be equal to $\ln E_t R_{c,t+1}^{e,(n)}$.

B.7. Exact term structure of welfare costs

The (continuously-compounded) n th component of the welfare term structure is

$$\begin{aligned} \ell_t^{(n)} &= \frac{1}{n} \ln E_t R_{t \rightarrow t+n}^{e,(n)} \\ &= \frac{1}{n} [f_t^{g,(n)} - f_t^{c,(n)} + f_t^{b,(n)}] \end{aligned}$$

Therefore,

$$\begin{aligned} E(\ell_t^{(n)}) &= \frac{1}{n} E(f_t^{g,(n)} - f_t^{c,(n)} + f_t^{b,(n)}) \\ \text{var}(\ell_t^{(n)}) &= \frac{1}{n^2} \text{var}(f_t^{g,(n)} - f_t^{c,(n)} + f_t^{b,(n)}) \end{aligned}$$

and the term structure of welfare costs can then be reconstructed using equation (B.3).

B.7.1. Formulas in Lettau and Wachter (2011)

Let $\Gamma \equiv A - BF'K$, and note the auxiliary result for the Lettau and Wachter (2011) model

$$\begin{aligned} B^{c,(n)} &= (C - H - DF'K) + B^{c,(n-1)}\Gamma \\ &= (C - H - DF'K) \sum_{j=0}^{n-1} \Gamma^j = (C - H - DF'K)[I - \Gamma]^{-1}[I - \Gamma^n] \\ B^{b,(n)} &= -H + B^{c,(n-1)}\Gamma \\ &= -H \sum_{j=0}^{n-1} \Gamma^j = -H[I - \Gamma]^{-1}[I - \Gamma^n] \\ B^{g,(n)} &= C + B^{g,(n-1)}A \\ &= C[I - A]^{-1}[I - A^n] \end{aligned}$$

where the last equality uses the boundary conditions $B^{(0)} = 0$. Recall that $A = \text{diag}(\phi_z, \phi_x, \phi_r)$, $B = [\sigma_z; \sigma_x; \sigma_r]$, $C = [1; 0; 0]'$, $D = \sigma_d$, $F = \sigma_d / \|\sigma_d\|$, $H = [0; 0; 1]'$, $K = [0; 1; 0]'$. Note that

$\sigma_{dx} = 0$ implies $KBF' = 0$. Then,

$$\Gamma^n = A^n + \begin{bmatrix} 0 & (\phi_z + \phi_x)^{n-1} \frac{\sigma_z \sigma'_d}{\|\sigma_d\|} & 0 \\ 0 & 0 & 0 \\ 0 & (\phi_r + \phi_x)^{n-1} \frac{\sigma_r \sigma'_d}{\|\sigma_d\|} & 0 \end{bmatrix}$$

$$\Rightarrow [I - \Gamma]^{-1}[I - \Gamma^n] = \begin{bmatrix} \frac{1-\phi_z^n}{1-\phi_z} & \left(\frac{\phi_x^n - \phi_z^n}{\phi_x - \phi_z} - \frac{1-\phi_x^n}{1-\phi_x} \right) \frac{1}{1-\phi_z} \frac{\sigma_z \sigma'_d}{\|\sigma_d\|} & 0 \\ 0 & \frac{1-\phi_x^n}{1-\phi_x} & 0 \\ 0 & \left(\frac{\phi_x^n - \phi_r^n}{\phi_x - \phi_r} - \frac{1-\phi_x^n}{1-\phi_x} \right) \frac{1}{1-\phi_r} \frac{\sigma_r \sigma'_d}{\|\sigma_d\|} & \frac{1-\phi_r^n}{1-\phi_r} \end{bmatrix}$$

Therefore,

$$\begin{aligned} \ell_t - \ell &= \{B^{g,(n)} - B^{c,(n)} + B^{b,(n)}\} \zeta_t \\ &= \{C[I - A]^{-1}[I - A^n] - (C - DF'K)[I - \Gamma]^{-1}[I - \Gamma^n]\} \zeta_t \\ &= \frac{1}{n} \left\{ \frac{1 - \phi_x^n}{1 - \phi_x} \sigma_d + \left(\frac{1 - \phi_x^n}{1 - \phi_x} - \frac{\phi_x^n - \phi_z^n}{\phi_x - \phi_z} \right) \frac{\sigma_z}{1 - \phi_z} \right\} \frac{\sigma'_d}{\|\sigma_d\|} \gamma_t \end{aligned}$$

B.8. Gabaix (2012) variable rare disaster mode

Consider a consumer with objective $E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$ with

$$\frac{C_{t+1}}{C_t} = \begin{cases} e^\mu & \text{with probability } 1 - p_t \\ e^\mu B_{t+1} & \text{with probability } p_t \end{cases}$$

where p_t is the probability of a disaster event at time $t + 1$, which would depress consumption growth by an amount B_{t+1} . Denote by $E_t^D[\cdot]$ an expectation conditional on disaster and $E_t^{ND}[\cdot]$ an expectation conditional on no disaster. The stochastic discount factor is

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} = \begin{cases} \beta e^{-\gamma\mu} & \text{with probability } 1 - p_t \\ \beta e^{-\gamma\mu} B_{t+1}^{-\gamma} & \text{with probability } p_t \end{cases}$$

Suppose that in the event of a disaster an asset pays off only a fraction $X_{t+1} > 0$ of the promised payoff. Namely, Gabaix (2012) assumes

$$\frac{D_{t+1}}{D_t} = \begin{cases} e^\mu (1 + \varepsilon_{t+1}^d) & \text{with probability } 1 - p_t \\ e^\mu (1 + \varepsilon_{t+1}^d) X_{t+1} & \text{with probability } p_t \end{cases}$$

where $\varepsilon_t^d > -1$ is a mean-zero shock that is independent of the disaster event. In the case of

consumption claims, $\varepsilon_t^d = 0$ and $X_{t+1} = B_{t+1}$; in the case of riskless real bonds, $D_t = X_t = 1$. Then,

$$\begin{aligned}
E_t M_{t+1} \frac{C_{t+1}}{C_t} &= \beta e^{(1-\gamma)\mu} \left\{ (1-p_t) + p_t E_t^D B_{t+1}^{1-\gamma} \right\} \\
&\equiv \beta e^{(1-\gamma)\mu} \{1 + H_{c,t}\} \\
E_t M_{t+1} \frac{D_{t+1}}{D_t} &= \beta e^{(1-\gamma)\mu} \left\{ (1-p_t) + p_t E_t^D B_{t+1}^{-\gamma} X_{t+1} \right\} \\
&\equiv \beta e^{(1-\gamma)\mu} \{1 + H_{d,t}\} \\
E_t M_{t+1} &= \beta e^{-\gamma\mu} \left\{ (1-p_t) + p_t E_t^D B_{t+1}^{-\gamma} \right\} \\
&\equiv \beta e^{-\gamma\mu} \{1 + H_{b,t}\} \\
E_t \frac{C_{t+1}}{C_t} &= (1-p_t)e^\mu + p_t e^\mu E_t^D B_{t+1} \\
&\equiv e^\mu \{1 + H_{g,t}\}
\end{aligned} \tag{B.12}$$

Next, let $\widehat{H}_{i,t} \equiv H_{i,t} - H_{i,*}$, where $H_{i,*} \equiv e^{h_{i,*}} - 1$ is a term independent of time. The state $\widehat{H}_{i,t}$ fully describes the price of the i th asset; Gabaix (2012) interprets it the resilience of the i th asset to disasters. Next, assume the exogenous near-AR(1) process to drive the state $\widehat{H}_{i,t}$ as

$$\begin{aligned}
\widehat{H}_{i,t+1} &= \frac{1 + H_{i,*}}{1 + H_{i,t}} e^{-\phi_i} \widehat{H}_{i,t} + \varepsilon_{i,t+1} \\
&= \frac{e^{h_{i,*} - \phi_i}}{1 + H_{i,t}} \widehat{H}_{i,t} + \varepsilon_{i,t+1}
\end{aligned}$$

where $\varepsilon_{i,t}$ is uncorrelated with either the disaster event or $\varepsilon_{d,t+1}$ and is s.th. $E_t \varepsilon_{i,t+1} = 0$ and $\widehat{H}_{i,t} \geq \max\{(1 + H_{i,*})(e^{-\phi_i} - 1), -p_t - H_{i,*}\}$. Therefore,

$$\begin{aligned}
E_t \left[M_{t+1} \frac{C_{t+1}}{C_t} \widehat{H}_{c,t+1} \right] &= E_t \left[M_{t+1} \frac{C_{t+1}}{C_t} \right] E_t \widehat{H}_{c,t+1} \\
&= \beta e^{(1-\gamma)\mu} (1 + H_{c,t}) \frac{e^{h_{c,*} - \phi_c}}{1 + H_{c,t}} \widehat{H}_{c,t} \\
&= e^{\ln(\beta) + (1-\gamma)\mu + h_{c,*} - \phi_c} \widehat{H}_{c,t} \\
E_t \left[M_{t+1} \frac{D_{t+1}}{D_t} \widehat{H}_{d,t+1} \right] &= e^{\ln(\beta) + (1-\gamma)\mu + h_{d,*} - \phi_d} \widehat{H}_{d,t} \\
E_t \left[M_{t+1} \widehat{H}_{b,t+1} \right] &= e^{\ln(\beta) - \gamma\mu + h_{b,*} - \phi_b} \widehat{H}_{b,t} \\
E_t \left[\frac{C_{t+1}}{C_t} \widehat{H}_{g,t+1} \right] &= e^{\mu + h_{g,*} - \phi_g} \widehat{H}_{g,t}
\end{aligned} \tag{B.13}$$

Next, guess the solution $F_{i,t}^{(n)} = a_i^{(n)} + b_i^{(n)} \widehat{H}_{i,t}$ and use equations (B.12) and (B.13) to solve for the unknown coefficients using the no-arbitrage properties of the yields of the term structure

components:

$$\begin{aligned}
a_c^{(n)} + b_c^{(n)} \widehat{H}_{c,t} &= E_t M_{t+1} \frac{C_{t+1}}{C_t} (a_c^{(n-1)} + b_c^{(n-1)} \widehat{H}_{c,t+1}) \\
&= a_c^{(n-1)} e^{\ln(\beta) + (1-\gamma)\mu + h_{c,*}} + (a_c^{(n-1)} + b_c^{(n-1)} e^{h_{c,*} - \phi_c}) e^{\ln(\beta) + (1-\gamma)\mu} \widehat{H}_{c,t} \\
&= e^{-n\delta_c} \left(1 + \frac{1 - e^{-n\phi_c}}{1 - e^{-\phi_c}} e^{-h_{c,*}} \widehat{H}_{c,t} \right) \\
a_d^{(n)} + b_d^{(n)} \widehat{H}_{d,t} &= e^{-n\delta_d} \left(1 + \frac{1 - e^{-n\phi_d}}{1 - e^{-\phi_d}} e^{-h_{d,*}} \widehat{H}_{d,t} \right) \\
a_b^{(n)} + b_b^{(n)} \widehat{H}_{b,t} &= e^{-n\delta_b} \left(1 + \frac{1 - e^{-n\phi_b}}{1 - e^{-\phi_b}} e^{-h_{b,*}} \widehat{H}_{b,t} \right) \\
a_g^{(n)} + b_g^{(n)} \widehat{H}_{g,t} &= e^{-n\delta_g} \left(1 + \frac{1 - e^{-n\phi_g}}{1 - e^{-\phi_g}} e^{-h_{g,*}} \widehat{H}_{g,t} \right)
\end{aligned}$$

where $\delta_c \equiv -\ln(\beta) - (1-\gamma)\mu - h_{c,*}$, $\delta_d \equiv -\ln(\beta) - (1-\gamma)\mu - h_{d,*}$, $\delta_b \equiv -\ln(\beta) + \gamma\mu - h_{b,*}$ and $\delta_g \equiv -\mu - h_{g,*}$.

Therefore,

$$\begin{aligned}
E_t R_{c,t+1}^{(n)} &= E_t \Delta C_{t+1} \frac{e^{-(n-1)\delta_c} \left[1 + \frac{1 - e^{-(n-1)\phi_c}}{1 - e^{-\phi_c}} e^{-h_{c,*}} \widehat{H}_{c,t+1} \right]}{e^{-n\delta_c} \left[1 + \frac{1 - e^{-n\phi_c}}{1 - e^{-\phi_c}} e^{-h_{c,*}} \widehat{H}_{c,t} \right]} \\
&= e^{\delta_c} \frac{\left[1 + \frac{e^{-\phi_c} - e^{-n\phi_c}}{1 - e^{-\phi_c}} \frac{\widehat{H}_{c,t}}{1 + \widehat{H}_{c,t}} \right]}{\left[1 + \frac{1 - e^{-n\phi_c}}{1 - e^{-\phi_c}} e^{-h_{c,*}} \widehat{H}_{c,t} \right]} E_t \Delta C_{t+1} \\
&= \frac{e^{\delta_c}}{1 + e^{-h_{c,*}} \widehat{H}_{c,t}} E_t \Delta C_{t+1} \\
&= e^{h_{b,*}} \frac{1 + H_{g,t}}{1 + H_{c,t}} \equiv e^{h_{b,*} + h_{g,t} - h_{c,t}} \\
E_t R_{b,t+1}^{(n)} &= e^{-\ln(\beta) + \gamma\mu} \frac{1}{1 + H_{b,t}} \equiv e^{-\ln(\beta) + \gamma\mu - h_{b,t}}
\end{aligned}$$

independent of n . The 1-period buy-and-hold expected return is the same across maturities, because strips of all maturities are exposed to the same risk in the event of a disaster. Hold-to-maturity returns load however differently on the respective state; this feature generates a non-trivial term structure of equity yields. In fact,

$$b_d^{(n)} - b_d^{(n-1)} = \frac{e^{-(n-1)\delta_d - h_{d,*}}}{1 - e^{-\phi_d}} [e^{-(n-1)\phi_d} (1 - e^{-(\phi_d + \delta_d)}) - (1 - e^{-\delta_d})]$$

which is negative $\forall n \geq N$ and positive $\forall n < N$, for some N large enough (i.e., long-duration strip yields load less on the priced state, $\widehat{H}_{d,t}$, then short-duration yields).

Therefore,

$$\begin{aligned}
l_t^{(n)} &= \frac{1}{n} \left(\frac{F_{g,t}^{(n)} F_{b,t}^{(n)}}{F_{c,t}^{(n)}} - 1 \right) \\
&= \frac{1}{n} \frac{a_b^{(n)} a_g^{(n)} - a_c^{(n)} + a_b^{(n)} b_g^{(n)} \widehat{H}_{g,t} + a_g^{(n)} b_b^{(n)} \widehat{H}_{b,t} - b_c^{(n)} \widehat{H}_{c,t} + b_b^{(n)} b_g^{(n)} \widehat{H}_{b,t} \widehat{H}_{g,t}}{a_c^{(n)} + b_c^{(n)} \widehat{H}_{c,t}}
\end{aligned}$$

Technical Appendix to Parts 2 and 3

This appendix derives the Real business cycle and the New Keynesian Q models.

I first determine the first-best optimum—a Real business cycle model with Tobin’s Q; then I depart from the assumption of perfect competition in goods and labor markets and assume instead monopolistic competition; then I add nominal rigidities, thus deriving the New Keynesian Q model.

I then derive the New Keynesian Q theory of investment that links stock prices (average Q), investment and inflation, and an equivalent formulation in differences, which links stock returns to investment growth and unexpected inflation.

Finally, I derive the welfare function in the New Keynesian Q model. Welfare losses are functions of the distance of some endogenous variables from their first-best (efficient) levels. I then use the welfare function to compute the second-best (constrained-efficient) optimum, i.e. the optimal monetary policy, and discuss its implementation.

C. Real business cycle Q model

$M_{t,t+h} \equiv \beta^h \frac{\Lambda_{t+h} P_{t+h}}{\Lambda_t P_t}$ is the stochastic discount factor for real payoffs. P_t is the price level at time t .

C.1. Consumer

The consumer solves the program

$$\max_{C_t, N_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad \text{s.t.} \quad P_t C_t + \frac{1}{1+i_t^f} B_t \leq W_t N_t + B_{t-1} + P_t D_t$$

C_t is real consumption and B_t are nominal risk-free bonds with return i_t^f . I assume a twice-differentiable utility function U_t , separable in consumption and labor, such that $U_{1,t} > 0$ and $U_{2,t} < 0$ on $[C_t; N_t] \in \mathbb{R}_+^2$, which ensures that the budget constraint is binding, and such that $U_{11,t} < 0$ and $U_{22,t} < 0$, which ensures a convex program whose solution is described by first-order conditions.

The dual program is

$$\min_{\Lambda_t \geq 0} \max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) + \beta^t \Lambda_t [W_t N_t + B_{t-1} - P_t C_t - \frac{1}{1+i_t^f} B_t + P_t D_t]$$

The first-order conditions (FOCs) for an interior solution are

$$\begin{aligned} U_{1,t} &= \Lambda_t P_t \\ -U_{2,t} &= \Lambda_t W_t \end{aligned} \Rightarrow MRS_t = \frac{W_t}{P_t}$$

$$1 = (1 + i_t^f) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \equiv (1 + i_t^f) E_t M_{t,t+1} \Pi_{t,t+1}^{-1}$$

besides the budget constraint

$$P_t C_t + \frac{1}{1 + i_t^f} B_t = W_t N_t + B_{t-1} + P_t D_t$$

and the terminality condition

$$\lim_{t \rightarrow \infty} E_0 M_{0,t} \frac{B_{t-1}}{P_t} = 0$$

C.2. Firm

The firm solves the program^{72,73}

$$\max_{N_t, I_t, K_{t+1}} E_0 \sum_{t=0}^{\infty} M_{0,t} [Y_t - \frac{W_t}{P_t} N_t - I_t] \quad \text{s.t.} \quad \begin{aligned} Y_t &= A_t K_t^\alpha N_t^{1-\alpha} \\ K_{t+1} &= (1 - \delta) K_t + \Phi(\frac{I_t}{K_t}) K_t \end{aligned}$$

where the constraints are the production function and a dynamic equation for the evolution of the capital stock. Y_t is real production, I_t is real investment, W_t is nominal wage rate per hour, N_t is hours worked, K_t is the stock of capital, and A_t is technology. Technology follows a stable autoregression with autoregressive coefficient ρ_a ,

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$$

with $\varepsilon_{a,t}$ a white noise process.

The adjustment cost function Φ is strictly increasing and concave, and such that, for some Z , $\Phi(Z) = \delta$, $\Phi'(Z) = 1$ and $\eta \equiv -\frac{\Phi''(Z)Z}{\Phi'(Z)}$.⁷⁴ The constant return to scale production function and the shape of the capital adjustment cost function ensure a convex problem whose solution is interior.

⁷²I adopt Neiss and Nelson (2003) definition of efficient capital level, rather than Woodford (2003, 372-8). To my purposes, the definition of efficient capital stock is relevant for determining the gap variables; and the gap variables are relevant because welfare is function of those gap terms. When the capital gap is expressed in terms of Neiss and Nelson (2003) definition of efficient capital stock, the welfare function is function of gap terms only. When the capital gap is expressed in terms of Woodford (2003) definition, the welfare function is function also of other terms.

⁷³The objective of the firm is discounted using the same discount factor as the firm's owner's, i.e., the consumer.

⁷⁴Thus, δ can be interpreted as the steady-state depreciation of capital, $Z = \delta$ as the steady-state investment-capital ratio and η as the inverse of the elasticity of the investment-capital ratio to changes in Q . Note that when $\eta = 0$, there are no capital-adjustment costs, while when $\eta \rightarrow \infty$ a positive investment adds nothing to capital, i.e. there are infinite capital-adjustment costs. $\eta \rightarrow \infty$ leads to the standard New Keynesian model with constant capital.

The dual program is

$$\min_{MC_t \geq 0, Q_t \geq 0} \max E_0 \sum_{t=0}^{\infty} M_{0,t} [Y_t - \frac{W_t}{P_t} N_t - I_t + MC_t [A_t K_t^\alpha N_t^{1-\alpha} - Y_t] + Q_t [(1-\delta)K_t + \Phi(\frac{I_t}{K_t})K_t - K_{t+1}]]$$

where the Lagrange multiplier MC_t is the average real marginal cost of production. The definition of marginal Q_t , is the real marginal value at t of a unit of capital.

FOCs:

$$\begin{aligned} MC_t &= 1 \\ MPN_t &= \frac{W_t}{P_t} \\ Q_t &= \left[\Phi'(\frac{I_t}{K_t}) \right]^{-1} \\ \Lambda_t P_t Q_t &= E_t \beta \Lambda_{t+1} P_{t+1} MC_{t+1} MPK_{t+1} + E_t \beta \Lambda_{t+1} P_{t+1} Q_{t+1} [(1-\delta) + \Phi(\frac{I_{t+1}}{K_{t+1}}) - \Phi'(\frac{I_{t+1}}{K_{t+1}}) \frac{I_{t+1}}{K_{t+1}}] \\ \Rightarrow 1 &= E_t M_{t+1} \left[\frac{MPK_{t+1} + Q_{t+1} [(1-\delta) + \Phi(\frac{I_{t+1}}{K_{t+1}}) - \Phi'(\frac{I_{t+1}}{K_{t+1}}) \frac{I_{t+1}}{K_{t+1}}]}{Q_t} \right] \end{aligned}$$

along with the production function

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \quad \Rightarrow \quad y_t = a_t + \alpha k_t + (1-\alpha)n_t \quad (C.1)$$

and the capital evolution equation

$$(1-\delta)K_t + \Phi(\frac{I_t}{K_t})K_t - K_{t+1} = 0$$

with terminality condition

$$\lim_{t \rightarrow \infty} E_0 M_{0,t} Q_t K_{t+1} = 0$$

The firm then distributes profits to consumers as dividends $P_t D_t = P_t Y_t - W_t N_t - P_t I_t = P_t C_t - W_t N_t$.

C.3. Market clearing

The market for goods clears

$$Y_t = C_t + I_t$$

C.4. Deterministic efficient steady state

Linearly detrended real variables are stationary in the deterministic steady state, i.e. $\frac{\Lambda_{t+1} P_{t+1}}{\Lambda_t P_t} = 1$, $Y_t = Y$, $K_t = K$, $C_t = C$, $I_t = I$. Thus, $I = \delta K$ and $Q = 1$, both by the normalization imposed on Φ ,

and $\Pi = \beta(1 + i^f)$. Thus, in logs,

$$\begin{aligned} q^e &= 0 & \pi^e &= -\rho + i^f \\ \omega^e &= \ln(1 - \alpha) + \frac{\alpha}{1 - \alpha} \ln\left(\frac{\alpha\beta}{1 - \beta(1 - \delta)}\right) & n^e &= y^e - \frac{\alpha}{1 - \alpha} \ln\left(\frac{\alpha\beta}{1 - \beta(1 - \delta)}\right) \\ k^e &= y^e + \ln\left(\frac{\alpha\beta}{1 - \beta(1 - \delta)}\right) & i^e &= y^e + \ln\left(\frac{\alpha\beta\delta}{1 - \beta(1 - \delta)}\right) \\ c^e &= y^e + \ln\left(\frac{1 - \beta(1 - \delta) - \alpha\beta\delta}{1 - \beta(1 - \delta)}\right) & y^e &= \frac{1}{\sigma + \varphi} \omega^e - \frac{\sigma}{\sigma + \varphi} (c^e - y^e) - \frac{\varphi}{\sigma + \varphi} (n^e - y^e) \end{aligned}$$

where $\omega_t \equiv w_t - p_t$ denotes the real wage rate.

Proof. $A = 1$ in the deterministic steady state. Consider the FOC for labor $(1 - \alpha)\frac{Y}{N} = \frac{W}{P}$, the production function $Y = K^\alpha N^{1-\alpha}$ and the FOC for capital accumulation $\frac{Y}{K} = \frac{1 - \beta(1 - \delta)}{\alpha\beta}$. Then, $\frac{1 - \beta(1 - \delta)}{\beta} = \alpha\frac{Y}{K} = \alpha\left(\frac{N}{Y}\frac{Y}{K}\right)^{1-\alpha} = \alpha\left(\frac{P}{(1 - \alpha)W} \frac{1 - \beta(1 - \delta)}{\alpha\beta}\right)^{1-\alpha}$, hence $\frac{W}{P} = (1 - \alpha)\left(\frac{\alpha\beta}{1 - \beta(1 - \delta)}\right)^{\frac{\alpha}{1-\alpha}}$. Then, $\frac{Y}{N} = \left(\frac{\alpha\beta}{1 - \beta(1 - \delta)}\right)^{\frac{\alpha}{1-\alpha}}$. Next, $\frac{I}{Y} = \frac{K}{Y} = \frac{\alpha\beta\delta}{1 - \beta(1 - \delta)}$. Next, from $Y = C + I$, $\frac{C}{Y} = 1 - \frac{I}{Y} = \frac{1 - \beta(1 - \delta) - \alpha\beta\delta}{1 - \beta(1 - \delta)}$. Finally, to determine all levels, consider $\frac{W}{P} = C^\sigma N^\varphi = \left(\frac{C}{Y}\right)^\sigma \left(\frac{N}{Y}\right)^\varphi Y^{\sigma+\varphi}$, hence $Y = \left(\frac{W}{P}\right)^{\frac{1}{\sigma+\varphi}} \left(\frac{C}{Y}\right)^{-\frac{\sigma}{\sigma+\varphi}} \left(\frac{N}{Y}\right)^{-\frac{\varphi}{\sigma+\varphi}}$. \square

C.5. Loglinearized efficient solution (first best)

The loglinearized dynamic equation for capital accumulation is

$$\begin{aligned} K_{t+1} &= (1 - \delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t \\ \Rightarrow \widehat{k}_{t+1} &= \delta \widehat{i}_t + (1 - \delta)\widehat{k}_t \end{aligned} \tag{C.2}$$

The curvature of utility is characterized by the constant relative risk aversion coefficients $\sigma \equiv -\frac{U_{11,t}C_t}{U_{1,t}}$ and $\varphi \equiv \frac{U_{22,t}N_t}{U_{2,t}}$. More specifically, utility is

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi}$$

whence $M_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$, and

$$MRS_t = C_t^\sigma N_t^\varphi \Rightarrow mrs_t = \sigma c_t + \varphi n_t$$

Then, the loglinearized intertemporal optimality condition is

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t^f - E_t \pi_{t+1} - \rho) \tag{C.3}$$

with

$$\widehat{m}_{t+1} = \sigma \Delta c_{t+1}$$

The loglinearized optimality condition for investment is

$$q_t = \eta(\widehat{i}_t - \widehat{k}_t) \quad (\text{C.4})$$

The loglinearized optimality condition for capital is

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (\vartheta E_t \widehat{mpk}_{t+1} + \beta E_t q_{t+1} - q_t) \quad (\text{C.5})$$

where $\vartheta \equiv 1 - \beta(1 - \delta)$.

The loglinearized goods market clearing condition is

$$\widehat{y}_t = \frac{C}{Y} \widehat{c}_t + \frac{I}{Y} \widehat{i}_t \equiv \gamma_c \widehat{c}_t + \gamma_i \widehat{i}_t \quad (\text{C.6})$$

with $\gamma_c \equiv \frac{1-\beta(1-\delta)-\alpha\beta\delta}{1-\beta(1-\delta)}$ and $\gamma_i \equiv \frac{\alpha\beta\delta}{1-\beta(1-\delta)}$.

The labor market clearing condition is

$$\begin{aligned} mrs_t &= mpn_t \\ \Rightarrow \sigma c_t + (1 + \varphi)n_t &= y_t + \ln(1 - \alpha) \end{aligned} \quad (\text{C.7})$$

Finally, combining (C.3) and (C.5), I get a no-arbitrage condition between the return on the risk-free asset and the return to investment

$$q_t = \beta E_t q_{t+1} + \vartheta E_t \widehat{mpk}_{t+1} - (i_t^f - E_t \pi_{t+1} - \rho)$$

C.6. A useful characterization of the efficient equilibrium

Using all (and only) the intratemporal optimality conditions,

$$\begin{aligned} 0 &\stackrel{(\text{C.7})}{=} \sigma \widehat{c}_t^e + (1 + \varphi) \widehat{n}_t^e - \widehat{y}_t^e \\ &\stackrel{(\text{C.1})}{=} \sigma \widehat{c}_t^e + \frac{\alpha + \varphi}{1 - \alpha} \widehat{y}_t^e - \frac{1 + \varphi}{1 - \alpha} a_t - \frac{\alpha(1 + \varphi)}{1 - \alpha} \widehat{k}_t^e \\ &\stackrel{(\text{C.4}), (\text{C.6})}{=} \left(\frac{\sigma - \gamma_c}{\gamma_c} + \frac{1 + \varphi}{1 - \alpha} \right) \widehat{y}_t^e - \frac{\gamma_i \sigma}{\gamma_c \eta} q_t^e - \frac{1 + \varphi}{1 - \alpha} a_t - \left(\frac{\sigma \gamma_i}{\gamma_c} + \frac{(1 + \varphi)\alpha}{1 - \alpha} \right) \widehat{k}_t^e \\ &\equiv \Xi_y \widehat{y}_t^e - \Xi_{yq} q_t^e - \Xi_{yk} \widehat{k}_t^e - \frac{1 + \varphi}{1 - \alpha} a_t \end{aligned} \quad (\text{C.8})$$

where $\Xi_y \equiv \frac{\sigma}{\gamma_c} + \frac{\alpha + \varphi}{1 - \alpha}$, $\Xi_{yq} \equiv \frac{\sigma \gamma_i}{\gamma_c \eta}$ and $\Xi_{yk} \equiv \frac{\sigma \gamma_i}{\gamma_c} + \frac{(1 + \varphi)\alpha}{1 - \alpha}$.

C.7. Solving the model

Combine the law of motion for a_t , (C.2), (C.5) and (C.8) to form a linear rational expectations model that can be solved with standard algorithms (e.g., Blanchard and Kahn, 1980; Klein, 2000).

Finally, use (C.3) to determine the efficient real interest rate, which will be relevant for determining the interest-rate rule implementing the optimal monetary policy,

$$r_t^{f,e} = \rho + \Psi E_t \Delta \widehat{y}_{t+1}^e - \Xi_{yq} E_t \Delta q_{t+1}^e - \eta \Xi_{yk} \Delta \widehat{k}_{t+1}^e$$

where $\Psi \equiv \sigma/\gamma_c$.

Notice that the real variables in the model are fully determined without having to consider monetary policy. Monetary policy is therefore neutral on the real variables (and on their steady-state values) in the efficient economy.

D. Real business cycle Q model with monopolistic competition

There is a continuum of goods, indexed by $i \in [0, 1]$, so that firm i is a monopolistic competitor offering its particular good.

There is a continuum of consumers indexed by $j \in [0, 1]$, offering different kinds of labor services, so that each j is a monopolistic competitor offering her particular labor.

The government levies lump-sum taxes, T_t , on households and firms to finance distortionary subsidies, τ , in goods and factor markets in order to offset any steady-state distortions due to monopolistic competition that make the competitive equilibrium inefficient.

D.1. Monopolistic competition in the labor market

The aggregate wage and hours worked are the Dixit-Stiglitz constant elasticity of substitution (CES) aggregates $N_t(i) \equiv [\int_0^1 N_t(i, j)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dj]^{\frac{\varepsilon_w}{\varepsilon_w-1}}$, and $W_t \equiv [\int_0^1 W_t(j)^{1-\varepsilon_w} dj]^{\frac{1}{1-\varepsilon_w}}$.

Then, each firm constructs an optimal labor unit, for a given expenditure for acquiring them, by maximizing their Dixit-Stiglitz CES aggregator, i.e. the i th firm solves

$$\max_{[N_t(i, j)]_0^1} N_t(i) \quad \text{s.t.} \quad \int_0^1 W_t(j) N_t(i, j) dj = Z_t(i)$$

FOCs:

$$N_t(i, j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t(i)$$

$$N_t(i) W_t = \int_0^1 W_t(j) N_t(i, j) dj$$

where the left-hand equation is a demand curve.

D.2. j th consumer

The j th consumer solves the program

$$\max_{C_t(j), W_t(j), N_t(j), B_t(j)} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t(j), \int_0^1 N_t(i, j) di)$$

s.t.

$$N_t(i, j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t(i)$$

$$\int_0^1 P_t(i) C_t(i, j) di + \frac{1}{1+i_t^f} B_t(j) \leq (1 - \tau_w) W_t(j) \int_0^1 N_t(i, j) di + B_{t-1}(j) + P_t D_t - P_t T_t^h$$

where $1 - \tau_w = \mathcal{M}_w$ is a labor income tax designed to neutralize the steady-state distortions due to the imperfection in the labor market and T_t^h are lump-sum governmental transfers.

$\int_0^1 P_t(i)C_t(i, j)di = P_t C_t(j)$ holds by construction of the optimal consumption unit. By definition the labor market clearing condition for labor type j , $N_t(j) = \int_0^1 N_t(i, j)di$. Thus, the dual program can be written as

$$\min_{\Lambda_t(j) \geq 0} \max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t(j), \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t) + \\ + \beta^t \Lambda_t(j) [(1 - \tau_w) W_t(j)^{1-\varepsilon_w} W_t^{\varepsilon_w} N_t + B_{t-1}(j) - P_t C_t(j) - \frac{1}{1+i_t^f} B_t(j) + P_t D_t - P_t T_t^h]$$

FOCs:

$$(1 - \tau_w) \Lambda_t(j) \frac{W_t(j)}{W_t} = -\mathcal{M}_w U_{2,t}(j) \\ = \mathcal{M}_w N_t(j)^{\varphi}$$

where $\mathcal{M}_w \equiv \frac{\varepsilon_w - 1}{\varepsilon_w}$.⁷⁵

$$\Lambda_t(j) P_t = U_{1,t}(j) = C_t(j)^{-\sigma} \\ 1 = (1 + i_t^f) E_t \beta \frac{\Lambda_{t+1}(j)}{\Lambda_t(j)}$$

Since each j faces an identical problem, they will all choose the same $W_t(j) = W_t$, $N_t(j) = N_t$, $C_t(j) = C_t$, $\Lambda_t(j) = \Lambda_t$, $B_t(j) = B_t$, and

$$MRS_t = \mathcal{M}_w^{-1} (1 - \tau_w) \frac{W_t}{P_t} \\ \Rightarrow mrs_t = w_t - p_t$$

where \mathcal{M}_w is the wedge driven in the condition for a Pareto optimum, due to imperfect competition in the labor market, and neutralized by the labor income tax $\tau_w = \varepsilon_w^{-1}$.

D.3. Monopolistic competition in the goods market

Consumption by consumer j and investment and capital by firm j are the CES aggregates $C_t(j) \equiv [\int_0^1 C_t(i, j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$, $I_t(j) \equiv [\int_0^1 I_t(i, j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$ and $K_t(j) \equiv \int_0^1 K_t(i, j) di$. The aggregate price level is the CES aggregator $P_t \equiv [\int_0^1 P_t(i)^{1-\varepsilon_p} di]^{\frac{1}{1-\varepsilon_p}}$.

Then, the j th buyer constructs an optimal consumption unit and an optimal investment unit, for

⁷⁵Note that, since $\frac{W_t(j)}{P_t} = \mathcal{M}_w MRS_t(j) \geq MRS_t(j)$, we may interpret \mathcal{M}_w as the markup by which the j th worker increases her wage because of monopolistic competition in the labor market.

a given expenditure for acquiring them, by maximizing their CES aggregator. They solve

$$\begin{aligned} \max_{[C_t(i,j)]_0^1} C_t(j) &= \left[\int_0^1 C_t(i,j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p-1}} \quad \text{s.t.} \quad \int_0^1 P_t(i) C_t(i,j) di = Z_t(j) \\ \max_{[I_t(i,j)]_0^1} I_t(j) &= \left[\int_0^1 I_t(i,j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p-1}} \quad \text{s.t.} \quad \int_0^1 P_t(i) I_t(i,j) di = Z_t(j) \end{aligned}$$

FOCs:

$$C_t(i,j) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} C_t(j) \quad P_t C_t(j) = \int_0^1 P_t(i) C_t(i,j) di \quad (\text{D.1})$$

$$I_t(i,j) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} I_t(j) \quad P_t I_t(j) = \int_0^1 P_t(i) I_t(i,j) di \quad (\text{D.2})$$

where the left-hand equations are demand curves.

The market clearing condition for the i th good, $Y_t(i) = C_t(i) + I_t^d(i) = \int_0^1 C_t(i,j) dj + \int_0^1 I_t(i,j) dj$, then implies that the i th firm faces the demand equation

$$\begin{aligned} Y_t(i) &= \int_0^1 C_t(i,j) dj + \int_0^1 I_t(i,j) dj \\ &= \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \int_0^1 [C_t(j) + I_t(j)] dj \\ &= \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t \end{aligned}$$

D.4. i th firm

The i th firm solves the program

$$\max_{P_t(i), N_t(i), I_t(i), K_{t+1}(i)} E_0 \sum_{t=0}^{\infty} M_{0,t} \left[\frac{P_t(i)}{P_t} Y_t(i) - (1 - \tau_p) \frac{W_t}{P_t} N_t(i) - (1 - \tau_p) \int_0^1 \frac{P_t(j)}{P_t} I_t(j,i) dj - T_t^f(i) \right]$$

s.t.

$$\begin{aligned} Y_t(i) &= \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t \\ P_t I_t(i) &= \int_0^1 P_t(j) I_t(j,i) dj \\ Y_t(i) &= A_t K_t(i)^\alpha N_t(i)^{1-\alpha} \\ K_{t+1}(i) &= (1 - \delta) K_t(i) + \Phi \left(\frac{I_t(i)}{K_t(i)} \right) K_t(i) \end{aligned}$$

where $1 - \tau_p = \mathcal{M}_p^{-1}$ is an employment and investment subsidy designed to neutralize the steady-state distortions due to the imperfection in the goods market and T_t^f are lump-sum governmental

transfers levied on firm's profits and used to finance the subsidy, i.e., $T_t^f(i) = T_t^f = \tau_p \frac{W_t}{P_t} N_t + \tau_p I_t$. Notice that these transfers depends on aggregate employment and investment; since each firm has zero mass they take the level of transfers as given.

The dual program is

$$\min_{MC_t(i), Q_t(i) \geq 0} \max E_0 \sum_{t=0}^{\infty} M_{0,t} \left[\left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon_p} Y_t - (1 - \tau_p) \frac{W_t}{P_t} N_t(i) - (1 - \tau_p) I_t(i) - T_t^f + MC_t(i) [A_t K_t(i)^\alpha N_t(i)^{1-\alpha} - \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t] + (1 - \tau_p) Q_t(i) [(1 - \delta) K_t(i) + \Phi\left(\frac{I_t(i)}{K_t(i)}\right) K_t(i) - K_{t+1}(i)] \right]$$

where I multiply by $1 - \tau_p$ the last Lagrange multiplier to remain with a unitary marginal Q in steady state.

FOCs:

$$\begin{aligned} \mathcal{M}_p^{-1} \frac{P_t(i)}{P_t} &= MC_t(i) \\ Q_t(i) &= \left[\Phi'\left(\frac{I_t(i)}{K_t(i)}\right) \right]^{-1} \\ \mathcal{M}_p^{-1} MPN_t(i) &= (1 - \tau_p) \frac{W_t}{P_t(i)} \\ Q_t(i) &= E_t M_{t+1} \left(\frac{\mathcal{M}_p^{-1}}{1 - \tau_p} MPK_{t+1}(i) + \right. \\ &\quad \left. + Q_{t+1}(i) \left[(1 - \delta) + \Phi\left(\frac{I_{t+1}(i)}{K_{t+1}(i)}\right) - \Phi'\left(\frac{I_{t+1}(i)}{K_{t+1}(i)}\right) \frac{I_{t+1}(i)}{K_{t+1}(i)} \right] \right) \end{aligned}$$

where $\mathcal{M}_p \equiv \frac{\varepsilon_p}{\varepsilon_p - 1}$.⁷⁶

Since each i faces an identical problem, each firm chooses the same quantities, so that $P_t(i) = P_t$, $Y_t(i) = Y_t$, $MC_t(i) = MC_t$, $N_t(i) = N_t$, $Q_t(i) = Q_t$, $I_t(i) = I_t$, $K_{t+1}(i) = K_{t+1}$, and

$$\begin{aligned} (1 - \tau_p) \frac{W_t}{P_t} &= \mathcal{M}_p^{-1} MPN_t \\ \Rightarrow mpn_t &= (w_t - p_t) \end{aligned}$$

where \mathcal{M}_p^{-1} is the wedge driven in the condition for a Pareto optimum, due to imperfect competition in the goods market, and neutralized by the employment subsidy $\tau_p = 1 - \mathcal{M}_p^{-1} = \varepsilon_p^{-1}$.

Firms then distribute to consumers profits as dividends $P_t D_t$.

⁷⁶Note that, since $P_t(i)/P_t = \mathcal{M}_p MC_t(i) \geq MC_t(i)$, \mathcal{M}_p can be interpreted as the markup that firm i charges over marginal costs because of monopolistic competition on the goods market.

D.5. Market clearing

The market for good i clears: $Y_t(i) = C_t(i) + I_t^d(i)$. Then, combine (D.1)-(D.2) and the market clearing condition for good i to find

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t$$

The labor market clears:

$$N_t(j) = \int_0^1 N_t(i, j) di$$

hence, since $N_t(j) = N_t$, $N_t = \int_0^1 N_t(i) di$.

D.6. Deterministic steady state

Since employment subsidies neutralize all steady-state distortions due to monopolistic competition in goods and labor markets (i.e. $1 - \tau_p = \mathcal{M}_p^{-1}$ and $1 - \tau_w = \mathcal{M}_w$), the deterministic steady state is the deterministic efficient steady state.

The general deterministic steady state is, in logs,

$$\begin{aligned} q^n &= 0 \\ \pi^n &= -\rho + i^f \\ \omega^n &= \frac{1}{1-\alpha} \ln \left(\frac{\mathcal{M}_p^{-1}}{1-\tau_p} \right) + \ln(1-\alpha) + \frac{\alpha}{1-\alpha} \ln \left(\frac{\alpha\beta}{1-\beta(1-\delta)} \right) \\ n^n &= y^n - \frac{\alpha}{1-\alpha} \ln \left(\frac{\mathcal{M}_p^{-1}\alpha\beta}{(1-\tau_p)[1-\beta(1-\delta)]} \right) \\ k^n &= y^n + \ln \left(\frac{\mathcal{M}_p^{-1}\alpha\beta}{(1-\tau_p)[1-\beta(1-\delta)]} \right) \\ i^n &= y^n + \ln \left(\frac{\mathcal{M}_p^{-1}\alpha\beta\delta}{(1-\tau_p)[1-\beta(1-\delta)]} \right) \\ c^n &= y^n + \ln \left(1 - \frac{\mathcal{M}_p^{-1}\alpha\beta\delta}{(1-\tau_p)[1-\beta(1-\delta)]} \right) \\ y^n &= \frac{1}{\sigma + \varphi} \ln \left(\frac{\mathcal{M}_w}{1-\tau_w} \right) + \frac{1}{\sigma + \varphi} \omega^n - \frac{\sigma}{\sigma + \varphi} (c^n - y^n) - \frac{\varphi}{\sigma + \varphi} (n^n - y^n) \end{aligned}$$

D.7. Loglinearized solution

With the efficient employment subsidies in place there are no steady-state distortions, and the loglinearized model with monopolistic competition (or natural economy) equals the loglinearized efficient economy, made by equations (C.1)-(C.7). The solution of the loglinearized model is therefore the same of the loglinearized efficient model.

The general rational expectations representation of the natural economy in deviations from the steady state equals the efficient economy. $\frac{\mathcal{M}_p^{-1}}{1-\tau_p}$ and $\frac{\mathcal{M}_w}{1-\tau_w}$ affect only the level of the steady state. Therefore, the natural steady-state levels are their efficient steady-state counterparts plus a constant.

E. New Keynesian Q model

E.1. Price and wage rigidities

The i th firm can readjust its price at each period t with probability $(1 - \theta_p)$. The j th consumer can readjust her wage at each period t with probability $(1 - \theta_w)$. Firms (consumers) do not all face an identical problem anymore, since some can adjust, some cannot. Price (wage) dispersion becomes relevant. Besides, an aggregate production function is no longer trivial to define.

I use notation $\bar{X}(i) \equiv X_t(i)/X_t$ to denote the ratio between an individual and the corresponding aggregate variable.

I use the subscript $\cdot_{t+h|t}$ to denote the variable at $t + h$ chosen by a firm (consumer) that last had the chance of resetting price (wage) at t . I then denote by \cdot_{t+h} the average value of the variable at $t + h$.

I assume a complete financial market and preferences separable in consumption and hours, so that consumers can always achieve the optimal level of consumption by substituting intertemporally; the distortions due to wage rigidities only affect their labor choice. Therefore, I still have that all agents choose the same consumption, i.e. $C_{t+h|t} = C_{t+h}$, hence $\Lambda_{t+h|t} = \Lambda_{t+h}$.

Lemma 4 (Price dispersion and price dynamics). *A firm can reset price at t with probability $1 - \theta_p$. I assume that the event ‘firm i resets at t ’, $s_{it} \sim B((1 - \theta_p))$, is independent across t and i . Let P_t^* the newly set price at t (if the firm gets the chance to reset). Then, the equation describing aggregate price dynamics is*

$$p_t = \theta_p p_{t-1} + (1 - \theta_p) p_t^* \quad (\text{E.1})$$

up to a first-order approximation around the zero-inflation steady state.

Proof. There are infinite firms, so that by the law of large numbers $1 - \theta_p$ is the fraction of firms that resets prices at t . Then,

$$\begin{aligned} \int e^{\bar{p}_t(i)} di &= \theta_p \int e^{\bar{p}_{t-1}(i) - \pi_t} di + (1 - \theta_p) \int e^{\bar{p}_t^*(i) - (p_t^* - p_t)} di \\ E_i \bar{p}_t(i) &= \theta_p \{E_i \bar{p}_{t-1}(i) - \pi_t\} + (1 - \theta_p) \{E_i \bar{p}_t^*(i) + (p_t^* - p_t)\} + O(\|\widehat{p}\|^2) \\ &\Rightarrow \theta_p \pi_t = (1 - \theta_p)(p_t^* - p_t) + O(\|\widehat{p}\|^2) \end{aligned}$$

which you can rearrange to find the result. □

Lemma 5 (Wage dispersion and wage dynamics). *A consumer can reset wage at t with probability $1 - \theta_w$. I assume that the event ‘worker j resets at t ’, $s_{jt} \sim B((1 - \theta_w))$, is independent across t and j . Let W_t^* the newly set wage at t (if the worker gets the chance to reset). Then, the equation describing aggregate wage dynamics is*

$$w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^*$$

up to a first-order Taylor approximation around zero-wage-inflation steady state.

Proof. Analogous to lemma 4. □

Lemma 6 (Aggregate production function). *Up to a first-order approximation,*

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}$$

is the aggregate production function.

Proof. Expanding the definition of output aggregator

$$\begin{aligned} 1 &= \int_0^1 e^{\frac{\varepsilon-1}{\varepsilon} \bar{y}_t(i)} di \\ 0 &= E_i \bar{y}_t(i) + O(\|\widehat{y}\|^2) \end{aligned}$$

and therefore $E_i \bar{y}_t(i) = O(\|\widehat{y}\|^2)$.

Analogous reasoning for the capital aggregator leads to

$$E_i \bar{k}_t(i) = O(\|\widehat{k}\|^2)$$

Furthermore,

$$\begin{aligned} \int N_t(i) di &= [A_t^{-1} Y_t K_t^{-\alpha}]^{\frac{1}{1-\alpha}} \int e^{\frac{1}{1-\alpha} \bar{y}_t(i) - \frac{\alpha}{1-\alpha} \bar{k}_t(i)} di \\ &= [A_t^{-1} Y_t K_t^{-\alpha}]^{\frac{1}{1-\alpha}} \left\{ 1 + \frac{1}{1-\alpha} E_i \bar{y}_t(i) - \frac{\alpha}{1-\alpha} E_i \bar{k}_t(i) \right\} \\ \Rightarrow (1-\alpha)n_t &= -a_t + y_t - \alpha k_t + O(\|\widehat{y}, \widehat{k}\|^2) \end{aligned}$$

which proves the result □

Consistent with lemma 6, I define the aggregate average marginal cost $MC_t \equiv \frac{W_t}{P_t MPN_t}$.

Also, I define the aggregate $MRS_t \equiv C_t^\sigma N_t^\varphi$.

E.2. Monopolistic competition in goods and labor markets

The demand equations in this economy are, as in the monopolistically competitive economy, $Y_{t+h}(i) = (\frac{P_{t+h}(i)}{P_{t+h}})^{-\varepsilon_p} Y_{t+h}$, and $N_{t+h}(j) = (\frac{W_{t+h}(j)}{W_{t+h}})^{-\varepsilon_p} N_{t+h}$.

E.3. j th consumer

Consumer j solves

$$\max_{C_{t+h}(j), W_{t+h}^*(j), N_{t+h}(j), B_{t+h}(j)} E_t \sum_{h=0}^{\infty} \beta^h U(C_{t+h}(j), N_{t+h}(j))$$

subject to

$$\begin{aligned}
N_{t+h}(j) &= \left(\frac{W_{t+h}(j)}{W_{t+h}} \right)^{-\varepsilon_w} N_{t+h} \\
P_{t+h}C_{t+h}(j) + \frac{1}{1+i_{t+h}^f} B_{t+h}(j) &\leq \left[\begin{array}{l} (1-\tau_w)W_{t+h}(j)N_{t+h}(j) + B_{t+h-1}(j) + \\ + P_{t+h}D_{t+h} - P_{t+h}T_{t+h}^h \end{array} \right] \\
W_{t+h}(j) &= \begin{cases} W_{t+h}^*(j), & \text{with probability } (1-\theta_w) \\ W_{t+h-1}(j), & \text{with probability } \theta_w \end{cases}
\end{aligned}$$

E.4. Resetting consumer

The consumers that gets the chance of resetting her wage at time t solves the program

$$\max_{C_{t+h}, W_t^*, N_{t+h|t}, B_{t+h}} \sum_{t=0}^{\infty} (\theta_w \beta)^h E_t U(C_{t+h}, N_{t+h|t})$$

s.t.

$$\begin{aligned}
N_{t+h|t} &= \left(\frac{W_t^*}{W_{t+h}} \right)^{-\varepsilon_w} N_{t+h} \\
P_{t+h}C_{t+h} + \frac{1}{1+i_{t+h}^f} B_{t+h} &\leq (1-\tau_w)W_t^* N_{t+h|t} + B_{t+h-1} + P_{t+h}D_{t+h} - P_{t+h}T_{t+h}^h
\end{aligned}$$

or the dual program

$$\begin{aligned}
\min_{\Lambda_{t+h} \geq 0} \max \quad & \sum (\theta_w \beta)^h E_t \left\{ U(C_{t+h}, (W_t^*)^{-\varepsilon_w} W_{t+h}^{\varepsilon_w} N_{t+h}) + \right. \\
& + \Lambda_{t+h} [(1-\tau_w)(W_t^*)^{1-\varepsilon_w} W_{t+h}^{\varepsilon_w} N_{t+h} + B_{t+h-1} - P_{t+h}C_{t+h} - \frac{1}{1+i_{t+h}^f} B_{t+h} + \\
& \left. + P_{t+h}D_{t+h} - P_{t+h}T_{t+h}^h] \right\}
\end{aligned}$$

FOCs (only those that differ from the corresponding conditions in section D):

$$\begin{aligned}
& \sum (\theta_w \beta)^h E_t \left[U_{2,t+h|t} \mathcal{M}_w \frac{N_{t+h|t}}{W_t^*} + (1-\tau_w) \Lambda_{t+h} N_{t+h|t} \right] = 0 \\
\Rightarrow & \sum (\theta_w \beta)^h E_t N_{t+h|t} U_{1,t+h} \left[(1-\tau_w) \frac{W_t^*}{P_{t+h}} - \mathcal{M}_w MRS_{t+h|t} \right] = 0 \quad (E.2)
\end{aligned}$$

where I used the other optimality conditions.

Also, the relation between the marginal rate of substitution of the resetting consumer and the aggregate marginal rate of substitution is, since $C_{t+h|t} = C_{t+h}$,

$$\begin{aligned}
mrs_{t+h|t} &= mrs_{t+h} + \varphi(n_{t+h|t} - n_{t+h}) \\
&= mrs_{t+h} - \varepsilon_w \varphi(w_t^* - w_{t+h})
\end{aligned}$$

As shown by Erceg et al. (2000), I can then approximate (E.2) to express the equation as the

New Keynesian wage Phillips curve

$$\pi_{w,t} = \beta E_t \pi_{w,t+1} - \lambda_w \widehat{\mu}_{w,t} \quad (\text{E.3})$$

where $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$.

Proof. A first-order approximation of (E.2) around the zero-inflation steady state yields $\widehat{w}_t^* = (1 - \beta\theta_w) \sum_{h=0}^{\infty} (\beta\theta_w)^h E_t [\widehat{mrs}_{t+h|t} + \widehat{p}_{t+h}] = \frac{1-\beta\theta_w}{1+\varepsilon_w\varphi} \sum_{h=0}^{\infty} (\beta\theta_w)^h E_t [\widehat{mrs}_{t+h} + \varepsilon_w\varphi \widehat{w}_{t+h} + \widehat{p}_{t+h}] = \frac{1-\beta\theta_w}{1+\varepsilon_w\varphi} \sum_{h=0}^{\infty} (\beta\theta_w)^h E_t [(1 + \varepsilon_w\varphi) \widehat{w}_{t+h} - \widehat{\mu}_{w,t+h}] = \beta\theta_w E_t \widehat{w}_{t+1}^* + (1 - \beta\theta_w) [\widehat{w}_t - (1 + \varepsilon_w\varphi)^{-1} \widehat{\mu}_{w,t}]$, by (E.15). Finally, use lemma 5 to find $\pi_{w,t} = \beta E_t \pi_{w,t+1} - \lambda_w \widehat{\mu}_{w,t}$. \square

E.5. *i*th firm

Firm *i* solves

$$\max_{P_{t+h}^*(i), Y_{t+h}(i), N_{t+h}(i), I_{t+h}(i)} E_t \sum_{h=0}^{\infty} M_{t,t+h} \left[\frac{P_{t+h}(i)}{P_{t+h}} Y_{t+h}(i) - (1 - \tau_p) \frac{W_{t+h}}{P_{t+h}} N_{t+h}(i) - (1 - \tau_p) I_{t+h}(i) - T_{t+h}^f \right]$$

subject to

$$\begin{aligned} Y_{t+h}(i) &= \left(\frac{P_{t+h}(i)}{P_{t+h}} \right)^{-\varepsilon_p} Y_{t+h} \\ Y_{t+h}(i) &= A_{t+h} K_{t+h}(i)^\alpha N_{t+h}(i)^{1-\alpha} \\ K_{t+h+1}(i) &= (1 - \delta) K_{t+h}(i) + \Phi \left(\frac{I_{t+h}(i)}{K_{t+h}(i)} \right) K_{t+h}(i) \\ P_{t+h}(i) &= \begin{cases} P_{t+h}^*(i), & \text{with probability } (1 - \theta_p) \\ P_{t+h-1}(i), & \text{with probability } \theta_p \end{cases} \end{aligned}$$

When firms accumulate firm-specific capital, the first-order conditions for an interior optimum are

$$\sum_{h=0}^{\infty} \theta^h E_t M_{t,t+h} P_{t+h}^{\varepsilon-1} Y_{t+h}(i) \left[P_t^*(i) - \mathcal{M}_p MC_{t+h}(i) P_{t+h} \right] = 0 \quad (\text{E.4})$$

$$\begin{aligned} MC_t(i) MPN_t(i) &= (1 - \tau_p) \frac{W_t}{P_t} \\ Q_t(i) &= \left[\Phi' \left(\frac{I_t(i)}{K_t(i)} \right) \right]^{-1} \\ E_t M_{t,t+1} R_{t+1}^I(i) &= 1 \end{aligned} \quad (\text{E.5})$$

with

$$Q_t(i) R_{t+1}^I(i) \equiv \widehat{MC}_{t+1}(i) MPK_{t+1}(i) + \left((1 - \delta) + \Phi \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) - \Phi' \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) \frac{I_{t+1}(i)}{K_{t+1}(i)} \right) Q_{t+1}(i)$$

where $Q_t(i), I_t(i), K_t(i)$ are chosen by the firm and where $\widehat{MC}_t(i) \equiv MC_t(i)/(1 - \tau_p)$.

The following derivation borrows heavily from Woodford (2005). For the generic firm i —independently of when it last had the chance of resetting prices—the optimal choice of labor implies $\overline{mc}_{t+h}(i) = -\overline{mpn}_{t+h}(i) = -\overline{y}_{t+h}(i) + \overline{n}_{t+h}(i) = \frac{\alpha}{1-\alpha}\overline{y}_{t+h}(i) - \frac{\alpha}{1-\alpha}\overline{k}_{t+h}(i) = -\frac{\alpha\varepsilon}{1-\alpha}\overline{p}_{t+h}(i) - \frac{\alpha}{1-\alpha}\overline{k}_{t+h}(i)$, where the second and third equalities used the production function and the fourth equality used the demand function. Note also that $\overline{mpk}_{t+h}(i) = \overline{y}_{t+h}(i) - \overline{k}_{t+h}(i) = \frac{1-\alpha}{\alpha}\overline{mc}_{t+h}(i)$. Summing up,

$$\begin{aligned}\overline{mpk}_t(i) &= -\varepsilon\overline{p}_t(i) - \overline{k}_t(i) \\ \overline{mc}_t(i) &= -\frac{\alpha\varepsilon}{1-\alpha}\overline{p}_t(i) - \frac{\alpha}{1-\alpha}\overline{k}_t(i)\end{aligned}\tag{E.6}$$

Next, note that not all firms that reset at t are equal, since they inherit a firm-specific capital stock, that depends on when they last had the chance of resetting prices. The optimal price chosen by the i th resetting firm (E.4) satisfies $\sum_{h=0}^{\infty}(\beta\theta)^h E_t^i[\overline{p}_{t+h}(i) - \widehat{mc}_{t+h}(i)] = 0$ with $E_t^i[p_{t+h}(i)] = p_t^*(i)$, where E^i denotes an expectation operator conditional on firm i not resetting in the future.⁷⁷ We can rewrite the last equation as $E_t^i\overline{p}_{t+h}(i) = \overline{p}_t^*(i) + (p_t^* - p_t) - E_t^i(p_{t+h} - p_t) = \overline{p}_t^*(i) + (p_t^* - p_t) - \sum_{j=1}^h E_t\pi_{t+j}$. I can then rewrite the optimal price setting equation as $\sum(\beta\theta)^h[\overline{p}_t^*(i) + (p_t^* - p_t) - \sum_{j=1}^h E_t\pi_{t+j} - \Theta\widehat{mc}_{t+h} + \frac{\alpha\Theta}{1-\alpha}E_t^i\overline{k}_{t+h}(i)] = 0$, where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$. Therefore,

$$(\overline{p}_t^*(i) + p_t^* - p_t) = \sum_{h=1}^{\infty}(\beta\theta)^h E_t\pi_{t+h} + (1-\beta\theta)\Theta \sum_{h=0}^{\infty}(\beta\theta)^h E_t\widehat{mc}_{t+h} - \frac{\alpha(1-\beta\theta)\Theta}{1-\alpha} \sum_{h=0}^{\infty}(\beta\theta)^h E_t^i\overline{k}_{t+h}(i) \tag{E.7}$$

where I used the fact that $\sum_{h=0}^{\infty}(\beta\theta)^h \sum_{j=1}^h \pi_{t+j} = \frac{1}{1-\beta\theta} \sum_{h=1}^{\infty}(\beta\theta)^h \pi_{t+h}$.

I then guess that the equilibrium solution for the distance between individual and aggregate capital has the form

$$\overline{k}_{t+1}(i) = \psi_2\overline{k}_t(i) + \psi_3\overline{p}_t(i) \tag{E.8}$$

which must be true because $\overline{p}_t(i)$ and $\overline{k}_t(i)$ are the only states at time t idiosyncratic to firm i .

Equation (E.8) implies $E_t^i\overline{k}_{t+h}(i) = \psi_2 E_t^i\overline{k}_{t+h-1}(i) + \psi_3 E_t^i\overline{p}_{t+h-1}(i) = \psi_2 E_t^i\overline{k}_{t+h-1}(i) + \psi_3[\overline{p}_t^*(i) + p_t^* - p_t - \sum_{j=1}^{h-1} E_t\pi_{t+j}]$ and therefore $(1-\beta\theta\psi_2) \sum_{h=0}^{\infty}(\beta\theta)^h E_t^i\overline{k}_{t+h}(i) = \overline{k}_t(i) + \psi_3 \sum_{h=1}^{\infty}(\beta\theta)^h [\overline{p}_t^*(i) + p_t^* - p_t - \sum_{j=1}^{h-1} E_t\pi_{t+j}] = \overline{k}_t(i) + \frac{\beta\theta\psi_3}{1-\beta\theta}[\overline{p}_t^*(i) + p_t^* - p_t - \sum_{h=1}^{\infty}(\beta\theta)^h E_t\pi_{t+h}]$. Plugging this expression into (E.7) and using the approximate aggregate price dynamics (E.1) we find

$$\kappa(\overline{p}_t^*(i) + \frac{\theta}{1-\theta}\pi_t) = \kappa \sum_{h=1}^{\infty}(\beta\theta)^h E_t\pi_{t+h} + (1-\beta\theta) \sum_{h=0}^{\infty}(\beta\theta)^h E_t\widehat{mc}_{t+h} - \frac{\alpha(1-\beta\theta)}{(1-\alpha)(1-\beta\theta\psi_2)}\overline{k}_t(i) \tag{E.9}$$

where $\kappa \equiv \frac{1}{\Theta} + \frac{\alpha\beta\theta\psi_3}{(1-\alpha)(1-\beta\theta\psi_2)}$.

⁷⁷Note that $E_t^i x_{t+j} = E_t x_{t+j}$ for an aggregate quantity x and $j \geq 0$.

I can then integrate equation (E.9) and solve the resulting difference equation to find⁷⁸

$$\begin{aligned}\frac{\kappa\theta}{1-\theta}\pi_t &= \frac{\kappa\theta^2}{1-\theta}\beta E_t\pi_{t+1} + \kappa\theta\beta E_t\pi_{t+1} + (1-\beta\theta)\widehat{mc}_t \\ &= \frac{\kappa\theta}{1-\theta}\beta E_t\pi_{t+1} + (1-\beta\theta)\widehat{mc}_t\end{aligned}\quad (\text{E.10})$$

which is the New Keynesian Phillips curve under firm-specific capital (Sveen and Weinke, 2005; Woodford, 2005). Moreover, subtracting the New Keynesian Phillips curve (E.10) from (E.9), I find the solution for newly set prices, which obey the linear function of the individual capital shock

$$\bar{p}_t^*(i) = \psi_1 \bar{k}_t(i) \quad (\text{E.11})$$

where $\psi_1 \equiv -\frac{\alpha(1-\beta\theta)}{\kappa(1-\alpha)(1-\beta\theta\psi_2)}$.

Next, the approximate optimality condition for capital for firm i is $-E_t\widehat{m}_{t+1} = \vartheta E_t(mc_{t+1}(i) + mpk_{t+1}(i)) + \frac{\beta\eta}{\delta}E_t\Delta k_{t+2}(i) - \frac{\eta}{\delta}\Delta k_{t+1}(i)$, where I used the first-order approximation of the individual capital accumulation equation $q_{t+h}(i) = \frac{\eta}{\delta}\Delta k_{t+h+1}(i)$. Integrating over i , the same equation holds also for the dynamics of the aggregate marginal Q as

$$-E_t\widehat{m}_{t+1} = \vartheta E_t[\widehat{mc}_{t+1} + \widehat{mpk}_{t+1}] + \beta E_t q_{t+1} - q_t \quad (\text{E.12})$$

Then, subtracting one from the other and using (E.6),

$$\Xi \bar{k}_{t+1}(i) = \beta E_t \bar{k}_{t+2}(i) + \bar{k}_t(i) - \varepsilon[\Xi - 1 - \beta]E_t \bar{p}_{t+1}(i) \quad (\text{E.13})$$

where $\Xi \equiv \frac{(1+\beta)(1-\alpha)\eta+\vartheta\delta}{\eta(1-\alpha)}$.

Note that, up to a first-order approximation, expected prices for the i th firm are the new price weighted by the probability of having the chance of resetting plus the old price weighted by the probability of not having the chance of resetting prices. Formally, $E_t\widehat{p}_{t+1}(i) = \theta\widehat{p}_t(i) + (1-\theta)E_t\widehat{p}_{t+1}^*(i)$, and therefore

$$\begin{aligned}E_t\bar{p}_{t+1}(i) &= \theta(\bar{p}_t(i) - E_t\pi_{t+1}) + (1-\theta)E_t(\bar{p}_{t+1}^*(i) + p_{t+1}^* - p_{t+1}) \\ &= \theta\bar{p}_t(i) + (1-\theta)E_t\bar{p}_{t+1}^*(i)\end{aligned}\quad (\text{E.14})$$

where the last equality uses the approximate aggregate price dynamics (E.1).

Then, I plug (E.8) into (E.13) and use (E.14) and (E.11) to identify ψ_2 and ψ_3 as

$$\begin{aligned}\Xi\psi_2 &= 1 + \beta\psi_2^2 + \{\beta\psi_3 - \varepsilon[\Xi - 1 - \beta]\}(1-\theta)\psi_1\psi_2 \\ \Xi\psi_3 &= \beta\psi_2\psi_3 + \{\beta\psi_3 - \varepsilon[\Xi - 1 - \beta]\}[\theta + (1-\theta)\psi_1\psi_3]\end{aligned}$$

which verifies the guess (E.8).

The problem of firms accumulating firm-specific capital under sticky prices adds therefore two idiosyncratic dynamic equations to the state-space system driving aggregate variables. Namely,

⁷⁸Note that $\int_0^1 \bar{x}_t(i)di = O(\|\bar{x}\|^2)$ independent of the shape of the aggregator for variable X_t .

equations (E.8) and (E.14) form the system

$$\begin{bmatrix} E_t \bar{p}_{t+1}(i) \\ \bar{k}_{t+1}(i) \end{bmatrix} = \begin{bmatrix} \theta + (1-\theta)\psi_1\psi_3 & (1-\theta)\psi_1\psi_2 \\ \psi_3 & \psi_2 \end{bmatrix} \begin{bmatrix} \bar{p}_t(i) \\ \bar{k}_t(i) \end{bmatrix}$$

which, to have a unique bounded solution, must have both eigenvalues within the unit circle.

E.5.1. Rental market

When firms purchase capital services on a competitive rental market, the first-order conditions for an interior optimum are

$$\begin{aligned} E_t M_{t,t+1} R_{t+1}^I &= 1 \\ Q_t &= \left[\Phi' \left(\frac{I_t}{K_t} \right) \right]^{-1} \\ \sum_{h=0}^{\infty} (\beta\theta)^h E_t M_{t,t+h} P_{t+h}^{\varepsilon-1} Y_{t+h}(i) \left[P_t^*(i) - \mathcal{M}_p MC_{t+h}(i) P_{t+h} \right] &= 0 \\ MC_t(i) MPN_t(i) &= (1 - \tau_p) \frac{W_t}{P_t} \\ MC_t(i) MPK_t(i) &= (1 - \tau_p) R_t^k \end{aligned}$$

with

$$Q_t R_{t+1}^I \equiv R_{t+1}^k + \left((1 - \delta) + \Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) - \Phi' \left(\frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right) Q_{t+1}$$

where Q_t , I_t , K_t are chosen by the representative consumer and the rental rate R_t^k is the same for all firms.

Since $MPN_t(i) = (1 - \alpha) A_t^{\frac{1}{1-\alpha}} \left(\frac{Y_t(i)}{K_t(i)} \right)^{-\frac{\alpha}{1-\alpha}}$, then $\overline{mc}_t(i) = 0$ and therefore $\overline{mpn}_t(i) = \overline{mpk}_t(i) = 0$. Therefore, $\bar{y}_t(i) = \bar{k}_t(i) = \bar{n}_t(i) = -\varepsilon \bar{p}_t(i)$; firms with higher demand for their goods purchase a proportionately higher quantity of capital and labor services. Note how each firm repurchases the required capital stock entirely at each period; therefore, all firms that reset at time t face an identical problem as they do not inherit different capital stocks. I may therefore introduce notation $x_{t+h|t}$ to denote the choice of variable x at time $t+h$ made by a firm that last reset prices at time t . Note how without capital adjustment costs I_t/K_t is a constant.

E.6. Market clearing

Combine (C.6) and (C.4):

$$\widehat{y}_t = \gamma_c \widehat{c}_t + \frac{\gamma_i}{\eta} q_t + \gamma_k \widehat{k}_t$$

E.7. Solving the model

There are two wedges responsible for potential departures of the competitive equilibrium from the social planner problem:

$$mc_t = (w_t - p_t) - mpn_t \quad \mu_{w,t} \equiv (w_t - p_t) - mrs_t \quad (\text{E.15})$$

I use the gap notation $\widetilde{\cdot}_t = \cdot_t - \cdot_t^e$. Then, use lemma 6, (C.4), (C.6) and (E.15):

$$\begin{aligned}
\widehat{mc}_t &= \widehat{\omega}_t - \widehat{mpn}_t \\
&= \widetilde{\omega}_t - \widetilde{y}_t + \widetilde{n}_t \\
&= \widehat{\mu}_{w,t} + \sigma \widetilde{c}_t - \widetilde{y}_t + (1 + \varphi) \widetilde{n}_t \\
&= \widehat{\mu}_{w,t} + \sigma \widetilde{c}_t + \frac{\alpha + \varphi}{1 - \alpha} \widetilde{y}_t - \frac{\alpha(1 + \varphi)}{1 - \alpha} \widetilde{k}_t \\
&= \widehat{\mu}_{w,t} + \Xi_y \widetilde{y}_t - \Xi_{yq} \widetilde{q}_t - \Xi_{yk} \widetilde{k}_t
\end{aligned}$$

Next, rewrite (C.3) and (E.12), recalling that $\Delta k_{t+1} = \frac{\delta}{\eta} q_t$, as

$$\begin{aligned}
\widehat{y}_t &= E_t \widehat{y}_{t+1} + \frac{\gamma_i(1 - \delta)}{\eta} q_t - \frac{\gamma_i}{\eta} E_t q_{t+1} - \frac{\gamma_c}{\sigma} (i_t^f - E_t \pi_{p,t+1} - \rho) \\
\left(1 + \frac{\sigma \gamma_i(1 - \delta)}{\gamma_c \eta}\right) q_t &= \left(\beta + \frac{\sigma \gamma_i}{\gamma_c \eta}\right) E_t q_{t+1} + \left(\vartheta - \frac{\sigma}{\gamma_c}\right) E_t \widehat{y}_{t+1} + \frac{\sigma}{\gamma_c} \widehat{y}_t + \vartheta E_t \widehat{mc}_{t+1} - \vartheta E_t \widehat{k}_{t+1}
\end{aligned}$$

Also, recall that $\widehat{y}_t^e = E_t \widehat{y}_{t+1}^e + \frac{\gamma_i(1 - \delta)}{\eta} q_t^e - \frac{\gamma_i}{\eta} E_t q_{t+1}^e - \frac{\gamma_c}{\sigma} (r_t^{f,e} - \rho)$ and $\left(1 + \frac{\sigma \gamma_i(1 - \delta)}{\gamma_c \eta}\right) q_t^e = \left(\beta + \frac{\sigma \gamma_i}{\gamma_c \eta}\right) E_t q_{t+1}^e + \left(\vartheta - \frac{\sigma}{\gamma_c}\right) E_t \widehat{y}_{t+1}^e + \frac{\sigma}{\gamma_c} \widehat{y}_t^e - \vartheta E_t \widehat{k}_{t+1}^e$. Thus, rewrite (C.3), (E.10) and (E.12) as

$$\pi_{p,t} = \beta E_t \pi_{p,t+1} + \lambda_p \Xi_y \widetilde{y}_t - \lambda_p \Xi_{yq} \widetilde{q}_t - \lambda_p \Xi_{yk} \widetilde{k}_t + \lambda_p \widehat{\mu}_{w,t} \quad (\text{E.16})$$

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} + \frac{\gamma_i(1 - \delta)}{\eta} \widetilde{q}_t - \frac{\gamma_i}{\eta} E_t \widetilde{q}_{t+1} - \frac{\gamma_c}{\sigma} (i_t^f - E_t \pi_{p,t+1} - r_t^{f,e}) \quad (\text{E.17})$$

$$\begin{aligned}
\left(1 + \Xi_{yq}(1 - \delta)\right) \widetilde{q}_t - \frac{\sigma}{\gamma_c} \widetilde{y}_t &= \beta \left(1 + \Xi_{yq}(1 - \delta)\right) E_t \widetilde{q}_{t+1} - \left(\frac{\sigma}{\gamma_c} - \vartheta(1 + \Xi_y)\right) E_t \widetilde{y}_{t+1} \\
&\quad - \vartheta(1 + \Xi_{yk}) \widetilde{k}_{t+1} + \vartheta E_t \widehat{\mu}_{w,t+1}
\end{aligned} \quad (\text{E.18})$$

Moreover, by the chosen utility function,

$$\widehat{\mu}_{w,t} = \widetilde{\omega}_t - \left(\frac{\sigma}{\gamma_c} + \frac{\varphi}{1 - \alpha}\right) \widetilde{y}_t + \Xi_{yq} \widetilde{q}_t + \left(\frac{\sigma \gamma_i}{\gamma_c} + \frac{\varphi \alpha}{1 - \alpha}\right) \widetilde{k}_t \quad (\text{E.19})$$

and note the identity

$$\widetilde{\omega}_t = \widetilde{\omega}_{t-1} - \pi_{p,t} + \pi_{w,t} - e_t \quad (\text{E.20})$$

where $e_t \equiv \Delta \widehat{\omega}_t^e$ is a policy-independent function of the states of the economy.

The last structural equation coming from consumers' optimization is the capital accumulation equation

$$\widetilde{k}_{t+1} = \frac{\delta}{\eta} \widetilde{q}_t + \widetilde{k}_t \quad (\text{E.21})$$

To fully determine all variables in the economy, I need only to specify an interest-rate rule, that this time is going to affect the real variables in the system. Then, combine the structural form of the

model solution, the interest-rate rule and the laws of motion of the exogenous processes in a linear rational expectations model of the kind discussed in Blanchard and Kahn (1980) or Klein (2000). I use the QZ algorithm in Klein (2000) to find the reduced form of the model solution.

E.8. The model (structural form)

We can now consider at least three alternative models:

E.8.1. Sticky prices and wages

The sextuplet [(E.3),(E.16),(E.17),(E.18),(E.19),(E.20)] embeds all relevant information (consumers' FOCs, firms' FOCs, market clearing conditions) for the New Keynesian model with endogenous capital accumulation, capital adjustment costs, and nominal rigidities both in prices and in wages.

E.8.2. Sticky prices

The quadruplet [(E.16),(E.17),(E.18),(E.21)] with $\widehat{\mu}_{w,t} = 0$ embeds all relevant information when $\theta_w = 0$, i.e. for the New Keynesian model with endogenous capital accumulation, capital adjustment costs, and nominal rigidities only in prices.

E.8.3. Sticky prices with exogenous variations in wage markups

The quadruplet [(E.16),(E.17),(E.18),(E.21)] embeds all relevant information when the efficiency wedge $\widehat{\mu}_{w,t}$ from wage rigidities is taken to be exogenous, i.e. for the New Keynesian model with endogenous capital accumulation, capital adjustment costs, endogenous nominal rigidities in prices, and exogenous nominal rigidities in wages. This is the simplest way within this framework to make an exogenous disturbance appear in the New Keynesian Phillips curve (Galí (2008)), which is what the literature calls a cost-push disturbance,

$$u_t = \lambda_p \mu_{w,t}$$

This cost-push disturbance allows me to break the divine coincidence that makes monetary policy be trivial, without complicating the analysis of policy because of endogenous wage rigidities affecting the loss function.

When I focus on the case of an exogenous cost-push disturbance, I assume the exogenous stable autoregression

$$u_t = \rho_u u_{t-1} + \varepsilon_{ut} \tag{E.22}$$

with ε_{ut} a white noise process.

F. New Keynesian Q theory of investment

F.1. Marginal and average Q

Along the lines of Hayashi (1982), I first derive the relation between the unobservable marginal Q and the theoretically observable average Q, that I denote by S_t . This result extends that in Hayashi

(1982) to a model with nominal rigidities, besides monopolistic competition in the goods market. I then define average Q as

$$S_t(i) \equiv \frac{E_t \sum_{h=1}^{\infty} M_{t,t+h} \left[\frac{P_{t+h}(i)}{P_{t+h}} Y_{t+h}(i) - (1 - \tau_p) \frac{W_{t+h}}{P_{t+h}} N_{t+h}(i) - I_{t+h}(i) - T_{t+h}^f \right]}{K_{t+1}(i)}$$

$$= \frac{E_t \sum_{h=1}^{\infty} M_{t,t+h} D_{t+h}(i)}{K_{t+1}(i)}$$

the ratio of two different valuations of the capital stock of firm i : the market price (expected discounted dividends) and the book value.

Proposition 5 (Relation between marginal and average Q s). *The relation between marginal Q , Q_t , and average Q , S_t , in a New Keynesian model with nominal rigidities is*

$$\widehat{s}_t = \beta E_t \widehat{s}_{t+1} + \kappa_1 q_t - \beta \kappa_2 E_t q_{t+1} + \kappa_3 E_t m_{t+1} - \kappa_4 E_t \widehat{m} c_{t+1} \quad (F.1)$$

where $\kappa_1 \equiv \left(1 + \frac{1-M^{-1}}{\alpha M^{-1}}\right) / \left(1 + \frac{(1-M^{-1})\vartheta}{\alpha(1-\beta)M^{-1}}\right)$, $\kappa_2 \equiv \left(1 + \frac{1-M^{-1}}{\alpha M^{-1}} \left(1 + \frac{\vartheta\delta}{(1-\beta)\eta}\right)\right) / \left(1 + \frac{(1-M^{-1})\vartheta}{\alpha(1-\beta)M^{-1}}\right)$, $\kappa_3 \equiv \frac{(1-M^{-1})}{\alpha M^{-1}} \left(1 + \frac{\vartheta}{1-\beta}\right) / \left(1 + \frac{(1-M^{-1})\vartheta}{\alpha(1-\beta)M^{-1}}\right)$, and $\kappa_4 \equiv \frac{\vartheta}{\alpha M^{-1}} / \left(1 + \frac{(1-M^{-1})\vartheta}{\alpha(1-\beta)M^{-1}}\right)$. The wedge \mathcal{M} that distorts the steady state is $\mathcal{M} \equiv \mathcal{M}_p(1 - \tau_p)$.

Besides, the zero-inflation steady state for average q , S , is weakly higher than the zero-inflation steady state for marginal q , $Q = 1$, i.e. $S = 1 + \frac{(1-M^{-1})\vartheta}{\alpha(1-\beta)M^{-1}} \geq 1$.

Proof. By the degree-one homogeneity of the production function, the optimality conditions $W_t = \frac{\mathcal{M}_p^{-1}}{1-\tau_p} P_t(i) M P N_t(i)$ and $MC_t(i) = \mathcal{M}_p^{-1} \bar{P}_t(i)$, and because lump-sum transfers finance aggregate employment and investment subsidies,

$$\begin{aligned} \bar{P}_t(i) Y_t(i) - (1 - \tau_p) \frac{W_t}{P_t} N_t(i) - T_t^f &= \bar{P}_t(i) Y_t(i) - \frac{W_t}{P_t} N_t(i) - \tau_p I_t(i) + \bar{T}_t(i) \\ &= \bar{P}_t(i) Y_t(i) - \widehat{MC}_t(i) M P N_t(i) N_t(i) - \tau_p I_t(i) + \bar{T}_t(i) \\ &= \widehat{MC}_t(i) M P K_t(i) K_t(i) + [\bar{P}_t(i) - \widehat{MC}_t(i)] Y_t(i) - \tau_p I_t(i) + \bar{T}_t(i) \end{aligned}$$

where $\bar{T}_t(i) \equiv \tau_p W_t [N_t(i) - N_t] + \tau_p [I_t(i) - I_t]$ and $\widehat{MC}_t(i) \equiv MC_t(i) / (1 - \tau_p)$.

Next, using $M_{t+1} \equiv \Lambda_{t+1} P_{t+1} / \Lambda_t P_t$, with Λ the marginal value of nominal income, rewrite the optimality condition (E.5) as

$$\begin{aligned} \beta E_t \Lambda_{t+1} P_{t+1} \widehat{MC}_{t+1}(i) M P K_{t+1}(i) K_{t+1}(i) &= \\ &= \Lambda_t P_t Q_t(i) K_{t+1}(i) - \beta E_t \Lambda_{t+1} P_{t+1} Q_{t+1}(i) \left[(1 - \delta) + \Phi\left(\frac{I_{t+1}(i)}{K_{t+1}(i)}\right) - \Phi'\left(\frac{I_{t+1}(i)}{K_{t+1}(i)}\right) \frac{I_{t+1}(i)}{K_{t+1}(i)} \right] K_{t+1}(i) \\ &= \Lambda_t P_t Q_t(i) K_{t+1}(i) - \beta E_t \Lambda_{t+1} P_{t+1} Q_{t+1}(i) K_{t+2}(i) + \beta E_t \Lambda_{t+1} P_{t+1} I_{t+1}(i) \end{aligned}$$

which I use to derive,

$$\begin{aligned}
S_t(i) &= \frac{\sum_{h=1}^{\infty} \beta^h E_t \frac{\Lambda_{t+h} P_{t+h}}{\Lambda_t P_t} [\bar{P}_{t+h}(i) Y_{t+h}(i) - (1 - \tau_p) \frac{W_{t+h}}{P_{t+h}} N_{t+h}(i) - (1 - \tau_p) I_{t+h}(i) - T_{t+h}^f]}{K_{t+1}(i)} \\
&= \frac{\sum_{h=1}^{\infty} \beta^h E_t \frac{\Lambda_{t+h} P_{t+h}}{\Lambda_t P_t} [\widehat{MC}_{t+h}(i) MPK_{t+h}(i) K_{t+h}(i) - I_{t+h}(i) + [\bar{P}_t(i) - \widehat{MC}_{t+h}(i)] Y_{t+h}(i) + \bar{T}_{t+h}(i)]}{K_{t+1}(i)} \\
&= \frac{\sum_{h=1}^{\infty} \beta^{h-1} E_t \Lambda_{t+h-1} P_{t+h-1} Q_{t+h-1}(i) K_{t+h}(i) - \beta^h E_t \Lambda_{t+h} P_{t+h} Q_{t+h}(i) K_{t+h+1}(i)}{\Lambda_t P_t K_{t+1}(i)} + \\
&\quad + \frac{\sum_{h=1}^{\infty} \beta^h E_t \Lambda_{t+h} P_{t+h} [\bar{P}_{t+h}(i) - \widehat{MC}_{t+h}(i)] Y_{t+h}(i)}{\Lambda_t P_t K_{t+1}(i)} + \frac{\sum_{h=1}^{\infty} \beta^h E_t \Lambda_{t+h} P_{t+h} \bar{T}_{t+h}(i)}{\Lambda_t P_t K_{t+1}(i)} \\
&= Q_t(i) + \frac{1}{K_{t+1}(i)} \sum_{h=1}^{\infty} E_t M_{t,t+h} [\bar{P}_{t+h}(i) - \widehat{MC}_{t+h}(i)] Y_{t+h}(i) + \frac{1}{K_{t+1}(i)} \sum_{h=1}^{\infty} E_t M_{t,t+h} \bar{T}_{t+h}(i)
\end{aligned}$$

The equation states that the market-book ratio is the marginal value of capital plus the expected discounted profits per unit of capital. The zero-inflation steady-state value is $S = 1 + \frac{\beta(1-\mathcal{M}^{-1})Y}{(1-\beta)K} \geq 1$. Disregarding the transfer terms, $\bar{T}_t(i)$, which aggregate to zero, this dynamic equation we can equivalently write as

$$S_t(i) - Q_t(i) = E_t M_{t+1} [S_{t+1}(i) - Q_{t+1}(i)] \frac{K_{t+2}(i)}{K_{t+1}(i)} + E_t M_{t+1} \frac{[\bar{P}_{t+1}(i) - \widehat{MC}_{t+1}(i)] Y_{t+1}(i)}{K_{t+1}(i)}$$

After a loglinearization around the zero-inflation steady state and aggregating,

$$\begin{aligned}
S \widehat{s}_t - q_t &= \beta E_t (S \widehat{s}_{t+1} - q_{t+1}) + \beta(S - 1) E_t (\widehat{m}_{t+1} + \Delta \widehat{k}_{t+2}) + \frac{\beta(1 - \mathcal{M}^{-1})Y}{K} E_t \widehat{m}_{t+1} + \\
&\quad - \frac{\beta \mathcal{M}^{-1}Y}{K} E_t \widehat{mc}_{t+1} + \frac{\beta(1 - \mathcal{M}^{-1})Y}{K} E_t \widehat{mpk}_{t+1} = \\
&= \beta E_t (S \widehat{s}_{t+1} - q_{t+1}) + \frac{(1 - \mathcal{M}^{-1})\vartheta}{\alpha(1 - \beta)\mathcal{M}^{-1}} E_t \widehat{m}_{t+1} + \frac{\beta(1 - \mathcal{M}^{-1})\vartheta\delta}{\alpha\eta(1 - \beta)\mathcal{M}^{-1}} E_t q_{t+1} \\
&\quad - \frac{\vartheta}{\alpha} E_t \widehat{mc}_{t+1} + \frac{(1 - \mathcal{M}^{-1})\vartheta}{\alpha\mathcal{M}^{-1}} E_t \widehat{mpk}_{t+1}
\end{aligned}$$

where I used $\frac{\beta(1-\mathcal{M}^{-1})Y}{K} = \frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha\mathcal{M}^{-1}}$ and $S = 1 + \frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha(1-\beta)\mathcal{M}^{-1}}$. Using (E.12), rewrite the equation as

$$\begin{aligned}
&\left(1 + \frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha(1-\beta)\mathcal{M}^{-1}}\right) \widehat{s}_t - \left(1 + \frac{1-\mathcal{M}^{-1}}{\alpha\mathcal{M}^{-1}}\right) q_t = \\
&= \beta \left(1 + \frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha(1-\beta)\mathcal{M}^{-1}}\right) E_t \widehat{s}_{t+1} - \beta \left(1 + \frac{1-\mathcal{M}^{-1}}{\alpha\mathcal{M}^{-1}}\right) \left(1 + \frac{\vartheta\delta}{(1-\beta)\eta}\right) E_t q_{t+1} + \\
&\quad + \frac{(1-\mathcal{M}^{-1})}{\alpha\mathcal{M}^{-1}} \left(1 + \frac{\vartheta}{1-\beta}\right) E_t \widehat{m}_{t+1} - \frac{\vartheta}{\alpha\mathcal{M}^{-1}} E_t \widehat{mc}_{t+1}
\end{aligned}$$

which ends the proof. \square

Note that, in the efficient economy, $\mathcal{M} = 1$, so that $s^e = 0$, $\kappa_1 = \kappa_2 = 1$, $\kappa_3 = 0$, $\kappa_4 = \frac{\vartheta}{\alpha}$ and

$\widehat{mc}_t = 0$. These facts imply $s_t^e = q_t^e$, the result in Hayashi (1982) that average and marginal Q are equal when there is no imperfection in goods markets.

F.2. Distorted steady state

In the monopolistically competitive model without employment subsidies, $\mathcal{M}^{-1} < 1$, so that marginal and average natural Qs generally differ, and (F.1) reads

$$\widehat{s}_t^n = \beta E_t \widehat{s}_{t+1}^n + \kappa_1 q_t^n - \beta \kappa_2 E_t q_{t+1}^n + \sigma \kappa_3 E_t \Delta c_{t+1}^n$$

Thus,

$$(s_t - s_t^n) = \beta E_t (s_{t+1} - s_{t+1}^n) + \kappa_1 (q_t - q_t^n) - \beta \kappa_2 E_t (q_{t+1} - q_{t+1}^n) + \sigma \kappa_3 E_t \Delta (c_{t+1} - c_{t+1}^n) - \kappa_4 E_t \widehat{mc}_{t+1}$$

hence, recalling that $\widehat{s}_t^e - \widehat{s}_t^n$ is a constant,

$$\begin{aligned} \widetilde{s}_t &= \beta E_t \widetilde{s}_{t+1} + \left(\kappa_1 + \frac{\sigma \gamma_i \kappa_3 (1-\delta)}{\gamma_c \eta} \right) \widetilde{q}_t + \left(\kappa_4 \Xi_{yq} - \frac{\sigma \gamma_i \kappa_3}{\gamma_c \eta} - \beta \kappa_2 \right) E_t \widetilde{q}_{t+1} + \\ &\quad - \frac{\sigma \kappa_3}{\gamma_c} \widetilde{y}_t + \left(\frac{\sigma \kappa_3}{\gamma_c} - \kappa_4 \Xi_y \right) E_t \widetilde{y}_{t+1} + \kappa_4 \Xi_{yk} \widetilde{k}_{t+1} - \frac{\kappa_4}{\lambda_p} E_t u_{t+1} \end{aligned}$$

where $\widetilde{s}_t \equiv \widehat{s}_t - \widehat{s}_t^e$ is a stock price gap (or average Q gap).

F.3. Undistorted steady state

Proposition 6 (Corollary to proposition 5). *Suppose that an employment subsidy is in place that corrects all steady-state distortions, i.e., $\mathcal{M}^{-1} = 1$. Then,*

$$s_t = q_t - \frac{\vartheta}{\alpha \lambda_p} E_t \pi_{t+1} \quad \Rightarrow \quad \widetilde{s}_t = \widetilde{q}_t - \frac{\vartheta}{\alpha \lambda_p} E_t \pi_{t+1} \quad (\text{F.2})$$

or, equivalently,

$$\pi_t = \frac{\alpha \beta \lambda_p \eta}{\vartheta} (\widehat{i}_t - \widehat{k}_t) - \frac{\alpha \beta \lambda_p}{\vartheta} s_t + \lambda_p \widehat{mc}_t \quad (\text{F.3})$$

Proof. Equation (F.1) with $\mathcal{M}^{-1} = 1$ is $s_t - q_t = \beta E_t (s_{t+1} - q_{t+1}) - \frac{\vartheta}{\alpha} E_t \widehat{mc}_{t+1} = -\frac{\vartheta}{\alpha} \sum_{j=0}^{\infty} \beta^j E_t \widehat{mc}_{t+j+1} = -\frac{\vartheta}{\alpha \lambda_p} E_t \pi_{t+1}$, where the last equality uses the New Keynesian Phillips curve (E.10). I can then rewrite the New Keynesian Phillips curve as $s_t = q_t - \frac{\vartheta}{\alpha \beta \lambda} \pi_t + \frac{\vartheta}{\alpha \beta} \widehat{mc}_t$. Rearrange and get $\pi_t = \frac{\alpha \beta \lambda \eta}{\vartheta} (\widehat{i}_t - \widehat{k}_t) - \frac{\alpha \beta \lambda}{\vartheta} s_t + \lambda \widehat{mc}_t$. \square

F.4. A first-differenced New Keynesian Q model

The gross real return to investment⁷⁹ is R_{t+1}^i , defined as

$$1 = E_t M_{t+1} \frac{\widehat{MC}_{t+1} MPK_{t+1} - \frac{I_{t+1}}{K_{t+1}} + \frac{K_{t+2}}{K_{t+1}} Q_{t+1}}{Q_t} \equiv E_t M_{t+1} R_{t+1}^I \quad (\text{F.4})$$

⁷⁹I disregard here the difference between aggregate and individual quantities, as it is irrelevant in view of the subsequent first-order approximation.

Then, using (E.12) for the aggregate quantities,

$$\widehat{MC}_{t+1} MPK_{t+1} K_{t+1} - I_{t+1} = R_{t+1}^I Q_t K_{t+1} - Q_{t+1} K_{t+2}$$

The gross real return to equity is R_{t+1}^m , defined as $R_{t+1}^m = \frac{E_{t+1} \sum_{h=1}^{\infty} M_{t+1,t+h} D_{t+h}}{E_t \sum_{h=1}^{\infty} M_{t,t+h} D_{t+h}}$, with $D_{t+h} \equiv Y_{t+h} - \frac{W_{t+h}}{P_{t+h}} N_{t+h} - I_{t+h} = \widehat{MC}_{t+h} MPK_{t+h} K_{t+h} + (1 - \widehat{MC}_{t+h}) Y_{t+h} - I_{t+h}$. Thus,

$$D_{t+h} = (1 - \widehat{MC}_{t+h}) Y_{t+h} + R_{t+h}^I Q_{t+h-1} K_{t+h} - Q_{t+h} K_{t+h+1} \quad (\text{F.5})$$

hence

$$\begin{aligned} R_{t+1}^m &= \frac{E_{t+1} \sum_{h=0}^{\infty} M_{t+1,t+h+1} D_{t+h+1}}{E_t \sum_{h=0}^{\infty} M_{t,t+h} D_{t+h} - D_t} \\ &= \frac{\Lambda_t P_t}{\beta \Lambda_{t+1} P_{t+1}} \times \\ &\quad \times \frac{E_{t+1} \sum_{h=1}^{\infty} \beta^h \Lambda_{t+h} P_{t+h} E_{t+h-1} (R_{t+h}^I Q_{t+h-1} K_{t+h} - Q_{t+h} K_{t+h+1} + (1 - \widehat{MC}_{t+h}) Y_{t+h})}{E_t \sum_{h=1}^{\infty} \beta^h \Lambda_{t+h} P_{t+h} E_{t+h-1} (R_{t+h}^I Q_{t+h-1} K_{t+h} - Q_{t+h} K_{t+h+1} + (1 - \widehat{MC}_{t+h}) Y_{t+h})} \end{aligned}$$

where the second equality uses (F.5) and the law of iterated expectations. Next, using (F.4), one has that $E_{t+h-1} \beta \Lambda_{t+h} P_{t+h} R_{t+h}^I - \Lambda_{t+h-1} P_{t+h-1} = 0$, so that the previous equation simplifies to

$$\begin{aligned} R_{t+1}^m &= \frac{\Lambda_t P_t}{\beta \Lambda_{t+1} P_{t+1}} \times \frac{\Lambda_{t+1} P_{t+1} R_{t+1}^I Q_t K_{t+1} + E_{t+1} \sum_{h=1}^{\infty} \beta^h \Lambda_{t+h} P_{t+h} (1 - \widehat{MC}_{t+h}) Y_{t+h}}{E_t \Lambda_{t+1} P_{t+1} R_{t+1}^I Q_t K_{t+1} + E_t \sum_{h=1}^{\infty} \beta^h \Lambda_{t+h} P_{t+h} (1 - \widehat{MC}_{t+h}) Y_{t+h}} \\ &= \frac{Q_t K_{t+1} R_{t+1}^I + E_{t+1} \sum_{h=1}^{\infty} M_{t+1,t+h} (1 - \widehat{MC}_{t+h}) Y_{t+h}}{Q_t K_{t+1} + E_t \sum_{h=1}^{\infty} M_{t,t+h} (1 - \widehat{MC}_{t+h}) Y_{t+h}} \end{aligned}$$

If $\widehat{MC}_{t+h} = 1$, then one has the prediction $R_{t+1}^m = R_{t+1}^I$.

When $\widehat{MC}_{t+h} \neq 1$, which is the case I am interested in, one can take a loglinear approximation around the steady state:

$$\begin{aligned} R^m(1 + \widehat{r}_{t+1}^m) &= R^i + \frac{QKR^i}{QK + \beta Y \frac{1 - M^{-1}}{1 - \beta}} \widehat{r}_{t+1}^I + \\ &\quad + \frac{(1 - M^{-1})Y}{QK + \beta Y \frac{1 - M^{-1}}{1 - \beta}} \left\{ (E_{t+1} - E_t) \sum_{h=1}^{\infty} \beta^{h-1} \left(-\frac{M^{-1}}{1 - M^{-1}} \widehat{mc}_{t+h} + \widehat{y}_{t+h} \right) + \right. \\ &\quad \left. + E_{t+1} \sum_{h=1}^{\infty} \beta^{h-1} \widehat{m}_{t+1,t+h} - E_t \sum_{h=1}^{\infty} \beta^{h-1} \widehat{m}_{t,t+h} \right\} \end{aligned}$$

where I used $1 = \beta R^i$ (which follows from (F.4)). Note that $R^m = R^i$. Next, using $\widehat{m}_{t+1,t+h} = -\gamma(c_{t+h} - c_{t+1})$,

$$\begin{aligned} \widehat{r}_{t+1}^m &= \kappa_1 \widehat{r}_{t+1}^I + \kappa_2 \gamma \Delta c_{t+1} + \kappa_2 (1 - \beta) (E_{t+1} - E_t) \sum_{h=1}^{\infty} \beta^{h-1} (\widehat{y}_{t+h} + \gamma c_{t+h}) + \\ &\quad - \kappa_3 (1 - \beta) (E_{t+1} - E_t) \sum_{h=1}^{\infty} \beta^{h-1} \widehat{mc}_{t+h} \end{aligned}$$

where $\kappa_1 \equiv \frac{QK(1-\beta)}{QK(1-\beta)+\beta(1-\mathcal{M}^{-1})Y}$, $\kappa_2 \equiv \frac{\beta(1-\mathcal{M}^{-1})Y}{QK(1-\beta)+\beta(1-\mathcal{M}^{-1})Y}$, and $\kappa_3 \equiv \frac{\beta\mathcal{M}^{-1}Y}{QK(1-\beta)+\beta(1-\mathcal{M}^{-1})Y}$.

Finally, use $\frac{\beta Y}{K} = \frac{\vartheta}{\alpha\mathcal{M}^{-1}}$ and $Q = 1$, to find $\kappa_1 = \frac{1-\beta}{1-\beta+\frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha\mathcal{M}^{-1}}}$, $\kappa_2 = \frac{\frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha\mathcal{M}^{-1}}}{1-\beta+\frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha\mathcal{M}^{-1}}}$, and $\kappa_3 = \frac{\frac{\vartheta}{\alpha}}{1-\beta+\frac{(1-\mathcal{M}^{-1})\vartheta}{\alpha\mathcal{M}^{-1}}}$.

Next, by the NKPC, $\pi_{t+1} = \beta E_{t+1}\pi_{t+2} + \lambda_p \widehat{mc}_{t+h} = \lambda_p \sum_{h=1}^{\infty} \beta^{h-1} \widehat{mc}_{t+h}$, so that $\kappa_3(1-\beta)(E_{t+1} - E_t) \sum_{h=1}^{\infty} \beta^{h-1} \widehat{mc}_{t+h} = \frac{\kappa_3(1-\beta)}{\lambda_p} (\pi_{t+1} - E_t \pi_{t+1})$. Suppose that some policy is in place that makes $\mathcal{M} = 1$, hence $\kappa_1 = 1$ and $\kappa_2 = 0$. Then,

$$\begin{aligned} \widehat{r}_{t+1}^m &= \widehat{r}_{t+1}^I - \frac{\vartheta}{\alpha\lambda_p} (E_{t+1} - E_t) \pi_{t+1} \\ \Rightarrow (E_{t+1} - E_t) \widehat{r}_{t+1}^m &= (E_{t+1} - E_t) \widehat{r}_{t+1}^I - \frac{\vartheta}{\alpha\lambda_p} (E_{t+1} - E_t) \pi_{t+1} \end{aligned} \quad (\text{F.6})$$

where $\widehat{r}_{t+1}^I = \vartheta(\widehat{mc}_{t+1} + \widehat{mpk}_{t+1}) + \eta\beta(\widehat{i}_{t+1} - \widehat{k}_{t+1}) - \eta(\widehat{i}_t - \widehat{k}_t)$.

Equation (F.6) is another way to state the New Keynesian Q theory of stock prices (F.2)-(F.3).

Also, $\widehat{r}_{t+1}^m = \vartheta(\widehat{mc}_{t+1} + \widehat{mpk}_{t+1}) + \beta\widehat{s}_{t+1} - \widehat{s}_t$.

F.5. Return-forecasting ability of the market-book ratio

You should remember that there is an identity linking the market return and market-book ratios: a version of the Campbell-Shiller (approximate) identity

$$r_{t+1}^m = \beta s_{t+1} - s_t + (1-\beta)dk_{t+1} \quad (\text{F.7})$$

where $dk_{t+1} \equiv d_{t+1} - k_{t+1} + \frac{\beta}{1-\beta}\Delta k_{t+2}$.

To derive this expression first note that the price of the market portfolio is $P_t^m = S_t K_{t+1}$, then consider the definition of market return $R_{t+1}^m \equiv (P_{t+1}^m + D_{t+1})/P_t^m$. Therefore, loglinearizing the last identity, $r_{t+1}^m = \beta p_{t+1}^m - p_t^m + (1-\beta)d_{t+1} = \beta s_{t+1} - s_t + (1-\beta)(d_{t+1} - k_{t+1}) + \beta\Delta k_{t+2} \equiv \beta s_{t+1} - s_t + (1-\beta)dk_{t+1}$.

Cochrane (2008a) provides strong economic and statistical arguments for return-forecasting ability of the price-dividend ratio. Since market-book and price-dividend ratios have nearly a perfect correlation, the same argument goes through here using the Campbell-Shiller identity (F.7) in its form

$$\text{var}(s_t) = (1-\beta)\text{cov}\left(\sum_{j=1}^{\infty} \beta^j dk_{t+j}, s_t\right) - \text{cov}\left(\sum_{j=1}^{\infty} \beta^j r_{t+j}^m, s_t\right) \Leftrightarrow 1 = b_d^{(\infty)} + b_r^{(\infty)}$$

where $b_d^{(\infty)}$ and $b_r^{(\infty)}$ are the OLS coefficients in the respective long-horizon forecasting equations, which says that the market-book ratio must forecast something.

G. Welfare function

I derive here a quadratic approximation to welfare, along the lines of Rotemberg and Woodford (1999), Woodford (2003) and Galí (2008).

Lemma 7 (Aggregate production function). *Up to a second-order approximation,*

$$(1 - \alpha)n_t = y_t - a_t - \alpha k_t + \frac{1}{2} \frac{\varepsilon_p}{\Theta} \text{var}_i p_t(i) + \frac{1}{2} \frac{\alpha}{1 - \alpha} \text{var}_i k_t(i) + \frac{\alpha \varepsilon_p}{1 - \alpha} \text{cov}_i(p_t(i), k_t(i)) + \frac{\varepsilon_w}{2} \text{var}_j w_t(j)$$

where $\Theta \equiv \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon_p}$.

Proof. Expanding the definition of output aggregator

$$\begin{aligned} 1 &= \int_0^1 e^{\frac{\varepsilon_p - 1}{\varepsilon_p} \bar{y}_t(i)} di \\ 0 &= E_i \bar{y}_t(i) + \frac{1}{2} \frac{\varepsilon_p - 1}{\varepsilon_p} E_i \bar{y}_t^2(i) + O(\|\widehat{y}\|^3) \\ &\Rightarrow [E_i \bar{y}_t(i)]^2 = O(\|\widehat{y}\|^4) \\ 0 &= E_i \bar{y}_t(i) + \frac{1}{2} \frac{\varepsilon_p - 1}{\varepsilon_p} \text{var}_i y_t(i) + O(\|\widehat{y}\|^3) \end{aligned}$$

Analogous reasoning for the capital and price aggregators leads to

$$\begin{aligned} 0 &= E_i \bar{k}_t(i) + \frac{1}{2} \text{var}_i k_t(i) + O(\|\widehat{k}\|^3) \\ 0 &= E_i \bar{p}_t(i) + \frac{1}{2} (1 - \varepsilon_p) \text{var}_i p_t(i) + O(\|\widehat{p}\|^3) \end{aligned}$$

Next, $N_t = \int_0^1 \int_0^1 N_t(i, j) dj di = \int_0^1 N_t(i) \int_0^1 \frac{N_t(i, j)}{N_t(i)} dj di = \int_0^1 \left(\frac{w_t(j)}{w_t}\right)^{-\varepsilon_w} dj \int_0^1 N_t(i) di = \int_0^1 N_t(i) di \left[1 + \frac{\varepsilon_w}{2} \text{var}_j w_t(j)\right]$. Furthermore,

$$\begin{aligned} \int N_t(i) di &= [A_t^{-1} Y_t K_t^{-\alpha}]^{\frac{1}{1-\alpha}} \int e^{\frac{1}{1-\alpha} \bar{y}_t(i) - \frac{\alpha}{1-\alpha} \bar{k}_t(i)} di \\ &= [A_t^{-1} Y_t K_t^{-\alpha}]^{\frac{1}{1-\alpha}} \left\{ 1 + \frac{1}{1-\alpha} E_i \bar{y}_t(i) + \frac{1}{2} \frac{1}{(1-\alpha)^2} \text{var}_i y_t(i) - \frac{\alpha}{1-\alpha} E_i \bar{k}_t(i) \right. \\ &\quad \left. + \frac{1}{2} \frac{\alpha^2}{(1-\alpha)^2} \text{var}_i k_t(i) - \frac{\alpha}{(1-\alpha)^2} \text{cov}_i(y_t(i), k_t(i)) + O(\|\widehat{y}, \widehat{k}\|^3) \right\} \end{aligned}$$

and therefore

$$\begin{aligned} (1 - \alpha)n_t &= -a_t + y_t - \alpha k_t + \frac{1}{2} \frac{1 - \alpha + \alpha \varepsilon_p}{\varepsilon_p (1 - \alpha)} \text{var}_i y_t(i) + \frac{1}{2} \frac{\alpha}{1 - \alpha} \text{var}_i k_t(i) \\ &\quad - \frac{\alpha}{1 - \alpha} \text{cov}_i(y_t(i), k_t(i)) + \frac{\varepsilon_w}{2} \text{var}_j w_t(j) + O(\|\widehat{y}, \widehat{k}, \widehat{w}\|^3) \\ &= -a_t + y_t - \alpha k_t + \frac{1}{2} \frac{(1 - \alpha + \alpha \varepsilon_p) \varepsilon_p}{1 - \alpha} \text{var}_i p_t(i) + \frac{1}{2} \frac{\alpha}{1 - \alpha} \text{var}_i k_t(i) \\ &\quad + \frac{\alpha \varepsilon_p}{1 - \alpha} \text{cov}_i(p_t(i), k_t(i)) + \frac{\varepsilon_w}{2} \text{var}_j w_t(j) + O(\|\widehat{k}, \widehat{p}, \widehat{w}\|^3) \end{aligned}$$

so that both price and capital dispersions are relevant to determine aggregate hours worked. \square

Lemma 8. *Up to a second-order approximation,*

$$\int_0^1 \widehat{n}_t(j) dj + \frac{1+\varphi}{2} \int_0^1 \widehat{n}_t(j)^2 dj = \widehat{n}_t + \frac{1+\varphi}{2} \widehat{n}_t^2 + \frac{\varepsilon_w^2 \varphi}{2} \text{var}_{j|w_t}(j)$$

Proof. Approximate the labor aggregator as $\widehat{n}_t + \frac{1+\varphi}{2} \widehat{n}_t^2 = \int_0^1 \widehat{n}_t(j) dj + \frac{1}{2} \int_0^1 \widehat{n}_t(j)^2 dj$. Next, $\int_0^1 \widehat{n}_t(j)^2 dj = \int_0^1 (\widehat{n}_t(j) - \widehat{n}_t)^2 dj + 2 \int_0^1 (\widehat{n}_t(j) - \widehat{n}_t) \widehat{n}_t dj + \widehat{n}_t^2 = \varepsilon_w^2 \int_0^1 \widehat{w}_t(j)^2 dj - 2\varepsilon_w \widehat{n}_t \int_0^1 \widehat{w}_t(j) dj + \widehat{n}_t^2 = \widehat{n}_t(j)^2 - w \widehat{n}_t \varepsilon_w (\varepsilon_w - 1) \text{var}_{j|w_t}(j) + \varepsilon_w^2 \text{var}_{j|w_t}(j) = \widehat{n}_t(j)^2 + \varepsilon_w^2 \text{var}_{j|w_t}(j)$, using the labor demand equation $\widehat{n}_t(j) - \widehat{n}_t = -\varepsilon_w \widehat{w}_t(j)$ and the second-order approximation to the wage aggregator derived in lemma 7, and disregarding third-order terms. \square

Proposition 7 (Quadratic approximation to utility function). *A quadratic approximation to the consumer's objective, integrated across j , around the deterministic zero-inflation steady-state value⁸⁰ is directly proportional to*

$$\begin{aligned} \mathbb{W}_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t & \left[\Phi \widetilde{y}_t - \frac{1}{2} \Xi_y \widetilde{y}_t^2 - \frac{1}{2} \Xi_q \widetilde{q}_t^2 + \Xi_{yq} \widetilde{y}_t \widetilde{q}_t - \frac{1}{2} \Xi_p \pi_{p,t}^2 - \frac{1}{2} \Xi_w \pi_{w,t}^2 \right. \\ & \left. - \alpha \Phi \widetilde{k}_t - \frac{1}{2} \Xi_k \widetilde{k}_t^2 + \Xi_{yk} \widetilde{y}_t \widetilde{k}_t - \Xi_{qk} \widetilde{q}_t \widetilde{k}_t \right] + t.i.p. \end{aligned} \quad (\text{G.1})$$

where $\Xi_y \equiv \frac{\sigma}{\gamma_c} + \frac{\alpha+\varphi}{1-\alpha}$, $\Xi_q \equiv \frac{\gamma_c}{\sigma} \Xi_{yq} (1 + \Xi_{yq})$, $\Xi_{yq} \equiv \frac{\sigma \gamma_i}{\gamma_c \eta}$, $\Xi_w \equiv \frac{\varepsilon_w \theta_w (1-\alpha)(1+\varepsilon_w \varphi)}{(1-\beta \theta_w)(1-\theta_w)}$, $\Xi_k \equiv \gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c} + \frac{(1+\varphi)\alpha^2}{1-\alpha}$, $\Xi_{yk} \equiv \frac{\sigma \gamma_i}{\gamma_c} + \frac{(1+\varphi)\alpha}{1-\alpha}$, $\Xi_{qk} \equiv \gamma_i \Xi_{yq}$. Lemma 9 describes coefficient Ξ_p .

As a corollary, the approximate average welfare loss per period is directly proportional to

$$\begin{aligned} \mathbb{L} \equiv & -\Phi E[\widetilde{y}_t] + \frac{1}{2} \Xi_y E[\widetilde{y}_t^2] + \frac{1}{2} \Xi_q E[\widetilde{q}_t^2] - \Xi_{yq} E[\widetilde{q}_t \widetilde{y}_t] + \frac{1}{2} \Xi_p E[\pi_{p,t}^2] + \frac{1}{2} \Xi_w E[\pi_{w,t}^2] \\ & + \alpha \Phi E[\widetilde{k}_t] + \frac{1}{2} \Xi_k E[\widetilde{k}_t^2] - \Xi_{yk} E[\widetilde{y}_t \widetilde{k}_t] + \Xi_{qk} E[\widetilde{q}_t \widetilde{k}_t] + t.i.p. \end{aligned} \quad (\text{G.2})$$

Proof. Approximate the integrated utility around the zero-inflation steady state (recall that $C_t(j) = C_t, \forall j$, and that a separable utility function implies $U_{12} = 0$), using the market clearing condition $C_t = Y_t - I_t$, and the optimality condition (C.4):

$$\begin{aligned} & \int_0^1 U(Y e^{\widehat{y}_t} - I e^{\frac{1}{\eta} q_t + \widehat{k}_t}, N e^{\widehat{n}_t(j)}) dj = \\ & = U + U_1 Y (\widehat{y}_t + \frac{1}{2} \widehat{y}_t^2) + \frac{1}{2} U_{11} Y^2 \widehat{y}_t^2 - U_1 \frac{I}{\eta} (q_t + \frac{1}{2} q_t^2) \\ & \quad - U_1 I (\widehat{k}_t + \frac{1}{2} \widehat{k}_t^2) + \frac{1}{2} U_{11} I^2 (\frac{1}{\eta} q_t + \widehat{k}_t)^2 - U_{11} Y I \widehat{y}_t (\frac{1}{\eta} q_t + \widehat{k}_t) \\ & \quad + U_2 N \int_0^1 (\widehat{n}_t(j) + \frac{1}{2} \widehat{n}_t(j)^2) dj + \frac{1}{2} U_{22} N^2 \int_0^1 \widehat{n}_t(j)^2 dj \end{aligned}$$

⁸⁰Woodford (2003, 383-92) explains the reasons for choosing this particular point of expansion. Note that this steady state is independent of policy.

Now, noting that $\frac{U_{11}}{U_1}C = -\sigma$, $\frac{U_{22}}{U_2}N = \varphi$, $Y = C\frac{1}{\gamma_c}$, $I = C\frac{\gamma_i}{\gamma_c}$, and using lemma 8,

$$\begin{aligned} \int_0^1 (U_t(j) - U) dj &= U_1 C \frac{1}{\gamma_c} \left[\widehat{y}_t + \frac{1}{2} \left(1 - \frac{\sigma}{\gamma_c} \right) \widehat{y}_t^2 \right] - U_1 C \frac{\gamma_i}{\gamma_c \eta} \left[q_t + \frac{1}{2} \left(1 + \frac{\sigma \gamma_i}{\gamma_c \eta} \right) q_t^2 \right] \\ &\quad - U_1 C \frac{\gamma_i}{\gamma_c} \left[\widehat{k}_t + \frac{1}{2} \left(1 + \frac{\sigma \gamma_i}{\gamma_c} \right) \widehat{k}_t^2 \right] + U_1 C \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t q_t + U_1 C \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t - U_1 C \frac{\sigma \gamma_i^2}{\gamma_c \eta} q_t \widehat{k}_t \\ &\quad + U_2 N \left[\widehat{n}_t + \frac{1+\varphi}{2} \widehat{n}_t^2 + \frac{\varepsilon_w^2 \varphi}{2} \text{var}_j w_t(j) \right] \end{aligned}$$

Recall that $-\frac{U_2}{U_1} = MRS = (1 - \Phi)MPN = (1 - \Phi)(1 - \alpha)\frac{\Upsilon}{N}$, where Φ is a measure of distortion of the steady state around which I am expanding. Thus,⁸¹ using lemma 7

$$\begin{aligned} U_t - U &\propto \gamma_c \frac{U_t - U}{U_1 C} \\ &= \left[\widehat{y}_t + \frac{1}{2} \left(1 - \frac{\sigma}{\gamma_c} \right) \widehat{y}_t^2 \right] - \frac{\gamma_i}{\eta} \left[q_t + \frac{1}{2} \left(1 + \frac{\sigma \gamma_i}{\gamma_c \eta} \right) q_t^2 \right] - \gamma_i \left[\widehat{k}_t + \frac{1}{2} \left(1 + \frac{\sigma \gamma_i}{\gamma_c} \right) \widehat{k}_t^2 \right] \\ &\quad + \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t q_t + \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t - \frac{\sigma \gamma_i^2}{\gamma_c \eta} q_t \widehat{k}_t - (1 - \Phi)(1 - \alpha) \left[\widehat{n}_t + \frac{1+\varphi}{2} \widehat{n}_t^2 + \frac{\varepsilon_w^2 \varphi}{2} \text{var}_j w_t(j) \right] \\ &= \widehat{y}_t - \gamma_i \widehat{i}_t + \frac{1}{2} \left(1 - \frac{\sigma}{\gamma_c} \right) \widehat{y}_t^2 - \frac{1}{2} \left(\frac{\gamma_i}{\eta} + \frac{\sigma \gamma_i^2}{\gamma_c \eta^2} \right) q_t^2 \\ &\quad - \frac{1}{2} \left(\gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c} \right) \widehat{k}_t^2 + \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t q_t + \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t - \frac{\sigma \gamma_i^2}{\gamma_c \eta} q_t \widehat{k}_t \\ &\quad - (1 - \Phi) \left[\widehat{y}_t - a_t - \alpha \widehat{k}_t + \frac{1}{2} w' \Delta_t + \frac{\Upsilon}{2} \text{var}_j w_t(j) + \frac{1+\varphi}{2(1-\alpha)} (\widehat{y}_t - a_t - \alpha \widehat{k}_t)^2 \right] \\ &\Rightarrow \Phi \widehat{y}_t - \alpha \Phi \widehat{k}_t - (\gamma_i \widehat{i}_t - \alpha \widehat{k}_t) + \frac{1}{2} \left(1 - \frac{\sigma}{\gamma_c} \right) \widehat{y}_t^2 - \frac{1}{2} \left(\frac{\gamma_i}{\eta} + \frac{\sigma \gamma_i^2}{\gamma_c \eta^2} \right) q_t^2 - \frac{1}{2} \left(\gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c} \right) \widehat{k}_t^2 \\ &\quad + \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t q_t + \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t - \frac{\sigma \gamma_i^2}{\gamma_c \eta} q_t \widehat{k}_t - (1 - \Phi) \left[\frac{\varepsilon_p}{2\Theta} \text{var}_i p_t(i) + \frac{\Upsilon}{2} \text{var}_j w_t(j) \right] \\ &\quad + \frac{1+\varphi}{2(1-\alpha)} \widehat{y}_t^2 + \frac{(1+\varphi)\alpha^2}{2(1-\alpha)} \widehat{k}_t^2 - \frac{(1+\varphi)\alpha}{1-\alpha} \widehat{y}_t \widehat{k}_t - \frac{1+\varphi}{1-\alpha} \widehat{y}_t a_t + \frac{(1+\varphi)\alpha}{1-\alpha} \widehat{k}_t a_t \end{aligned}$$

where $\Upsilon \equiv \varepsilon_w(1 - \alpha)(1 + \varepsilon_w \varphi)$ and $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}$, $\Delta_t \equiv [\text{var}_i p_t(i); \text{var}_i k_t(i); \text{cov}_i(p_t(i), k_t(i))]$ and $w \equiv [\frac{1}{2} \frac{\varepsilon}{\Theta}; \frac{1}{2} \frac{\alpha}{1-\alpha}; \frac{\alpha\varepsilon}{1-\alpha}]$.

Now, using (C.2) and recalling that $\gamma_i = \frac{\alpha\beta\delta}{\vartheta}$,

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t (\gamma_i \widehat{i}_t - \alpha \widehat{k}_t) &= \sum_{t=0}^{\infty} \beta^t \frac{\alpha}{\vartheta} (\beta \widehat{k}_{t+1} - \widehat{k}_t) \\ &= -\frac{\alpha}{\vartheta} k_0 \end{aligned}$$

which is a term independent of policy, so that, as I sum the period-by-period utilities, I can cancel out the linear terms in $[\widehat{i}_t, \widehat{k}_t]$.

Next, under the small distortions assumption that Φ times second-order terms is of order higher than two,⁸²

$$\begin{aligned} &\Rightarrow \Phi \widehat{y}_t - \alpha \Phi \widehat{k}_t - \frac{1}{2} \left[\frac{\varepsilon_p}{\Theta} \text{var}_i p_t(i) + \Upsilon \text{var}_j w_t(j) \right] - \frac{1}{2} \times \\ &\quad \times \left[\left(\frac{\sigma}{\gamma_c} - 1 \right) \widehat{y}_t^2 + \left(\frac{\gamma_i}{\eta} + \frac{\sigma \gamma_i^2}{\gamma_c \eta^2} \right) q_t^2 + \left(\gamma_i + \frac{\sigma \gamma_i^2}{\gamma_c} \right) \widehat{k}_t^2 - 2 \frac{\sigma \gamma_i}{\gamma_c \eta} \widehat{y}_t q_t - 2 \frac{\sigma \gamma_i}{\gamma_c} \widehat{y}_t \widehat{k}_t + 2 \frac{\sigma \gamma_i^2}{\gamma_c \eta} q_t \widehat{k}_t + \right. \\ &\quad \left. + \frac{1+\varphi}{1-\alpha} \widehat{y}_t^2 + \frac{(1+\varphi)\alpha^2}{1-\alpha} \widehat{k}_t^2 - 2 \frac{(1+\varphi)\alpha}{1-\alpha} \widehat{y}_t \widehat{k}_t - 2 \widehat{y}_t \frac{1+\varphi}{1-\alpha} a_t + 2 \alpha \widehat{k}_t \frac{1+\varphi}{1-\alpha} a_t \right] \end{aligned}$$

⁸¹By ‘ \Rightarrow ’ I mean that I disregard both terms independent of policy, and terms of order higher than two.

⁸²See Woodford (2003, 383-92) for a comment on this assumption.

I focus now on the last two rows of the last expression, which is what I want to express in terms of gaps. Let the output gap with respect to the efficient economy $\widetilde{y}_t \equiv \widehat{y}_t - \widehat{y}_t^e \equiv (y_t - y_t^e) - (y^n - y^e)$, the marginal Q gap $\widetilde{q}_t \equiv q_t - q_t^e$ and the capital gap $\widetilde{k}_t \equiv \widehat{k}_t - \widehat{k}_t^e$.⁸³

I next add and subtract the quantities needed to make gaps appear, and use (C.8) to substitute out $\frac{1+\varphi}{1-\alpha}a_t$:

$$\begin{aligned} \Rightarrow & \left[\left(\frac{\sigma}{\gamma_c} - 1 + \frac{1+\varphi}{1-\alpha} \right) (\widehat{y}_t^2 - 2\widehat{y}_t\widehat{y}_t^e) + \left(\frac{\gamma_i}{\eta} + \frac{\sigma\gamma_i^2}{\gamma_c\eta^2} \right) (q_t^2 - 2q_tq_t^e) \right. \\ & + \left(\gamma_i + \frac{\sigma\gamma_i^2}{\gamma_c} + \frac{(1+\varphi)\alpha^2}{1-\alpha} \right) (\widehat{k}_t^2 - 2\widehat{k}_t\widehat{k}_t^e) - 2\frac{\sigma\gamma_i}{\gamma_c\eta} (\widehat{y}_tq_t - \widehat{y}_tq_t^e - q_t\widehat{y}_t^e) \\ & - 2\left(\frac{\sigma\gamma_i}{\gamma_c} + \frac{(1+\varphi)\alpha}{1-\alpha} \right) (\widehat{y}_t\widehat{k}_t - \widehat{y}_t\widehat{k}_t^e - \widehat{k}_t\widehat{y}_t^e) + 2\frac{\sigma\gamma_i^2}{\gamma_c\eta} (q_t\widehat{k}_t - q_t\widehat{k}_t^e - \widehat{k}_tq_t^e) \Big] \\ & + \left[2\left(\frac{\sigma}{\gamma_c} - 1 + \frac{1+\varphi}{1-\alpha} \right) \widehat{y}_t\widehat{y}_t^e + 2\left(\frac{\gamma_i}{\eta} + \frac{\sigma\gamma_i^2}{\gamma_c\eta^2} \right) q_tq_t^e + 2\left(\gamma_i + \frac{\sigma\gamma_i^2}{\gamma_c} + \frac{(1+\varphi)\alpha^2}{1-\alpha} \right) \widehat{k}_t\widehat{k}_t^e - 2\frac{\sigma\gamma_i}{\gamma_c\eta} \widehat{y}_tq_t^e \right. \\ & - 2\frac{\sigma\gamma_i}{\gamma_c\eta} q_t\widehat{y}_t^e - 2\left(\frac{\sigma\gamma_i}{\gamma_c} + \frac{(1+\varphi)\alpha}{1-\alpha} \right) \widehat{y}_t\widehat{k}_t^e - 2\left(\frac{\sigma\gamma_i}{\gamma_c} + \frac{(1+\varphi)\alpha}{1-\alpha} \right) \widehat{k}_t\widehat{y}_t^e + 2\frac{\sigma\gamma_i^2}{\gamma_c\eta} q_t\widehat{k}_t^e + 2\frac{\sigma\gamma_i^2}{\gamma_c\eta} \widehat{k}_tq_t^e \Big] \\ & + \left[-2\left(\frac{\sigma}{\gamma_c} - 1 + \frac{1+\varphi}{1-\alpha} \right) \widehat{y}_t\widehat{y}_t^e + 2\frac{\sigma\gamma_i}{\gamma_c\eta} \widehat{y}_tq_t^e + 2\left(\frac{\sigma\gamma_i}{\gamma_c} + \frac{(1+\varphi)\alpha}{1-\alpha} \right) \widehat{y}_t\widehat{k}_t^e \right. \\ & \left. + 2\left(\frac{\sigma}{\gamma_c} - 1 + \frac{1+\varphi}{1-\alpha} \right) \alpha\widehat{k}_t\widehat{y}_t^e - 2\frac{\sigma\gamma_i}{\gamma_c\eta} \alpha\widehat{k}_tq_t^e - 2\left(\frac{\sigma\gamma_i}{\gamma_c} + \frac{(1+\varphi)\alpha}{1-\alpha} \right) \alpha\widehat{k}_t\widehat{k}_t^e \right] \end{aligned}$$

Some terms in the last four rows wipe each other out. After straightforward simplification,

$$\begin{aligned} & \left[\Xi_y\widehat{y}_t^2 + \Xi_q\widehat{q}_t^2 + \Xi_k\widehat{k}_t^2 - 2\Xi_{yq}\widehat{y}_t\widehat{q}_t - 2\Xi_{yk}\widehat{y}_t\widehat{k}_t + 2\Xi_{qk}\widehat{q}_t\widehat{k}_t \right] \\ & + 2q_t \left[\left(\frac{\gamma_i}{\eta} + \frac{\sigma\gamma_i^2}{\gamma_c\eta^2} \right) q_t^e - \frac{\sigma\gamma_i}{\gamma_c\eta} \widehat{y}_t^e + \frac{\sigma\gamma_i^2}{\gamma_c\eta} \widehat{k}_t^e \right] \\ & + 2\widehat{k}_t \left[\left(\frac{\sigma\gamma_i^2}{\gamma_c\eta} - \frac{\sigma\gamma_i\alpha}{\gamma_c\eta} \right) q_t^e - \left(\frac{\sigma\gamma_i}{\gamma_c} - \frac{\sigma\alpha}{\gamma_c} + \alpha \right) \widehat{y}_t^e + \left(\gamma_i + \frac{\sigma\gamma_i^2}{\gamma_c} - \frac{\sigma\gamma_i\alpha}{\gamma_c} \right) \widehat{k}_t^e \right] \\ \equiv & \left[\Xi_y\widehat{y}_t^2 + \Xi_q\widehat{q}_t^2 + \Xi_k\widehat{k}_t^2 - 2\Xi_{yq}\widehat{y}_t\widehat{q}_t - 2\Xi_{yk}\widehat{y}_t\widehat{k}_t + 2\Xi_{qk}\widehat{q}_t\widehat{k}_t \right] + 2q_t\zeta_{q,t} + 2\widehat{k}_t\zeta_{k,t} \end{aligned}$$

Next,

$$\begin{aligned} \zeta_{q,t} &= \frac{\gamma_i}{\eta} \widehat{q}_t^e + \Xi_{yq} \left[\frac{\gamma_i}{\eta} \widehat{q}_t^e - \widehat{y}_t^e + \gamma_i \widehat{k}_t^e \right] \\ &= \frac{\gamma_i}{\eta} \widehat{q}_t^e - \Xi_{yq} \gamma_c \widehat{c}_t^e \\ &= \frac{\gamma_i}{\eta} (\widehat{q}_t^e - \sigma \widehat{c}_t^e) \\ \zeta_{k,t} &= (\gamma_i - \alpha) \frac{\sigma}{\gamma_c} (\gamma_i \widehat{k}_t^e - \widehat{y}_t^e + \frac{\gamma_i}{\eta} \widehat{q}_t^e) + \gamma_i \widehat{k}_t^e - \alpha \widehat{y}_t^e \\ &= \gamma_i (\widehat{k}_t^e - \sigma \widehat{c}_t^e) - \alpha (\widehat{y}_t^e - \sigma \widehat{c}_t^e) \end{aligned}$$

⁸³Note that $\widehat{y}_t^2 = (\widehat{y}_t^2 - \widehat{y}_t^e)^2 \Leftarrow \widehat{y}_t - 2\widehat{y}_t\widehat{y}_t^e$, $\widehat{q}_t^2 \Leftarrow (q_t^2 - 2q_tq_t^e)$ and $\widehat{k}_t^2 \Leftarrow (\widehat{k}_t^2 - 2\widehat{k}_t\widehat{k}_t^e)$; also, $\widehat{y}_t\widehat{q}_t \Leftarrow (\widehat{y}_tq_t - \widehat{y}_tq_t^e - q_t\widehat{y}_t^e)$, $\widehat{k}_t\widehat{q}_t \Leftarrow (\widehat{k}_tq_t - \widehat{k}_tq_t^e - q_t\widehat{k}_t^e)$, and $\widehat{y}_t\widehat{k}_t \Leftarrow (\widehat{y}_t\widehat{k}_t - \widehat{y}_t\widehat{k}_t^e - \widehat{k}_t\widehat{y}_t^e)$.

so that,

$$\begin{aligned} 2q_t\zeta_{q,t} + 2\widehat{k}_t\zeta_{k,t} &= 2\frac{\gamma_i}{\eta}q_t[q_t^e - \sigma\widehat{c}_t^e] + 2\gamma_i\widehat{k}_t[\widehat{k}_t^e - \sigma\widehat{c}_t^e] - 2\alpha\widehat{k}_t[\widehat{y}_t^e - \sigma\widehat{c}_t^e] \\ &= 2(\alpha\widehat{k}_t - \gamma_i\widehat{i}_t)\sigma\widehat{c}_t^e + 2\frac{\gamma_i}{\eta}q_tq_t^e + 2\gamma_i\widehat{k}_t\widehat{k}_t^e - 2\alpha\widehat{k}_t\widehat{y}_t^e \end{aligned}$$

Now, note that the intertemporal optimality conditions (C.2) and (C.5) imply that

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t (\alpha\widehat{k}_t - \gamma_i\widehat{i}_t) \sigma\widehat{c}_t^e &= \sum_{t=0}^{\infty} \beta^t \frac{\alpha}{\vartheta} (\widehat{k}_t - \beta\widehat{k}_{t+1}) \sigma\widehat{c}_t^e \\ &\Rightarrow \frac{\alpha}{\vartheta} \sum_{t=1}^{\infty} \beta^t \widehat{k}_t \sigma \Delta\widehat{c}_t^e \\ &= \frac{\alpha}{\vartheta} \sum_{t=1}^{\infty} \beta^t \widehat{k}_t [\vartheta m p \widehat{k}_t^e + \beta\widehat{q}_t^e - \widehat{q}_{t-1}^e] \end{aligned}$$

Plugging this expression into the discounted sum $\sum_{t=0}^{\infty} 2[q_t\zeta_{q,t} + \widehat{k}_t\zeta_{k,t}]$, I get

$$\begin{aligned} 2 \sum_{t=0}^{\infty} \beta^t [q_t\zeta_{q,t} + \widehat{k}_t\zeta_{k,t}] &= \\ &= 2 \sum_{t=0}^{\infty} \beta^t [\frac{\gamma_i}{\eta}q_tq_t^e + \gamma_i\widehat{k}_t\widehat{k}_t^e - \alpha\widehat{k}_t\widehat{y}_t^e - (\gamma_i - \alpha)k_t\sigma c_t^e - \gamma_i(\widehat{i}_t - k_t)\sigma c_t^e] \\ &= 2 \sum_{t=0}^{\infty} \beta^t [\frac{\alpha}{\vartheta}\widehat{k}_t(\vartheta\widehat{y}_t^e - \vartheta\widehat{k}_t^e + \beta q_t^e - q_{t-1}^e) + \frac{\gamma_i}{\eta}q_tq_t^e + \gamma_i\widehat{k}_t\widehat{k}_t^e - \alpha\widehat{k}_t\widehat{y}_t^e] \\ &\Rightarrow 2 \sum_{t=0}^{\infty} \beta^t [\alpha\widehat{k}_t\widehat{y}_t^e - \alpha\widehat{k}_t\widehat{k}_t^e + \frac{\alpha\beta}{\vartheta}\widehat{k}_tq_t^e - \frac{\alpha\beta}{\vartheta}\widehat{k}_{t+1}q_t^e + \frac{\gamma_i}{\eta}q_tq_t^e + \gamma_i\widehat{k}_t\widehat{k}_t^e - \alpha\widehat{k}_t\widehat{y}_t^e] \\ &= 2 \sum_{t=0}^{\infty} \beta^t [(\gamma_i - \alpha)\widehat{k}_t\widehat{k}_t^e + (\frac{\gamma_i}{\eta}q_t - \frac{\gamma_i}{\delta}\Delta\widehat{k}_{t+1})q_t^e] \\ &= 2(\gamma_i - \alpha) \sum_{t=0}^{\infty} \beta^t \widehat{k}_t\widehat{k}_t^e \\ &\Rightarrow -2\frac{\alpha\beta}{\vartheta} \sum_{t=0}^{\infty} \beta^t (\widehat{k}_{t+1}\widehat{k}_{t+1}^e - \widehat{k}_t\widehat{k}_t^e) \end{aligned}$$

where I used the facts that $\frac{\alpha\beta}{\vartheta} = \frac{\gamma_i}{\delta}$, $\Delta\widehat{k}_{t+1} = \delta(\widehat{i}_t - \widehat{k}_t) = \frac{\delta}{\eta}q_t$, and that $\gamma_i - \alpha = -\frac{\alpha(1-\beta)}{\vartheta}$. Now, by stationarity of \widehat{k}_t and \widehat{k}_t^e , $\sum_{t=0}^{\infty} \beta^t (E[\widehat{k}_{t+1}\widehat{k}_{t+1}^e] - E[\widehat{k}_t\widehat{k}_t^e]) = 0$, so that the last term has zero unconditional expectation, so that it does not appear in \mathbb{L} .

I can thus compute the discounted sum of losses, applying lemma 9, to end up with the expressions for \mathbb{W}_0 and \mathbb{L} . \square

I next follow Sveen and Weinke (2009) in deriving the relationship between the dispersion term Δ_t and inflation, both under firm-specific capital accumulation and under a rental market for capital services.

Lemma 9. Firm-specific capital Up to a second-order approximation, Sveen and Weinke (2009)

show how

$$\begin{aligned}\Delta_t^p &= \theta_p \Delta_{t-1}^p + (1 - \theta_p) \psi_1 \Delta_t^k + \frac{\theta_p}{1 - \theta_p} \pi_t^2 \\ \Delta_t^k &= \psi_2^2 \Delta_{t-1}^k + \psi_3^2 \Delta_{t-1}^p \\ \Delta_t^{pk} &= \theta_p \psi_2 \Delta_{t-1}^{pk} + \theta_p \psi_3 \Delta_{t-1}^p + (1 - \theta_p) \psi_1 \Delta_t^k\end{aligned}$$

I can represent this system of equations in equivalent vector notation,

$$\begin{bmatrix} 1 & -(1 - \theta_p) \psi_1^2 & 0 \\ 0 & 1 & 0 \\ 0 & -(1 - \theta_p) \psi_1 & 1 \end{bmatrix} \Delta_t = \begin{bmatrix} \theta_p & 0 & 0 \\ \psi_3^2 & \psi_2^2 & 0 \\ \theta_p \psi_3 & 0 & \theta_p \psi_2 \end{bmatrix} \Delta_{t-1} + \begin{bmatrix} \frac{\theta_p}{1 - \theta_p} \\ 0 \\ 0 \end{bmatrix} \pi_t^2 \quad \Leftrightarrow \quad B_\Delta \Delta_t = A_\Delta \Delta_{t-1} + C_\Delta \pi_t^2$$

which allows me to express the discounted sum of the dispersion term in welfare as

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t w' \Delta_t &= \sum_{t=0}^{\infty} \beta^t w' \sum_{h=0}^{t-1} (B_\Delta^{-1} A_\Delta)^h B_\Delta^{-1} C_\Delta \pi_{t-h}^2 + t.i.p. \\ &= \sum_{t=1}^{\infty} \beta^t w' \Delta_t = \frac{1}{2} \Xi_p \pi_t^2 + t.i.p.\end{aligned}$$

where $\Xi_p \equiv w' [B_\Delta - \beta A_\Delta]^{-1} C_\Delta$.

Rental market When firms purchase capital services on a competitive rental market, we have $\bar{y}_t(i) = \bar{k}_t(i)$ and therefore $\Delta_t^y = \Delta_t^k = \Delta_t^{yk} = \varepsilon^2 \Delta_t^p$. The recursive formulation for the cross-sectional dispersion of prices becomes (Woodford, 2003)

$$\Delta_t^p = \theta_p \Delta_{t-1}^p + \frac{\theta_p}{1 - \theta_p} \pi_t^2$$

because there is no dispersion in $p_t^*(i)$. Therefore, the dispersion component in the welfare loss measure is

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t w' \Delta_t &= \frac{\varepsilon}{2} \sum_{t=0}^{\infty} \beta^t \Delta_t^p \\ &= \frac{1}{2} \frac{\varepsilon_p \theta_p}{(1 - \theta_p)(1 - \beta \theta_p)} \sum_{t=1}^{\infty} \beta^t \pi_t^2 + t.i.p.\end{aligned}$$

H. Optimal monetary policy

I focus on the New Keynesian Q model with sticky prices and an exogenous cost-push disturbance.

H.1. The linear rational expectations model

Consider the quadruplet [(E.16),(E.17),(E.18),(E.21)]:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_p \Xi_y \bar{y}_t - \lambda_p \Xi_{yq} \bar{q}_t - \lambda_p \Xi_{yk} \bar{k}_t + u_t \quad (\text{H.1})$$

$$\Lambda \bar{q}_t = \beta \Lambda E_t \bar{q}_{t+1} + \Psi \bar{y}_t - \beta \Omega E_t \bar{y}_{t+1} - \vartheta (1 + \Xi_{yk}) \bar{k}_{t+1} + \frac{\vartheta}{\lambda_p} E_t u_{t+1} \quad (\text{H.2})$$

$$\bar{k}_{t+1} = \frac{\delta}{\eta} \bar{q}_t + \bar{k}_t \quad (\text{H.3})$$

$$i_t^f = r_t^{f,e} - \Psi \bar{y}_t - (1 - \Lambda) \bar{q}_t + E_t \pi_{t+1} + \Psi E_t \bar{y}_{t+1} - \Xi_{yq} E_t \bar{q}_{t+1} \quad (\text{H.4})$$

where $\Lambda \equiv 1 + \Xi_{yq}(1 - \delta)$, $\Psi \equiv \frac{\sigma}{\gamma_c}$ and $\Omega \equiv \beta^{-1}(\Psi - \vartheta(1 + \Xi_y))$.

Next, write the quadruplet [(H.1),(H.2),(H.3),(H.4)] and the law of motion for the cost-push disturbance (E.22) as the linear rational expectations model

$$\begin{aligned} & \begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\delta}{\eta} & 0 \\ -1 & \lambda_p \Xi_{yk} & 1 & \lambda_p \Xi_{yq} & -\lambda_p \Xi_y \\ 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & 0 & 0 & 1 - \Lambda & \Psi \end{bmatrix} \begin{bmatrix} u_t \\ \bar{k}_t \\ \pi_t \\ \bar{q}_t \\ \bar{y}_t \end{bmatrix} = \\ & = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ \frac{\vartheta}{\lambda_p} & -\vartheta(1 + \Xi_{yk}) & 0 & \beta \Lambda & -\beta \Omega \\ 0 & 0 & 1 & -\Xi_{yq} & \Psi \end{bmatrix} E_t \begin{bmatrix} u_{t+1} \\ \bar{k}_{t+1} \\ \pi_{t+1} \\ \bar{q}_{t+1} \\ \bar{y}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} (i_t^f - r_t^{f,e}) \end{aligned} \quad (\text{H.5})$$

This is the structural form of the model solution. We next have to specify an equation for monetary policy to close the model.

H.2. Optimal monetary policy under commitment (second best)

The policymaker can determine the best feasible dynamics for the variables under full commitment by choosing the stochastic processes $\{\pi_t, \bar{q}_t, \bar{y}_t, \bar{k}_{t+1}, t \geq 0\}$ to

$$\max_{\{\pi_t, \bar{q}_t, \bar{y}_t, \bar{k}_{t+1}\}} \mathbb{W}_t \quad \text{s.t.} \quad (\text{H.1}), (\text{H.2}), (\text{H.3})$$

i.e. to

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\Phi \widetilde{y}_t + \frac{1}{2} \Xi_y \widetilde{y}_t^2 + \frac{1}{2} \Xi_q \widetilde{q}_t^2 - \Xi_{yq} \widetilde{y}_t \widetilde{q}_t + \frac{1}{2} \Xi_p \pi_t^2 + \right. \\ & \quad + \alpha \Phi \widetilde{k}_t + \frac{1}{2} \Xi_k \widetilde{k}_t^2 - \Xi_{yk} \widetilde{y}_t \widetilde{k}_t + \Xi_{qk} \widetilde{q}_t \widetilde{k}_t + \\ & \quad + \gamma_t [\pi_t + \lambda_p \Xi_{yq} \widetilde{q}_t - \lambda_p \Xi_y \widetilde{y}_t - \beta \pi_{t+1} + \lambda_p \Xi_{yk} \widetilde{k}_t - u_t] + \\ & \quad + \varphi_t [\Lambda \widetilde{q}_t - \Psi \widetilde{y}_t - \beta \Lambda \widetilde{q}_{t+1} + \beta \Omega \widetilde{y}_{t+1} + \vartheta (1 + \Xi_{yk}) \widetilde{k}_{t+1} - \frac{\vartheta}{\lambda_p} E_t u_{t+1}] + \\ & \quad \left. + \nu_t [\frac{\delta}{\eta} \widetilde{q}_t + \widetilde{k}_t - \widetilde{k}_{t+1}] \right\} \end{aligned}$$

The FOCs for an interior optimum, along with (H.1), (H.2) and (H.3), are

$$\begin{aligned} \nu_t &= \alpha \beta \Phi + \beta \Xi_k \widetilde{k}_{t+1} + \vartheta (1 + \Xi_{yk}) \varphi_t + \beta \lambda_p \Xi_{yq} E_t \gamma_{t+1} + \beta E_t \nu_{t+1} + \beta \Xi_{qk} E_t \widetilde{q}_{t+1} - \beta \Xi_{yk} E_t \widetilde{y}_{t+1} \\ -\gamma_t &= \Xi_p \pi_t - \gamma_{t-1} \\ \Xi_{qk} \widetilde{k}_t + \Xi_q \widetilde{q}_t - \Xi_{yq} \widetilde{y}_t - \Lambda \varphi_{t-1} + \frac{\delta}{\eta} \nu_t &= -\lambda_p \Xi_{yq} \gamma_t - \Lambda \varphi_t \\ -\Xi_{yk} \widetilde{k}_t - \Xi_{yq} \widetilde{q}_t + \Xi_y \widetilde{y}_t + \Omega \varphi_{t-1} &= \Phi + \lambda_p \Xi_y \gamma_t + \Psi \varphi_t \end{aligned} \quad (\text{H.6})$$

I can write (H.1), (H.2), (H.3) and (H.6) as the linear rational expectations model

$$\begin{aligned} & \begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{\delta}{\eta} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & \lambda_p \Xi_{yq} & 0 & 0 & 0 & 0 & 1 & \lambda_p \Xi_{yq} & -\lambda_p \Xi_y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & \Xi_p & 0 & 0 \\ 0 & \Xi_{qk} & 0 & -\Lambda & 0 & \frac{\delta}{\eta} & 0 & \Xi_q & -\Xi_{yq} \\ 0 & -\Xi_{yk} & 0 & \Omega & 0 & 0 & 0 & -\Xi_{yq} & \Xi_y \end{bmatrix} \begin{bmatrix} u_t \\ \widetilde{k}_t \\ \gamma_{t-1} \\ \varphi_{t-1} \\ \gamma_t \\ \nu_t \\ \pi_t \\ \widetilde{q}_t \\ \widetilde{y}_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha \beta \Phi \\ 0 \\ 0 \\ \Phi \end{bmatrix} + \\ & + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta & 0 \\ \frac{\vartheta}{\lambda_p} & -\vartheta(1 + \Xi_{yk}) & 0 & 0 & 0 & 0 & 0 & \beta \Lambda & -\beta \Omega \\ 0 & \beta \Xi_k & 0 & \vartheta(1 + \Xi_{yk}) & \beta \lambda_p \Xi_{yq} & \beta & 0 & \beta \Xi_{qk} & -\beta \Xi_{yk} \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_p \Xi_{yq} & -\Lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_p \Xi_y & \Psi & 0 & 0 & 0 & 0 & 0 \end{bmatrix} E_t \begin{bmatrix} u_{t+1} \\ \widetilde{k}_{t+1} \\ \gamma_t \\ \varphi_t \\ \gamma_{t+1} \\ \nu_{t+1} \\ \pi_{t+1} \\ \widetilde{q}_{t+1} \\ \widetilde{y}_{t+1} \end{bmatrix} \end{aligned}$$

or, identically,

$$B_c Y_t = A_c E_t Y_{t+1} + C_c$$

The system has five nonpredetermined, $[\pi_{t+1}, \widetilde{q}_{t+1}, \widetilde{y}_{t+1}, \gamma_{t+1}, \nu_{t+1}]$, and four predetermined variables, $[u_{t+1}, \widetilde{k}_{t+1}, \gamma_t, \varphi_t]$, with initial conditions $\gamma_{-1} = \varphi_{-1} = k_0 = u_0 = 0$.

I solve the system with the algorithm in Klein (2000). As shown by Blanchard and Kahn (1980), if the matrix $B_c^{-1} A_c$ has five eigenvalues inside and four outside the unit circle, then its solution is

unique, and has reduced form

$$\begin{bmatrix} \gamma_t^{*c} \\ \nu_t^{*c} \\ \pi_t^{*c} \\ \widetilde{q}_t^{*c} \\ \widetilde{y}_t^{*c} \end{bmatrix} = G_0^c + G_1^c \begin{bmatrix} u_t \\ \widetilde{k}_t \\ \gamma_{t-1} \\ \varphi_{t-1} \end{bmatrix} \quad (\text{H.7})$$

$$\begin{bmatrix} u_{t+1} \\ \widetilde{k}_{t+1}^{*c} \\ \gamma_t^{*c} \\ \varphi_t^{*c} \end{bmatrix} = \begin{bmatrix} 0 \\ H_{0,2}^c \\ H_{0,3}^c \\ H_{0,4}^c \end{bmatrix} + \begin{bmatrix} \rho_u & 0 & 0 & 0 \\ H_{1,21}^c & H_{1,22}^c & H_{1,23}^c & H_{1,24}^c \\ H_{1,31}^c & H_{1,32}^c & H_{1,33}^c & H_{1,34}^c \\ H_{1,41}^c & H_{1,42}^c & H_{1,43}^c & H_{1,44}^c \end{bmatrix} \begin{bmatrix} u_t \\ \widetilde{k}_t \\ \gamma_{t-1} \\ \varphi_{t-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \varepsilon_{t+1}^u$$

which allows me to study the responses of the variables to a cost-push shock and compute the welfare criterion (G.1) for the optimal monetary policy under commitment.

Assuming the determinacy of (H.7)—which holds under the baseline calibration—we can next rearrange (H.6) to substitute out all Lagrange multipliers. We thus get the targeting rule under commitment that implements second best, which links the endogenous variables as

$$\begin{aligned} 0 = & \Xi_p \left(1 + \lambda_p \frac{\Xi_y \Lambda - \Xi_{yq} \Omega}{\Xi_y (\Gamma - \Lambda) + \Xi_{yq} (\Omega - \Psi)} (1 - L) \right) \pi_t \\ & + \frac{\beta \delta \Xi_{qk}}{\eta} (\Psi - \Omega L) (1 - L) E_t \widetilde{q}_{t+1} - \left(\frac{\Xi_{yq} \Gamma - \Psi (\Xi_q + \frac{\beta \delta^2}{\eta^2} \Xi_k)}{\Xi_y (\Gamma - \Lambda) + \Xi_{yq} (\Omega - \Psi)} - \frac{\Xi_{yq} \Lambda - \Omega (\Xi_q + \frac{\beta \delta^2}{\eta^2} \Xi_k)}{\Xi_y (\Gamma - \Lambda) + \Xi_{yq} (\Omega - \Psi)} L \right) (1 - L) \widetilde{q}_t \\ & - \frac{\beta \delta \Xi_{yk}}{\eta} (\Psi - \Omega L) (1 - L) E_t \widetilde{y}_{t+1} + \left(\frac{\Xi_y \Gamma - \Xi_{yq} \Psi}{\Xi_y (\Gamma - \Lambda) + \Xi_{yq} (\Omega - \Psi)} - \frac{\Xi_y \Lambda - \Xi_{yq} \Omega}{\Xi_y (\Gamma - \Lambda) + \Xi_{yq} (\Omega - \Psi)} L \right) (1 - L) \widetilde{y}_t \\ & - \left(\frac{\Xi_{yk} \Gamma - \Psi (\Xi_{qk} + \frac{\beta \delta}{\eta} \Xi_k)}{\Xi_y (\Gamma - \Lambda) + \Xi_{yq} (\Omega - \Psi)} - \frac{\Xi_{yk} \Lambda - \Omega (\Xi_{qk} + \frac{\beta \delta}{\eta} \Xi_k)}{\Xi_y (\Gamma - \Lambda) + \Xi_{yq} (\Omega - \Psi)} L \right) (1 - L) \widetilde{k}_t \end{aligned} \quad (\text{H.8})$$

where L is the lag operator, that lags also the conditional expectation operator, and $\Gamma \equiv \Lambda + \frac{\delta \theta}{\eta} (1 + \Xi_{yk})$.

We can also implement second best with an interest-rate rule. Two examples are the fundamentals-based and the expectations-based interest-rate rules of Evans and Honkapohja (2003, 2006) and Woodford (2003, 527-33).

H.2.1. Fundamentals-based interest-rate rule

Using the reduced form (H.7), I can express the interest rate in (H.4) in terms of $u_t, \widetilde{k}_t, \gamma_{t-1}, \varphi_{t-1}$. This is the fundamentals-based interest-rate rule, which is consistent with the solution (H.7). However, this interest-rate rule may well be consistent with other solutions, as the rational expectations model under the fundamentals-based interest-rate rule may be indetermined. Indeed, under the baseline calibration the system has multiple solutions.

H.2.2. Expectations-based interest-rate rule

Invert (H.5) as

$$\begin{aligned} \begin{bmatrix} u_t \\ \widetilde{k}_t \\ \pi_t \\ \widetilde{q}_t \\ \widetilde{y}_t \end{bmatrix} &= \begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\delta}{\eta} & 0 \\ -1 & \lambda_p \Xi_{yk} & 1 & \lambda_p \Xi_{yq} & -\lambda_p \Xi_y \\ 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & 0 & 0 & 1 - \Lambda & \Psi \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ \frac{\theta}{\lambda_p} & -\vartheta(1 + \Xi_{yk}) & 0 & \beta\Lambda & -\beta\Omega \\ 0 & 0 & 1 & -\Xi_{yq} & \Psi \end{bmatrix} E_t \begin{bmatrix} u_{t+1} \\ \widetilde{k}_{t+1} \\ \pi_{t+1} \\ \widetilde{q}_{t+1} \\ \widetilde{y}_{t+1} \end{bmatrix} \\ &+ \begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\delta}{\eta} & 0 \\ -1 & \lambda_p \Xi_{yk} & 1 & \lambda_p \Xi_{yq} & -\lambda_p \Xi_y \\ 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & 0 & 0 & 1 - \Lambda & \Psi \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} (i_t^f - r_t^{f,e}) \end{aligned} \quad (\text{H.9})$$

Then, plug the expressions for $[\widetilde{k}_t, \pi_t, \widetilde{y}_t, \widetilde{q}_t]$ of (H.9) into the targeting rule (H.8) and solve for i_t^f , to find the expectations-based interest-rate rule. This rule implements the optimal monetary policy under commitment and by construction leads the system to determinacy.

Proof. By construction, this interest-rate rule, once combined with [(H.1),(H.2),(H.3),(H.4)], implies that the targeting rule, in the structural form (H.8), holds at all periods. And I already know that the targeting rule, combined with (H.1), (H.2) and (H.3), yields the solution (H.7). \square

H.2.3. Optimal monetary policy under commitment, naïve welfare criterion

If the central bank used the naïve inflation-output welfare criterion $\Phi \widetilde{y}_t - \frac{1}{2} \Xi_y \widetilde{y}_t^2 - \frac{1}{2} \Xi_p \pi_t^2$, the system it would consider is

$$\begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{\delta}{\eta} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & \lambda_p \Xi_{yk} & 0 & 0 & 0 & 0 & 1 & \lambda_p \Xi_{yq} & -\lambda_p \Xi_y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & \Xi_p & 0 & 0 \\ 0 & 0 & 0 & -\Lambda & 0 & \frac{\delta}{\eta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega & 0 & 0 & 0 & 0 & \Xi_y \end{bmatrix} \begin{bmatrix} u_t \\ \widetilde{k}_t \\ \gamma_{t-1} \\ \varphi_{t-1} \\ \gamma_t \\ v_t \\ \pi_t \\ \widetilde{q}_t \\ \widetilde{y}_t \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \Phi \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta & 0 & 0 \\ \frac{\vartheta}{\lambda_p} & -\vartheta(1 + \Xi_{yk}) & 0 & 0 & 0 & 0 & 0 & \beta\Lambda & -\beta\Omega \\ 0 & 0 & 0 & \vartheta(1 + \Xi_{yk}) & \beta\lambda_p\Xi_{yk} & \beta & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_p\Xi_{yq} & -\Lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_p\Xi_y & \Psi & 0 & 0 & 0 & 0 & 0 \end{bmatrix} E_t \begin{bmatrix} u_{t+1} \\ \tilde{k}_{t+1} \\ \gamma_t \\ \varphi_t \\ \gamma_{t+1} \\ \nu_{t+1} \\ \pi_{t+1} \\ \tilde{q}_{t+1} \\ \tilde{y}_{t+1} \end{bmatrix}$$

H.3. Optimal monetary policy under discretion

The policymaker operating under discretion can determine the best feasible dynamics for the variables by choosing the random variables $\{\pi_t, \tilde{q}_t, \tilde{y}_t, \tilde{k}_{t+1}\}$ to

$$\max_{\pi_t, \tilde{q}_t, \tilde{y}_t, \tilde{k}_{t+1}} \mathbb{W}_t \quad \text{s.t.} \quad (\text{H.1}), (\text{H.2}), (\text{H.3})$$

i.e. to

$$\begin{aligned} & -\Phi\tilde{y}_t + \frac{1}{2}\Xi_y\tilde{y}_t^2 + \frac{1}{2}\Xi_q\tilde{q}_t^2 - \Xi_{yq}\tilde{y}_t\tilde{q}_t + \frac{1}{2}\Xi_p\pi_t^2 \\ & + \alpha\beta\Phi\tilde{k}_{t+1} + \frac{1}{2}\beta\Xi_k\tilde{k}_{t+1}^2 - \Xi_{yk}\tilde{y}_t\tilde{k}_t - \beta\Xi_{yk}\tilde{k}_{t+1}E_t\tilde{y}_{t+1} + \Xi_{qk}\tilde{q}_t\tilde{k}_t + \beta\Xi_{qk}\tilde{k}_{t+1}E_t\tilde{q}_{t+1} \\ \max_{[\gamma, \varphi, \nu] \geq 0} \min & + \gamma[\pi_t + \lambda_p\Xi_{yq}\tilde{q}_t - \lambda_p\Xi_y\tilde{y}_t - \beta E_t\pi_{t+1} + \lambda_p\Xi_{yk}\tilde{k}_t - u_t] \\ & + \varphi[\Lambda\tilde{q}_t - \Psi\tilde{y}_t - \beta\Lambda E_t\tilde{q}_{t+1} + \beta\Omega E_t\tilde{y}_{t+1} + \vartheta(1 + \Xi_{yk})\tilde{k}_{t+1} - \frac{\vartheta}{\lambda_p}E_t u_{t+1}] \\ & + \nu[\frac{\delta}{\eta}\tilde{q}_t + \tilde{k}_t - \tilde{k}_{t+1}] \end{aligned}$$

The FOC for an interior optimum, along with (H.1), (H.2) and (H.3), is

$$\begin{aligned} & -(\Xi_{yk}\Gamma - \Xi_{qk}\Psi)\tilde{k}_t + \lambda_p\Xi_p(\Xi_y\Gamma - \Xi_{yq}\Psi)\pi_t - (\Xi_{yq}\Gamma - \Xi_q\Psi)\tilde{q}_t + (\Xi_y\Gamma - \Xi_{yq}\Psi)\tilde{y}_t = \\ & = (\Gamma - \frac{\alpha\beta\delta}{\eta}\Psi)\Phi - \frac{\beta\delta\Xi_k\Psi}{\eta}\tilde{k}_{t+1} - \frac{\beta\delta\Xi_{qk}\Psi}{\eta}E_t\tilde{q}_{t+1} + \frac{\beta\delta\Xi_{yk}\Psi}{\eta}E_t\tilde{y}_{t+1} \end{aligned} \quad (\text{H.10})$$

Equation (H.10) can be interpreted as a targeting rule for the output gap.

I can now write (E.22), (H.1), (H.2), (H.3) and (H.10) as the linear rational expectations model

$$\begin{aligned} & \begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\delta}{\eta} & 0 \\ -1 & \lambda_p\Xi_{yk} & 1 & \lambda_p\Xi_{yq} & -\lambda_p\Xi_y \\ 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & -(\Xi_{yk}\Gamma - \Xi_{qk}\Psi) & \lambda_p\Xi_p(\Xi_y\Gamma - \Xi_{yq}\Psi) & -(\Xi_{yq}\Gamma - \Xi_q\Psi) & \Xi_y\Gamma - \Xi_{yq}\Psi \end{bmatrix} \begin{bmatrix} u_t \\ \tilde{k}_t \\ \pi_t \\ \tilde{q}_t \\ \tilde{y}_t \end{bmatrix} = \\ & = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ \frac{\vartheta}{\lambda_p} & -\vartheta(1 + \Xi_{yk}) & 0 & \beta\Lambda & -\beta\Omega \\ 0 & -\frac{\beta\delta\Xi_k\Psi}{\eta} & 0 & -\frac{\beta\delta}{\eta}\Xi_{qk}\Psi & \frac{\beta\delta}{\eta}\Xi_{yk}\Psi \end{bmatrix} E_t \begin{bmatrix} u_{t+1} \\ \tilde{k}_{t+1} \\ \pi_{t+1} \\ \tilde{q}_{t+1} \\ \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ (\Gamma - \frac{\alpha\beta\delta}{\eta}\Psi)\Phi \end{bmatrix} \end{aligned}$$

or, identically,

$$B_d Y_t = A_d E_t Y_{t+1} + C_d$$

which is a system with three nonpredetermined, $[\pi_{t+1}, \tilde{q}_{t+1}, \tilde{y}_{t+1}]$, and two predetermined variables, u_{t+1}, \tilde{k}_{t+1} , with initial condition $\tilde{k}_0 = u_0 = 0$. I solve the system with the algorithm in Klein (2000). As shown by Blanchard and Kahn (1980), if the matrix $B_d^{-1} A_d$ has three eigenvalues inside and two outside the unit circle, then the solution is unique, and has reduced form

$$\begin{bmatrix} \pi_t^{*d} \\ \tilde{q}_t^{*d} \\ \tilde{y}_t^{*d} \end{bmatrix} = G_0^d + G_1^d \begin{bmatrix} u_t \\ \tilde{k}_t \end{bmatrix}$$

$$\begin{bmatrix} u_{t+1} \\ \tilde{k}_{t+1}^{*d} \end{bmatrix} = \begin{bmatrix} 0 \\ H_{0,2}^d \end{bmatrix} + \begin{bmatrix} \rho_u & 0 \\ H_{1,21}^d & H_{1,22}^d \end{bmatrix} \begin{bmatrix} u_t \\ \tilde{k}_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \varepsilon_{t+1}^u$$

which allows me to study the responses of the variables to a cost-push shock and compute the welfare criterion (G.1) for the optimal monetary policy under discretion.

H.3.1. Optimal monetary policy under discretion, naïve welfare criterion

If the central bank used the naïve inflation-output welfare criterion $\Phi \tilde{y}_t - \frac{1}{2} \Xi_y \tilde{y}_t^2 - \frac{1}{2} \Xi_p \pi_t^2$, the system it would consider is

$$\begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\delta}{\eta} & 0 \\ -1 & \lambda_p \Xi_{yk} & 1 & \lambda_p \Xi_{yq} & -\lambda_p \Xi_y \\ 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & 0 & \lambda_p \Xi_p (\Xi_y \Gamma - \Xi_{yq} \Psi) & 0 & \Xi_y \Gamma \end{bmatrix} \begin{bmatrix} u_t \\ \tilde{k}_t \\ \pi_t \\ \tilde{q}_t \\ \tilde{y}_t \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ \frac{\vartheta}{\lambda_p} & -\vartheta(1 + \Xi_{yk}) & 0 & \beta \Lambda & -\beta \Omega \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} E_t \begin{bmatrix} u_{t+1} \\ \tilde{k}_{t+1} \\ \pi_{t+1} \\ \tilde{q}_{t+1} \\ \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Gamma \Phi \end{bmatrix}$$

H.4. Divine coincidence

Consider the family of Taylor rules

$$i_t^f = r_t^{f,e} + \phi_\pi \pi_t$$

Then, (H.5) becomes

$$\begin{bmatrix} \rho_u & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\delta}{\eta} & 0 \\ -1 & \lambda_p \Xi_{yk} & 1 & \lambda_p \Xi_{yq} & -\lambda_p \Xi_y \\ 0 & 0 & 0 & \Lambda & -\Psi \\ 0 & 0 & \phi_\pi & 1 - \Lambda & \Psi \end{bmatrix} \begin{bmatrix} u_t \\ \tilde{k}_t \\ \pi_t \\ \tilde{q}_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ \frac{\vartheta}{\lambda_p} & -\vartheta(1 + \Xi_{yk}) & 0 & \beta \Lambda & -\beta \Omega \\ 0 & 0 & 1 & -\Xi_{yq} & \Psi \end{bmatrix} E_t \begin{bmatrix} u_{t+1} \\ \tilde{k}_{t+1} \\ \pi_{t+1} \\ \tilde{q}_{t+1} \\ \tilde{y}_{t+1} \end{bmatrix}$$

or, identically,

$$B_\pi Y_t = A_\pi E_t Y_{t+1}$$

which is a system with three nonpredetermined, $[\pi_{t+1}, \widetilde{q}_{t+1}, \widetilde{y}_{t+1}]$, and two predetermined variables, $u_{t+1}, \widetilde{k}_{t+1}$, with initial condition $\widetilde{k}_0 = u_0 = 0$. As shown by Blanchard and Kahn (1980), if the matrix $B_\pi^{-1}A_\pi$ has three eigenvalues inside and two outside the unit circle, then the solution is unique, and has reduced form

$$\begin{bmatrix} \pi_t^* \\ \widetilde{q}_t^* \\ \widetilde{y}_t^* \end{bmatrix} = G_0 + G_1 \begin{bmatrix} u_t \\ \widetilde{k}_t \end{bmatrix}$$

$$\begin{bmatrix} u_{t+1} \\ \widetilde{k}_{t+1}^* \end{bmatrix} = \begin{bmatrix} 0 \\ H_{0,2} \end{bmatrix} + \begin{bmatrix} \rho_u & 0 \\ H_{1,21} & H_{1,22} \end{bmatrix} \begin{bmatrix} u_t \\ \widetilde{k}_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \varepsilon_{t+1}^u$$

Suppose that $\varepsilon_{u,t} = 0$ at all t . Then, since $\widetilde{k}_0 = 0$, and as long as the matrix $B_\pi^{-1}A_\pi$ is stable, the unique reduced form solution of the model is $\pi_t^* = \widetilde{q}_t^* = \widetilde{y}_t^* = \widetilde{k}_{t+1}^* = 0$ at all t . What Blanchard and Galí (2007) dub ‘divine coincidence’ thus holds in the particular case of $\varepsilon_{u,t} = 0$ at all t .

I. Simple interest-rate rules

I consider the simple generalized Taylor rules

$$\begin{aligned} i_t^f &= \rho + \phi_\pi \pi_t + \phi_q q_t \equiv r_t^{f,e} + \phi_\pi \pi_t + \phi_s \widetilde{q}_t + v_t \\ i_t^f &= \rho + \phi_\pi \pi_t + \phi_q \widehat{s}_t \equiv r_t^{f,e} + \phi_\pi \pi_t + \phi_s \widetilde{s}_t + v_t \end{aligned} \tag{I.1}$$

with the implementation disturbance $v_t = -\widehat{r}_t^e + \phi_s q_t^e$, where I used $s_t^e = q_t^e$.

To plug (I.1) into the linear rational expectations model (H.5), I have to insert equation (F.2) and a law of motion for v_t in (H.5).

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