

The Term Structure of Credit Spreads and the Economic Activity

Maurizio Luisi

Submitted for the degree of Ph.D. in Economics at

Faculty of Economics
University of Lugano
Lugano, Switzerland

Thesis Committee:

Prof. G. Barone-Adesi, University of Lugano
Prof. A. Berardi, University of Verona
Prof. E. G. De Giorgi, University of Lugano

24th October 2008

© 2008 by Maurizio Luisi

1 Preface

In recent years there has been a significant surge of interest in the modelling and pricing of credit risk. This can be attributed to several factors: the greater role played by financial markets, as opposed to intermediaries, in the contracting of credit; rapid growth in credit derivatives markets; and regulatory developments, notably the recently signed Basel II accord governing bank capital requirements, supervision and disclosure. It is surprising, then, that relatively little work has been done on the empirical relationship between credit spreads and the macroeconomy. Casual observation points to a close link between credit spreads and the business cycle. But apart from a few studies, the relationship between spreads and macroeconomic variables has largely been left unexamined. Some recent papers have focused on explaining changes in spreads in a regression framework.¹ By contrast, the main objective of this study is to provide new empirical evidence on the role of macroeconomic factors in an *arbitrage-free* affine model of the term structure of credit spreads.

While the literature on spreads and macroeconomic variables is relatively sparse, more work has been done examining the link between default risk and the macroeconomy.² From a theoretical perspective, a systematic relationship between, on the one hand, financial conditions and spreads, and on the other hand, output and inflation, can be explained by a general equilibrium model with a financial accelerator in investment and nominal price rigidities.³ In sum, both the empirical and theoretical literature suggest that real economic activity and inflation have a role to play in determining corporate yields and credit spreads.

1.1 Detailed outline of the chapters

1.1.1 Chapter 1 - The Term Structure of Credit Spreads and the Economic Activity: A Literature Review

In one regard, our work builds on recent studies of affine term structure models of default-free yields with macroeconomic factors. In the first chapter we review the recent contribution of the literature in this area, with a particular emphasis on the role of macroeconomic variables in a no-arbitrage affine model specification. We explore also the interaction between credit spreads and economic activity, providing an introduction to the existing empirical work on reduced-form intensity models.

As the models we implement belong to the affine family, the second part of the

¹See Collin-Dufresne, Goldstein and Martin (2001) and Morris, Neal and Rolph (2001).

²For example, default probabilities depend upon macroeconomic variables in two well-known risk management models, McKinsey's CreditPortfolioView (see Wilson (1997a, 1997b)) and Algorithmic's Mark to Future (see Dembo, Aziz, Rosen and Zerbs (2000)). Cantor and Mann (2003) document the procyclicality of credit quality changes using a long history of Moody's data. Altman, Brady, Resti and Sironi (2005) show that there is a relationship between the correlation of default rates and loss in the event of default and the business cycle.

³See Bernanke, Gertler and Gilchrist (1999).

first chapter provides more details on the theoretical and empirical results achieved so far within the affine framework. The contributions of Dai and Singleton (2000, 2002) and Duffee (2002), all building on the Duffie and Kan (1996) affine framework, have provided unprecedented impetus to researchers in this area. The first chapter mainly focus on various types of affine parameterization when applied to the Government term structures. To carry out this exercise, we examine essentially affine just-yields models, i.e. models where factors are unobservable but conceivable as linear combinations of yields, as well as essentially affine models where the state variables include macroeconomic factors beyond unobserved components. We also outline how term-structure models are estimated through a Maximum Likelihood via the Kalman filter.

1.1.2 Chapter 2 - Macro Factors in the Term Structure of Credit Spreads

In the second chapter we specify and estimate multi-factor affine term structure models of Treasury yields and corporate spreads using monthly data for the United States over the period 1992-2004. For corporate spreads we estimate doubly-stochastic intensity-based models. In the light of the poor performance of firm-value (i.e. Merton-type) models in explaining spreads,⁴ intensity-based models have become increasingly popular for pricing defaultable debt.⁵ Following recent work on Treasury yield curves, we introduce observable macroeconomic variables, along with latent factors, into the state vector of an affine model. Since our focus is on the role of macro variables in affecting yields, the fact that the data used here spans more than one complete business cycle is an important advantage in this study. Second, there is significant empirical evidence to support the conclusion that the Federal Reserve systematically responds to movements in inflation and real activity in setting its target for the federal funds rate. As a consequence, we allow the instantaneous risk-free rate to depend upon both of these variables, whereas other studies before omitted these variables from the model specification. It turns out that Treasury yields, especially at the short end of the maturity spectrum, are strongly affected by the macro factors. Third, we allow for a third latent factor to affect corporate spreads, and we examine the relationship between financial conditions and the factors driving corporate spreads.

We estimate arbitrage-free term structure models of US Treasury yields and spreads on BBB and B-rated corporate bonds in a doubly-stochastic intensity-based framework. A novel feature of our analysis is the inclusion of macroeconomic variables – indicators of real activity, inflation and financial conditions – as well as latent factors, as drivers of term structure dynamics. Our results point to three key roles played by macro factors in the term structure of spreads: they have a significant impact on the level, and particularly the slope, of the curves; they are largely responsible for variation in the prices of systematic risk; and speculative grade spreads exhibit greater sensitivity to macro shocks than high grade spreads.

⁴See, e.g., Huang and Huang (2003) and Eom, Helwege and Huang (2004)

⁵See Jarrow and Turnbull (1995), Lando (1998), and Duffie and Singleton (1999).

Unlike in previous empirical studies on multi-factor intensity-based models of corporate spreads, we do not model the default process at the firm level. Instead, we focus on spread dynamics at the sector level and across credit rating categories. One advantage of focussing on sector-level spreads is that the noise of idiosyncratic firm-level shocks is eliminated, allowing for more efficient estimation of the role of macroeconomic variables in the term structure of spreads. In addition, we are able to better document key differences across sectors and ratings. The main results are based on spreads of BBB-rated industrial firms, which is one of the sector-rating classes with the largest number of outstanding issues in the market, but we also report how our conclusions change when we examine speculative grade industrial firms.

1.1.3 Chapter 3 - Estimates of physical and risk-neutral default intensities using data on EDFsTM and spreads

In the third and last chapter, in addition to estimating risk-neutral default intensities, we provide estimates of physical default intensities using data on Moody's KMV EDFsTM as a forward-looking proxy for default risk. We find that the real and financial activity indicators, along with filtered estimates of the latent factors from our term structure model, explain a large portion of the variation in EDFsTM across time. Furthermore, measures of the price of default event risk implied by estimates of physical and risk-neutral intensities indicate that compensation for default event risk is countercyclical, varies widely across the cycle, and is higher on average and more variable for higher-rated bonds.

Chapter 1 - The Term Structure of Credit Spreads and the Economic Activity: A Literature Review

Maurizio Luisi
University of Lugano

Abstract

In this study we review the literature covering dynamic term structure models of default-free yields, with a particular emphasis on the role of macroeconomic variables in a no-arbitrage affine model specification. We explore also the interaction between credit spreads and economic activity, providing an overview of the existing empirical work on reduced-form intensity models.

As the models we focus on belong to the affine family, part of this study provides more details on the theoretical and empirical results achieved so far within the affine framework. The contributions of Dai and Singleton (2000, 2002) and Duffee (2002) have provided unprecedented impetus to researchers in this area. We focus on various types of affine parameterization: we examine essentially affine just-yields models as well as essentially affine models where the state variables include macroeconomic factors beyond unobserved components.

JEL Classification Numbers: C13, C32, E44, E52, G12, G13, G14

Keywords: corporate bonds, default intensity, event risk, risk premia, interest rate rule, term structure models, state space models, Kalman filter

1 Introduction

In one regard, our work builds on recent studies of affine term structure models of default-free yields with macroeconomic factors (e.g. Ang and Piazzesi (2003), Dewachter and Lyrio (2006), Hordhal, Tristani and Vestin (2006)). In this first chapter we review the recent contribution of the literature in this area, with a particular emphasis on the role of macroeconomic variables in a no-arbitrage affine model specification. We explore also the interaction between credit spreads and economic activity, providing an introduction and overview to the existing empirical work on reduced-form intensity models.

As the models we implement belong to the affine family, the second part of this chapter provides more details on the theoretical and empirical results achieved so far within the affine framework. The contributions of Dai and Singleton (2000, 2002) and Duffee (2002), all building on the Duffee and Kan (1996) affine framework, have provided unprecedented impetus to researchers in this area. In this chapter we mainly focus on various types of affine parameterization when applied to the Government term structures. To carry out this exercise, we examine essentially affine just-yields models, i.e. models where factors are unobservable but conceivable as linear combinations of yields, as well as essentially affine models where the state variables include macroeconomic factors beyond unobserved components. We also outline how term-structure models are estimated through a Maximum Likelihood via the Kalman filter (see Lund, J (1997)).

The rest of this paper is organized as follows. Section 2 focuses on recent studies of interest rate models where the entire term structure is described as a combination of macroeconomic and latent factors. Section 3 introduces studies that have examined the empirical relationship between default risk and macroeconomic conditions. Section 4 describes how reduced-form models with fractional recovery of market value have been implemented empirically and reviews recent contributions to the existing literature linking physical default probabilities to macroeconomic variables. Section 5 introduces briefly the family of affine models which we will use extensively in the remain of this work. In section 6 we outline how term-structure models are estimated through a Maximum Likelihood via the Kalman filter.

2 Term Structure and Economic Activity

In the last decades, starting with the seminal work of Vasicek (1977) and Cox, Ingersoll, and Ross (1985), researchers have built increasingly sophisticated no-arbitrage models of the term structure. These models completely describe the dynamic behavior of yields at all maturities, specifying the evolution of state variables under both the physical and equivalent martingale measures. Much of this research focuses on latent factor settings, in which the state variables are not directly observable; assuming implicitly that the evolution of yields is described in terms of yields themselves.

The influential contribution of Piazzesi (2003) and Ang and Piazzesi (2003) extends this rather restrictive view by including macroeconomic variables in the general affine setting of Duffie and Kan (1996). As a matter of fact a classic macroeconomic perspective would look at the short-term interest rate as a policy instrument under the direct control of the central bank, which adjusts the rate to achieve its economic stabilization goals; on the other hand, from a finance perspective, the short rate is an essential building block for yields of other maturities, which are just risk-adjusted averages of expected future short rates. As illustrated by much recent research, a joint macro-finance modelling strategy provides a more comprehensive understanding of the term structure of interest rates. This new approach gives a better perspective to examine aspects at the boundaries of macroeconomics and finance, since combines in a unique framework this two views.

Intensive research focuses on these issues using models that describe the entire term structure with a combination of macroeconomic and latent factors. Recent work includes Dewachter, Lyrio, and Maes (2006), Dewachter and Lyrio (2006), Hördahl, Tristani, and Vestin (2006), Ang and Bekaert (2003), Ang, Piazzesi, and Wei (2003), and Rudebusch and Wu (2003).

Yield curve models almost invariably employ a structure that consists of a small set of factors and the associated factor loadings that relate yields of different maturities to those factors. Besides providing a useful compression of information, a factor structure is also consistent with the celebrated “parsimony principle”. For example, to capture the time series variation in yields, one or two factors may suffice since the first two principal components account for almost all (99%) of the variation in yields. Also, for forecasting yields, using just a few factors may often provide the greatest accuracy.

However, more than two factors will invariably be needed in order to obtain a close fit to the entire yield curve at any point in time, say, for pricing derivatives. There are a variety of methods employed in the literature.

One general approach places structure only on the estimated factors. For example, the factors could be the first few principal components, which are restricted to be mutually orthogonal, while the loadings are relatively unrestricted. Indeed, the first three principal components typically closely match simple empirical proxies for level (e.g., the long rate), slope (e.g., a long minus short rate), and curvature (e.g., a mid-maturity rate minus a short and long rate average).

A second approach, which is popular among market and central bank practitioners, is a fitted Nelson-Siegel curve. As described by Diebold and Li (2006), this representation is effectively a dynamic three-factor model of level, slope, and curvature. However, the Nelson-Siegel factors are unobserved, or latent, which allows for measurement error, and the associated loadings have plausible economic restrictions (forward rates are always positive, and the discount factor approaches zero as maturity increases).

A third approach is the no-arbitrage dynamic latent factor model, which is the model of choice in finance. The most common subclass of these models postulates flexible linear or affine forms for the latent factors and their loadings along with restrictions that rule out arbitrage strategies involving various bonds. Both the Nelson-Siegel and affine no-arbitrage dynamic latent factor models provide useful statistical descriptions of the yield curve, but they offer little insight into the nature of the underlying economic forces that drive its movements. To shed some light on the fundamental determinants of interest rates, researchers have begun to incorporate macroeconomic variables into these yield curve models. For example, Diebold, Rudebusch, and Aruoba (2006) provide a macroeconomic interpretation of the Nelson-Siegel representation by combining it with VAR dynamics for the macroeconomy.

2.1 The role of macroeconomic variables in a no-arbitrage affine model

Recently, several authors have explored the role of macroeconomic variables in a no-arbitrage affine model.

The interactions between macroeconomic and term structure dynamics have also

been left unexplored in the macroeconomic literature, in spite of the fact that simple “policy rules” have often scored well in describing the dynamics of the short-term interest rate (see Clarida, Galí and Gertler, 2000). An attempt to bridge this gap within an estimated, arbitrage-free framework has recently been made by Ang and Piazzesi (2003). These authors estimate a term structure model based on the assumption that the short term rate is affected partly by macroeconomic variables, as in the literature on simple monetary policy rules, and partly by unobservable factors, as in the affine term-structure literature. Ang and Piazzesi’s results suggest that macroeconomic variables have an important explanatory role for yields and that the inclusion of such variables in a term structure model can improve its one-step ahead forecasting performance. Nevertheless, unobservable factors without a clear economic interpretation still play an important role in their model.

In Ang and Piazzesi (2003) and Ang, Dong, and Piazzesi (2007), the macroeconomic factors are measures of inflation and real activity. The joint dynamics of these macro factors and additional latent factors are captured by VARs. In related papers, Dewachter and Lyrio (2006) and Dewachter, Lyrio and Maes (2006) also estimate jointly a term structure model built on a continuous time VAR. In Ang and Piazzesi (2003), the measures of real activity and inflation are each constructed as the first principal component of a large set of candidate macroeconomic series, to avoid relying on specific macro series. These papers explore various methods to identify structural shocks. In Piazzesi (2005), the key observable factor is the Federal Reserve’s interest rate target. The target follows a step function or pure jump process, with jump probabilities that depend on the schedule of policy meetings and three latent factors, which also affect risk premiums. The short rate is modeled as the sum of the target and short-lived deviations from target. The empirical results show that relative to standard latent factor models using macroeconomic information can substantially lower pricing errors. In particular, including the Fed’s target as one of four factors allows the model to match both the short and the long end of the yield curve. Finally, Rudebusch and Wu (2003) provide an example of a macro-finance specification that employs more macroeconomic structure and includes both rational expectations and inertial elements. They obtain a good fit to the data with a model that combines an affine no-arbitrage dynamic specification for yields and a small fairly standard macro model, which consists of a monetary policy reaction function, an output Euler equation, and an inflation equation. Diebold,

Rudebusch, and Aruoba (2006) examine the correlations between Nelson-Siegel yield factors and macroeconomic variables. They find that the level factor is highly correlated with inflation, and the slope factor is highly correlated with real activity. The curvature factor appears unrelated to any of the main macroeconomic variables. Similar results with a more structural interpretation are obtained in Rudebusch and Wu (2003); in their model, the level factor reflects market participants' views about the underlying or medium-term inflation target of the central bank, and the slope factor captures the cyclical response of the central bank, which manipulates the short rate to fulfill its dual mandate to stabilize the real economy and keep inflation close to target. In addition, shocks to the level factor feed back to the real economy through an *ex ante* real interest rate. Piazzesi (2005), Ang and Piazzesi (2003) and Ang, Dong, and Piazzesi (2007) examine the structural impulse responses of the macro and latent factors that jointly drive yields in their models. For example, Piazzesi (2005) documents that monetary policy shocks change the slope of the yield curve, because they affect short rates more than long ones. Ang and Piazzesi (2003) find that output shocks have a significant impact on intermediate yields and curvature, while inflation surprises have large effects on the level of the entire yield curve. They also find that better interest rate forecasts are obtained in an affine model in which macro factors are added to the usual latent factors. For estimation tractability, Ang and Piazzesi (2003) only allow for unidirectional dynamics in their arbitrage-free model, specifically, macro variables help determine yields but not the reverse. Diebold, Rudebusch, and Aruoba (2006) consider a general bidirectional characterization of the dynamic interactions and find that the causality from the macroeconomy to yields is indeed significantly stronger than in the reverse direction but that interactions in both directions can be important. Ang, Dong, and Piazzesi (2007) also allow for bidirectional macro-finance links but impose the no-arbitrage restriction as well, which poses a severe estimation challenge that is solved via Markov Chain Monte Carlo methods. The authors find that the amount of yield variation that can be attributed to macro factors depends on whether or not the system allows for bidirectional linkages. When the interactions are constrained to be unidirectional (from macro to yield factors), macro factors can only explain a small portion of the variance of long yields. In contrast, the bidirectional system attributes over half of the variance of long yields to macro factors.

Hördahl, Tristani and Vestin (2006) present a unified empirical framework where a

small structural model of the macro economy is combined with an arbitrage-free model of bond yields. Their model proposes to derive bond prices using no-arbitrage conditions based on an explicit structural macroeconomic model, including both forward-looking and backward-looking elements. The fact that they use a structural macroeconomic framework, rather than a reduced-form VAR representation of the data, is one of the main innovative features of their paper. Their approach allow to relax Ang and Piazzesi's restriction that inflation and output be independent of the policy interest rate, facilitating an economic interpretation of the results. The authors document the ability of the model to account for deviations from the expectations hypothesis in the observed term structure, arguing that the model's success is due to both the inclusion of macroeconomic variables in the information set and to the imposition of a large number of no-arbitrage and structural restrictions.

The assumption of no arbitrage ensures that, after accounting for risk, the dynamic evolution of yields over time is consistent with the cross-sectional shape of the yield curve at any point in time. Ang and Piazzesi (2003) present some empirical evidence favorable to imposing no-arbitrage restrictions because of improved forecasting performance.

Both economic and econometric considerations motivate this analysis. The goodness-of-fits of dynamic term structure models (DTSM) depend critically on the specification of the market price of risk (see, e.g., Duffee (2002), Dai and Singleton (2002), Duarte (2004), and Ahn, Dittmar, and Gallant (2002)). Furthermore, as suggested by many statistical tests in the literature, these risk premiums on nominal bonds appear to vary over time, contradicting the assumption of risk-neutrality. To model these premiums, Ang and Piazzesi (2003), Rudebusch and Wu (2003) and Hördahl, Tristani and Vestin (2006) specify time-varying "prices of risk," which translate a unit of factor volatility into a term premium. This time variation is modeled using business cycle indicators such as the slope of the yield curve or measures of real activity. However, Diebold, Rudebusch, and Aruoba (2006) suggest that the importance of the statistical deviations from the expectations hypothesis may depend on the application. However, the functional forms of the market price of risk in these studies are quite restrictive, reflecting a trade-off in continuous-time formulations of dynamic term structure models between generality in pricing and tractability of estimation.

Dai, Le, and Singleton (2006) develop a rich class of discrete-time, nonlinear dy-

dynamic term structure models. Under the risk-neutral measure, the distribution of the state vector resides within a family of discrete-time affine processes that nests the exact discrete-time counter-parts of the entire class of continuous-time models in Duffie and Kan (1996) and Dai and Singleton (2000). By allowing the researcher almost complete freedom in specifying the dependence of the market price of risk on the state vector, this new formulation facilitate empirical investigation of much richer specifications of risk premiums than have so far been examined empirically. Furthermore, the development of the exact discrete-time counterparts to the entire family of affine models examined by Dai and Singleton (2000) substantially expands the family of models within which the macroeconomic underpinnings of the latent risk factors in dynamic term structure models can be tractably studied empirically. To date, the literature on integrating dynamic term structure models with dynamic macroeconomic models (e.g., Rudebusch and Wu (2003), Hordahl, Tristani, and Vestin (2006), and Ang, Dong, and Piazzesi (2007)) has focused exclusively on discrete-time Gaussian dynamic term structure models thereby ruling out a role for either nonlinearity or time-varying second moments in modeling macroeconomic risks.

Neftci (1984) and Hamilton (1989) illustrate how the introduction of multiple regimes with state-dependent probabilities of switching regimes accommodates the asymmetric nature of business cycles, where recoveries tend to take longer than contractions. Following this observation, another body of work introduces multiple regimes into affine DTSMs with latent risk factors. Common to all these studies is the finding that the switches in regimes are closely matched with recessions and expansions in the US economy. Ang and Bekaert (2002) examine the performance of regime-switching VAR models for interest rate data from the US, Germany and UK and find that these models provide better out of sample forecast than one regime models, although do not always perform better in terms of moment matching. Ang, Bekaert and Wei (2008) introduce inflation into a regime-switching model in order to extract measures of ex ante real interest rates within a DTSM. Dai, Singleton, and Yang (2007) estimate a model with two regimes in which the risk of shifting across regimes is priced. In the empirical analysis conducted on US Treasury zero-coupon bond yields they find that the market prices of regime-shift risk vary over time with the stage of the business cycle. More recently, Boyarchenko (2008) considers a regime switching model where the short rate is modelled as a quadratic function of the macroeconomic factors and an affine function

of the unobservable factors. In this model the regime switching is over the parameters of the dependence on the macroeconomic factors, while it is assumed that the behavior of the factors themselves is regime-independent. The latent factors and the parameters of the affine dependence are assumed to be regime-independent as well.

3 Credit Spreads and Economic Activity

There are compelling reasons to expect that spreads are influenced by the macroeconomy. Theoretical models of default risk, as well as general equilibrium models with financial frictions and nominal rigidities, predict systematic relationships between spreads, output and/or inflation (e.g. Bernanke, Gertler and Gilchrist (1999)). Estimates of unconditional correlations indicate a close empirical link between credit spreads, the state of financing conditions faced by borrowers and the business cycle. Indeed, several past studies have examined the empirical relationship between default risk and macroeconomic conditions. Jonsson and Fridson (1996), Chava and Jarrow (2004) and Duffie, Saita and Wang (2007), amongst others, have shown there is a countercyclical relationship between default risk and economic activity. In a study of default loss rates, Altman, Brady, Resti and Sironi (2005) estimate negative correlations between default rates, loss given default and the business cycle. Cantor and Mann (2003) document the procyclicality of credit quality changes using a long history of Moody's credit ratings data.¹ But apart from a few studies, the relationship between corporate bond spreads and macroeconomic variables has been left largely unexamined. Previous work has mainly focused on explaining changes in spreads using regression analysis (e.g. Collin-Dufresne, Goldstein and Martin (2001), Morris, Neal and Rolph (2001)).

One of our main objectives in what follows is to assess the separate impact of the macroeconomy on risk-free rates, expected losses from default and the prices of systematic risk. This leads us to new insights about bond risk premia and the relationship between risk-free rates and spreads. First, recent macro-finance models of the term structure have shown that Treasury bond risk premia are driven by macroeconomic variables. Since default risk tends to rise in recessions when investors' incomes are

¹For a more complete review of how macroeconomic factors have been incorporated into credit risk models, see Allen and Saunders (2003).

relatively low, we would also expect business cycle risk to be priced in spreads. In fact, we find that movements in risk premia on corporate bonds can be largely attributed to our observable macro factors, especially output and inflation risk. Second, an advantage of our approach is that we can shed further light on the source of the negative unconditional correlation between risk-free rates and spreads documented in previous studies (e.g. Duffee (1999)).

3.1 Reference Curves for Spreads

Though the concept of credit spread is easily described, numerous data problems compromise the informativeness of spreads about default risk. With the limitations of each data set in mind, we attempt to identify patterns that are unlikely to have been spuriously induced by measurement problems. Fortunately, some of these problems can be reduced by careful use of the data.

3.1.1 Reference Curves using Corporate Bonds

Using U.S. corporate bond credit spreads in the empirical implementation of corporate bond pricing models involves some consideration on the historical properties of these type of data. One of the main concern in using these data is that many corporate are relatively illiquid, so reliable transactions data for individual bonds are not readily available; therefore, credit spreads can reflect such non-credit factors as liquidity risk. Furthermore many corporate bonds have embedded options, so changes in spreads are not purely associated with changes in default risk. Our strategy for dealing with these data limitations is to present results for a specific data base, which is constructed from historical spreads on corporate bonds compiled from the Bloomberg data base. An attractive feature of this data set is its relatively long history. Matching the peaks in yields to business cycle activity, we see that credit spreads tended to be larger during recessions, a pattern that is consistent with the intuition that default probabilities (or default risk premia, or both) increase during weak economic times. The magnitudes of the fluctuation, at least as a reflection of credit risk, must be interpreted with some caution however, because many of the bonds entering the Bloomberg index have embedded call options. For each bond, at each month end, we computed the difference between the yield on the bond and a constant maturity-Treasury (CMT) yield for a

nearby maturity. The data are monthly, from May, 1992 through April, 2004. Looking ahead, in the second chapter we use this data set to explore the relationships between spreads, observed macro-economic factors and latent variables.

One advantage of using aggregate index data, in contrast to firm-level data (Duffee (1999), Driessen (2005)), is that noise from idiosyncratic firm-level shocks is eliminated, thereby allowing more efficient estimation of the role of macroeconomic variables in the term structure. One disadvantage is that we are unable to assess the relative importance of firm-level versus aggregate shocks in the pricing of individual bonds.

Recent empirical evidence generally supports negative correlations between credit spreads and yields on Treasury bonds of comparable maturities; see, for example, Duffee (1998). Three possible interpretations of the negative sign of these correlations are (i) the effects of macroeconomic business cycles on spreads, (ii) the illiquidity of corporate bonds relative to Treasury bonds, and (iii) supply responses of issuers to changing market conditions. If the likelihood of default increases during recessions and if Treasury rates tend to fall during cyclical downturns, then we might expect a negative correlation between Treasury rates and spreads. This result is also consistent with the implications of a model in which default occurs the first time that the total value of the firm falls below a default-triggering boundary, such as the face value of its liabilities. An increase in the default-free rate implies a higher risk-neutral mean growth rate of assets, and, fixing the initial value of the firm and the default boundary for assets, risk-neutral survival probabilities go up, lowering spreads. One can easily take issue with the idea of holding the market value of the firm fixed for purposes of this calculation. In theory, the market value of the firm is the risk-neutral expectation of the discounted present value of its future cash flows, using market interest rates for discounting. If we instead take the firm's cash-flow process as given and raise interest rates, then the entire path of the market value of firm is lowered, thus advancing its default time and widening spreads. Of course, the effect of market rates on the cash-flow process itself is also to be considered. An alternative explanation for negative correlations is that corporate bond markets are less liquid than Treasury markets. In the Bloomberg data, this problem is mitigated somewhat by the use of actively traded bonds to construct the par curves. Nevertheless, there may be some stale prices, in which case an increase in the Treasury rate may be associated with a decline in spreads, at least until the corporate market reacts to changing conditions. Thirdly, we hear anecdotally that issuers tend to

reduce their supply of new corporate debt when Treasury rates rise. If the demand for corporate debt remains largely unchanged or increases when Treasury rates rise, then corporate yield spreads would tend to narrow. More generally, there are pronounced bond "issuance cycles," suggesting a business-cycle component to supply and demand effects on yield spreads. The performance of equity prices also reflects business-cycle developments, both through changes in discount rates and through expectations about future corporate earnings. Furthermore, the likelihood of default for a firm depends on the value of the firm's assets relative to its liabilities, which is obviously a property of equity prices. Both of these observations suggest that equity returns should be negatively correlated with corporate spreads. These correlations also change with the stage of the business cycle and the credit quality of the issuers, with high-yield firms showing more correlation with equity returns than do low-leverage firms (see Shane (1994)). Credit spreads also show evidence substantial persistence over time. Therefore, in developing a model of risk-neutral default intensities based on historical bond yield spreads, it is not sufficient to know the contemporaneous correlations among spreads, interest rate levels, and other variables that might influence default intensities. In addition, it is helpful to have information about the temporal interactions among these variables.

More extensive studies of the dynamic correlations among corporate bond yields and macroeconomic variables are presented in Morris, Neal, and Rolph (2001) and Collin-Dufresne, Goldstein, and Martin (2001). The latter study finds that a large proportion of the variation in yield spreads is unexplained by the macro information included in their statistical analyses. Their principal-component analysis also suggests that a single, corporate market specific factor also explains most of the variation in spreads that is not accounted for by the macro variables. This suggests that there are important economic factors underlying spread variability that are not well proxied by any of the three predictor variables used in their time-series analysis. In other words, when modeling the market and credit risks of corporate bond portfolios, it seems essential to allow for variation that is idiosyncratic to the corporate-bond market relative to the Treasury and equity markets.

3.1.2 Reference Curves using CDS

In what follow this study focuses only on corporate bond data, but it is worth to mention that in the last few years the CDS market, one of the fastest growing segments of the global financial system, is becoming a suitable source of data for examining credit spreads. A CDS is an insurance contract that protects the buyer against losses from a credit event associated with an underlying reference entity. In exchange for credit protection, the buyer of a default swap pays a regular premium to the seller of protection for the duration of the contract. Most of the initial development in the CDS market was in single-name contracts. However, since late 2003 there has also been increasing activity in contracts related to CDS indices. The notional amount of debt covered by default swaps has been roughly doubling each year for the past decade, and in 2005 is estimated to be over 12 trillion U.S. dollars, according to the British Bankers Association. While the net value exposures is much smaller in CDS space, trading volumes are estimated to be significantly greater than in the underlying bond markets. There are several reasons to focus on the CDS market instead of the cash market. One is that default swaps now play a central role in credit markets: a broad range of investors use default swaps to express credit views; banks use them for hedging purposes; and default swaps are a basic building block in synthetic credit structures. Another is that the relatively high liquidity in the default swap market means that CDS spreads are presumably a fairly clean measure of default and recovery risk compared to spreads on most corporate bonds. This facilitates the identification of credit risk premia.

Blanco, Brennan, and Marsh (2005) show that CDS rates represent somewhat fresher price information than do bond yield spreads. This may be due to the fact that default swaps are “unfunded exposures,” in the language of dealers, meaning that in order to execute a trade, neither cash nor the underlying bonds need to be immediately sourced and exchanged. Default swap rates are therefore less likely to be affected by market illiquidity than are bond yield spreads. The extent of this difference in liquidity is explored in Longstaff, Mithal, and Neis (2005) (Longstaff et al (2005)).

Given the relatively short life of the CDS market, most research on spreads has been conducted using bond data; recent studies of CDS spreads using intensity models include Berndt, Douglas, Duffie, Ferguson and Schranz (2005) (Berndt et al (2005)), Longstaff et al (2005) and Pan and Singleton (2005).

4 Parametric Reduced-Form Models with Macroeconomic and Latent Variables

Reduced-form models with fractional recovery of market value have been implemented empirically for corporate bonds under various assumptions about the determinants of intensities and the reference curves.

In Chapter 2, we will analyse spreads in a multi-factor term structure model subject to restrictions imposed by the absence of arbitrage opportunities. Default risk is modelled using a doubly-stochastic intensity-based framework (Lando (1998), Duffie and Singleton (1999)), where risk-neutral instantaneous default loss rates (“instantaneous spreads”) are assumed to be affine functions of the state variables. One innovation of our approach is that the state vector is comprised of both observable macroeconomic variables – indicators of real activity, inflation and financial activity – and unobserved latent factors. Most of the existing empirical work on reduced-form term structure models has been based on latent factors only (e.g. Duffee (1999), Driessen (2005)). Thus, our paper seeks to extend these earlier studies by drawing additional insights from the inclusion of observable variables as factors. Ideally, we would like to specify a completely observable state space to model yields and spreads, but our findings point to a crucial role played by latent factors in improving the fit of our model to market data. This result may be due to an overly restrictive state space (we include three macro factors), or it may reflect a well-known finding by Collin-Dufresne, Goldstein and Martin (2001) that, in addition to macroeconomic variables, there appears to be a common “unknown” factor in corporate bond returns. Moreover, since we do not explicitly account for liquidity or tax effects on corporate bond prices, such as in Driessen (2005), latent factors in our model may implicitly pick up these other influences on spreads.

In one regard, our work builds on recent studies of affine term structure models of default-free yields with macroeconomic factors (e.g. Ang and Piazzesi (2003), Dewachter and Lyrio (2006), Hordhal, Tristani and Vestin (2006)).² In these models,

²Affine models have been the workhorse in the empirical term structure literature on default-free debt. It is impossible to cite all of the relevant contributions here. See Dai and Singleton (2001) and Piazzesi (2003) for a broad overview of these types of models.

real activity and consumer price inflation are amongst the drivers of government bond yields due to their influence on the risk-free rate and the discount factors agents use to price assets. Motivated by firm-value (Merton-type) models of default risk and the ratings methodologies of rating agencies (e.g. Standard & Poor's (2003)), our model of defaultable bond pricing also includes a measure of financial activity along with real output and inflation as observable state variables. Despite the poor performance of specific formulations of firm-value models in explaining spreads (Huang and Huang (2003), Eom, Helwege and Huang (2004)), regression estimates presented in the second chapter suggest that key drivers of default risk in these models have substantial explanatory power for spreads. Our particular indicator of financial conditions combines information on leverage, interest coverage, cash flow and asset volatility.

4.1 Accommodating Observable Economic and Credit Factors

Up to this point, the econometric models that we have examined treat the short spread s_t as an unobservable state variable. Alternatively, one or more of the state variables could be an observable economic time series that is thought to be related to the degree of credit risk.

Yang (2003) also examines the role of output and inflation in the term structure of spreads. By comparison, Bakshi, Madan and Zhang (2006) (Bakshi et al (2006)) add several firm-specific risk factors, including leverage and volatility, to latent factor models, but do not examine the potential role of output or inflation as factors. They find that including leverage as an observable state variable helps to significantly reduce model pricing errors for high-yield, but not investment grade, bonds.³

Within a *recovery of market value* specification, Bakshi et al (2006) assumed that the short spread s_t is an affine function of the reference short rate r_t and an observed credit related state variable, say X_t . In their alternative formulations, X_t is one of: (i) the ratio of the book value of debt to the total of the book value of debt and the market value of equity, (ii) the ratio of the book value of equity to the market value of equity, (iii) the ratio of operating income to net sales, (iv) a lagged credit spread, and (v) the logarithm of the firm's stock price. Bakshi et al (2006) took the reference

³In other contemporaneous work, Wu and Zhang (2005) examine the role of output, inflation and market volatility in term structure models applied to bond data on individual firms.

short rate r_t to be determined by a two-factor Gaussian state vector. The observable credit factor X was assumed to be a mean reverting Gaussian diffusion, so that the state vector upon which r_t and the short spread s_t depends is a three-factor affine process. The various versions of the model, differing only in terms of the choice of the observable credit variable, were estimated using Lehman Brothers data on individual U.S. corporate issuers. For cases in which X_t is a leverage measure, Bakshi et al (2006) found that, after controlling for interest-rate risk, higher leverage indeed implies a higher short spread s_t . They estimated that the sensitivity of the short spread to leverage is largest among AA-rated firms and lowest among utilities. They also found that leverage-related credit risk is more pronounced for long-maturity than for short-maturity corporate bonds. Similar results were found when X is the book-to-market ratio.

4.2 Physical default probabilities and macroeconomic variables

The default intensity of firms in intensity models typically plays the role of a latent variable which we cannot observe, a role similar to that of the short rate in the classical models of the term structure.

By methods outlined in the next sections, we propose an intensity model for corporate bond prices, we can use filtering techniques to estimate from bond prices the evolution of the intensity. To keep the argument simple we assume in what follows that prices have no liquidity component.

Moreover, the risk-neutral intensity $h_{j,t}^Q$ can be split into the physical intensity $h_{j,t}^P$ and the market price of default event risk $\Gamma_{j,t}$:

$$h_{j,t}^Q = h_{j,t}^P \cdot [1 + \Gamma_{j,t}] \quad (1)$$

The prices of default event risk, which may be bond-specific, differ from the prices of systematic risk, though they could be determined by the same underlying risk factors. Our approach is to derive estimates of the market price of default event risk from estimates of physical and risk-neutral intensities.

Understanding the market prices of risk fully in intensity-based models requires a pairing of empirical default data or default intensities with pricing intensities. Only then can we separate risk premiums for event risk from variations in default risk and obtain correct estimates for both.

The results introduced later in this work add to a small but growing literature on the empirical properties of credit spreads and the risk aversion of credit investors. The most closely related study to this approach is the paper by Berndt et al (2005), who estimate risk premia using CDS data on a set of US firms in different industries and Moody's KMV's Expected Default Frequencies (EDFsTM) as measures of default probabilities. They identify default risk premia by estimating fully specified dynamic credit risk models for each entity.

Berndt et al (2005) find that the average price of default event risk is approximately between one and two, which means that risk-neutral default probabilities are more than twice the size of physical default probabilities even in the absence of systematic risk. To our knowledge, in what follow, we are the first to estimate the market price of default event risk across the business cycle and, in particular, to assess how it is related to observable measures of macroeconomic activity. We find that the price of default event risk is countercyclical, varies significantly across the cycle, and is higher and more variable for higher rated debt.

Elton, Gruber, Agrawal, and Mann (2001) examine how much of the variation over time in spreads (less expected loss and taxes) can be explained by the Fama-French factors, and then calculate a risk premium based on these contributions. Driessen (2005) estimates a dynamic term structure model by dividing spreads into several components. He finds evidence of large and time-varying default risk premia, as well as liquidity premia.

Estimates of physical and risk-neutral default intensities obtained using data on EDFsTM and spreads, respectively, provide new evidence on the size and evolution of the price of default *event* risk. If investors can conditionally diversify credit portfolios – that is, investors can eliminate their exposure to individual defaults – then the default event itself will not be priced (Jarrow, Lando and Yu (2005)). Recent evidence indicates this not to be true and that the market price of default event risk has been large and highly volatile over time (Driessen (2005), Berndt et al (2005), Amato and Remolona (2005)).

Our study contributes to the existing literature linking physical default probabilities to macroeconomic variables. As noted above, several studies on default prediction point to a large negative correlation between default probabilities and the business cycle. While the use of spreads data in the term structure model presented in Chapter 2 only

enables us to uncover *risk-neutral* instantaneous loss rates (see Jarrow, Lando and Yu (2005) for further discussion of this issue), by using an additional source of data on default risk we can also estimate *physical* instantaneous loss rates. In our case, this is accomplished by fitting one-year default probabilities implied by a doubly-stochastic intensity model to Expected Default Frequencies (EDFsTM) from Moody's KMV, which are assumed to be proxies for real world default probabilities. By assuming that physical default intensities are driven by the same factors determining spreads, we are able to explain a large portion of the time series variation in EDFsTM on both BBB and B-rated industrial bonds. As we will show later, the real and financial activity indicators, in particular, have significant marginal predictive power for future default risk.

Bohn (2000), Delianedis and Geske (1998), Delianedis, Geske, and Corzo (1998), and Huang and Huang (2003) use structural approaches to estimating the relationship between actual and risk-neutral default probabilities, generally assuming that the Black-Scholes-Merton model applies to the asset value process, and assuming constant volatility. Eom, Helwege, and Huang (2004) have found that these structural models tend to fit the data rather poorly, and typically underestimate credit spreads, especially for shorter maturity bonds. Chen, Collin-Dufresne, and Goldstein (2005) show an improvement in fit by incorporating an assumption of counter-cyclical default boundaries. Preliminary new work by Saita (2005) estimates the high levels of risk premia that can be obtained for portfolios of corporate debt through diversification.

While Fisher (1959) took a simple regression approach to explaining yield spreads on corporate debt in terms of various credit-quality and liquidity related variables, Fons (1987) gave an earlier empirical analysis of the relationship between actual and risk-neutral default probabilities. Driessen (2005) estimated the relationship between actual and risk-neutral default probabilities, using U.S. corporate bond price data (rather than CDS data), and assuming that conditional default probabilities are equal to average historical default frequencies by credit rating.

Driessen reported an average ratio of risk-neutral to actual default intensities of 1.89, after accounting for tax and liquidity effects, roughly in line with Berndt et al (2005). While the conceptual foundations of Driessen's study are similar to Berndt et al (2005), there are substantial differences in their respective data sources and methodology. First, the time periods covered are different. Second, the corporate bonds underlying Driessen's study are less homogeneous with respect to their sectors, and have

significant heterogeneity with respect to maturity, coupon, and time period. In Berndt et al (2005) each CDS in their rate observations, on the other hand, is effectively a new 5-year par-coupon credit spread on the underlying firm that is not as corrupted, by tax and liquidity effects, as are corporate bond spreads. Third, and most importantly when considering variation of default risk premia over time, Berndt et al (2005) do not assume that current conditional default probabilities are equal to historical average default frequencies by credit rating. Kavvathas (2001) and others have shown that, for a given firm at a given time, the historical default frequency by firms of the same rating is a stale and coarse-grained estimator of conditional default probability. Moody's KMV EDF measures of default probability provide significantly more power to discriminate among the default probabilities of firms (Kealhofer (2003), Bohn, Arora, and Korablev (2005)).

Estimates obtained in this way would need to be tested for robustness to model specification. Recent work by Pan and Singleton (2005) on sovereign CDS spreads, for instance, indicates that estimates of risk aversion can be sensitive to the form of the model. Second, it would be desirable to relate measures of risk aversion and risk premia estimated using CDS data to those obtained from other credit instruments or asset classes, such as equities and government bonds. This would help further our understanding of the extent to which prices on assets in different markets are driven by common forces.

5 Affine modeling

In what follow we introduce briefly the family of affine models which we will use extensively in the remain of this work.

Well after the initial great success encountered by the latent single factor model of Cox et al. (1985), term structure modeling has received huge and renewed attention. The contributions of Dai and Singleton (2000, 2002) and Duffee (2002), all building on the Duffie and Kan (1996) affine framework, have provided unprecedented impetus to researchers in this area. What is the reason for such a change of interest? First of all, the first two authors rationalised the variety of available models, by showing how to derive canonical structures, where all relevant parameters are exactly identified.

Their work helped to show that many pre-existing models often had restrictive and unnecessary assumptions for the yields' dynamics, an example of this being the absence of correlation typically placed on the factors in multivariate applications of the Cox et al. (1985) model. The impact of these restrictions has revealed to be non-trivial: it is precisely the lack of this correlation feature which brings to a dramatic deterioration in the fit of the multivariate CIR model. Second, fitting multivariate affine model to US yields they highlighted the existence of a trade-off between the desire of matching the future mean of the yields (which amounts to satisfying the expectations hypothesis) instead than their second moments or, more generally, other characteristics of the yields' distribution. In other words, when the simple Gaussian affine model, where factors have fixed variance, is enriched by the introduction of stochastic volatility of the type allowed by CIR-like dynamics (i.e. variances proportional to the level of the factors), then the interest rates' distribution is caught with more precision. However, since additional restrictions are placed on the term premia generated by the model, the fit gets worse in terms of other characteristics of the yields. Among these, and more central to typical utilizations of the yield curve among practitioners and central bankers, especially the ability to reproduce the historical relation between future yield changes and the slope of the yield curve, which is well-known to deviate from the prediction of the expectations hypothesis. Gaussian models seem to be best suited to respect this prediction, given the high flexibility allowed to the dynamics of the risk premia required for the fluctuation of the factors, especially the possibility of generating changes in their sign. Allowing for conditional heteroskedasticity, i.e. for time variation in conditional variances, weakens this ability since, in very loose terms, trying to fit conditional second order moments makes the model 'lose track' of the conditional first moments. There is of course of huge number of issues, both theoretical and empirical, that arose in the affine modeling framework, summarised among the others in Piazzesi (2003), which is almost impossible to list. Among these i) the predictability issue, which is dealt with especially in Dai and Singleton (2002) and in a different but related context in Diebold and Li (2006) and Diebold et al. (2006) and ii) the modelisation of term premia, for which the main references are Dai and Singleton (2002), Duarte (2004) and Cheridito, Filipović, and Kimmel (2007)).

5.1 The affine class

Dai and Singleton (2000, 2002), Duffee (2002), Duarte (2004) and Cheridito, Filipović, and Kimmel (2007) present applications and extensions of the affine term structure class developed in Duffee and Kan (1996). Dai and Singleton (2000) provide a classification for exponentially affine models where a set of yields is driven by a given number of factors, N , with a subset of these factors, m , determining the conditional volatility of all of the N factors as well as of the yields. For $N = 3$, the most employed class in empirical analyses concerning the US term structure, the Gaussian case is the simplest model and is named $A_0(3)$, since no factors enter the volatility dynamics, which is conditionally fixed; at the other extreme of the class, the Cox et al. (1985) model, named $A_3(3)$, has each of the factors representing also a volatility factor. In the 3-factor class there are 4 minimal models depending on the value taken by m .

Many papers have clarified that beyond fitting the observed interest rates with a reasonable accuracy, a term structure model should match other relevant features of the data. As concerns bonds only, great emphasis has been placed on matching the expectations hypothesis. Despite the fact that the vast majority of the recent yield-curve literature has focused on richer and richer specifications of the conditional volatility of the yields, following the widespread findings of ARCH effects in asset returns (see Andersen, Benzoni and Lund (2002)), Dai and Singleton showed that to the aim of respecting the expectations hypothesis a Gaussian model, with fixed volatility, overperforms alternative specifications with stochastic volatility. Duffee (2002) also shows that essentially affine models (see next subsection) with fixed volatility forecast yields better than OLS regressions based on the actual slope of the yield curve (so called Fama-Bliss regression) as well as better than the random walk for the 6-month, 2-year and 10-year rates at 3, 6 and 12-month horizons. This finding, which at a first glance may seem counterintuitive, stems from a more realistic, less constrained specification of the risk premia. It can be useful to think of it in the following way: volatilities must be positive while term premia have no sign restrictions (and the ex-post yield differential between long-term bonds and short-term bonds is frequently negative). Ultimately the conclusion is that there exists a trade-off between matching higher order moments of the interest rate process and getting the right values for some characteristics of the first moments. It is important to recall that this trade-off arises only because of the

structure of affine models, but can be easily relaxed under different conditions.

There are also other indications, deriving from trying to apply models estimated on bonds to pricing and hedging interest rate derivatives, which need to be taken into due considerations. As said, there is established evidence that models with stochastic volatility are not so successful in pricing bonds, a finding which originates from the constraints imposed by volatility to the dynamics of risk premia. In addition, Collin-Dufresne, Goldstein and Jones (2004) illustrate very clearly that affine models with stochastic volatility generate time series of bond yields volatility that are negatively correlated with GARCH-based estimates; this seems to happen because in such models volatility is forced to be also a pricing factor for bonds, i.e. to play a cross-sectional role, which leads it to 'lose' its time series properties. It is also unclear if models fitted to bond yields only are able to price accurately derivatives such as swaptions or interest rate caps and floors. This issue relates to the concept of unspanned stochastic volatility put forward by Collin-Dufresne and Goldstein (2002), which amounts to saying that portfolios of derivatives built so to be exposed only to volatility risk are not affected by bonds' volatility. This evidence outlines another fundamental characteristic of term structure models, i.e. that there is a substantial difference between pricing and hedging: even if a model fitted to bond yields were able to produce accurate prices for interest rate derivatives, still hedging performance could be rather poor, as shown among the others by Fan, Gupta and Ritchken (2003).

The general affine term structure model assumes that yields are driven by the state variables $X_{i,t}$, $i = 1, 2, \dots, N$, assumed to follow the N -dimensional physical stochastic process

$$dX_t = K(\Theta - X_t)dt + \Sigma\sqrt{S_t}dW_t,$$

where $X_t = (X_{1,t}, X_{2,t}, \dots, X_{N,t})'$, K , Σ and $K\Theta$ are $N \times N$ matrices, possibly non-diagonal and non-symmetric, S_t is a diagonal matrix with the i^{th} element given by $[S_t]_{ii} = \sqrt{\alpha_i + \beta_i' X_t}$. Risk premia are determined either as

$$\Lambda_t = S_t \cdot \lambda, \tag{2}$$

which is referred to as the completely affine specification or as

$$\Lambda_t = S_t \cdot \lambda_0 \frac{1}{2\mathbf{1}} S_t^- \cdot \lambda_1 \cdot X_t \tag{3}$$

with $[S_t]_{ii}^- = \frac{1}{\sqrt{\alpha_i + \beta'_i X_t}}$ when $\alpha_i + \beta'_i X_t > 0$ and nil elsewhere in the essentially affine scheme of Duffee (2002), where $\lambda_0 = (\lambda_1, \dots, \lambda_N)'$ is a N -vector of constants and λ_1 a $N \cdot N$ matrix of real parameters.

The time t price $P(t, \tau)$ of a zero coupon bond with maturity τ is given by the usual relation

$$P(t, \tau) = E_t^Q \left[e^{-\int_t^{t+\tau} r(u) du} \right],$$

where $E_t^Q[\cdot] = E_t^Q[\cdot | I_t]$ is the expectation with respect to the risk-neutral measure Q , absolutely continuous with respect to the physical density P . By Girsanov theorem it is straightforward to show that the state variables have the following Q -dynamics

$$dX_t = \tilde{K} \left(\tilde{\Theta} - X_t \right) dt + \Sigma \cdot S_t d\tilde{W}(t),$$

where $\tilde{W}(t)$ is an N -dimensional vector of independent Brownian motions, $\tilde{K} = K + \Sigma\Phi$, $\tilde{\Theta} = \tilde{K}^{-1}(K\Theta - \Sigma\psi)$, the i^{th} row of Φ is given by $\lambda_i\beta_i^i$, and ψ is a N -vector whose i^{th} element is given by $\lambda_i\alpha_i$. The risk neutral drift $\mu(t) = \tilde{K}(\tilde{\Theta} - X_t)$ and variance $\sigma'_t\sigma_t = (\Sigma \cdot S_t)'(\Sigma \cdot S_t)$ of Y_t are both affine functions of Y_t , so that zero coupon bond prices are log linear in the state vector X_t .

It is interesting to observe that with risk premia specified as in (2) the state variables are affine under both the P and Q measure; when the risk premia are instead as in (3) then

$$\begin{aligned} dX_t &= (K(\Theta - X_t)dt + \Lambda_t \cdot \Sigma \cdot S_t)dt + \Sigma \cdot S_t dW_t \\ &\quad (K\Theta - KX_t + [\lambda_0 S_t \Sigma + \lambda_1 S_t^2 \Sigma X_t])dt + (\cdot) \\ &= [K\Theta + \lambda_0 \sqrt{\alpha_i + \beta'_i X_t} - (K - [\lambda_1 \Sigma(\alpha_i + \beta'_i X_t)])]dt + (\cdot) \end{aligned}$$

and the drift becomes nonlinear under the physical measure. As an example, in the well-known Cox et al. (1985) case, the physical dynamics is

$$\begin{aligned} dr_t &= (k\theta - kr_t)dt + \sigma\sqrt{r_t}dW_t \\ &= (k\theta - kr_t + \lambda_1\sigma\sqrt{r_t}\sqrt{r_t} + \frac{\lambda_2}{\sqrt{r_t}}\sigma r_t)dt + \sigma\sqrt{r_t}dW_t \\ &= (k\theta - [k - \lambda_1\sigma]r_t + \lambda_2\sigma\sqrt{r_t})dt + \sigma\sqrt{r_t}dW_t \end{aligned}$$

with $\mu_t(\cdot)$ displaying a dependence on the square root of r_t beyond its level. This characteristic does not represent a technical problem but can produce shortcomings

with particular estimation techniques, such as with the Kalman Filter employed in this paper, which could lose its optimality property. On this respect some authors find that despite losing its optimality, the filter still maintains an excellent performance when compared to appropriate estimation procedures, while others choose to adopt a linearization of the drift term, say $\mu_t(\cdot)$, around a chosen value as, in the case of CIR:

$$\frac{\partial \mu_t}{\partial r_t} \Big|_{r=r_0} = [(-k + \lambda_1 \sigma) + \lambda_2 \sigma \frac{\partial r_t^{0.5}}{\partial r_t} \Big|_{r=r_0}.$$

The affine framework states that the short term rate is determined as:

$$r_t = \delta_0 + \sum_{i=1}^N \delta_i X_{i,t} = \delta_0 + \delta'_x \cdot X_t \quad (4)$$

where δ_0 is a scalar and $\delta_x = (\delta_1, \dots, \delta_N)'$. Duffie and Kan (1996) showed that bond prices can be obtained in closed form and guessed a solution of the following exponential type

$$P(t, \tau) = e^{A(\tau) - B(\tau)' X(t)},$$

where the functions of time to maturity, τ , A and B , satisfy the following set of ordinary differential equations

$$\frac{dA(\tau)}{d\tau} = -\tilde{\Theta}' \tilde{K}' B(\tau) + \frac{1}{2} \sum_{i=1}^N [\Sigma' B(\tau)]_i^2 \alpha_i - \delta_0,$$

$$\frac{dB(\tau)}{d\tau} = -\tilde{K}' B(\tau) - \frac{1}{2} \sum_{i=1}^N [\Sigma' B(\tau)]_i^2 \beta_i + \delta_y.$$

which require numerical integration starting from the initial conditions $A(0) = 0$, $B(0) = 0_{N \times 1}$ in specifications with time-varying volatility (see Appendix). Under the Gaussian $A_0(N)$ model, instead, they become difference equations. In fact, while the pricing of a 1-period (or instantaneous) bond is determined by the Taylor rule-type equation (4) $r_t^{(1)} = \delta_0 + \delta'_x \cdot X_t$, n -period bonds are priced via the usual no-arbitrage relation

$$p_t^n = E_t[m_{t+1} p_{t+1}^{n-1}]$$

where p_t^n is the price at time t of a bond with a maturity equal to n . The pricing kernel, m_{t+1} has the standard representation

$$m_{t+1} = \exp(-r_t) \frac{\xi_{t+1}}{\xi_t}$$

so that

$$p_t^1 = E[m_{t+1} \cdot 1] = \exp(-r_t)$$

and since in the exponential affine class of models

$$p_t^1 = \exp(A_1 + B_1 x_t) = \exp(-\delta_0 - \Delta x_t)$$

equating the terms gives:

$$\begin{aligned} A_1 &= -\delta_0 \\ B_1 &= -\Delta \end{aligned}$$

In general, for a n-period bond

$$\begin{aligned} p_t^n &\equiv \exp(A_n + B_n x_t) = Et[m_{t+1} \cdot p_{t+1}^{n-1}] \equiv \\ &\equiv E_t \exp(-r_t - 0.5\lambda'_t \lambda_t - 0.5\lambda'_t \varepsilon_{t+1} + A_{n-1} + B_{n-1} x_{t+1}) \\ &= \exp(-r_t - 0.5\lambda'_t \lambda_t + A_{n-1}) \cdot E_t(\exp(-0.5\lambda'_t \varepsilon_{t+1} + B_{n-1} x_{t+1})) \\ &= \exp(-\delta_0 - \Delta \cdot x_t - 0.5\lambda'_t \lambda_t + A_{n-1}) \cdot \\ &\quad E_t(\exp(-0.5\lambda'_t \varepsilon_{t+1} + B_{n-1}(\Phi \cdot x_t + \Sigma \cdot \varepsilon_{t+1}))) \\ &= \exp(-\delta_0 - \Delta \cdot x_t - 0.5\lambda'_t \lambda_t + A_{n-1} + B_{n-1} \cdot \Phi \cdot x_t) \cdot \\ &\quad E_t(\exp(-0.5\lambda'_t + \Sigma B_{n-1}) \varepsilon_{t+1}) \\ &= \exp(-\delta_0 - \Delta \cdot x_t + A_{n-1} + B_{n-1} \cdot \Phi \cdot x_t) \cdot \\ &\quad \exp(0.5 \cdot B'_{n-1} \cdot \Sigma' \cdot \Sigma \cdot B_{n-1} - B_{n-1} \cdot \Sigma \cdot (\lambda_0 + \lambda_1 x_t)) \end{aligned}$$

Hence

$$\begin{aligned} B_{n+1} &= B_n[\Phi - \lambda_1 \cdot \Sigma] - \Delta \\ A_{n+1} &= A_n - B_n \cdot \Sigma \cdot \lambda_0 + 0.5 B'_n \cdot \Sigma' \cdot \Sigma \cdot B_n - \delta_0 \end{aligned}$$

which is a recursive structure. The presence of unconditional means in the physical factor dynamics, $\mu = K\Theta$, is accounted for by replacing the equation for A_{n+1} as

$$A_{n+1} = A_n - B_n \cdot \Sigma \cdot \lambda_0 + 0.5 \cdot B'_n \cdot \Sigma' \cdot \Sigma \cdot B_n - \delta_0 - B'_n \cdot \mu$$

where the vector μ collects the unconditional means of the factors.

Unrestricted affine models are not the rule since for an arbitrary choice of the parameter vector $\xi \equiv (K, \Theta; \Sigma; \beta, \alpha)$, the conditional variances $[S_t]_{ii}$ may not be not be

positive over the domain of X . The requisite of positive variances, admissibility, calls for a trade off in flexibility between the drift parameters (K and Θ) and the diffusion coefficients (Σ and β), which clarifies that admissibility is intimately related to the presence of stochastic volatility in the states. There is no admissibility problem if $\beta_i = 0$ while it becomes more and more stringent as the number of state variables determining $[S_t]_{ii}$ increases. In the following subsections we present the structure of the models that we wish to employ. To keep the link with the existing empirical term structure literature, we fix $N = 3$, i.e. we will only work with three-factor models.

5.2 Essentially affine three-factor models

Nearly all of the papers dealing with the US term structure have employed a 3-factor framework. For this magic number, 'three', are responsible above all the papers by Litterman and Scheinkman (1991) and Knez, Litterman, and Scheinkman (1994), who first interpreted the three factors as level, slope and curvature of the term structure and showed that, by so doing, they were able to explain great part of yields' variability. The affine class allows us to maintain the three factor approach but also provides for additional flexibility, since different specifications for volatility and risk premia can be chosen.

When $m = 0$ none of the X_t affects the volatility of X_t , so the state variables are homoskedastic and X_t follows a three-dimensional Gaussian diffusion. The one period rate is determined as

$$r_t = \delta_0 + \delta_1 X_{t,1} + \delta_2 X_{t,2} + \delta_3 X_{t,3},$$

while the physical dynamics of X_t is

$$d \begin{pmatrix} X_{t,1} \\ X_{t,2} \\ X_{t,3} \end{pmatrix} = \begin{bmatrix} (K\theta)_1 \\ (K\theta)_2 \\ (K\theta)_3 \end{bmatrix} - \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \begin{pmatrix} X_{t,1} \\ X_{t,2} \\ X_{t,3} \end{pmatrix} dt + \Sigma S_t dW_t,$$

$$S_{t(ii)} = \sqrt{\alpha_i + (\beta_{i1} \beta_{i2} \beta_{i3}) X_t},$$

with risk premia

$$\Lambda_t = S_t \begin{pmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \end{pmatrix} + S_t^- \begin{pmatrix} \lambda_{1(11)} & \lambda_{1(12)} & \lambda_{1(13)} \\ \lambda_{1(21)} & \lambda_{1(22)} & \lambda_{1(23)} \\ \lambda_{1(31)} & \lambda_{1(32)} & \lambda_{1(33)} \end{pmatrix} X_t. \quad (5)$$

The elements of the diagonal matrix S_t^- are such that:

$$S_{t(ii)}^- = \begin{cases} \left(\alpha_i + \beta'_i X_t\right)^{-1/2}, & \text{if } \inf\left(\alpha_i + \beta'_i X_t\right) > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

In the $E_0(3)$, $\alpha_i = 1$ ($i = 1, 2, 3$) and $\beta_{ij} = 0$ ($i, j = 1, 2, 3$) so that the risk premia become a linear function of the unobservable factors, i.e. $\Lambda_t = \lambda_0 + \lambda_1 \cdot X_t$. It is easy to verify that despite having fixed volatility, the Gaussian $E_0(3)$ model has not necessarily less flexibility in fitting the yield dynamics compared, for example, to the $E_1(3)$ (a model in which one of the three factors represents the conditional volatility of the system). Rewriting the above model assuming that X_3 plays the role of the volatility factor, then

$$\Lambda_t = \begin{pmatrix} \sqrt{\alpha_1 + \beta_1 X_{3,t}} & & \\ & \sqrt{\alpha_2 + \beta_2 X_{3,t}} & \\ & & \sqrt{X_{3,t}} \end{pmatrix} \begin{pmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \end{pmatrix} + \begin{pmatrix} \sqrt{\alpha_1 + \beta_1 X_{3,t}} & & \\ & \sqrt{\alpha_2 + \beta_2 X_{3,t}} & \\ & & \sqrt{X_{3,t}} \end{pmatrix}^{-1} \begin{pmatrix} \lambda_{1(11)} & \lambda_{1(12)} & \lambda_{1(13)} \\ \lambda_{1(21)} & \lambda_{1(22)} & \lambda_{1(23)} \\ 0 & 0 & 0 \end{pmatrix} X_t$$

so that the risk premium for the fluctuation of the first state variable becomes:

$$\Lambda_{1,t} = \frac{1}{\sqrt{\alpha_1 + \beta_1 X_{3,t}}} [\lambda_{01} + \lambda_{1(11)} X_{1,t} + \lambda_{1(12)} X_{2,t} + \lambda_{1(13)} X_{3,t}]$$

which evidences the additional constraint imposed over the $E_0(3)$ obtained from (5) after setting $\alpha_i = 0, \beta_i = 0$ in (6).

$$\Lambda_{1,t} = \lambda_{01} + \lambda_{1(11)} X_{1,t} + \lambda_{1(12)} X_{2,t} + \lambda_{1(13)} X_{3,t}$$

More compactly, in the $E_1(3)$ and the $E_2(3)$ risk premia follow more and more constrained processes, assuming X_1 in the $E_1(3)$ case and X_1 and X_2 in the $E_2(3)$ case to display stochastic volatility, as:

$$\Lambda_t(E_1(3)) = \begin{bmatrix} \frac{\lambda_{10}}{\sqrt{X_{1,t}}} + \lambda_{11} \cdot \sqrt{X_{1,t}} \\ \frac{\lambda_{20} + \lambda_{21} \cdot X_{1,t} + \lambda_{22} \cdot X_{2,t} + \lambda_{23} \cdot X_{3,t}}{\sqrt{\alpha_2 + \beta_{21} X_{1,t}}} \\ \frac{\lambda_{30} + \lambda_{31} \cdot X_{1,t} + \lambda_{32} \cdot X_{2,t} + \lambda_{33} \cdot X_{3,t}}{\sqrt{\alpha_3 + \beta_{31} X_{1,t}}} \end{bmatrix}$$

and

$$\Lambda_t(E_2(3)) = \begin{bmatrix} \frac{\lambda_{10} + \lambda_{11} \cdot X_{1,t} + \lambda_{12} \cdot X_{2,t}}{\sqrt{X_{1,t}}} \\ \frac{\lambda_{20} + \lambda_{21} \cdot X_{1,t} + \lambda_{22} \cdot X_{2,t}}{\sqrt{X_{2,t}}} \\ \frac{\lambda_{30} + \lambda_{31} \cdot X_{1,t} + \lambda_{32} \cdot X_{2,t} + \lambda_{33} \cdot X_{3,t}}{\sqrt{\alpha_3 + \beta_{31} X_{1,t} + \beta_{32} X_{2,t}}} \end{bmatrix}$$

As for why different models in the $E_m(3)$ class fit certain characteristics of yields, it is also important to note that, being the price of risk:

$$\Lambda_t = (\sigma_t)^{-1} \cdot (\mu_X^P - \mu_X^Q)$$

the specification of the completely affine framework,

$$\Lambda_t = S_t \cdot \lambda_0$$

implies i) that risk premia are proportional to interest rate volatility and ii) that, since the instantaneous expected excess return on a τ - period zero-coupon bond over the riskless rate ($r_t^{(1)}$) is

$$E[\mu_X - r^{(1)}] = -B(\tau) \cdot \Sigma \cdot S_t \cdot S_t \cdot \lambda_0 \quad (7)$$

where $B(\tau)$ is the factor loading on the state vector X_t from the affine pricing relation $P(t, \tau) = e^{A(\tau) - B(\tau)'X(t)}$, and since in Gaussian models $\Sigma = S_t = I$, that risk premia will be constant through time while iii) in the other three cases ($E_1(3)$, $E_2(3)$, $E_3(3)$) they will have the same sign over time. In the essentially affine specification suggested by Duffee (2002) (see also Dai and Singleton (2002) and Duarte (2004)):

$$\Lambda_t = S_t \lambda_0 + S_t^- \lambda_1 X_t,$$

$$E[\mu_X - r] = -B(\tau) \cdot \Sigma \cdot S_t \cdot (S_t \lambda_0 + S_t^- \lambda_1 X_t)$$

and the risk factors, X_t , have a direct effect on the market prices of risk in addition to the effect that passes through factor volatilities. In this specification, ex-ante excess returns on bond are time-varying and are allowed to switch sign over time, according to a feature found in ex-post data. Note that beyond the term premia in (7) the model allows to calculate forward premia, which in the case of the n -maturity rate and the k -month horizon are defined as

$$fw_{t,k}^{(n)} = f_{t,k}^{(n)} - \frac{1}{2} [B(n)' \cdot E_t(r_{t+\tau})] \quad (8)$$

where $fw_{t,k}^{(n)}$ is the k-period forward for a rate of maturity n , as well as yield premia, as

$$yp_{t,k}^{(n)} = \sum_{i=1}^n fw_{t,i}^{(1)} \quad (9)$$

An increasing strand of literature has recently started to move away from the just-yields model, trying to specify affine schemes in which the unobservable factors affect yields along with suitably chosen macroeconomic variables. Albeit other authors have addressed this issue, the first contribution within the affine framework is due to Ang and Piazzesi (2003). In this paper, two observable factors, denotable as inflation and growth, coexist with three standard unobserved factors. Though causality runs only in one direction, i.e. there is no feedback between the short term rate (i.e. the monetary policy stance) and the macroeconomic variables, the paper provides a framework in which the impact of the two macro variables on the shape of the yield curve can be formally assessed. The monetary policy feedback is instead present in Hordhal, Tristani and Vestin (2006) in which a rational expectations forward looking model is nested within an affine framework and solved, yielding a vector autoregressive structure of the first order. Similarly, Dewachter and Lyrio (2006) as well as Dewachter, Lyrio and Maes (2006) use inflation, growth and the short rate itself as relevant pricing factors; in addition, they allow for time-varying means (i.e. time-varying central tendencies) for each of the factors and show that most of the fitting ability of their model comes indeed from this last feature.

Over very long samples, as those employed in this study, it is important to try and disentangle the contribution of macroeconomic variables from the contribution of factors which generally bear a correlation with some macroeconomic variable but whose identification is always a matter of subjectiveness. Here we introduce an example which parallels the scheme employed by Ang and Piazzese (2003), where the state variables are composed by three unobservables factors named $X_{t,i}^{un}$ ($i=1,2,3$) and two observables $X_{t,j}^{ob}$ ($j=1,2$) which are identified as inflation and growth. The states $X_t = [X_{1,t}^{un}, X_{2,t}^{un}, X_{3,t}^{un}, X_{1,t}^{ob}, X_{2,t}^{ob}]$ evolve as before as a VAR(1) under the physical measure.

$$d \begin{pmatrix} X_{t,1}^{un} \\ X_{t,2}^{un} \\ X_{t,3}^{un} \\ X_{t,1}^{ob} \\ X_{t,2}^{ob} \end{pmatrix} = \begin{bmatrix} (K\theta)_1 \\ (K\theta)_2 \\ (K\theta)_3 \\ (K\theta)_4 \\ (K\theta)_5 \end{bmatrix} - \begin{pmatrix} k_{11} & k_{12} & k_{13} & 0 & 0 \\ k_{21} & k_{22} & k_{23} & 0 & 0 \\ k_{31} & k_{32} & k_{33} & 0 & 0 \\ 0 & 0 & 0 & \bar{k}_{44} & \bar{k}_{45} \\ 0 & 0 & 0 & \bar{k}_{54} & \bar{k}_{55} \end{pmatrix} \begin{pmatrix} X_{t,1}^{un} \\ X_{t,2}^{un} \\ X_{t,3}^{un} \\ X_{t,1}^{ob} \\ X_{t,2}^{ob} \end{pmatrix} dt + \Sigma S_t dW_t,$$

$$S_{t(ii)} = \sqrt{\alpha_i + (\beta_{i1} \beta_{i2} \beta_{i3}) X_t},$$

and the short rate is determined according to the following Taylor-rule type equation

$$r_t = \bar{\delta}_0 + \bar{\delta}_y X_{t,y}^{ob} + \bar{\delta}_\pi X_{t,\pi}^{ob} + \delta_1 X_{t,1}^{un} + \delta_2 X_{t,2}^{un} + \delta_3 X_{t,3}^{un},$$

The risk premia are proportional to the factors

$$\Lambda_t = S_t \begin{pmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \\ 0 \\ 0 \end{pmatrix} + S_t^- \begin{pmatrix} \lambda_{1(11)} & \lambda_{1(12)} & \lambda_{1(13)} & 0 & 0 \\ \lambda_{1(21)} & \lambda_{1(22)} & \lambda_{1(23)} & 0 & 0 \\ \lambda_{1(31)} & \lambda_{1(32)} & \lambda_{1(33)} & 0 & 0 \\ 0 & 0 & 0 & \lambda_{1(44)} & \lambda_{1(45)} \\ 0 & 0 & 0 & \lambda_{1(54)} & \lambda_{1(55)} \end{pmatrix} X_t.$$

where as before the diagonal matrix S_t^- is

$$S_{t(ii)}^- = \begin{cases} \left(\alpha_i + \beta'_i X_t \right)^{-1/2}, & \text{if } \inf \left(\alpha_i + \beta'_i X_t \right) > 0 \\ 0, & \text{otherwise.} \end{cases}$$

In this macro essentially affine scheme the VAR(1) which determines the dynamics of the observables is estimated in a first step and the relevant parameters $[(k\theta)_4, (k\theta)_5, k_{44}, k_{45}, k_{54}, k_{55}]$ are kept fixed at these values. It is only the compensation for the fluctuation of growth and inflation that is left to be estimated, i.e. $\lambda_{1(44)}, \lambda_{1(55)}, \lambda_{1(45)}$ and $\lambda_{1(54)}$, in addition to the factor loadings $B_4^{(\tau)}$ and $B_5^{(\tau)}$ in the pricing relation $P_t^{(n)} = \exp(A^{(n)} + B_1^{(n)} X_{1,t}^{un} + \dots + B_4^{(n)} X_{1,t}^{ob} + B_5^{(n)} X_{2,t}^{ob})$.

Allowing for macroeconomic factors to affect the term structure could also mitigate the presence of model misspecification coming from regime shifts which typically contaminate long samples. Some papers have recently addressed the problem of allowing for regime shifts in affine schemes, as Ang and Bekaert (2002), Dai, Singleton and Yang (2007) and Bansal and Zhou (2002). The last authors show that a 2-factor model with regime shifts is able to outperform a 3-factor affine scheme from many standpoints. They first focus on predictability, intended as getting a slope of unity in the following regression

$$R_{t+1}^{n-1} - R_t^n = \alpha + \beta[R_t^n - r_t^{(1)}] + \frac{1}{n-1} e_t^{(n)} + \varepsilon_{t+1}$$

where $e_t^{(n)}$ is the expected term premium for a n-period bond over the riskless rate, and then also consider reproducing the tent-shaped pattern found by Cochrane and

Piazzesi (2005) when regressing changes in yields on 1-year forward rates

$$R_{t+1}^n - R_t^n = \alpha + \beta_1 f_{1,1,t} + \beta_2 f_{1,2,t} + \dots + \beta_k f_{1,k,t} + \varepsilon_{t+1}$$

In addition to predictability they also examine the dynamic characteristics of the data, as conditional variances and correlations of yields, obtained by simulating (reprojecting) their model. To both aims, predictability and dynamic features, the 2-factor regime switching model is superior to any other specification based on US data observed monthly between 1964 and 2001.

It is important to notice that one expects the regimes identified by the term structure model to behave much like business cycle regimes or as monetary policy regimes, the two being eventually related. However the correlation between the regimes and business cycles, albeit positive, is generally very low, as in Dai, Singleton and Yang (2007) or in Ang and Bekaert (2002). The problem with unidentified regimes is that it becomes difficult to understand why the switch in the pricing relation occurs; at least unobserved factors can always be thought of in terms of linear combination of yields, but regimes are difficult to conceive if not in relation with changes in well defined macro variables. However, if one is interested in term structure models where the regime is identified ex-post, then it is very easy to modify the macroeconomic affine model described above to account for shifts in parameters as a function of the state of the economy. Of course, since there is no uncertainty in the identification of the business cycle regime, there will be no compensation for business cycle risk (i.e. there is no risk premium for jumps in the business cycle phase) although the pricing function as well as the risk premia will be allowed to differ across regimes.

A Estimation Methods for Affine Models

In the attempt to estimate dynamic models of the term structure, different econometric approaches have been proposed and used by researchers. Among these the most popular are the Generalized Method of Moments (Singleton (2001), Gibbons and Ramaswamy (1993)), the Simulated Maximum Likelihood (Pedersen (1995), Santa-Clara (1995), Piazzesi (2001)), the Hermite Expansions (Ait-Sahalia (2001)), the Quasi-Maximum

Likelihood (Lo (1988), Fisher and Gilles (1996), Duffee (2002)), and the Fourier Inversion of the Characteristic Function (Singleton (2001)). A comprehensive review of these methods is in Piazzesi (2003). Affine models can be naturally cast as state space systems, where the observation equation links observable yields to the state vector and the transition equation describes the dynamics of the state. As the underlying state variables are unobservable, problems related to employing proxies of the factors are ruled out.

In this section we outline a technique for estimating the parameters of intensity models for bond prices which exploits the similarities between ordinary term structure modeling and the modeling of corporate bonds.

A.1 Estimation methodology: Estimation using the Kalman Filter

The Kalman filter is a standard tool for estimating latent-factor models for the term structure of government bonds and for corporate bonds. The Kalman filter approach has been used in the affine term-structure literature in Duan and Simonato (1995), Lund (1997), Geyer and Pichler (1998), de Jong (2000), and Babbs and Nowman (1999). More recently Duffee and Stanton (2004) show that a linearized Kalman filter is a reasonably accurate estimation technique for dynamics term structure models (henceforth DTSM). Suppose to have time series of length T for M zero-coupon bond yields with constant maturities $Y_t = (y_t(\tau_1), \dots, y_t(\tau_M))$, where $y_{it} = -\ln P_{it}/\tau_i$ and τ_i is the time to maturity of bond i . From Duffee and Kan (1996), zero-coupon bond prices are given by

$$P_t(\tau) = e^{A(\tau) - B(\tau)^\top X(t)},$$

hence the following relationship between zero-coupon yields and zero-coupon bond prices exists

$$y_t(\tau) = -\frac{\ln P_t(\tau)}{\tau} = \frac{1}{\tau} \left(-A(\tau) + B(\tau)^\top X(t) \right). \quad (10)$$

The absence of arbitrage is imposed through the values taken by A and B . For the general class of affine model the functions $A(\tau)$ and $B(\tau)$ can be obtained as solution

of Riccati ordinary differential equations (ode)

$$\begin{aligned}\frac{dA(\tau)}{d\tau} &= -\tilde{\Theta}' \tilde{K}' B(\tau) + \frac{1}{2} \sum_{i=1}^N [\Sigma' B(\tau)]_i^2 \alpha_i - \delta_0, \\ \frac{dB(\tau)}{d\tau} &= -\tilde{K}' B(\tau) - \frac{1}{2} \sum_{i=1}^N [\Sigma' B(\tau)]_i^2 \beta_i + \delta_X\end{aligned}$$

where $K = \tilde{K} - \Sigma \begin{pmatrix} \lambda_{01}\beta'_1 & \dots & \lambda_{0n}\beta'_n \end{pmatrix}' - \Sigma \sqrt{S_t} \sqrt{S_t'} \lambda_1$ and $K\Theta = \begin{pmatrix} \tilde{K}\tilde{\Theta} \end{pmatrix} - \Sigma \begin{pmatrix} \alpha_1\lambda_{01} & \dots & \alpha_1\lambda_{01} \end{pmatrix}'$.

For simple Gaussian and CIR type affine models with uncorrelated factors, closed form solution for the functions $A(\tau)$ and $B(\tau)$ are available.⁴ The ODE were solved using standard numerical integration methods based on an explicit Runge-Kutta formula.

Bond yields are then affine functions of the state vector:

$$y_t(\tau) = -a(\tau) + b^\top(\tau) X(t), \quad (11)$$

for $a(\tau) = A(\tau)/\tau$ and $b(\tau) = B(\tau)/\tau$.

The vector of observable interest rates at time t , Y_t , is assumed to be related to the vector of unobserved state variables X_t via the measurement equation:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_M) \end{pmatrix} = \begin{pmatrix} \frac{A_i(\tau_1)}{\tau_1} \\ \frac{A_i(\tau_2)}{\tau_2} \\ \vdots \\ \frac{A_i(\tau_M)}{\tau_M} \end{pmatrix} - \begin{pmatrix} \frac{B_1(\tau_1)}{\tau_1} & \frac{B_2(\tau_1)}{\tau_1} & \frac{B_3(\tau_1)}{\tau_1} \\ \frac{B_1(\tau_2)}{\tau_2} & \frac{B_2(\tau_2)}{\tau_2} & \frac{B_3(\tau_2)}{\tau_2} \\ \vdots & \vdots & \vdots \\ \frac{B_1(\tau_M)}{\tau_M} & \frac{B_2(\tau_M)}{\tau_M} & \frac{B_3(\tau_M)}{\tau_M} \end{pmatrix} \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Mt} \end{pmatrix}, \quad (12)$$

We can rewrite the measurement equation system as,

$$Y_t(X_t; \Psi, \tau) = d(\Psi, \tau) + Z(\Psi, \tau) X_t + \varepsilon_t, \quad (13)$$

⁴For an essentially affine gaussian three-factor term structure model $A_0(3)$, the absence of arbitrage is imposed by computing A and B from the following difference equations:

$$A(\tau+1) - A(\tau) = -B^\top(\tau) \lambda_0 + \frac{1}{2} B^\top(\tau) \Sigma \Sigma^\top B(\tau) + A(1),$$

$$B(\tau+1) = -\lambda_1^\top \Sigma B(\tau) - K^\top B(\tau) + B(1), \quad \text{for } \tau = 1, 2, \dots, M.$$

where the initial conditions are given by $A_1 = \frac{1}{32}\delta_0$ and $B_1 = -\delta_y$.

where $d(\Psi, \tau) = \frac{A(\tau)}{\tau}$ and $Z(\Psi, \tau) = -\frac{B(\tau)}{\tau}$.

The measurement error ε_t is assumed to be normally distributed:

$$\varepsilon_t \sim N(0, H(\Psi)), \quad (14)$$

where the variance-covariance matrix of the measurement errors is assumed to be diagonal as $H = \text{diag}(h_1, \dots, h_M)$. In our notation, the vector Ψ contains the unknown parameters of the model including those related to measurement errors. The i^{th} rows of the matrices $d(Mx1)$ and $Z(Mx3)$ are given by $a(\Psi, \tau)$ and $b(\Psi, \tau)$, respectively. Since H has been assumed to be diagonal, there are no serial correlation and cross correlation in these measurement errors for the bond rates. Of course elements on the diagonal are allowed to differ, so that the variance of measurement errors depends on maturity.

In general, we only require one zero-coupon rate for each factor used in the estimation. That is, considering a three-factor model, only three observed zero-coupon yields are needed. By adding rates in the estimation, however, we provide cross-sectional information about the term structure of interest rates at each observed point in time. This information is particularly helpful in detecting the price of risk parameters as well as to improve the precision of the estimates. Geyer and Pichler (1998) show that for short and long maturities error variances turn out to be higher than for the other maturities and other studies highlight that error variances have a U-shaped pattern. To accommodate this U-shape pattern and to reduce the number of parameters to be estimated, Brandt and He (2006) suggest to parameterize the variance of the errors as a quadratic function of the time-to-maturity

$$h^2(\tau) = \exp(p_0 + p_1\tau + p_2\tau^2),$$

which allows for heterogeneity in the measurement errors without requiring a separate parameter for each maturity. In what follows we use both approaches to model the measurement errors of the estimated models.

The vector of unobservable state variables X_t follows a discrete time Markov process, i.e. the transition equation is:

$$X_t = c_t(\Psi) + \Phi_t(\Psi) X_{t-1} + u_t \quad (15)$$

where the standardized innovations u_t are i.i.d. with zero mean and variance⁵, given

⁵From now on we indicate with I_t the filtration generated by the measurement system or, more

by

$$u_t \mid I_{t-1} \sim i.i.d. (0, V(X_{t-1}, \Psi)). \quad (16)$$

The conditional mean of the unobservables is $E[X_t \mid X_{t-1}] = c_t(\Psi) + \Phi_t(\Psi) X_{t-1}$ and their covariance matrix $Var[X_t \mid X_{t-1}] = v_0 + v_1 X_{t-1}$, which are both linear in X_{t-1} , due to the affine form of the continuous time process. The Appendix reports analytical expressions for the first two conditional moments.

In one factor models the K parameters determine the rate of mean reversion. In the multivariate case the functional forms for $\Phi(\Psi)$ is given by e^{-Kh} , that is the matrix exponential function, defined as $\exp[-Kh] = \sum_{k=0}^{\infty} \frac{1}{k!} (-Kh)^k$, where h is the size of the time interval over which the data are sampled (here $h = 1/12$ for monthly data). If the k_{ii} parameter is close to zero, the state variable is a random walk. The covariance matrix for the error terms in the measurement and transition equations can be written as

$$E_{t-1} \left[\begin{pmatrix} u_t \\ \epsilon_t \end{pmatrix} \begin{pmatrix} u_t \\ \epsilon_t \end{pmatrix}' \right] = \begin{bmatrix} V_t & 0 \\ 0 & H \end{bmatrix} \quad (17)$$

noting that, in the general case, V_t is time varying as it depends on X_{t-1} . The error terms of the measurement (ϵ_t) and transition equations (u_t) are assumed to be not correlated.

Given the measurement equation

$$Y_t(\Psi) = d(\Psi) + Z(\Psi) X_t + \epsilon_t, \quad (18)$$

the transition equation is obtained from the exact discrete-time distribution of the state variables. In the Gaussian case, the discrete-time distribution is a VAR(1) model with Gaussian innovations (see Lund (1997) for derivations). For the general affine DTSM and for $h=1$, we have that

$$X_t = c_t(\Psi) + \Phi_t(\Psi) X_{t-h} + u_t, \quad (19)$$

where the discrete time factors dynamics is driven by the matrix

$$\Phi_t(\Psi) = \exp(-K\Delta_t),$$

formally,

$$I_t = \sigma \{y_0, y_1, \dots, y_M\},$$

is the information set available at time t .

and

$$c_t(\Psi) = \int_{t-h}^t \exp(-K(t-v)) K \Theta dv = [I - \exp(-K\Delta_t)] \Theta.$$

In the general case the factors dynamics are time-varying, and the covariance matrix of the innovations is given by

$$V(X_{t-h}, \Psi) = \int_{t-h}^t \exp(-K(t-s)) F(t, s) \exp(-K'(t-s)) ds. \quad (20)$$

Note that in the $E_0(3)$ model $c_t(\Psi)$ is set to zero and V_t is not time varying as it does not depend on X_{t-1} .⁶

For each model the approximate Kalman filter recursion is initialized with the stationary mean and variance of the unobserved state variables. The conditional forecast of the measurement equation is:

$$E[Y_t | I_{t-h}] = d + ZE[X_t | I_{t-h}] \quad (21)$$

where

$$\hat{X}_{t|t-h} = E[X_t | I_{t-h}] = c_t + \Phi_t \hat{X}_{t-h}$$

The associated conditional variance is,

$$\hat{\Omega}_{t|t-h} = E \left[(X_t - \hat{X}_{t|t-h}) (X_t - \hat{X}_{t|t-h})' | I_{t-h} \right] = \Phi_t \hat{\Omega}_{t-h} \Phi_t^\top + V_t \quad (22)$$

We now observe the true value of the measurement system, v_t , which is

$$v_t = Y_t - E[Y_t | I_{t-h}] = Y_t - d - Z\hat{X}_{t|t-h} \quad (23)$$

the covariance matrix of the prediction errors is given by

$$F_t = Cov[Y_t | I_{t-h}] = E[v_t v_t^\top | I_{t-h}] = Z\hat{\Omega}_{t|t-h} Z^\top + H \quad (24)$$

At this point the prediction error v_t updates the inference about the unobserved transition system, providing the unknown values of the state system for the next time

⁶In particular, the conditional variance of the transition system for a gaussian model has the following form:

$$V_t(\Psi) = \int_{t-h}^t \exp[-K(t-s)] \exp[-K'(t-s)] ds$$

period conditioning on the updated values for the previous period. The update step uses the additional information to revise the conditional expectation of X_t and obtain the filtered estimation as the expression

$$\hat{X}_t = E[X_t | I_{t-h}] = \hat{X}_{t|t-h} + \hat{\Omega}_{t|t-h} Z^\top F_t^{-1} v_t \quad (25)$$

where the gain matrix F_t determines the weight given to the new observation (as summarized by the prediction error, v_t). The conditional variance of the state is updated as

$$\begin{aligned} \hat{\Omega}_t &= E \left[(X_t - \hat{X}_t) (X_t - \hat{X}_t)' | I_t \right] = \hat{\Omega}_{t|t-h} - \hat{\Omega}_{t|t-h} Z^\top F_t^{-1} Z \hat{\Omega}_{t|t-h} = \\ &= \left(I - \hat{\Omega}_{t|t-h} Z^\top F_t^{-1} Z \right) \hat{\Omega}_{t|t-h} = \left(\hat{\Omega}_{t|t-h}^{-1} + Z^\top F_t^{-1} Z \right)^{-1}. \end{aligned} \quad (26)$$

The exact log-likelihood function for a linear, Gaussian state space model is given by:

$$\log L(Y_t, \Psi) = \sum_{t=1}^T -\frac{N}{2} \log(2\pi) - \frac{1}{2} \log |F_t| - \frac{1}{2} v_t^\top F_t^{-1} v_t.$$

Generalizing the Kalman filter to non-Gaussian exponentially-affine models turns out to be somewhat problematic (when $\beta \neq 0$). The structure of the measurement equation is as in the Gaussian case, but the transition dynamics are no longer a Gaussian VAR(1). With a non-Gaussian distribution for the state variables the linear Kalman filter is no longer optimal, and we do not obtain the exact likelihood function. We adopt the same approach used among others by Lund (1997) and Duan and Simonato (1995) that propose a Gaussian QML, where is assumed a normal distribution for the innovations in the transition equation.⁷

⁷The transition equation is obtained from the first and second conditional moments of the state variables:

$$X_t = c_t(\Psi) + \Phi_t(\Psi) X_{t-h} + u_t$$

where

$$Cov[u_t | X_{t-h}] = V(X_{t-h}, \Psi)$$

is obtained from (20). Note that there are two differences between this transition equation and the Gaussian counterpart. First, u_t is not normally distributed in (19), not even conditionally. Second, in the conditional covariance matrix of u_t is an affine function of the lagged state vector, X_{t-h} . The

To find the optimal parameter set we maximize the log-likelihood function using successive applications of a simplex search method. When the Kalman filter recursion produce negative estimates of the state vector $\hat{X}_{i,t}$ (for $i \leq m$), in models with stochastic volatility for the variables that represent the variance, we modify the standard filter by replacing any negative element of the state estimate \hat{X}_t , with a small positive number.⁸

For quasi-maximum-likelihood estimates White (1982) proposes a robust approximation of the variance-covariance matrix given by:

$$E \left(\hat{\Psi} - \Psi \right) \left(\hat{\Psi} - \Psi \right)' \simeq T^{-1} \{ I_{2D} I_{OP}^{-1} I_{2D} \}^{-1}$$

where I_{2D} is computed by evaluating the actual second derivatives matrix of the log-likelihood function at the maximum likelihood estimates; I_{OP} is equal to outer product of the first derivatives of the log-likelihood function at the maximum likelihood estimates,

$$I_{OP} = \left[\sum_{t=1}^T \left(\frac{\partial \log L(Y_t, \hat{\Psi})}{\partial \hat{\Psi}} \right) \left(\frac{\partial \log L(Y_t, \hat{\Psi})}{\partial \hat{\Psi}} \right)' \right].$$

Given that the numerical computation of first and second derivatives can result in relevant approximation errors, we choose to derive the standard errors of our estimates using the Berndt, Hall, Hall, and Hausman (1974) estimator, based on the outer product of the gradients.⁹ For the Gaussian case $E_0(3)$, comparable standard errors were

linear Kalman filter recursions are modified in two respects. First, since V_t is now state dependent, we evaluate V_t at t . Second, in a non-Gaussian exponential-affine model, the number of factors, m , affecting the conditional volatility of the system are restricted to be non-negative. The linear update step in the Kalman filter recursions does not take such restrictions into account, and negative estimates of X_t could occur. If V_{t+1} is evaluated at a negative \hat{X}_t , the result may not be positive definite. Chen and Scott (2002) propose a solution to the problem that involves replacing negative estimates of $X_{t,j}$ with zero.

⁸In this case we would implement an algorithm to allow a maximum number of violations of the zero lower bound no greater than 2% of the total number of observations used in the estimations.

⁹We estimated the first and second derivatives numerically, using the symmetric central difference method (or two sided numerical derivative) in most cases; we opted for a one side numerical derivative only when the estimates for the Ψ were close to the boundary condition.

obtained using the sample Hessian matrix.

In case of coupon bond the yield is not an affine function of the state variable. This is handled in practice by letting the price function be approximated by a first-order Taylor approximation around the estimate state.

A.2 Conditional mean of the factors

$$E[X_s | X_t] = (I - e^{-K(s-t)})\theta + e^{-K(s-t)}X_t$$

A.3 Conditional variance of the factors

When K is diagonalizable we can write K as QDQ^{-1} , where D is the diagonal matrix of eigenvalues $diag(d_1 \dots d_N)$, and Q is an invertible matrix whose columns are the eigenvectors of K .

$$K = QDQ^{-1}$$

$$D = diag(d_1 \dots d_N)$$

Fisher and Gills (1996) show that when K is diagonalizable, we can write the conditional variance as

$$Var[X_t | X_{t+\tau}] = Q \left\{ \int_0^\tau e^{-D(\tau-s)} F(t, s) e^{-D(\tau-s)} ds \right\} Q^\top$$

where $F(t, s)$ is equal to the instantaneous covariance matrix of the transformed factor:

$$\begin{aligned} F(t, s) &= G^0 + \sum_{i=1}^N G^i [E(X_s^+ | X_t^+)]_i = \\ &= G^0 + \sum_{i=1}^N G^i [[Q^{-1}\theta]_i + e^{d_i(s-t)} ([Q^{-1}X_t]_i - [Q^{-1}\theta]_i)] \end{aligned}$$

where G^0 and G^i are equal to:

$$\begin{aligned} G^0 &= (Q^{-1}\Sigma) diag(\alpha) (Q^{-1}\Sigma)^\top \\ G^i &= (Q^{-1}\Sigma) diag([\beta Q]_i) (Q^{-1}\Sigma)^\top \end{aligned}$$

where $[\beta Q]_i$ is the i^{th} column of the matrix $\beta \cdot Q$ and G_0 and G_i are N by N matrices. The conditional variance can then be written as:

$$Var [X_t | X_{t+\tau}] = Q \left\{ \begin{aligned} & \int_0^\tau e^{-D(\tau-s)} G^0 e^{-D(\tau-s)} ds + \\ & \sum_{i=1}^N [Q^{-1}\theta]_i \left[\int_0^\tau e^{-D(\tau-s)} G^i e^{-D(\tau-s)} ds \right] + \\ & \sum_{i=1}^N ([Q^{-1}X_t]_i - [Q^{-1}\theta]_i) \left[\int_0^\tau e^{-D(\tau-s)} G^i e^{-D(\tau-s)} e^{-d_i(\tau-s)} ds \right] \end{aligned} \right\} Q^\top.$$

Solving the integrals and collecting the terms, we express the $(j, k) - th$ element of $Var [X_t | X_{t+\tau}]$ in two component v_0 and v_l , respectively the constant and the time varying component of the covariance matrix. We write v_0 as:

$$\begin{aligned} v_0 = & (d_j + d_k)^{-1} G_{j,k}^0 (1 - e^{-(d_j+d_k)(s-t)}) + \\ & \sum_{i=1}^N [Q^{-1}\theta]_i [(d_j + d_k)^{-1} G_{j,k}^i (1 - e^{-(d_j+d_k)(s-t)}) + \\ & - (d_j + d_k - d_i)^{-1} G_{j,k}^i (e^{-d_i(s-t)} - e^{-(d_j+d_k)(s-t)})], \end{aligned}$$

and v_l as:

$$v_l = (d_j + d_k - d_i)^{-1} G_{j,k}^i (e^{-d_i(s-t)} - e^{-(d_j+d_k)(s-t)}), \text{ for } l = 1, \dots, N.$$

We then write the closed-form solution for the unconditional variance as:

$$Var [X_t | X_{t+\tau}] = Q \left\{ v_0 + \sum_{i=1}^N v_i [Q^{-1}X_t]_i \right\} Q^\top,$$

so that the conditional variance of the original factors is given by

$$Var [X_s | X_t] = Q v_0 Q^\top + \sum_{i=1}^N \left(\sum_{l=1}^N Q v_l Q^\top Q_{l,i}^{-1} \right) X_{t,i}.$$

References

- [1] Ahn, D H, R F Dittmar, and A R Gallant.(2002): "Quadratic term structure models: theory and evidence." *Review of Financial Studies*, 16, 459-485.
- [2] Allen, L and A Saunders (2003): "A survey of cyclical effects in credit risk measurement models", BIS Working Paper no. 126.
- [3] Altman, E I, B Brady, A Resti and A Sironi (2005): "The link between default and recovery rates: theory, empirical evidence and implications", *Journal of Business*, 78(6), pp 2203–2227.
- [4] Andersen, T G, L Benzoni and J Lund (2002): "An empirical investigation of continuous-time equity return models", *Journal of Finance*, 57, pp 1239-1284.
- [5] Amato, J and T Laubach (1999): "The value of interest rate smoothing: how the private sector helps the Federal Reserve", *Federal Reserve Bank of Kansas City Economic Review*, Third Quarter, pp 47–64.
- [6] Amato, J and E Remolona (2005): "The pricing of unexpected credit losses", BIS Working Paper no. 190.
- [7] Ang, A and G Bekaert (2002): "Regime switches in interest rates", *Journal of Business and Economic Statistics*, 20, pp 163-182.
- [8] Ang, A, G Bekaert and M Wei (2008): "The term structure of real rates and expected inflation", *Journal of Finance*, 63(2), pp 797-849.
- [9] Ang, A and M Piazzesi (2003): "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables", *Journal of Monetary Economics*, 50, pp 745–787.
- [10] Ang, A, S Dong and M Piazzesi (2007): "No-arbitrage Taylor rules", NBER Working paper,13448.
- [11] Ang, A, M Piazzesi and M Wei (2006): "What does the yield curve tell us about GDP growth?", *Journal of Econometrics*, 131, pp 359-403.

- [12] Babbs, S H and K B Nowman (1999): "Kalman Filtering of generalized vasicek term structure models", *Journal of Financial and Quantitative Analysis*, 34, pp 115-130.
- [13] Bansal, R and H Zhou (2002): "Term structure of interest rates with regimes shifts" *Journal of Finance*, 57, 1997-2043
- [14] Bakshi, G, D Madan and F Zhang (2001): "Understanding the role of recovery in default risk models: empirical comparisons of implied recovery rates", working paper, University of Maryland.
- [15] Bakshi, G, D Madan and F Zhang (2006): "Investigating the role of systematic and firm-specific factors in default risk: lessons from empirically evaluating credit risk models", *Journal of Business*, 79, pp 1955-1988.
- [16] Bernanke, B S, M Gertler and S Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework", in J B Taylor and M Woodford (eds), *Handbook of Macroeconomics*, vol. 1, Amsterdam: North-Holland, pp 1341-93.
- [17] Berndt, E K, B H Hall, R E Hall, and J A Hausman (1974): "Estimation and inference in nonlinear structural models," *Annals of Economic and Social Measurement*, 3, pp 653-665.
- [18] Berndt, A, R Douglas, D Duffie, M Ferguson and D Schranz (2005): "Measuring default risk premia from default swap rates and EDFs", working paper, Stanford University.
- [19] Bhon, J, N Arora and I Korablev (2005): "Power and level validation of the Moody's KMV EDF credit measure in the U.S. market", Moody's KMV White paper.
- [20] Blanco, R, S Brennan, and I W Marsh (2005): "An empirical analysis of the dynamic relation between investment-grade bonds and credit default swaps" ,*Journal of Finance*, 60, pp 2255-2281.
- [21] Bohn, J (2000): "An empirical assessment of a simple contingent-claims model for the valuation of risky debt", *Journal of Risk Finance*, pp 55-77.

- [22] Boyarchenko, N (2008): "Are analysts right? Macroeconomic factors and regime switching in the term structure of interest rates", working paper, University of Chicago.
- [23] Brandt, M W and P He (2006): "Simulated Likelihood Estimation of Affine Term Structure Models from Panel Data", working paper, Fuqua School of Business, Duke University.
- [24] Cantor, R and C Mann (2003): "Measuring the performance of corporate bond ratings", Moody's Special Comment, April.
- [25] Chava, S and R Jarrow (2004): "Bankruptcy prediction with industry effects", *Review of Finance*, 8, pp 537–569.
- [26] Chen, R and L Scott (2002): "Cox-Ingersoll-Ross models of the term structure: estimates and tests from a Kalman filter model", working paper, Rutgers Business School, Rutgers University.
- [27] Chen, L, P Collin-Dufresne, and R Goldstein (2005): "On the relationship between credit spread puzzles and the equity premium puzzle", working paper, University of California, Berkeley.
- [28] Cheridito, P, D Filipović, and R Kimmel (2007): "Market price of risk specifications for affine models: theory and evidence," *Journal of Financial Economics*, 83, pp 123-170.
- [29] Clarida, R, J Galí and M Gertler (2000): "Monetary policy rules and macroeconomic stability: evidence and some theory", *Quarterly Journal of Economics*, 115, pp 147-180.
- [30] Cochrane, J and M Piazzesi (2005): "Bond risk premia", *American Economic Review*, 95, pp 138-160.
- [31] Collin-Dufresne, P and R Goldstein (2002): "Do bonds span the fixed income markets? Theory and evidence for unspanned stochastic volatility", *Journal of Finance*, 57, pp 1685-1730.

- [32] Collin-Dufresne, P, R Goldstein and J Spencer Martin (2001): "The determinants of credit spread changes", *Journal of Finance*, 56, pp 2177–2207.
- [33] Collin-Dufresne, P, R Goldstein and C Jones (2004): "Can interest rate volatility be extracted from the cross section of bond yields? An investigation of unspanned stochastic volatility", working paper, NBER 10756.
- [34] Cox, J C, J E Ingersoll and S A Ross (1985): "A theory of the term structure of interest rates", *Econometrica*, 53, pp 385–407.
- [35] Dai, Q, A Le, and K Singleton (2006): "Discrete-time dynamic term structure models with generalized market prices of risk", working paper, Stanford University.
- [36] Dai, Q and K Singleton (2000): "Specification analysis of affine term structure models", *Journal of Finance* 55, pp 1943–1978.
- [37] Dai, Q and K Singleton (2002): "Expectation puzzles, time-varying risk premia, and affine models of the term structure", *Journal of Financial Economics*, 63, pp 415–441.
- [38] Dai, Q, K Singleton, and W Yang (2007): "Regime shifts in a dynamic term structure model of U.S. treasury bond yields", *Review of Financial Studies*, forthcoming.
- [39] De Jong, F (2000): "Time-series and cross-section information in affine term structure models", *Journal of Business & Economics Statistics*, 18, pp 300–314.
- [40] Delianedis, G and R Geske (1998): "Credit risk and risk neutral default probabilities: information about rating migrations and defaults", working paper, 19-98, Anderson Graduate School of Business, University of California.
- [41] Delianedis, G and R Geske (2001): "The components of corporate credit spreads: default, recovery, tax, jumps, liquidity and market factors", Working Paper 22–01, Anderson Graduate School of Management, UCLA.
- [42] Delianedis, G, R Geske, and T Corzo (1998): "Credit risk analysis with option models: estimation and comparison of actual and risk-neutral default probabilities", working paper, Anderson Graduate School of Business, University of California.

- [43] Dewachter, H and M Lyrio (2006): “Macro factors and the term structure of interest rates”, *Journal of Money, Credit, and Banking*, 38, pp 119-140.
- [44] Dewachter H, M Lyrio and K Maes (2006): ”A joint model for the term structure of interest rates and the macroeconomy”, *Journal of Applied Econometrics*, 21, pp 439-462.
- [45] Diebold, F X, and C Li (2006): “Forecasting the term structure of government bond yields,” *Journal of Econometrics*, 130, pp 337-364.
- [46] Diebold, F X, G D Rudebusch and B Aruoba (2006): “The macroeconomy and the yield curve: a dynamic latent factor approach,” *Journal of Econometrics*, 131, pp 309-338.
- [47] Driessen, J (2005): “Is default event risk priced in corporate bonds?”, *Review of Financial Studies*, 18, pp 165–195.
- [48] Duan, J and J Simonato (1995): “Estimating and testing exponentially affine term structure models by Kalman filter”, working paper, McGill University.
- [49] Duarte, J (2004): “Evaluating an alternative risk preference in affine term structure models,” *Review of Financial Studies*, 17, pp 379-404.
- [50] Duffee, G (1999): “Estimating the price of default risk”, *Review of Financial Studies*, 12, pp 197–226.
- [51] Duffee, G (2002): “Term premia and interest rate forecasts in affine models”, *Journal of Finance* 57, pp 405–443.
- [52] Duffee, G and R H Stanton (2004): ”Estimation of dynamic term structure models”, working paper, Haas School of Business, University of California, Berkeley.
- [53] Duffie, D and R Kan (1996): “A yield-factor model of interest rates”, *Mathematical Finance*, 6, pp 379–406.
- [54] Duffie, D and K Singleton (1999): “Modeling term structures of defaultable bonds”, *Review of Financial Studies*, 12, pp 687-720.

- [55] Duffie, D and K Singleton (2003): *Credit Risk: Pricing, Measurement and Management*, Princeton University Press.
- [56] Duffie, D, L Saita and K Wang (2007): "Multi-period corporate default prediction with stochastic covariates", *Journal of Financial Economics*, 83, pp 635-665.
- [57] Elton, E, M Gruber, D Agrawal and C Mann (2001): "Explaining the rate spread on corporate bonds", *Journal of Finance*, 56, pp 247-77.
- [58] Eom, Y, J Helwege and J Huang (2004): "Structural models of corporate bond pricing: an empirical analysis", *Review of Financial Studies*, 17, pp 499-544.
- [59] Fan, R, A Gupta and P Ritchken (2003): "Hedging in the possible presence of unspanned stochastic volatility: evidence from swaption markets", *Journal of Finance*, 58, pp 2219-2248.
- [60] Fisher, L (1959): "Determinants of the risk premium on corporate bonds", *Journal of Political Economy*, 67, pp 217-237.
- [61] Fisher, M, and C Gilles (1996): "Estimating exponential-affine models of the term structure", working paper, Federal Reserve Atlanta.
- [62] Fons, J (1987): "The default premium and corporate bond experience", *Journal of Finance* 42, pp 81-97.
- [63] Gertler, M and C Lown (1999): "The information in the high-yield bond spread for the business cycle: evidence and some implications", *Oxford Review of Economic Policy*, 15, pp 132-150.
- [64] Gibbons, M R, and K Ramaswamy (1993): "A test of the Cox, Ingersoll and Ross model of the term structure", *Review of Financial Studies*, 6, pp 619-58.
- [65] Geyer, A L J, and S. Pichler (1998): "A state-space approach to estimate and test multifactor Cox-Ingersoll-Ross models of the term structure", working paper, University of Economics, Vienna.
- [66] Hamilton, J (1989): "A new approach to the economic analysis of nonstationary time series and the business cycle", *Econometrica*, 57, pp 357-384.

- [67] Hamilton, J (1994): Time Series Analysis, Princeton: Princeton University Press.
- [68] Harvey, A (1991): Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press.
- [69] Hordhal, P, O Tristani and D Vestin (2006): “A joint econometric model of macroeconomic and term structure dynamics”, Journal of Econometrics, 131, pp 405-444.
- [70] Huang, J and M Huang (2003): “How much of the corporate-treasury yield spread is due to credit risk?: a new calibration approach”, working paper, Penn State University.
- [71] Kavvathas, D (2001): ”Estimating credit rating transition probabilities for corporate bonds”, working paper, University of Chicago.
- [72] Kealhofer, S (2003): “Quantifying credit risk I: default prediction”, Financial Analysts Journal, January–February, pp 30–44.
- [73] Knez, P K, R Litterman and J A Scheinkman (1994): ”Explorations into factors explaining money market returns”, Journal of Finance, 49, pp 1861-1882.
- [74] Janosi, T, R Jarrow and Y Yildirim (2001): “Estimating expected losses and liquidity discounts implicit in debt prices”, working paper, Cornell University.
- [75] Jarrow, R, D Lando and F Yu (2005): “Default risk and diversification: theory and applications”, Mathematical Finance, 15, pp 1–26.
- [76] Jonsson, J and M Fridson (1996): “Forecasting default rates on high-yield bonds”, Journal of Fixed Income, June, pp 69–77.
- [77] Lando, D (1998): “Cox processes and credit-risky securities”, Review of Derivatives Research, 2, pp 99-120.
- [78] Litterman, R (1983): “A random walk, markov model for the distribution of time series”, Journal of Business and Economic Statistics, 1, pp 169-173.
- [79] Litterman, R and J Scheinkman (1991): “Common factors affecting bond returns”, Journal of Fixed Income, 1, pp 51–61.

- [80] Lo, A W (1988): “Maximum likelihood estimation of generalized Ito processes with discretely-sampled data”, *Econometric Theory*, 4, pp 231-47.
- [81] Longstaff, F, S Mithal and E Neis (2005): “Corporate yield spreads: default risk or liquidity? New evidence from the credit default swap market”, *Journal of Finance*, 60, pp 2213–2253.
- [82] Lund, J (1997): “Econometric analysis of continuous-time arbitrage-free models of the term structure of interest rates”, working paper, Aarhus School of Business.
- [83] Morris, C, R Neal and D Rolph (2001): “Interest rates and credit spreads”, working paper, Indiana University.
- [84] Neftci, S. (1984): ”Are economic time series asymmetric over the business cycle?”, *Journal of Political Economy*, 92, pp 307-328.
- [85] Pan, J and K Singleton (2005): “Default and recovery implicit in the term structure of sovereign CDS spreads”, working paper, Stanford University.
- [86] Pedersen, A R (1995): “A new approach to maximum likelihood estimation for stochastic differential equations based on discrete observations”, *Scandinavian Journal of Statistics*, 22, pp 55-71.
- [87] Piazzesi, M (2003): “Affine term structure models”, working paper, UCLA.
- [88] Piazzesi, M (2005): ”Bond yields and the Federal Reserve”, *Journal of Political Economy*, 113, pp 311-344.
- [89] Rudebusch, G and T Wu (2003): “A macro-finance model of the term structure”, working paper, Federal Reserve Bank of San Francisco.
- [90] Saita, L. (2005): ”The puzzling price of corporate credit risk” working paper, Stanford University.
- [91] Santa-Clara, P (1995): “Simulated likelihood estimation of diffusions with an application to the short term interest rate”, Ph.D. Dissertation, Insead, France.
- [92] Schultz, P (2001): “Corporate bond trading costs: a peek behind the curtain”, *Journal of Finance*, 56, pp 677–98.

- [93] Shane, H (1994): "Comovements of low-grade debt and equity returns of highly leveraged firms," *Journal of Fixed Income*, 3, pp 79-89.
- [94] Singleton, K (2001): "Estimation of affine asset pricing models using the empirical characteristic function", *Journal of Econometrics*, 102, pp 111-141.
- [95] Singleton, J K (2006): *Empirical Dynamic Asset Pricing*, Princeton University Press.
- [96] Vasicek, O (1977): "An equilibrium characterization of the term structure", *Journal of Financial Economics*, 5, pp 177–188.
- [97] Wu, T (2000): "Macro factors and the affine term structure of interest rates", working paper, Yale University.
- [98] Wu, L and F Zhang (2005): "A no-arbitrage analysis of economic determinants of the credit spread term structure", working paper, Federal Reserve Board.
- [99] Yang, J (2003): "An empirical investigation of the effects of macroeconomic variables on default risk", working paper, University of Toronto.

Chapter 2 - Macro Factors in the Term Structure of Credit Spreads

Maurizio Luisi
University of Lugano

Abstract

In this chapter we estimate arbitrage-free term structure models of US Treasury yields and spreads on BBB and B-rated corporate bonds in a doubly-stochastic intensity-based framework. A novel feature of our analysis is the inclusion of macroeconomic variables – indicators of real activity, inflation and financial conditions – as well as latent factors, as drivers of term structure dynamics. Our results point to three key roles played by macro factors in the term structure of spreads: they have a significant impact on the level, and particularly the slope, of the curves; they are largely responsible for variation in the prices of systematic risk; and speculative grade spreads exhibit greater sensitivity to macro shocks than high grade spreads.

JEL Classification Numbers: C13, C32, E44, E52, G12, G13, G14

Keywords: corporate bonds, default intensity, event risk, risk premia, interest rate rule

1 Introduction

In this study we provide new evidence on the impact of macroeconomic conditions on corporate bond spreads. There are compelling reasons to expect that spreads are influenced by the macroeconomy. Theoretical models of default risk, as well as general equilibrium models with financial frictions and nominal rigidities, predict systematic relationships between spreads, output and/or inflation (e.g. Bernanke, Gertler and Gilchrist (1999)). Estimates of unconditional correlations indicate a close empirical link between credit spreads, the state of financing conditions faced by borrowers and the business cycle. For example, the monthly correlation between five-year US BBB-rated industrial spreads and real output is -0.52. Indeed, several past studies have examined the empirical relationship between default risk and macroeconomic conditions. Jonsson and Fridson (1996), Chava and Jarrow (2004) and Duffie, Saita and Wang (2007), amongst others, have shown there is a countercyclical relationship between default risk and economic activity. In a study of default loss rates, Altman, Brady, Resti and Sironi (2005) estimate negative correlations between default rates, loss given default and the business cycle. Cantor and Mann (2003) document the procyclicality of credit quality changes using a long history of Moody's credit ratings data.¹ But apart from a few studies, the relationship between corporate bond spreads and macroeconomic variables has been left largely unexamined. Previous work has mainly focused on explaining changes in spreads using regression analysis (e.g. Collin-Dufresne, Goldstein and Martin (2001), Morris, Neal and Rolph (2001)).

In this chapter, by contrast, we analyse spreads in a multi-factor term structure model subject to restrictions imposed by the absence of arbitrage opportunities. Default risk is modelled using a doubly-stochastic intensity-based framework (Lando (1998), Duffie and Singleton (1999)), where risk-neutral instantaneous default loss rates ("instantaneous spreads") are assumed to be affine functions of the state variables. One innovation of our approach is that the state vector is comprised of both observable macroeconomic variables – indicators of real activity, inflation and financial activity – and unobserved latent factors. Most of the existing empirical work on reduced-form term structure models has been based on latent factors only (e.g. Duffee (1999),

¹For a more complete review of how macroeconomic factors have been incorporated into credit risk models, see Allen and Saunders (2003).

Driessen (2005)). Thus, our study seeks to extend these earlier studies by drawing additional insights from the inclusion of observable variables as factors. Ideally, we would like to specify a completely observable state space to model yields and spreads, but our findings point to a crucial role played by latent factors in improving the fit of our model to market data. This result may be due to an overly restrictive state space (we include three macro factors), or it may reflect a well-known finding by Collin-Dufresne, Goldstein and Martin (2001) that, in addition to macroeconomic variables, there appears to be a common “unknown” factor in corporate bond returns. Moreover, since we do not explicitly account for liquidity or tax effects on corporate bond prices, such as in Driessen (2005), latent factors in our model may implicitly pick up these other influences on spreads.

In one regard, our work builds on recent studies of affine term structure models of default-free yields with macroeconomic factors (e.g. Ang and Piazzesi (2003), Dewachter and Lyrio (2006), Hordhal, Tristani and Vestin (2006)).² In these models, real activity and consumer price inflation are amongst the drivers of government bond yields due to their influence on the risk-free rate and the discount factors agents use to price assets. Motivated by firm-value (Merton-type) models of default risk and the ratings methodologies of rating agencies (e.g. Standard & Poor’s (2003)), our model of defaultable bond pricing also includes a measure of financial activity along with real output and inflation as observable state variables. Despite the poor performance of specific formulations of firm-value models in explaining spreads (Huang and Huang (2003), Eom, Helwege and Huang (2004)), regression estimates presented below suggest that key drivers of default risk in these models have substantial explanatory power for spreads. Our particular indicator of financial conditions combines information on leverage, interest coverage, cash flow and asset volatility.

Yang (2003) also examines the role of output and inflation in the term structure of spreads, although there are several important differences in the scope, methodology and results of our respective studies. For example, we allow the risk-free rate to depend upon macroeconomic variables (in contrast to Yang), we examine the relationship between financial activity variables and spreads, and our data covers a longer time

²Affine models have been the workhorse in the empirical term structure literature on default-free debt. It is impossible to cite all of the relevant contributions here. See Dai and Singleton (2001) and Piazzesi (2003) for a broad overview of these types of models.

span. By comparison, Bakshi, Madan and Zhang (2006) add several firm-specific risk factors, including leverage and volatility, to latent factor models, but do not examine the potential role of output or inflation as factors. They find that including leverage as an observable state variable helps to significantly reduce model pricing errors for high-yield, but not investment grade, bonds.³

We concentrate our analysis on spread dynamics at the sector-rating level, specifically, BBB and B-rated industrial firms.⁴ Our chosen sector-rating classes are amongst those with the largest number of outstanding issues in the investment grade and high-yield markets, respectively. Of particular interest in this study are potential differences in the sensitivities of investment grade and high-yield spreads to macroeconomic conditions. Economic theory suggests that lower-rated firms likely face tighter financing constraints, especially in cyclical downturns, and that they generally suffer greater adverse effects from financial market imperfections. Consequently, we expect speculative grade spreads to be more sensitive, all else equal, to aggregate economic activity (see, e.g., Gertler and Lown (1999)). By estimating a model on both BBB and B-rated bonds, we can examine whether spreads on lower-rated debt respond differently to the macroeconomy.

Our study begins by presenting regression estimates of Treasury yields and corporate spreads on macroeconomic variables. In general, we find that both yields and spreads are strongly related to real economic activity and financial conditions, and less so to inflation. Moreover, as anticipated, spreads on lower-rated debt are affected more by macroeconomic variables than those on investment grade bonds. The regression results are a good indicator of what we find in our affine term structure model; after all, our model predicts that yields and spreads are affine functions of the state variables. However, as noted elsewhere (Duffee (2002), Piazzesi (2003)), there are many insights to be gained from a no-arbitrage term structure model that cannot be inferred from

³In other contemporaneous work, Wu and Zhang (2005) examine the role of output, inflation and market volatility in term structure models applied to bond data on individual firms.

⁴One advantage of using aggregate index data, in contrast to firm-level data (Duffee (1999), Driessen (2005)), is that noise from idiosyncratic firm-level shocks is eliminated, thereby allowing more efficient estimation of the role of macroeconomic variables in the term structure. One disadvantage is that we are unable to assess the relative importance of firm-level versus aggregate shocks in the pricing of individual bonds.

simple regressions.

One of our main objectives is to assess the separate impact of the macroeconomy on risk-free rates, expected losses from default and the prices of systematic risk. This leads us to new insights about bond risk premia and the relationship between risk-free rates and spreads. First, recent macro-finance models of the term structure have shown that Treasury bond risk premia are driven by macroeconomic variables (see references above). Since default risk tends to rise in recessions when investors' incomes are relatively low, we would also expect business cycle risk to be priced in spreads. In fact, we find that movements in risk premia on corporate bonds can be largely attributed to our observable macro factors, especially output and inflation risk. Second, an advantage of our approach is that we can shed further light on the source of the negative unconditional correlation between risk-free rates and spreads documented in previous studies (e.g. Duffee (1999)). Our results indicate that real activity is primarily responsible for the negative correlation between these variables: the risk-free rate rises in response to an increase in output, whereas spreads, especially at short-medium maturities, decline.

Finally, our results also shed new light on the role of macroeconomic variables in the term structure of Treasury yields. For instance, our data sample covers a period when inflation was relatively low and stable, and therefore our results provide a test of the stability of estimates obtained elsewhere over a longer sample period that includes the high and variable inflation of the 1970s and early 1980s (e.g. Ang and Piazzesi (2003)). In contrast to their results, we find that shocks to real activity have a much stronger initial effect on the entire yield curve compared to inflation shocks. In addition, we show that our financial conditions indicator affects the entire Treasury curve, including the risk-free rate.

2 Data

2.1 Treasury Yields

We use data on zero-coupon constant maturity US Treasury yields to estimate the benchmark default-free curve in our model and to construct the spreads data. Data at various maturities is taken from interpolated yield curves available in the BIS DBS database, which have been constructed based on closing market bid yields on actively

traded Treasury securities obtained by the Federal Reserve Bank of New York. The sources of all data series used in this chapter are summarised in Table 1. In estimation, we use maturities of one, three, 12, 36, 60 and 120 month(s) (denoted 1M, 3M, 12M, 36M, 60M and 120M, respectively). A monthly time series of yields is assembled by taking month-end observations. Our sample period is dictated by the availability of data on corporate bond yields, and runs from 1992:05 to 2004:04, giving 144 observations in total.

Table 2 reports summary statistics on US Treasury yields, and the top panel of Figure 1 shows plots of these yields at 1M, 60M and 120M maturities. The unconditional means point to an upward-sloping yield curve on average — from 3.71% at 1M to 5.72% at 120M. The term structure of the unconditional volatilities is hump-shaped, increasing from 1M to 12M, and then decreasing at longer maturities. Yield levels are highly persistent, with first-order serial correlations equal to or greater than 0.95. There is some evidence that yields are platykurtic and have negative skewness, but the departures from normality are not strong.

Pairwise correlations in Treasury yields at all maturities are high, with contemporaneous correlations for adjacent maturities often in excess of 0.95. Table 3 shows the percentage of variation in yields explained by the six ordered principal components, on a marginal and cumulative basis. Most of the variability is accounted for by the first two components (over 99%). This suggests that a small number of common factors determine movements across the whole yield curve, consistent with many previous studies (see Litterman and Scheinkman (1991)).

2.2 Corporate Bond Yields

Corporate spreads are constructed using data on corporate bond yields extracted from Bloomberg’s Fair Market Value yield curves. These curves are constructed on a daily basis for various sectors and rating classes from a sample of Bloomberg Generic bond prices at market closing. Bonds with embedded options are adjusted to create option-adjusted yields. We utilise data on the curves for BBB and B-rated industrial firms.⁵ As with Treasury yields, we create monthly time series using month-end observations.

⁵Credit ratings are based on the Bloomberg composite rating, which is a blend of ratings of the major agencies.

Corporate spreads are calculated as the differences between the industrial yields and Treasury yields. In estimation, we utilise maturities of 12M, 36M, 60M, 84M and 120M.

Table 2 reports summary statistics on corporate spreads and Figure 1 plots time series of these variables. As with Treasuries, the unconditional means of BBB and B-rated spreads are increasing in maturity. By contrast, higher moments display some differences. The term structure of unconditional volatility is upward sloping for BBB-rated spreads, while it slopes downward for B-rated spreads. BBB-rated spreads appear to be platykurtic, whereas B-rated spreads exhibit excess kurtosis. Spreads in both rating classes are positively skewed, especially at long maturities. In summary, there is some evidence of non-normality in the distribution of spreads, but as with Treasury yields, the departures from the Gaussian assumption are not dramatic.

Corporate spreads are also highly correlated across maturities. Table 3 shows that the first principal component accounts for almost 93% of the variation in the five BBB-rated spreads included in our study, and over 99% of the variation is captured by the first three components together; the corresponding values for B-rated spreads are even larger.

One reason we concentrate our analysis on the BBB and B-rated industrial sectors is that these are amongst the broadest and deepest rating-sector categories in the US corporate bond market. Figure 2 shows a breakdown by industry of the number of bonds used to create the BBB-rated industrial curve on 24 August 2004. The industries with the greatest representation are transportation, food, forest products, and oil and gas.

2.3 Macroeconomic Factors

To construct the real activity, inflation and financial conditions factors, we adopt the methodology of Ang and Piazzesi (2003); details are given in Appendix A. Each of these variables is computed as a common factor – specifically, the first principal component – from a set of observable macroeconomic time series. The main purpose of utilising common factors in our model, instead of the observable variables directly, is to reduce the dimensionality of the state space. Table 1 summarises the data series used. The series corresponding to real activity and inflation are the same as in Ang and Piazzesi (2003), though our respective sample periods differ. For real activity, these are the index of Help Wanted Advertising in Newspapers (HELP), unemployment rate (UE),

the growth rate of employment (EMPLOY) and the growth rate of industrial production (IP). The inflation measures are the growth rates of the Consumer Price Index (CPI), Producer Price Index of finished goods (PPI) and a broad-based Commodity Prices Index (PCOM). All growth rates are measured as the 12-month difference in logs of the index.

The financial activity factor is based on variables that represent leverage, interest coverage, cash flow and assets volatility. These quantities play a key role in firm-value models of credit risk and the ratings methodologies of the major ratings agencies. In fact, in several well-known Merton-type models, “distance-to-default” – which is essentially a volatility-adjusted measure of leverage – is a sufficient statistic for default risk. Even though our financial activity indicator does not explicitly depend upon an aggregate distance-to-default measure, it does incorporate information on both aggregate leverage and volatility. Leverage is measured as $DEBT/PRO$, where $DEBT$ is Credit Market Debt and PRO is Profit After Tax; interest coverage is set equal to INT/GDP , where INT is Net Interest Payments and GDP is real Gross Domestic Product; a proxy for the ability of firms to generate cash flow is $PRO/SALES$, where $SALES$ is Final Sales of Domestic Product; and the volatility of assets is proxied by call implied volatility on the S&P 500 (IMPVOL) obtained from Bloomberg.⁶ Data on $DEBT$, PRO , INT , and GDP refer to non-financial corporate business and are in real terms.

Plots of the macro factors are shown in Figure 3. They are normalised to have mean zero and a standard deviation of one. As would be expected, the factors display relatively little high frequency volatility; instead, most of the movement is at business cycle frequencies. For example, the real activity factor increases for several years on the heels of the 1991 recession, and later falls significantly at the onset of the recession in 2001. The financial variable reflects the deleveraging undertaken by firms at the start of the recovery in the early 1990s, and the subsequent rebuilding up of leverage in the latter stages of the 90s boom, only to fall sharply again with the winding down of the recent recession.

⁶Bloomberg’s data on call implied volatility begins in 1994. IMPVOL is extended back to 1992 using the VIX index. Values of the VIX index and implied volatility are almost identical in the month following the start of the latter, so there is no apparent break in the longer series.

3 Regression Analysis

The model we construct in the next section implies that Treasury yields and corporate spreads are affine functions of the state variables. Thus, a natural starting point is to investigate the relationship between yields, spreads and macroeconomic variables using linear regression analysis. Unrestricted regression equations do not impose the necessary cross-equation restrictions implied by the absence of arbitrage in the bond pricing model. Nonetheless, the partial correlations uncovered in regression estimates should indicate the nature of the relationships we should expect to find in estimation of the no-arbitrage model.

Unconditional linear correlations of selected variables are reported in Table 4. Both real activity and inflation are positively correlated with Treasury yields at all maturities, with much higher correlations for real activity (0.7-0.9) than inflation (about 0.2) in our sample period. While the correlation of real activity and spreads is large and negative, it is almost nil for inflation and spreads. The financial activity factor has a high and negative correlation with Treasuries, and a positive, and higher in absolute value, correlation with spreads.

Table 5 reports regression estimates of yields and spreads on the macro factors. In addition to reporting results for various maturities, the table also gives estimates of equations for, in the terminology of Litterman and Scheinkman (1991), the “level”, “slope” and “curvature” of the term structures.⁷

Looking first at the Treasury yield estimates, three points are worth emphasising. First, the results for the 1M yield confirm the finding in many other studies of a strong link between the short rate and standard macro variables (e.g. Amato and Laubach (1999)). The estimated coefficients on real activity and inflation are positive, although the latter is not significant. A new result is the finding of a positive and significant relationship between the 1M yield and the financial activity variable. If we interpret this equation as a proxy for the monetary policy reaction function, then these estimates suggest that the Federal Reserve has tightened monetary policy in response to

⁷For the Treasury curve, in terms of yields at given maturities, the level is defined as $(1M+36M+120M)/3$; the slope as $(120M-1M)$; and the curvature as $(1M+120M)-2 \times 36M$. For the corporate curves, the 12M and 60M spreads replace the 1M and 36M spreads, respectively, in the above formulae.

developments in the financial sector beyond what they imply for output and inflation. Second, the large adjusted- R^2 statistics in the multiple regressions provide the basis for including macro variables in the yield curve model. Interestingly, both real activity and financial conditions, as opposed to inflation, seem to be more important for higher-maturity Treasury yields as well, in contrast to the larger role played by inflation in Ang and Piazzesi's (2003) analysis of yield curve dynamics (see below). Third, macro variables capture a significant portion of the variation in the level and slope of the Treasury curve. An increase in the financial activity indicator, for example, leads to a flattening of the yield curve.

Now consider the regressions for BBB and B-rated spreads. First, the fit of the regressions are similar to or better than they are for Treasury yields in many cases. Thus, this is compelling evidence for including macro factors in a corporate spreads model. Second, financial conditions have the strongest impact on spreads. Spreads widen with an increase in financial activity, with the size of the impact increasing in maturity. By contrast, real activity tends to have a larger (negative) impact on spreads at short maturities. Third, as in the case of Treasury yields, macro variables capture a large portion of the variation in the level and slope of the spreads curves; for instance, over half of the variation in the slope of the term structure of B-rated spreads can be explained by our three economic indicators.

4 Term Structure Model

In this section we describe our model of the joint dynamics of Treasury yields, BBB-rated and B-rated corporate spreads. We specify processes for the risk-free rate, the risk-neutral instantaneous spreads on bonds of both rating classes and the prices of systematic risk to be affine functions of the state variables.

The state vector X_t consists of a set of six risk factors, the three macro factors and three latent factors:

$$X_t \equiv \begin{bmatrix} X_{1,t} \\ X_{2,t} \\ X_{3,t} \\ X_{f,t} \\ X_{y,t} \\ X_{\pi,t} \end{bmatrix}$$

We assume that X_t evolves according to a multivariate Gaussian diffusion process under the physical measure P :

$$dX_t = -KX_t dt + \Sigma dW_t \quad (1)$$

where W_t is a vector of independent Brownian motions.⁸ We have imposed the long-run means of all factors to be zero. This is done without any loss in generality, as the means cannot be separately identified from the constants in the equations for the risk-free rate and instantaneous spreads given below. Similarly, we have normalised the unconditional variances of the factors to equal one, as these are not separately identified from the factor loadings on these variables in the equations for the risk-free rate and instantaneous spreads. Restrictions are placed on the elements of Σ such that the innovations to the latent factors are mutually independent and independent of the innovations to the macro factors. Finally, the matrix governing mean-reversion is specified as:

$$K = \begin{pmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ k_{21} & k_{22} & 0 & 0 & 0 & 0 \\ k_{31} & k_{32} & k_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{ff} & k_{fy} & k_{f\pi} \\ 0 & 0 & 0 & k_{yf} & k_{yy} & k_{y\pi} \\ 0 & 0 & 0 & k_{\pi f} & k_{\pi y} & k_{\pi\pi} \end{pmatrix} \quad (2)$$

The zero-restrictions in the off-diagonal blocks of (2) are imposed to reduce the dimensionality of the parameter space.

The instantaneous risk-free rate r_t is determined according to:

$$r_t = \delta_0 + \delta_1 X_{1,t} + \delta_2 X_{2,t} + \delta_3 X_{3,t} + \delta_f X_{f,t} + \delta_y X_{y,t} + \delta_\pi X_{\pi,t} \quad (3)$$

This specification is similar to that used in recent studies on the role of macroeconomic factors in the term structure (Wu (2000), Ang and Piazzesi (2003), Rudebusch and Wu (2003), Hordahl, Tristani and Vestin (2006)), and encompasses standard latent factor models with Gaussian factors (e.g. Vasicek (1977)). Equation (3) also takes the form of a monetary policy reaction function or Taylor-type rule, although there are two main differences between (3) and standard monetary policy rules. First, composite indicators of real economic activity and inflation are used instead of observable variables such as real GDP and CPI inflation. Since the Federal Reserve is generally regarded

⁸In the terminology of Duffee (2002), our model is part of the essentially affine ($EA_0(6)$) class of term structure models.

as responding to forecasts of inflation (rather than current inflation), one advantage of using the composite indicator $X_{\pi,t}$ in (3) is that it appears to provide more accurate forecasts of future consumer price inflation than predictions based on the current value of consumer price inflation itself (see appendix A). Second, a term representing financial conditions is rarely included in models of the risk-free short rate, yet the results in the previous section pointed to a strong negative correlation between Treasury yields and $X_{f,t}$, suggesting that its inclusion in (3) may help our understanding of short-rate dynamics and their implications for the yield curve.⁹

Default is modelled in a doubly-stochastic intensity-based framework. Specifically, the default time τ_j on a bond with rating $j = \{BBB, B\}$ arrives according to a Poisson process with associated physical default intensity $h_{j,t}^P$. For pricing purposes, we are interested in the *risk-neutral* intensity $h_{j,t}^Q$. The difference between $h_{j,t}^P$ and $h_{j,t}^Q$ depends upon the price of default event risk, which is analysed in a later section. Even though recent work by Duffie, Saita and Wang (2007) rejects the doubly-stochastic model using firm-level data, it still may be a reasonable assumption for modelling spreads at the sector-wide level, as interdependence amongst firms tends to be strongest within sector.

The pricing of defaultable securities depends upon the treatment of recovery in the event of default. We follow Duffie and Singleton (1999) and assume that recovery is determined as a fixed fraction of the market value of the bond just prior to default (known as “Recovery of Market Value” (RMV))¹⁰. This assumption allows us to work in terms of the risk-neutral instantaneous default loss rate, or *instantaneous spread*, defined as $s_{j,t}^Q \equiv h_{j,t}^Q \cdot L_{j,t}^Q$ for bonds with rating j , where $L_{j,t}^Q$ is the risk-neutral rate of loss given default. As with the risk-free rate, we assume that $s_{j,t}^Q$ is an affine function of the state:

$$s_{j,t}^Q = \gamma_0^j + \gamma_1^j X_{1,t} + \gamma_2^j X_{2,t} + \gamma_3^j X_{3,t} + \gamma_f^j X_{f,t} + \gamma_y^j X_{y,t} + \gamma_\pi^j X_{\pi,t} \quad (4)$$

⁹The risk-free rate is a highly persistent process, even after conditioning on persistent macroeconomic variables, as in (3). This can be handled by explicitly including lagged interest rates or, as we have done, persistent latent factors in the short rate equation. See Ang, Dong and Piazzesi (2005) for a discussion on the observational equivalence of models with latent factors and lagged observable variables in short rate equations.

¹⁰Two other common recovery assumptions in the literature are “Recovery of Face Value” and “Recovery of Treasury”. See Bakshi, Madan and Zhang (2001) and Duffie and Singleton (2003) for empirical analysis and a discussion of the relative attributes of these alternatives.

for $j = \{BBB, B\}$. The loadings on the factors in (4) are allowed to differ across rating categories. The inclusion of macro factors in (4) extends intensity-based models that contain only latent factors, such as Duffee (1999) who allowed three latent factors to drive the risk-neutral intensity, two of which were the determinants of the risk-free rate. The specifications (3) and (4) are sufficiently general to allow all three latent factors to affect Treasury yields and spreads. Whether such generality is necessary, given the inclusion of macro variables in these equations, as well as our findings above that only a few factors are necessary to capture most of the variation in yields and spreads, will be borne out by our estimates.

Note that equations (1), (3) and (4) imply that r_t and $s_{j,t}^Q$ could become negative, depending upon the configuration of realised values for the Gaussian state variables. Of course, it is desirable to have processes for interest rates and spreads that are always positive. In the results reported below, it turns out that r_t , $s_{BBB,t}^Q$ and $s_{B,t}^Q$ remain positive throughout the sample.

Finally, we assume that the prices of bonds are arbitrage-free, which implies the existence of a stochastic discount factor and an associated equivalent martingale measure Q .¹¹ In line with the affine term structure literature, we assume that the market prices of systematic risk Λ_t are affine in the factors:

$$\Lambda_t = \lambda_0 + \lambda_1 X_t \quad (5)$$

where

$$\lambda_0 = \begin{pmatrix} \lambda_{0,1} \\ \lambda_{0,2} \\ \lambda_{0,3} \\ \lambda_{0,f} \\ \lambda_{0,y} \\ \lambda_{0,\pi} \end{pmatrix} \text{ and } \lambda_1 = \begin{pmatrix} \lambda_{1,(1,1)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{1,(2,2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{1,(3,3)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{1,(f,f)} & \lambda_{1,(f,y)} & \lambda_{1,(f,\pi)} \\ 0 & 0 & 0 & \lambda_{1,(y,f)} & \lambda_{1,(y,y)} & \lambda_{1,(y,\pi)} \\ 0 & 0 & 0 & \lambda_{1,(\pi,f)} & \lambda_{1,(\pi,y)} & \lambda_{1,(\pi,\pi)} \end{pmatrix}$$

The structure of λ_1 is chosen to allow for rich interactions amongst the macro factors in the pricing of macroeconomic risk, while at the same time achieving a manageable dimensionality of the parameter space.¹²

¹¹In the current context where markets are incomplete, it is not guaranteed that this measure would be unique.

¹²In estimation of alternative parameterisations of the model, we have found that imposing more zero restrictions in λ_0 or λ_1 leads to a much poorer fit of the data.

Under our assumptions, the price of a zero-coupon Treasury bond with N periods left to maturity at time t is:

$$P_t(N) = E_t^Q \left[\exp \left(- \int_{u=t}^{t+N} r_u du \right) \right] \quad (6)$$

where $E_t^Q(\cdot) \equiv E^Q(\cdot|I_t)$ is the expectation under Q conditional on the information set at time t . The price of a zero-coupon defaultable bond with rating j is given by:

$$V_{j,t}(N) = E_t^Q \left[\exp \left(- \int_{u=t}^{t+N} (r_u + s_{j,u}^Q) du \right) \right] \quad (7)$$

Using results in Duffie and Kan (1996), the expectations in (6) and (7) can be solved to give the following expressions:

$$P_t(\tau) = \exp \left(A_T(N) + B_T(N)^\top X_t \right) \quad (8)$$

and

$$V_{j,t}(\tau) = \exp \left(\tilde{A}_j(N) + \tilde{B}_j(N)^\top X_t \right) \quad (9)$$

where $A(\tau)$ and $B(\tau)$ are obtained as solutions to a set of ordinary differential equations (see Appendix B). Yields on zero-coupon Treasury and corporate bonds are therefore given by:

$$y_{T,t}(N) = -\frac{\ln P_t(N)}{N} = -\frac{1}{N} \left(A_T(N) + B_T(N)^\top X_t \right) \quad (10)$$

and

$$y_{j,t}(N) = -\frac{\ln V_{j,t}(N)}{N} = -\frac{1}{N} \left(\tilde{A}_j(N) + \tilde{B}_j(N)^\top X_t \right) \quad (11)$$

which implies that the corporate bond spread at maturity N is:

$$\begin{aligned} S_{j,t}(N) &\equiv y_{j,t}(N) - y_{T,t}(N) \\ &= -\frac{1}{N} \left(\left[\tilde{A}_j(N) - A_T(N) \right] + \left[\tilde{B}_j(N) - B_T(N) \right]^\top X_t \right) \\ &\equiv -\frac{1}{N} \left(A_j(N) + B_j(N)^\top X_t \right) \end{aligned} \quad (12)$$

5 Estimation Results

5.1 Estimation Procedure

One of the novel features of our approach is that we conduct joint estimation of the model for Treasury yields and spreads in both rating categories. The typical approach

taken in the literature has been to impose orthogonality conditions in the model that permits estimation on a firm-by-firm basis or by rating-sector category. In addition, the parameters related to the Treasury portion of these models are usually estimated in a first step before estimating the corporate term structure. In our setting, each of the latent factors can affect the valuation of all securities, and we also allow for rich interactions in the joint evolution of the latent factors and in the prices of systematic risk. By estimating the model jointly across all bonds, we hope to obtain more efficient estimates. Furthermore, we test our assumption that a common set of latent factors, in addition to macroeconomic variables, are needed to explain prices across Treasury and corporate bond markets.

The macro factors are assumed to be exogenous with respect to yields and spreads, so we can estimate the model in two steps. First, since a discretized version of the process for X_t in (1) is a vector autoregression (VAR) of order one, we estimate these parameters by OLS. In addition, we estimate the coefficients on the macro factors in the equations for the instantaneous risk-free rate and instantaneous spreads by OLS. We use the 1M Treasury yield to proxy for the risk-free rate.¹³ Similarly, we utilise the lowest maturity spread available (3M) to estimate the coefficients on the macro factors in (4) for both BBB and B-rated bonds.

In the second step we estimate the remaining parameters using maximum likelihood estimation. This sequential procedure is similar to the method used by Ang and Piazzesi (2003) for estimating a Treasury curve model with macro factors. We assume that all yields and spreads are observed with measurement error, and so the likelihood function and estimates of the latent factors are constructed using the Kalman filter (see, e.g., Duan and Simonato (1995) and Lund (1997)). Appendix B gives further details on the estimation procedure.

5.2 Parameter Estimates

Table 6 reports estimates of the parameters. The parameters are grouped into those governing the persistence and cross-dynamics of the factors; the loadings in the risk-

¹³If we use the Federal Funds Rate as the regressand, coefficient estimates and the R^2 statistic are similar to those obtained for the 1M Treasury yield. This suggests that the equation for the 1M Treasury yield resembles the Federal Reserve’s reaction function.

free rate and instantaneous spreads; and the market prices of systematic risk. Consider these in turn.

First, OLS estimates of the VAR coefficients, expressed in the table in continuous time as elements of K , indicate a high degree of persistence in the macro factors. Similarly, each of the latent factors exhibits a high degree of autocorrelation, with the latent factor labelled Latent 3 ($X_{3,t}$) being the least persistent.

Second, as already noted in the discussion of the regression results of the 1M yield shown in Table 5, an increase in each of the macro factors raises the risk-free rate. Since the standard deviations of all factors are normalised to one, the magnitudes of the coefficients are directly comparable. Thus, real activity has the largest impact on the risk-free rate. Real activity also has the biggest effect on the instantaneous spread for B-rated bonds ($\gamma_y^B = -0.00052$), whereas the financial factor has the largest impact in the case of BBB-rated bonds ($\gamma_f^{BBB} = 0.00011$). An increase in real activity lowers instantaneous spreads, while increases in the other two macro factors raise instantaneous spreads. Estimates of the loadings on the latent factors in the risk-free rate are positive, whereas the signs are mixed on these terms in the equations for instantaneous spreads. Below we give an interpretation of these factors. For now, note that all of the latent factors have statistically significant coefficients in r_t , $s_{BBB,t}^Q$ and $s_{B,t}^Q$.

Third, the estimates of λ_0 suggest that all of the risk factors contribute to average systematic risk premia. There is substantial time variation in systematic risk premia, which in our model is driven solely by variation in the prices of risk as determined by λ_1 (factor conditional variances are constants). All of the estimated elements of λ_1 are statistically significant. The values of the lower three diagonal elements in λ_1 – the “own loadings” on each of the macro factors – are negative. In the case of the financial activity factor, for example, the negative value of $\lambda_{1,(ff)}$ implies that positive innovations to this factor lead to an increase in risk premia and a widening of corporate spreads, with the impact increasing in maturity. The off-diagonal terms in the lower-right block of λ_1 indicate that there are important interactions amongst the macro factors in the pricing of macroeconomic risk.

Estimated time series of the prices of systematic risk are plotted in Figure 4. Overall, the prices of risk on observable macroeconomic variables exhibit much greater time variation than those associated with latent factors. One of the most important episodes in terms of real output risk was when its market price became strongly negative prior

to the recession in 2001. This episode illustrates the interdependence among the macro factors in the pricing of risk. Even though the downturn in output had not yet transpired, the price of output risk had nonetheless been changing by early 2000 due to rising inflation and leverage. The increase in inflation also had a marginally negative impact on the price of inflation risk, although the (still) elevated level of real output meant that the total price of inflation risk remained positive prior to the recession (the value of $\lambda_{1,(\pi y)}$ is large and positive).

One drawback of our study is that our estimates of systematic risk premia may be distorted by liquidity and tax effects. Several papers point to the presence of significant liquidity premia in corporate bond spreads (Delianedis and Geske (2001), Janosi, Jarrow and Yildirim (2001), Driessen (2005), Longstaff, Mithal and Neis (2005)), although estimates vary widely. Liquidity effects are arguably less severe in our corporate yield data as a result of the procedure used by Bloomberg to construct the credit curves. Nonetheless, to the extent that one or more of the latent factors incorporate liquidity risk, our estimates suggest that time variation in liquidity premia is dominated by variation in premia arising from macroeconomic risk.¹⁴ Regarding the implications of taxes, Elton, Gruber, Agarwal and Mann (2001) and Driessen (2005) argue that spreads should include compensation for the differential treatment of taxes on interest income from corporate bonds relative to US Treasuries. While spreads probably reflect taxes to some extent, marginal tax rates on corporate bond income vary widely across jurisdictions, and, therefore, the impact of taxes on spreads will depend upon where the marginal investor resides.

5.3 Loadings on Macro Factors

The factor loadings, denoted by $-B_i(N)/N$, give the initial impact of an innovation to a factor on Treasury yields and corporate spreads at maturity N . Figure 5 displays these loadings for maturities $N = 1, \dots, 120$. Examining Treasuries first (left-hand panel), a shock to real activity generates the largest impact on yields at short maturities, while financial activity has a bigger effect at maturities beyond 60 months. While the effect of real activity monotonically declines towards zero as maturity increases, the

¹⁴It is also possible that liquidity premia in the corporate bond market is driven, in part, by our macro factors.

sign on financial activity switches sign at about the two-year maturity. This implies that a positive innovation to the financial factor has an inversion effect on the yield curve, with short-term rates rising and long-term rates falling. The large impact of financial activity on Treasury yields is a new finding that has not been documented in previous term structure studies. Inflation, by contrast, has a relatively muted effect on Treasury yields, with its impact increasing slightly with maturity. This result differs sharply from estimates obtained in many previous studies, including Ang and Piazzesi (2003).¹⁵ They estimate that shocks to inflation, versus those to real activity, had a much stronger initial effect on the yield curve. One key difference is that our model is estimated over a sample period of relatively low and stable inflation, whereas Ang and Piazzesi's data sample covered the 1970s and early 1980s, a period when both inflation and Treasury yields were high and highly volatile.

The centre and right-hand panels in Figure 5 report the factor loadings in BBB and B-rated spreads, respectively. Positive shocks to the financial activity indicator have a positive impact on spreads that is largely increasing in size with maturity. This is consistent with standard structural models of default, in which an increase in leverage or volatility, for example, raises the probability of default and, hence, spreads. An increase in real activity reduces BBB-rated spreads at short maturities, but the sign of the loadings changes for maturities greater than about 60 months. In any case, the loadings on real activity are much smaller in magnitude compared to financial activity. By contrast, a rise in real activity lowers B-rated spreads at most of the maturities considered. A rise in inflation leads to a widening of short-maturity B-rated spreads, though it has a negligible impact on BBB-rated spreads. The relatively stronger impact of real activity and inflation on high-yield versus investment grade debt is consistent with theories that attribute a greater impact to cyclical fluctuations on lower-rated debt, possibly due to sharper financial frictions faced by these firms.

Previous literature has documented a negative unconditional correlation between proxies for the risk-free rate and corporate spreads. In empirical work on corporate term structure models with latent factors, Duffee (1999) found that the two latent factors determining the risk-free rate in his model had negative loadings, on average, in the risk-neutral intensities of the 169 firms in his sample. In our sample, the unconditional correlations between the one-month Treasury yield and 60-month spreads are -0.36

¹⁵Our findings on inflation are closer to the results in the VAR(12) model in Ang and Piazzesi (2003).

and -0.45 for BBB and B-rated bonds, respectively. One advantage of our modelling approach is that we can determine the contribution of macroeconomic variables to these correlations from the loadings plotted in Figure 5. Recall that all three macro factors have a positive impact on the risk-free rate in our model. Furthermore, it is evident that real activity is the only macro factor that has a strong negative impact on 60-month B-rated spreads. Thus, the differential response of risk-free rates and spreads to real activity is one source of the observed negative correlation between these variables. For BBB-rated spreads, by contrast, it seems that innovations to Latent 1, which could be interpreted as monetary policy shocks, are the primary source of this negative unconditional correlation, whereas the contribution of macro factors is minor.

5.4 Latent Factors

Turning to the latent factors, it can be seen in Figure 5 that a positive shock to Latent 1 or Latent 2 raises Treasury yields at all maturities, with the size of the effect decreasing (increasing) with the former (latter). Latent 3 has little impact on the Treasury curve. An increase in Latent 1 leads to a narrowing of BBB-rated spreads at all maturities. By contrast, positive shocks to Latent 2 or Latent 3 lead to a widening of spreads, with the impact of Latent 2 declining monotonically across the BBB curve and the opposite for Latent 3. The loadings on the latent factors are relatively large (in absolute value) compared to real activity and inflation, and similar in size to those on financial activity. For B-rated bonds, an increase in Latent 3 also leads to a widening of spreads, whereas the impact of the other latent factors differs from BBB-rated bonds, specifically, an increase in Latent 2 lowers the term structure of B-rated spreads and Latent 1 has only a minor impact across the curve.

We would like to relate the latent factors to the shapes of the term structures. Figure 6 plots filtered estimates of the latent factors with the levels, slopes and curvatures of the Treasury and corporate curves (as defined in the regression analysis above). Table 7 reports estimates from univariate regressions of the filtered latent factors on the curve variables. As foreshadowed by the factor loadings in Figure 5, Latent 1 is closely related to the level of the Treasury curve. One interpretation of this result is that Latent 1 is capturing interest rate smoothing by the Federal Reserve. Since our specification of the risk-free rate explicitly omits *lagged* risk-free rate terms, this latent factor picks up much

of the persistence in the three-month Treasury bill rate (see Table 2). There is also a strong relationship between Latent 1 and the level and curvature of the BBB-rated curve, suggesting a link between the smoothing behaviour of the Fed and movements in the term structure of investment grade spreads. In contrast, the link between Latent 1 and the B-rated curve is much weaker.

The regression results also indicate that Latent 3 explains a larger portion of the variation in the level of the credit curves than does Latent 2. Yet while Latent 2 also appears to be related to the level of the BBB-rated curve, there is no relationship between Latent 2 and the level of the B-rated curve. Finally, note that the latent factors capture relatively little of the variation in the *slopes* of the term structures of spreads. This is where the macro factors have a relatively bigger impact on the shapes of the curves (see Table 5, as discussed above).

5.5 Variance Decompositions

Evidence on the proportion of the variance in conditional forecast errors due to each of the factor innovations is given in Table 8. The table reports variance decompositions of Treasury yields and spreads at 3M, 12M and 60M maturities and forecast horizons of 3, 12 and 60 months.

Both macro and latent factors contribute significantly to the conditional variability of Treasury yields. At a 3-month horizon, one-third of the variation in the 3M Treasury yield is due to real activity and two-thirds to Latent 1 and Latent 2. For higher maturities, the latent factors account for a greater percentage of the variation. As the forecast horizon increases, the financial activity factor accounts for a greater fraction of the variation in Treasury yields, for example, 38% of the 12M Treasury at a 60-month horizon. Inflation accounts for virtually none of the conditional variances of Treasury yields at the horizons considered. By contrast, Ang and Piazzesi (2003) found that inflation accounts for about 60-70% of the variation in 12M yields.

Most of the conditional variances of spreads is driven by the financial activity and latent factors. Innovations to Latent 2 are responsible for most of the variation in BBB-rated spreads, particularly for shorter maturities. Latent 1 becomes more relevant at a 60M maturity. For B-rated spreads, Latent 3 is the main driver of conditional variances. As anticipated from Figure 5, the financial factor also contributes to the conditional

variances of spreads, particularly at longer horizons.

The dominance of the latent factors in driving variation in spreads of both rating categories recalls one of the main conclusions in the study by Collin-Dufresne, Goldstein and Martin (2003); namely, that an unknown “corporate bond market factor” seems to be the principal source of fluctuations in spreads. Our results point to one dominant factor per rating category, which suggests that independent local demand and supply factors may be operating in different segments of the corporate bond markets. A topic for future research is to assess whether a richer state space of observable macroeconomic variables would attach less weight to latent factors in the conditional variances of spreads.

6 Conclusion

In this study we estimated arbitrage-free term structure models of US Treasury yields and spreads on BBB and B-rated corporate bonds in a doubly-stochastic intensity-based framework. A novel feature of our approach is the inclusion of macroeconomic variables – indicators of real activity, inflation and financial conditions – as well as latent factors, as drivers of term structure dynamics.

In our model we allowed the instantaneous risk-free rate to depend upon both of these variables. It turned out that Treasury yields, especially at the short end of the maturity spectrum, are strongly affected by the macro factors. Furthermore, we allowed for a third latent factor to affect corporate spreads, and we examined the relationship between financial conditions and the factors driving corporate spreads. Our results point to three key roles played by macro factors in the term structure of spreads: they have a significant impact on the level, and particularly the slope, of the curves; they are largely responsible for variation in the prices of systematic risk; and speculative grade spreads exhibit greater sensitivity to macro shocks than high grade spreads.

Since our focus was on the role of macro variables in affecting yields, the fact that the data used here spans more than one complete business cycle is an important advantage in this study. We focused only on corporate bond data, although in the last few years the CDS market is becoming a suitable source of data for examining credit spreads. Given the relatively short life of the CDS market, most research on spreads has been

conducted using bond data. In the near future, when CDS data will be available over a longer time period, the framework proposed here could be easily extended to such datasets. There are several reasons to focus on the CDS market instead of the corporate bond market. One is that default swaps now play a central role in credit markets: a broad range of investors use default swaps to express credit views; banks use them for hedging purposes; and default swaps are a basic building block in synthetic credit structures. Another is that the relatively high liquidity in the default swap market means that CDS spreads are presumably a fairly clean measure of default and recovery risk compared to spreads on most corporate bonds. CDS rates are therefore less likely to be affected by market illiquidity than are bond yield spreads. This facilitates the identification of credit risk premia.

To date, the literature on integrating dynamic term structure models with dynamic macroeconomic has not yet built on recent work of Dai, Le, and Singleton (2006), who developed a rich class of discrete-time, nonlinear dynamic term structure models. Our methodology focused exclusively on discrete-time Gaussian dynamic term structure models, thereby ruling out a role for either nonlinearity or time-varying second moments in modeling macroeconomic risks. Further research in this area seems warranted, since this new approach would allow the investigation of much richer specifications of risk premiums than the ones presented in this study.

A Construction of Macro Factors

We follow Ang and Piazzesi (2003) in constructing the macro factors for output and inflation, as well as the financial factor analysed in section 4. The approach taken is to utilise information on several related variables in order to construct a single indicator variable each of real activity ($X_{y,t}$), inflation ($X_{\pi,t}$) and financial conditions ($X_{f,t}$), and thereby reduce the dimensionality of the state space. This is done using principal components analysis.

The sets of observable variables used to estimate the macro and financial factors are described in section 3. Group together the variables by type, as follows:

$$X_t^y = [\text{HELP}_t \text{ UE}_t \text{ EMPLOY}_t \text{ IP}_t]$$

$$X_t^\pi = [\text{CPI}_t \text{ PPI}_t \text{ PCOM}_t]$$

$$X_t^f = \left[\frac{DEBT_t}{PROF_t} \quad \frac{INT_t}{GDP_t} \quad \frac{PRO_t}{SALES_t} \quad IMPVOL_t \right]$$

Table A.1 reports summary statistics on these variables.

All of the individual series in X_t^y and X_t^π are published at a monthly frequency. By contrast, only $IMPVOL_t$ is available (at least) on a monthly basis among the components in X_t^f . To construct the monthly financial conditions indicator, we first transform the quarterly series in X_t^f into monthly series. This is done using the approach in Litterman (1983). First, we impose the constraint that the average of within-quarter monthly values equals the observed quarterly value. Second, the monthly values $y_{t,i}^m$ are assumed to be linearly related to a set of P observable monthly variables $x_{t,i}^{m,p}$:

$$y_{t,i}^m = \beta_1 x_{t,i}^{m,1} + \beta_2 x_{t,i}^{m,2} + \dots + \beta_p x_{t,i}^{m,p} + u_{t,i}^m$$

where

$$u_{t,i}^m = u_{t,i-1}^m + \epsilon_{t,i}^m$$

and

$$\epsilon_{t,i}^m = \alpha \epsilon_{t,i-1}^m + e_{t,i}^m$$

where $e_{t,i}^m$ is a white noise process with mean 0 and variance σ^2 . The random walk assumption for the monthly error term $u_{t,i}^m$ defines a filter that removes all serial correlation in the quarterly residuals when the model is correct; when this is not true, then our specification of the dynamics of $\epsilon_{t,i}^m$ provide more accurate results. In our case, we use just one instrument $x_{t,i}^{m,1}$, the observed monthly values of $IMPVOL_t$.

We define $X_{i,t}$ ($i = y, \pi, f$) to be the first principal component of X_t^i , namely, $X_{i,t} = \Omega_1^\top X_t^i$, where Ω_1 is the eigenvector corresponding to the largest eigenvalue Λ_1 of $var(X_t^i) = \Omega \Lambda \Omega^\top$. Table A.2 shows the loadings of the observable variables on the ordered principal components for each factor. Over 67% of the variance of the real activity variables is explained by the first principal component, 62% for inflation and 69% for financial conditions. The loadings on the first principal component have the expected sign in all cases. To aid intuition, we multiply the factors by -1 so that, for example, an increase in industrial production leads to an increase in $X_{y,t}$. Table A.3 reports the correlations between the factors and the underlying observable variables.

B Estimation using the Kalman Filter

Affine models can be naturally cast as state-space systems, where the observation equation links observable yields and factors to the state vector and the transition equation describes the dynamics of the state. The Kalman filter has been used to estimate affine term-structure models in many studies; early examples are Duan and Simonato (1995) and Lund (1997). In this appendix, we layout the state-space form of our model and provide further details on our estimation technique.

As stated in (10) and (12), zero-coupon Treasury yields and corporate spreads are a linear function of the state:

$$y_{T,t}(N) = -\frac{1}{N} \left(A_T(N) + B_T(N)^\top X_t \right) \quad (13)$$

$$S_{j,t}(N) = -\frac{1}{N} \left(A_j(N) + B_j(N)^\top X_t \right) \quad (14)$$

As shown in Duffie and Kan (1996), the functions $A_T(N)$ and $B_T(N)$ in (13) and (14) can be obtained as solutions to the following set of ordinary differential equations (ODEs):

$$\begin{aligned} \frac{dA(N)}{dN} &= -\left(\tilde{K}\tilde{\Theta}\right)^\top B(N) + \frac{1}{2} \sum_{i=1}^N [\Sigma^\top B(N)]_i^2 - \delta_0, \\ \frac{dB(N)}{dN} &= -\tilde{K}^\top B(N) - \delta \end{aligned}$$

where

$$\delta = (\delta_T \quad \delta_f \quad \delta_y \quad \delta_\pi)^\top$$

$$\tilde{K} = K - \Sigma\lambda_1$$

$$\tilde{K}\tilde{\Theta} = -\Sigma\lambda_0$$

Similar expressions obtain for the loadings in spreads.

In estimation, we utilise time series data of length T_N for zero-coupon Treasury bond yields at maturities 1M, 3M, 12M, 36M, 60M and 120M and corporate bond spreads at maturities 12M, 36M, 60M, 84M and 120M. We assume that each of the yields and spreads is observed with measurement error. Let Y_t denote the vector of observable variables:

$$Y_t \equiv \left(Y_{T,t}^\top \quad Y_{BB,t}^\top \quad \underset{23}{Y_{B,t}^\top} \quad X_{f,t} \quad X_{y,t} \quad X_{\pi,t} \right)^\top$$

where

$$Y_{T,t} \equiv \begin{pmatrix} y_{T,t}(1M) & \cdots & y_{T,t}(120M) \end{pmatrix}^\top$$

$$Y_{j,t} \equiv \begin{pmatrix} S_{j,t}(12M) & \cdots & S_{j,t}(120M) \end{pmatrix}^\top$$

Similarly, let ε_t denote the vector of measurement errors:

$$\varepsilon_t \equiv \begin{pmatrix} \varepsilon_{T,t}^\top & \varepsilon_{BBB,t}^\top & \varepsilon_{B,t}^\top & 0 & 0 & 0 \end{pmatrix}^\top$$

The measurement equations of the state-space system can thus be written as:

$$Y_t = d + ZX_t + \varepsilon_t \quad (15)$$

where d and Z are defined implicitly in (13) and (14). ε_t is assumed to be normally distributed with mean 0 and diagonal variance-covariance matrix H .¹⁶

$$\varepsilon_t \sim N(0, H)$$

A discretised version of the state dynamics in (1) is:

$$X_t = \Phi X_{t-h} + \eta_t \quad (16)$$

where $\Phi = \exp(-Kh)$ and

$$\eta_t \sim N(0, I)$$

We utilise data at a monthly frequency, and so $h = 1/12$. Equations (15) and (16) form our state-space model.

In our baseline model, we use the method of maximum likelihood to estimate the parameters in step two of our estimation procedure conditional on OLS estimates of a subset of parameters obtained in step 1. More specifically, let Ψ_1 and Ψ_2 denote the vectors of parameters estimated in steps 1 and 2, respectively. In step two we maximize the conditional log-likelihood function:

$$\ln L(Y_t, \Psi_2) = \sum_{t=1}^{T_N} f(Y_t; \Psi_2, \hat{\Psi}_1)$$

¹⁶Since H has been assumed to be diagonal, there is no serial correlation and cross correlation in the measurement errors. Elements on the diagonal are allowed to differ, so that the variance of measurement error depends on maturity.

where $\hat{\Psi}_1$ denotes the OLS estimate of Ψ_1 . The log-likelihood is constructed using the Kalman filter. The Kalman filter recursions are initialized with the stationary mean and variance of the unobserved state variables. Standard errors are obtained numerically by evaluating the inverse Hessian matrix at the maximum likelihood estimates and under the assumption that parameters estimated in step 1 are estimated without error.

References

- [1] Allen, L and A Saunders (2003): “A survey of cyclical effects in credit risk measurement models”, BIS Working Paper no. 126.
- [2] Altman, E I, B Brady, A Resti and A Sironi (2005): “The link between default and recovery rates: theory, empirical evidence and implications”, *Journal of Business*, 78(6), pp 2203–2227.
- [3] Amato, J and T Laubach (1999): “The value of interest rate smoothing: how the private sector helps the Federal Reserve”, *Federal Reserve Bank of Kansas City Economic Review*, Third Quarter, pp 47–64.
- [4] Amato, J and E Remolona (2005): “The pricing of unexpected credit losses”, BIS Working Paper no. 190.
- [5] Ang, A and M Piazzesi (2003): “A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables”, *Journal of Monetary Economics*, 50, pp 745–787.
- [6] Ang, A, S Dong and M Piazzesi (2007): “No-arbitrage Taylor rules”, NBER Working paper, 13448.
- [7] Bakshi, G, D Madan and F Zhang (2001): “Understanding the role of recovery in default risk models: empirical comparisons of implied recovery rates”, working paper, University of Maryland.
- [8] Bakshi, G, D Madan and F Zhang (2006): “Investigating the role of systematic and firm-specific factors in default risk: lessons from empirically evaluating credit risk models”, *Journal of Business*, 79, pp 1955–1988.
- [9] Bernanke, B S, M Gertler and S Gilchrist (1999): “The financial accelerator in a quantitative business cycle framework”, in J B Taylor and M Woodford (eds), *Handbook of Macroeconomics*, vol. 1, Amsterdam: North-Holland, pp 1341–93.
- [10] Berndt, A, R Douglas, D Duffie, M Ferguson and D Schranz (2005): “Measuring default risk premia from default swap rates and EDFs”, working paper, Stanford University.

- [11] Cantor, R and C Mann (2003): “Measuring the performance of corporate bond ratings”, Moody’s Special Comment, April.
- [12] Chava, S and R Jarrow (2004): “Bankruptcy prediction with industry effects”, *Review of Finance*, 8, pp 537–569.
- [13] Collin-Dufresne, P, R Goldstein and J Spencer Martin (2001): “The determinants of credit spread changes”, *Journal of Finance*, 56, pp 2177–207.
- [14] Dai, Q and K Singleton (2000): “Specification analysis of affine term structure models”, *Journal of Finance* 55, pp 1943–1978.
- [15] Dai, Q, A Le, and K Singleton (2006): ”Discrete-time dynamic term structure models with generalized market prices of risk”, working paper, Stanford University.
- [16] Delianedis, G and R Geske (2001): “The components of corporate credit spreads: default, recovery, tax, jumps, liquidity and market factors”, Working Paper 22–01, Anderson Graduate School of Management, UCLA.
- [17] Dewachter, H and M Lyrio (2006): “Macro factors and the term structure of interest rates”, *Journal of Money, Credit, and Banking*, 38, pp 119–140.
- [18] Driessen, J (2005): “Is default event risk priced in corporate bonds?”, *Review of Financial Studies*, 18, pp 165–195.
- [19] Duan, J and J Simonato (1995): “Estimating and testing exponentially affine term structure models by Kalman filter”, working paper, McGill University.
- [20] Duffee, G (1999): “Estimating the price of default risk”, *Review of Financial Studies*, 12, pp 197–226.
- [21] Duffee, G (2002): “Term premia and interest rate forecasts in affine models”, *Journal of Finance* 57, pp 405–443.
- [22] Duffie, D. and R Kan (1996): “A yield-factor model of interest rates”, *Mathematical Finance*, 6, pp 379–406.
- [23] Duffie, D., and K Singleton (1999): “Modeling term structures of defaultable bonds”, *Review of Financial Studies*, 12, pp 687–720.

- [24] Duffie, D and K Singleton (2003): *Credit Risk: Pricing, Measurement and Management*, Princeton University Press.
- [25] Duffie, D, L Saita and K Wang (2007): “Multi-period corporate default prediction with stochastic covariates”, *Journal of Financial Economics*, 83, pp 635-665.
- [26] Elton, E, M Gruber, D Agrawal and C Mann (2001): “Explaining the rate spread on corporate bonds”, *Journal of Finance*, 56, pp 247–77.
- [27] Eom, Y, J Helwege and J Huang (2004): “Structural models of corporate bond pricing: an empirical analysis”, *Review of Financial Studies*, 17, pp 499–544.
- [28] Gertler, M and C Lown (1999): “The information in the high-yield bond spread for the business cycle: evidence and some implications”, *Oxford Review of Economic Policy*, 15, pp 132–150.
- [29] Harvey, A (1991): *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press.
- [30] Hordhal, P, O Tristani and D Vestin (2006): “A joint econometric model of macroeconomic and term structure dynamics”, *Journal of Econometrics*, 131, pp 405-444.
- [31] Huang, J and M Huang (2003): “How much of the corporate-treasury yield spread is due to credit risk?: a new calibration approach”, working paper, Penn State University.
- [32] Kealhofer, S (2003): “Quantifying credit risk I: default prediction”, *Financial Analysts Journal*, January–February, pp 30–44.
- [33] Janosi, T, R Jarrow and Y Yildirim (2001): “Estimating expected losses and liquidity discounts implicit in debt prices”, working paper, Cornell University.
- [34] Jarrow, R, D Lando and F Yu (2005): “Default risk and diversification: theory and applications”, *Mathematical Finance*, 15, pp 1–26.
- [35] Jonsson, J and M Fridson (1996): “Forecasting default rates on high-yield bonds”, *Journal of Fixed Income*, June, pp 69–77.

- [36] Lando, D (1998): “Cox processes and credit-risky securities”, *Review of Derivatives Research*, 2, pp 99-120.
- [37] Litterman, R (1983) “A random walk, markov model for the distribution of time series”, *Journal of Business and Economic Statistics*, 1, pp 169-173.
- [38] Litterman, R and J Scheinkman (1991): “Common factors affecting bond returns”, *Journal of Fixed Income*, 1, pp 51–61.
- [39] Longstaff, F, S Mithal and E Neis (2005): “Corporate yield spreads: default risk or liquidity? New evidence from the credit default swap market”, *Journal of Finance*, 60, pp 2213–2253.
- [40] Lund, J (1997): “Econometric analysis of continuous-time arbitrage-free models of the term structure of interest rates”, working paper, Aarhus School of Business.
- [41] Neal, R, D Rolph and C Morris (2001): “Interest rates and credit spreads”, working paper, Indiana University.
- [42] Pan, J and K Singleton (2005): “Default and recovery implicit in the term structure of sovereign CDS spreads”, working paper, Stanford University.
- [43] Piazzesi, M (2003): “Affine term structure models”, working paper, UCLA.
- [44] Rudebusch, G and T Wu (2003): “A macro-finance model of the term structure”, working paper, Federal Reserve Bank of San Francisco.
- [45] Singleton, J K (2006): *Empirical Dynamic Asset Pricing*, Princeton University Press.
- [46] Schultz, P (2001): “Corporate bond trading costs: a peek behind the curtain”, *Journal of Finance*, 56, pp 677–98.
- [47] Vasicek, O (1977): “An equilibrium characterization of the term structure”, *Journal of Financial Economics*, 5, pp 177–188.
- [48] Wu, T (2000): “Macro factors and the affine term structure of interest rates”, working paper, Yale University.

- [49] Wu, L and F Zhang (2005): “A no-arbitrage analysis of economic determinants of the credit spread term structure”, working paper, Federal Reserve Board.
- [50] Yang, J (2003): “An empirical investigation of the effects of macroeconomic variables on default risk”, working paper, University of Toronto.

Table 1
Data Description

Variable	Data Series	Frequency, Unit
Real Activity		
HELP	Index of Help Wanted Advertising in Newspapers	Monthly, sa, Index 1987=100
UE	Unemployment rate	Monthly, sa, Per cent
EMPLOY	Employment, civilian	Monthly, sa, Persons Thousands
IP	Industrial Production Index	Monthly, sa, Index 1997=100
Inflation		
CPI	Consumer Price Index, All Urban Consumers: All Items	Monthly, sa, Index 1982-84=100
PPI	Producer Price Index, Finished Goods	Monthly, sa, Index 1982=100
PCOM	Market Commodity Prices Index	Monthly, sa, Index 1996=100
Financial Activity		
SALES	Final Sales of Domestic Product	Quarterly, sa, US \$ Billion
DEBT	Credit Market Debt for NFCB	Quarterly, sa, US \$ Billion
PRO	Profit After Tax for NFCB	Quarterly, sa, US \$ Billion
INT	Net Interest Payments for NFCB	Quarterly, sa, US \$ Billion
GDP	GDP for NFCB	Quarterly, sa, US \$ Billion
IMPVOL	Implied Volatility on S&P 500 (extended back using VIX index)	Monthly
Yields		
Treasury	Zero-Coupon Bond Yields, Constant Maturity U.S. Treasury	Monthly, annualized
Industrial BBB	Fair Market Curve Index, Sector: Industrial, Rating: BBB	Monthly, annualized
Industrial B	Fair Market Curve Index, Sector: Industrial, Rating: B	Monthly, annualized

Notes: "sa" denotes seasonally adjusted; "NFCB" denotes Non-Financial Corporate Business.

Table 2
Summary Statistics on Treasury Yields and Corporate Spreads

Maturity (Months)							Autocorrelation			
							Lag 1	Lag 2	Lag 3	Lag 4
Treasury Yields										
1	3.71	1.54	-0.51	2.00	0.80	6.06	0.96	0.93	0.90	0.87
3	3.98	1.66	-0.59	2.00	0.90	6.38	0.98	0.96	0.93	0.89
12	4.30	1.69	-0.58	2.07	1.09	7.20	0.98	0.95	0.92	0.88
36	4.93	1.49	-0.64	2.51	1.58	7.80	0.97	0.93	0.88	0.84
60	5.30	1.26	-0.55	2.62	2.30	7.83	0.96	0.90	0.86	0.81
120	5.72	1.03	-0.21	2.42	3.37	7.91	0.95	0.89	0.84	0.78
BBB-rated Spreads										
12	1.05	0.36	0.15	1.69	0.49	1.80	0.95	0.89	0.84	0.80
36	1.06	0.44	0.24	1.81	0.38	1.96	0.97	0.94	0.91	0.88
60	1.11	0.45	0.21	1.66	0.45	1.97	0.97	0.94	0.91	0.88
84	1.17	0.46	0.42	1.85	0.56	2.13	0.97	0.94	0.92	0.89
120	1.24	0.47	0.51	1.96	0.61	2.31	0.98	0.95	0.92	0.89
B-rated Spreads										
12	3.86	1.43	0.57	3.52	1.45	7.99	0.96	0.91	0.86	0.81
36	3.94	1.38	0.76	3.39	1.73	7.99	0.96	0.91	0.86	0.81
60	4.08	1.21	0.88	3.27	2.31	7.53	0.94	0.88	0.83	0.77
84	4.11	1.12	1.04	3.30	2.64	7.33	0.93	0.86	0.81	0.74
120	4.21	1.06	0.93	3.03	2.75	7.12	0.93	0.85	0.79	0.72

Notes: This table reports summary statistics for US Treasury yields (Panel A), BBB-rated industrial spreads (Panel B) and B-rated industrial spreads (Panel C). A spread is calculated as the difference between an industrial yield and the Treasury yield with the same maturities. Data are at a monthly frequency, using month-end observations, over the period 1992:05-2004:04 (144 observations).

Table 3
Principal Components of Yields and Spreads

Maturity	Principal Components Loadings					
(Months)	1st	2nd	3rd	4th	5th	6th
Treasury Yields						
1	-0.42	-0.45	0.67	-0.38	0.13	0.00
3	-0.47	-0.39	-0.10	0.66	-0.43	0.04
12	-0.49	-0.10	-0.53	-0.05	0.66	-0.18
36	-0.43	0.28	-0.27	-0.45	-0.36	0.58
60	-0.34	0.46	0.09	-0.13	-0.33	-0.73
120	-0.25	0.59	0.42	0.45	0.36	0.29
Variance (marg)	92.11	7.19	0.50	0.16	0.05	0.01
Variance (cum)	92.11	99.29	99.79	99.95	99.99	100
BBB-rated Spreads						
12	-0.32	0.83	-0.41	-0.20	-0.01	
36	-0.46	0.21	0.60	0.34	0.53	
60	-0.47	-0.07	-0.01	0.53	-0.70	
84	-0.48	-0.21	0.33	-0.75	-0.22	
120	-0.48	-0.47	-0.61	0.05	0.42	
Variance (marg)	92.61	5.71	1.11	0.35	0.21	
Variance (cum)	92.61	98.32	99.43	99.79	100	
B-rated Spreads						
12	-0.51	0.59	0.61	-0.14	0.09	
36	-0.50	0.27	-0.52	0.44	-0.46	
60	-0.44	-0.13	-0.36	0.00	0.81	
84	-0.41	-0.34	-0.13	-0.77	-0.34	
120	-0.36	-0.67	0.46	0.45	-0.10	
Variance (marg)	94.31	4.72	0.76	0.15	0.06	
Variance (cum)	94.31	99.03	99.79	99.94	100	

Notes: This table contains the principal components loadings for US Treasury yields, and BBB-rated and B-rated industrial spreads. The rows labeled *Variance (marg)* (*Variance (cum)*) display the marginal (cumulative) variance explained by each of the principal components. The numbers in these rows are the percentage variation in variables explained by the first k principal components computed as $100 \times \frac{\sum_{i=1}^k \Lambda_i}{tr(\Lambda)}$. Sample period is 1992:05 - 2004:04.

Table 4
Correlations of Macro Factors with Yields and Spreads

Variables	Real	Infl	Fin	1M ^T	12M ^T	60M ^T	12M ^{BBB}	60M ^{BBB}	12M ^B	60M ^B
Real	1	0.14	-0.63	0.83	0.88	0.72	-0.53	-0.50	-0.69	-0.64
Infl	0	1	-0.11	0.18	0.17	0.20	-0.04	-0.03	0.16	0.07
Fin	0	0	1	-0.45	-0.55	-0.58	0.72	0.90	0.68	0.76
1M ^T	0	0	0	1	0.95	0.79	-0.41	-0.36	-0.53	-0.45
12M ^T	0	0	0	0	1	0.91	-0.43	-0.46	-0.60	-0.56
60M ^T	0	0	0	0	0	1	-0.32	-0.55	-0.55	-0.62
12M ^{BBB}	0	0	0	0	0	0	1	0.81	0.74	0.66
60M ^{BBB}	0	0	0	0	0	0	0	1	0.74	0.81
12M ^B	0	0	0	0	0	0	0	0	1	0.93
60M ^B	0	0	0	0	0	0	0	0	0	1

Notes: This table reports unconditional linear correlations over the sample period 1992:05 - 2004:04.

Table 5
Regressions of Yields and Spreads on Macro Factors

Maturity	Real Activity		Inflation		Financial		Adj. R ²
(Months)	Estim.	(Std. Err.)	Estim.	(Std. Err.)	Estim.	(Std. Err.)	
Treasury Yields							
1	0.0139	(0.0009)	0.0010	(0.0007)	0.0020	(0.0009)	0.69
60	0.0074	(0.0009)	0.0012	(0.0007)	-0.0025	(0.0009)	0.55
120	0.0033	(0.0008)	0.0016	(0.0006)	-0.0039	(0.0008)	0.44
Level	0.0094	(0.0008)	0.0011	(0.0006)	-0.0011	(0.0008)	0.66
Slope	-0.0106	(0.0009)	0.0006	(0.0007)	-0.0060	(0.0009)	0.47
Curvature	-0.0047	(0.0007)	0.0010	(0.0006)	-0.0013	(0.0002)	0.28
BBB-rated Spreads							
12	-0.0004	(0.0003)	0.0002	(0.0002)	0.0023	(0.0003)	0.52
60	0.0005	(0.0002)	0.0003	(0.0002)	0.0043	(0.0002)	0.81
120	0.0012	(0.0002)	0.0003	(0.0002)	0.0050	(0.0002)	0.83
Level	0.0004	(0.0002)	0.0003	(0.0001)	0.0039	(0.0002)	0.81
Slope	0.0017	(0.0003)	0.0002	(0.0002)	0.0026	(0.0003)	0.35
Curvature	-0.0001	(0.0002)	-0.0000	(0.0002)	-0.0013	(0.0002)	0.28
B-rated Spreads							
12	-0.0066	(0.0009)	0.0038	(0.0007)	0.0059	(0.0009)	0.63
60	-0.0034	(0.0008)	0.0022	(0.0006)	0.0072	(0.0008)	0.64
120	0.0004	(0.0007)	0.0003	(0.0006)	0.0087	(0.0007)	0.62
Level	-0.0032	(0.0008)	0.0021	(0.0006)	0.0073	(0.0008)	0.65
Slope	0.0070	(0.0006)	-0.0035	(0.0005)	0.0028	(0.0006)	0.53
Curvature	0.0007	(0.0005)	-0.0002	(0.0004)	0.0001	(0.0005)	0.00

Table 6
Estimates of Term Structure Model Parameters

	Index i :						
	0	1	2	3	f	y	π
$\delta_{i \times 100}$	0.310 (0.006)	0.049 (0.000)	0.042 (0.000)	0.000 (0.000)	0.017 (0.008)	0.116 (0.008)	0.008 (0.006)
$\gamma_i^{BBB} \times 100$	0.108 (0.002)	-0.015 (0.000)	0.016 (0.000)	0.007 (0.000)	0.011 (0.002)	-0.005 (0.002)	0.002 (0.002)
$\gamma_i^B \times 100$	0.337 (0.006)	-0.009 (0.000)	-0.029 (0.000)	0.064 (0.000)	0.041 (0.008)	-0.052 (0.008)	0.032 (0.006)
k_{1i}	—	0.321 (0.031)	0	0	0	0	0
k_{2i}	—	0.082 (0.006)	0.5238 (0.075)	0	0	0	0
k_{3i}	—	-0.043 (0.004)	0.0062 (0.004)	1.797 (0.087)	0	0	0
k_{fi}	—	0	0	0	0.405 (0.033)	0.924 (0.017)	0.161 (0.034)
k_{yi}	—	0	0	0	-0.233 (0.034)	0.712 (0.018)	-0.234 (0.034)
$k_{\pi i}$	—	0	0	0	-0.260 (0.026)	0.229 (0.014)	0.662 (0.027)
$\lambda_{0,i}$	—	-0.060 (0.004)	-0.057 (0.003)	-0.035 (0.002)	-0.006 (0.003)	-0.010 (0.012)	-0.034 (0.010)
$\lambda_{1,(1i)}$	—	-0.033 (0.002)	0	0	0	0	0
$\lambda_{1,(2i)}$	—	0	-0.043 (0.002)	0	0	0	0
$\lambda_{1,(3i)}$	—	0	0	-0.073 (0.002)	0	0	0
$\lambda_{1,(fi)}$	—	0	0	0	-0.121 (0.006)	0.039 (0.012)	0.111 (0.007)
$\lambda_{1,(yi)}$	—	0	0	0	-0.426 (0.013)	-0.205 (0.016)	-0.146 (0.010)
$\lambda_{1,(\pi i)}$	—	0	0	0	0.081 (0.021)	0.255 (0.034)	-0.052 (0.022)
Log-likelihood = 1279; AIC = -25510; BIC = -25385; Number of parameters = 42							

Notes: Asymptotic standard errors are reported in parentheses.

Table 7
Regressions of Latent Factors on Curve Dynamics

	Dependent Variable:					
	Latent 1		Latent 2		Latent 3	
	Estim. (Std. Err.)	R ²	Estim. (Std. Err.)	R ²	Estim. (Std. Err.)	R ²
Treasury Level	0.81 (0.05)	0.66	0.39 (0.08)	0.16	-0.06 (0.08)	0.00
Treasury Slope	-0.04 (0.08)	0.00	0.07 (0.08)	0.00	-0.16 (0.08)	0.03
Treasury Curvature	-0.66 (0.06)	0.44	-0.43 (0.08)	0.18	0.22 (0.08)	0.05
BBB Spreads Level	-0.63 (0.07)	0.40	0.43 (0.08)	0.18	0.58 (0.07)	0.34
BBB Spreads Slope	-0.43 (0.08)	0.18	-0.07 (0.08)	0.00	0.24 (0.08)	0.06
BBB Spreads Curvature	0.72 (0.06)	0.52	0.22 (0.08)	0.05	-0.24 (0.08)	0.06
B Spreads Level	-0.48 (0.07)	0.23	0.16 (0.08)	0.02	0.81 (0.05)	0.66
B Spreads Slope	0.23 (0.08)	0.05	0.04 (0.08)	0.00	-0.27 (0.08)	0.07
B Spreads Curvature	0.28 (0.08)	0.08	0.53 (0.07)	0.28	-0.02 (0.08)	0.00

Table 8
Variance Decompositions of Treasury Yields and Corporate Spreads

Maturity	Horizon	Financial	Real Activity	Inflation	Latent 1	Latent 2	Latent 3
Treasury Yields							
3	3	0.01	0.33	0.00	0.37	0.29	0.00
	12	0.19	0.23	0.02	0.32	0.24	0.00
	60	0.40	0.11	0.11	0.23	0.15	0.00
12	3	0.02	0.25	0.00	0.39	0.34	0.00
	12	0.19	0.18	0.01	0.34	0.28	0.00
	60	0.38	0.09	0.10	0.25	0.18	0.00
60	3	0.10	0.02	0.01	0.37	0.50	0.01
	12	0.20	0.02	0.00	0.34	0.44	0.00
	60	0.28	0.01	0.04	0.30	0.37	0.00
BBB-rated Spreads							
3	3	0.08	0.02	0.01	0.05	0.66	0.19
	12	0.13	0.01	0.03	0.06	0.66	0.11
	60	0.13	0.01	0.06	0.12	0.61	0.07
12	3	0.08	0.01	0.00	0.07	0.61	0.23
	12	0.14	0.01	0.02	0.08	0.63	0.13
	60	0.13	0.01	0.04	0.14	0.59	0.08
60	3	0.11	0.01	0.01	0.16	0.28	0.44
	12	0.16	0.01	0.01	0.19	0.35	0.29
	60	0.14	0.01	0.01	0.28	0.36	0.19
B-rated Spreads							
3	3	0.04	0.01	0.13	0.08	0.23	0.51
	12	0.18	0.01	0.19	0.07	0.21	0.34
	60	0.27	0.01	0.25	0.05	0.18	0.23
12	3	0.04	0.01	0.08	0.07	0.25	0.54
	12	0.19	0.01	0.13	0.07	0.24	0.36
	60	0.30	0.01	0.20	0.05	0.20	0.24
60	3	0.11	0.00	0.00	0.05	0.29	0.55
	12	0.27	0.00	0.02	0.04	0.29	0.38
	60	0.37	0.01	0.06	0.03	0.26	0.27

Notes: In this table we report the percentage of the conditional variances of Treasury yields (top panel), BBB-rated industrial spreads (middle panel) and B-rated industrial spreads (bottom panel) explained by each of the factors at various forecast horizons.

Table A.1
Summary Statistics on Macro Variables

Variable	Mean	SD	Skew	Kurt	Min	Max	Autocorrelation			
							Lag 1	Lag 2	Lag 3	Lag 4
Real Activity										
HELP	70.92	17.86	-0.66	2.05	35	93	0.98	0.96	0.94	0.92
UE	5.56	1.06	0.22	2.10	3.8	7.80	0.99	0.98	0.96	0.95
EMPLOY	1.17	1.04	-0.59	2.71	-1.52	3.19	0.93	0.87	0.81	0.74
IP	2.91	3.18	-0.82	3.04	-5.55	8.05	0.96	0.92	0.88	0.82
Inflation										
CPI	2.57	0.71	0.22	3.48	1.06	4.91	0.91	0.80	0.73	0.64
PCOM	0.73	11.24	0.55	2.70	-18.65	29.22	0.94	0.86	0.78	0.69
PPI	1.41	1.71	-0.38	2.46	-2.79	4.70	0.93	0.85	0.79	0.69
Financial										
DEBT/PRO	15.16	5.16	1.78	8.73	9.49	38.41	0.82	0.70	0.64	0.58
INT/GDP	0.03	0.01	1.06	3.59	0.03	0.047	0.88	0.74	0.58	0.43
PRO/SALES	0.03	0.01	-0.01	2.19	0.01	0.042	0.93	0.86	0.80	0.73
IMPVOL	17.84	5.12	0.59	2.90	9.83	31.50	0.73	0.60	0.55	0.50

Notes: This table reports summary statistics for the variables used in the construction of the macro factors. Panel A reports the variables that capture real activity: the index of Help Wanted Advertising in Newspapers (HELP), unemployment (UE), the growth rate of employment (EMPLOY) and the growth rate of industrial production (IP). Panel B reports various inflation measures which are based on the Consumer Price Index (CPI), the Producer Price Index of finished goods (PPI), and Market Commodity Prices Index (PCOM). Panel C reports the data used in the construction of the financial activity factors are Final Sales of Domestic Product (SALES), Credit Market Debt (DEBT), Profit After Tax (PRO), Net Interest Payments (INT), GDP, and implied volatility on the SP500 (IMPVOL). Data on DEBT, PRO, INT, and GDP refer to Non Financial Corporate Business. Data on IMPVOL are obtained joining the Bloomberg historical call implied volatility observed on the SP500 index for the period 1994-2004 with the VIX index for the early sample 1992-1994. All growth rates (including inflation) are measured as the difference in logs of the index at time t and $t - 12$, t in months. The variables EMPLOY, IP, CPI, PPI, and PCOM are measured by annual growth rates, where IP is the annual industrial production growth rate and CPI is the annual inflation rate. We collected data on SALES, PRO, INT, and GDP from the Bureau of Economic Analysis; data on DEBT are from the Flow of Funds (Liabilities). Data for UE, EMPLOY, CPI, and PPI are from U.S. Department of Labor, Bureau of Labor Statistics; while IP and HELP are from the Board of Governors of the Federal Reserve System.

Table A.2
Principal Components of Macro Factors

Principal Components				
Variables	1st	2nd	3rd	4th
Real Activity				
HELP	-0.54	0.01	-0.78	-0.32
UE	0.34	0.90	-0.15	-0.21
EMPLOY	-0.55	0.19	0.60	-0.55
IP	-0.54	0.39	0.07	0.74
Variance (marg)	67.56	20.41	8.42	3.60
Variance (cum)	67.56	87.97	96.40	100
Inflation				
CPI	-0.59	0.54	-0.60	
PCOM	-0.43	-0.84	-0.33	
PPI	-0.68	0.058	0.73	
Variance (marg)	62.60	29.89	7.50	
Variance (cum)	62.60	92.50	100	
Financial				
DEBT/PRO	-0.56	0.11	-0.54	-0.63
INT/GDP	-0.52	0.20	0.81	-0.20
PRO/SALES	0.56	-0.25	0.24	-0.75
IMPVOL	-0.32	-0.94	0.05	0.08
Variance (marg)	69.39	19.98	8.46	2.17
Variance (cum)	69.39	89.37	97.83	100

Notes: This table reports the eigenvectors corresponding to the eigenvalues of the covariance matrices of the three groups of variables used to construct the Real Activity factor (Panel A), the Inflation factor (Panel B), and the Financial factor (Panel C). The rows labeled *Variance (marg)* (*Variance (cum)*) display the marginal (cumulative) variance explained by each of the principal components. The numbers in these rows are the percentage variation in variables explained by

the first k principal components computed as $100 \times \frac{\sum_{i=1}^k \Lambda_i}{\text{tr}(\Lambda)}$.

Table A.3
Correlations of Macro Factors with Macro Variables

	HELP	UE	EMPLOY	IP
Real Activity	0.88	-0.57	0.90	0.90
	CPI	PCOM	PPI	
Inflation	0.81	0.58	0.94	
	DEBT/PRO	INT/GDP	PRO/SALES	IMPVOL
Financial	0.93	0.86	-0.94	0.54

Notes: This table reports the correlations between the macro factors and the variables used to extract the first principal components.

Figure 1
Treasury Yields and Corporate Spreads

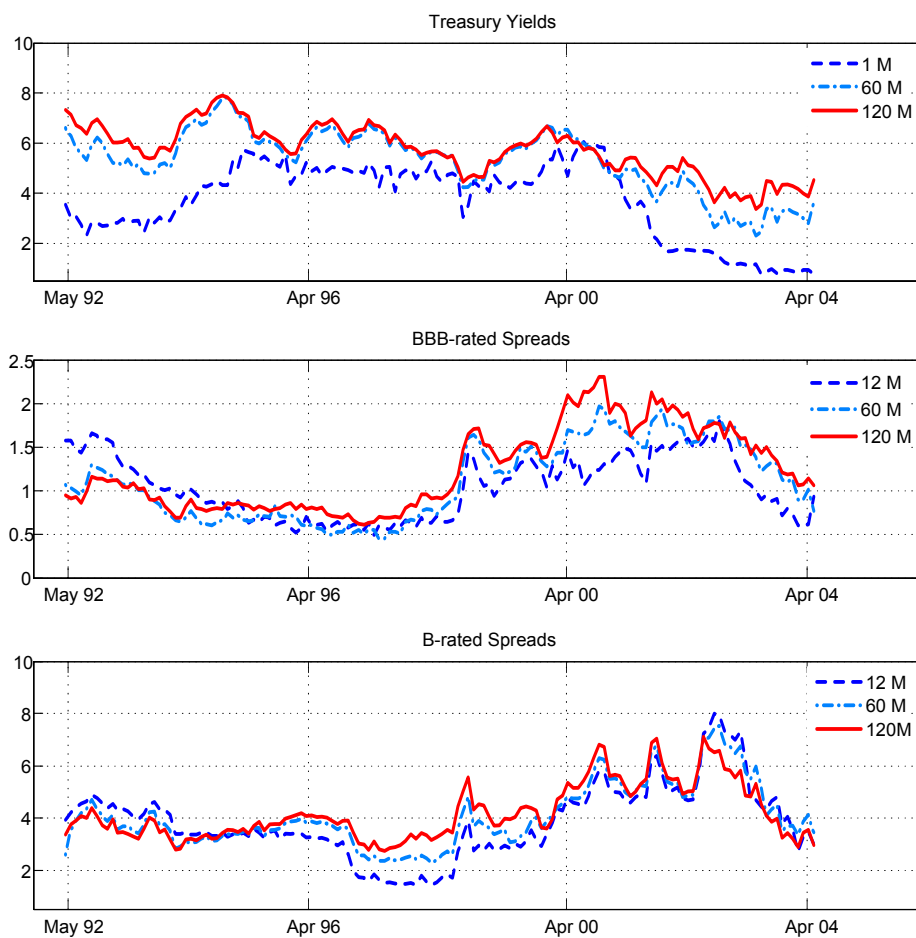
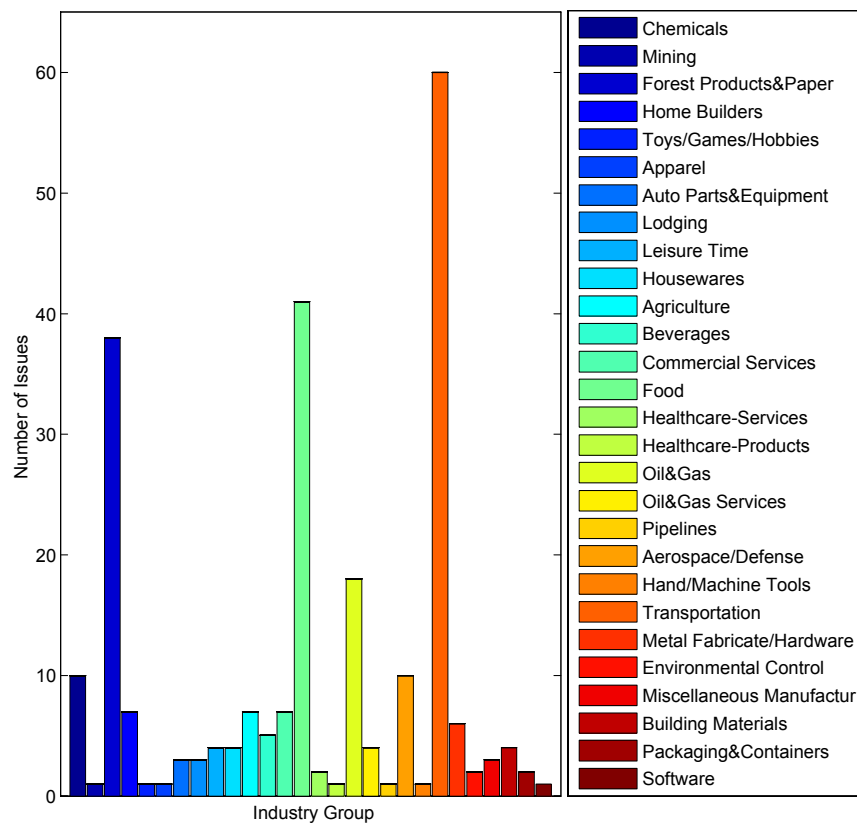


Figure 2
Industry Composition of BBB-rated Industrial Yield Curve



Notes: This graph reports the composition of the basket of bonds used to construct Bloomberg's BBB-rated industrial bond yield index on 24 August 2004.

Figure 3
Macroeconomic Factors

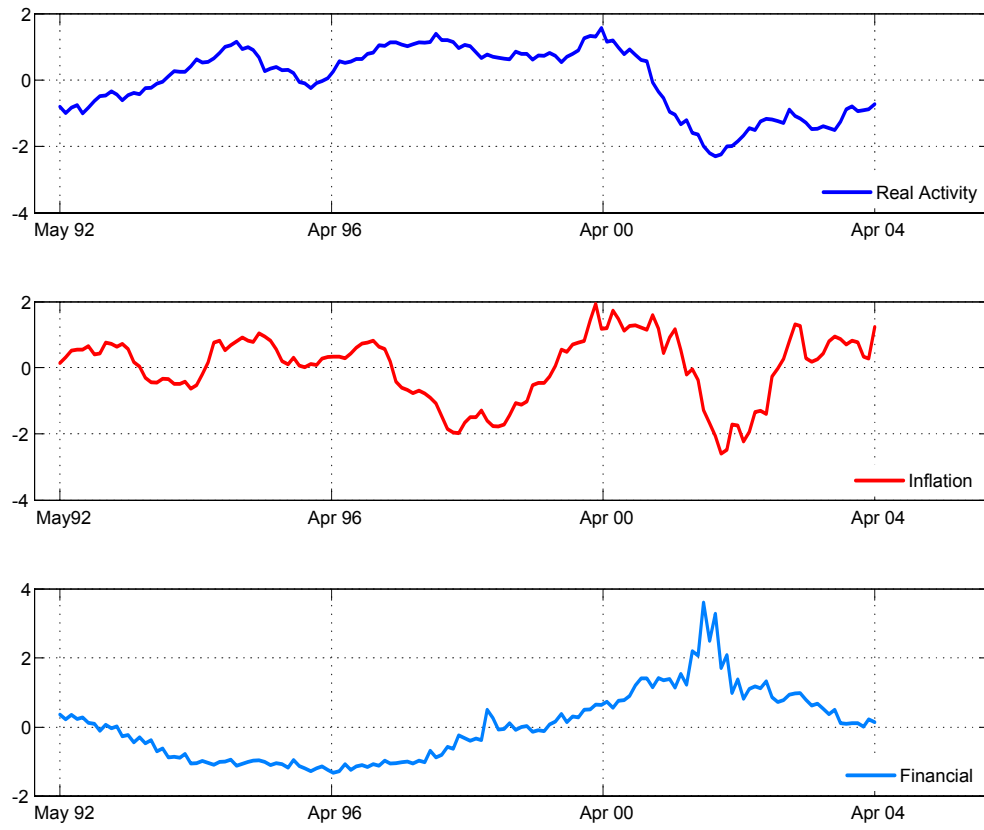


Figure 4
Market Prices of Systematic Risk

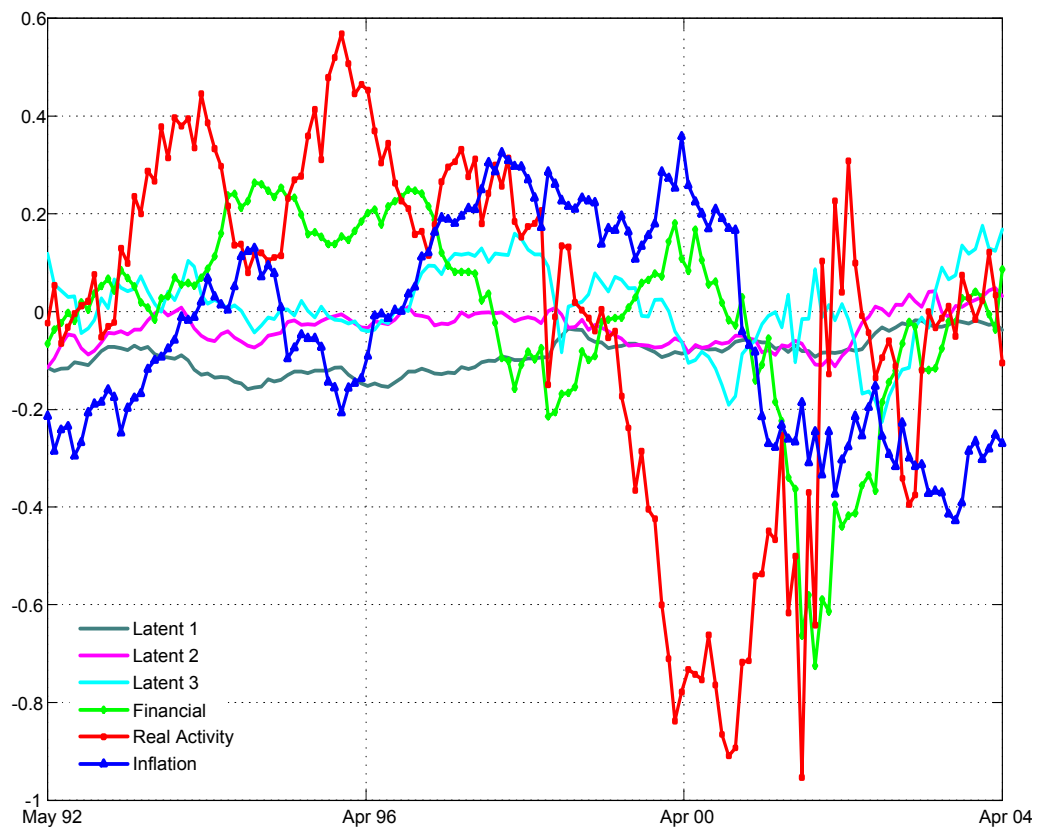
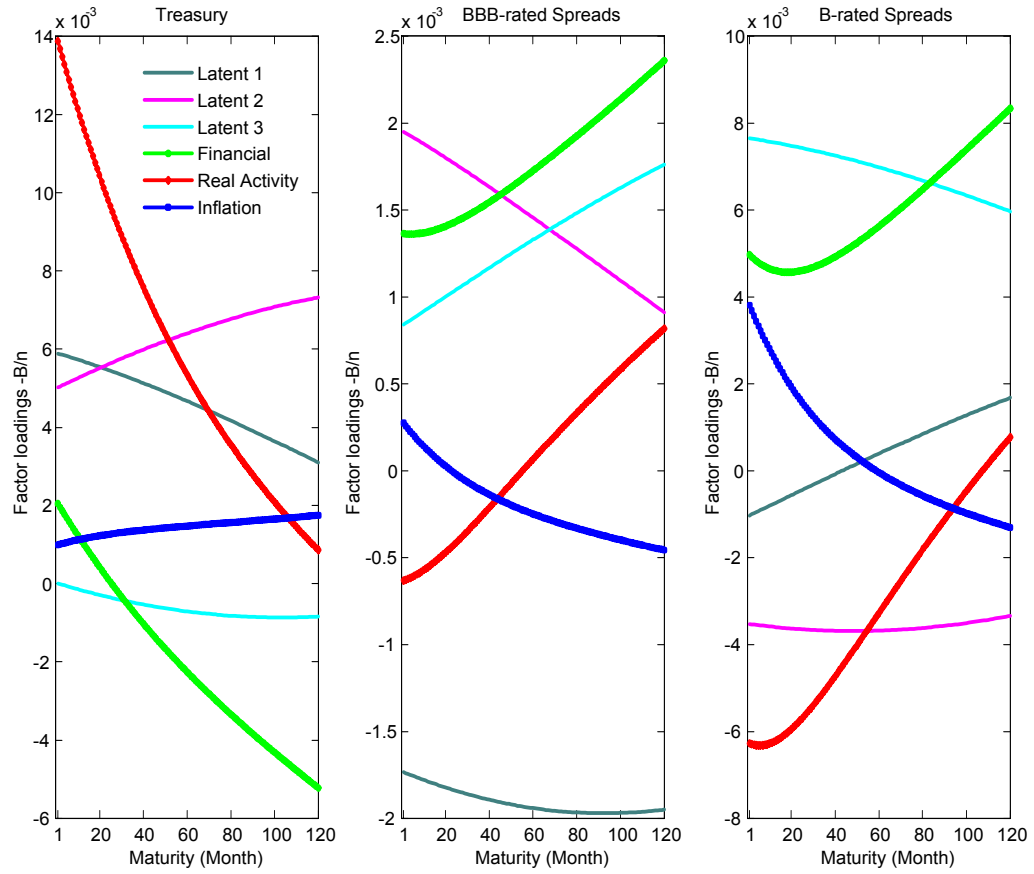
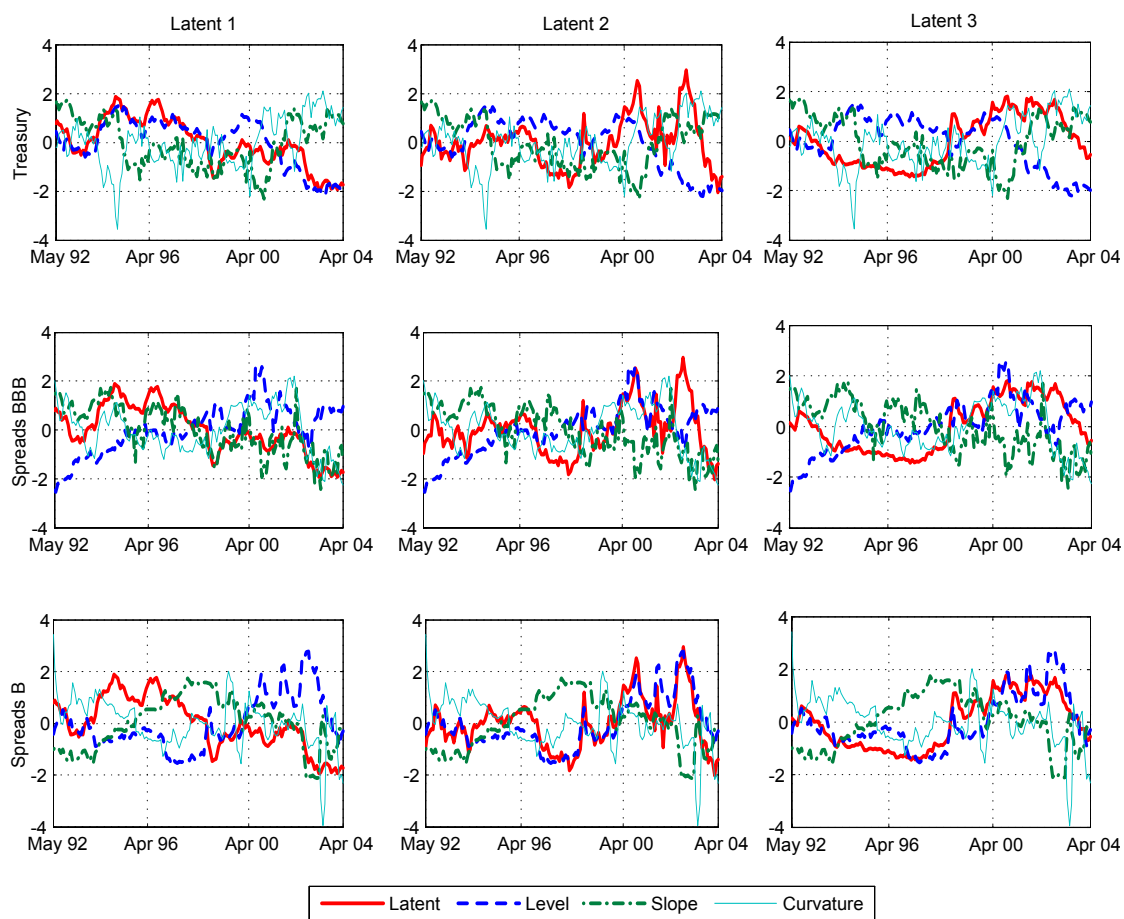


Figure 5
Factor Loadings



Notes: This figure displays the estimates of the factor loadings $-B(N)/N$ in the affine expressions for bond yields and spreads.

Figure 6
Latent Factors



Chapter 3 - Estimates of physical and risk-neutral default intensities using data on EDFsTM and spreads

Maurizio Luisi
University of Lugano

Abstract

In this chapter, in addition to estimating risk-neutral default intensities, we provide estimates of physical default intensities using data on Moody's KMV EDFsTM as a forward-looking proxy for default risk. We find that the real and financial activity indicators, along with filtered estimates of the latent factors from our term structure model, explain a large portion of the variation in EDFsTM across time. Furthermore, measures of the price of default event risk implied by estimates of physical and risk-neutral intensities indicate that compensation for default event risk is countercyclical, varies widely across the cycle, and is higher on average and more variable for higher-rated bonds.

JEL Classification Numbers: C13, C32, E44, E52, G12, G13, G14

Keywords: corporate bonds, default intensity, event risk, risk premia, interest rate rule

1 Introduction

This study contributes to the literature linking physical default probabilities to macroeconomic variables. Several studies on default prediction point to a large negative correlation between default probabilities and the business cycle. While the use of spreads data in the term structure model presented in Chapter 2 only enables us to uncover *risk-neutral* instantaneous loss rates (see Jarrow, Lando and Yu (2005) for further discussion of this issue), by using an additional source of data on default risk we can also estimate *physical* instantaneous loss rates. In our case, this is accomplished by fitting one-year default probabilities implied by a doubly-stochastic intensity model to Expected Default Frequencies (EDFsTM) from Moody's KMV, which are assumed to be proxies for real world default probabilities. By assuming that physical default intensities are driven by the same factors determining spreads, we are able to explain a large portion of the time series variation in EDFsTM on both BBB and B-rated industrial bonds. The real and financial activity indicators, in particular, have significant marginal predictive power for future default risk.

Estimates of physical and risk-neutral default intensities obtained using data on EDFsTM and spreads, respectively, provide new evidence on the size and evolution of the price of default *event* risk. If investors can conditionally diversify credit portfolios – that is, investors can eliminate their exposure to individual defaults – then the default event itself will not be priced (Jarrow, Lando and Yu (2005)). Recent evidence indicates this not to be true and that the market price of default event risk has been large and highly volatile over time (Driessen (2005), Berndt, Douglas, Duffie, Ferguson and Schranz (2005), Amato and Remolona (2005)). For instance, using data on credit default swaps, Berndt, Douglas, Duffie, Ferguson and Schranz (2005) find that the average price of default event risk is approximately between one and two, which means that risk-neutral default probabilities are more than twice the size of physical default probabilities even in the absence of systematic risk. To our knowledge, we are the first to estimate the market price of default event risk across the business cycle and, in particular, to assess how it is related to observable measures of macroeconomic activity. We find that the price of default event risk is countercyclical, varies significantly across the cycle, and is higher and more variable for higher rated debt.

The rest of this paper is organized as follows. Section 2 discusses our data sources for

conditional default probabilities. Section 3 introduces the main theoretical implications in decomposing instantaneous spreads in risk-neutral intensity and risk-neutral loss given default. Section 4 reports the estimation of physical intensities using EDFsTM and the estimates of the market price of default event risk. Section 5 presents the conclusions that can be drawn from our empirical analysis.

2 Data

In this section we discuss data sources for conditional default probabilities, including an overview on the construction of the measure of default probability we choose to use in our empirical analysis.

2.1 A Practical Approach in Measuring Default Probabilities

A standard structural model of default timing assumes that a corporation defaults when its assets drop to a sufficiently low level relative to its liabilities. For example, the models of Black and Scholes (1973), Merton (1974), Fisher, Heinkel, and Zechner (1989), and Leland (1994) take the asset process to be a geometric Brownian motion. In these models, a firm’s conditional default probability is completely determined by its distance to default, which is the number of standard deviations of annual asset growth by which the asset level (or expected asset level at a given time horizon) exceeds the firm’s liabilities.¹ Estimates of current assets and the current standard deviation of asset growth (“volatility”) are calibrated from historical observations of the firm’s equity-market capitalization and of the liability measure. The calibration of these models, explained for example in Vassalou and Xing (2004), is based on the model of Black and Scholes (1973) and Merton (1974), by which the price of a firm’s equity may be viewed as the price of an option on assets struck at the level of liabilities. Vasicek and Kealhofer have extended the Black-Scholes-Merton framework to produce a model of default probability known as the Vasicek-Kealhofer (VK) model. To overcome the regular problems encountered by structural models due to the assumption of normality (see Eom, Helwege, and Huang (2004) for details of the discussion), the VK model uses

¹The liability measure is, in the current implementation of the EDFsTM model, the firm’s short-term book liabilities plus one half of its long-term book liabilities.

an empirical mapping based on actual default data to get the default probabilities. This model assumes the firm’s equity is a perpetual option with the default point acting as the absorbing barrier for the firm’s asset value. When the asset value hits the default point, the firm is assumed to default. Multiple classes of liabilities are modeled: short-term liabilities, long-term liabilities, convertible debt, preferred equity, and common equity. When the firm’s asset value becomes very large, the convertible securities are assumed to convert and dilute the existing equity. In addition, cash payouts such as dividends are explicitly used in the VK model. A default database is used to derive an empirical distribution relating the distance-to-default to a default probability. In this way, the relationship between asset value and liabilities can be captured without resorting to a substantially more complex model characterizing a firm’s liability process. This default covariate, using market equity data and accounting data for liabilities, has been adopted in industry practice by Moody’s KMV, a leading provider of estimates of default probabilities.

2.1.1 Moody’s KMV conditional default probabilities

Moody’s KMV produces time series of one-year and five-year conditional default probabilities known as Expected Default Frequencies (EDFsTM), which are available at the firm level for publicly traded companies in the United States and elsewhere. Our use of EDFsTM as a proxy for default probabilities is predicated on the assumption that they (approximately) represent the market’s view of default risk. By construction, EDFsTM are normalised to equal, on average, historical default rates of firms in a similar rating-sector category.² Relative to ratings, however, EDFsTM vary much more through time in an attempt to capture short-term changes in default risk. See Kealhofer (2003) for further discussion of EDFsTM and the methodology employed by Moody’s KMV.

Dwyer and Korablev (2007) assess the performance of Moody’s KMV EDFsTM in its timeliness of default prediction, ability to discriminate good firms from bad firms, and accuracy of levels. They compare the performance of EDFsTM to that of other popular alternatives, such as agency ratings, Altman’s equity-based Z-Score, and a simpler

²Crosbie and Bohn (2002) and Kealhofer (2003) provide more details on the KMV model and the fitting procedures for distance to default and EDF. Bharath and Shumway (2004) show that the fitting procedure is relatively robust.

version of the Merton model. They find that EDFsTM over the sample period of 1996-2006 perform consistently well across different time horizons, and different subsamples based on firm size and credit quality. They also find that EDFsTM leads agency rating in timely default prediction. EDFsTM measure substantially outperforms Merton model's implied default probability and Z-Score in their ability to discriminate good firms from bad firms. In terms of accuracy of levels the predicted default rate tracks the realized default rate very well as reported by Bhon, Arora and Korablev (2005).

Some of the credit risk measures mentioned above cannot be directly interpreted as physical default probabilities, henceforth they cannot be compared against EDFsTM. Altman's Z-Scores are not directly interpreted as default probabilities, but as ordinal measures of financial health based on observable accounting and market ratios (see Altman (1968) and Altman, Haldeman, and Narayanan (1977)). Similarly, agency ratings are not intended to be a measure of default probability as rating agencies tend to adjust ratings only gradually to new information; this tendency has been documented in several empirical studies as Behar and Nagpal (2001), Lando and Skødeberg (2002), Kavvathas (2001), and Nickell, Perraudin, and Varotto (2000). On the other hand, while the Merton model's implied probabilities can be interpreted as default probabilities, these are usually too small, with the default probabilities decreasing very sharply with the declining risk of the firm. Therefore it is difficult to interpret Merton's default probabilities on the same scale as EDFsTM.

A common feature of most structural models is that firm value does not depend directly on macroeconomic information. The interplay between the dependence of firm-value dynamics on macro information and credit spreads is explored by Hackbarth, Miao and Morellecet (2006) and Tang and Yan (2006). Theory as well as empirical studies suggest that enriching structural models with additional state variables (beyond the distance to default), such as macroeconomic variables, could lead to an improved prediction for such extended structural models. Duffie, Saita, and Wang (2007) provide evidence that the predictive power of conditional probabilities of corporate default can be improved incorporating the dynamics of firm-specific and macroeconomic covariates to model based on distance to default. They estimate a model where the term structure of conditional future default probabilities depends on a firm's distance to default, on the firm's trailing stock return, on trailing S&P 500 returns, and on US interest rates. The out-of-sample predictive performance of this model is an improvement over that of

other approaches based on structural models.

While one could criticize the EDF^{TM} measure as an estimator of the “true” conditional default probability, it has some important merits for business practice and for our study relative to the other approaches mentioned above. In this specific case performance should be measured along several dimensions including discrimination power, ability to adjust to the credit cycle and the ability to quickly reflect any deterioration in credit quality. The EDF^{TM} value generated from the equity market and financial statement information of a firm does all of these things well. The dynamics of the EDF^{TM} credit measure come mostly from the dynamics of the equity value. It is simply very hard to hold the equity price of a firm up as it heads towards default. The ability to discriminate between high and low default risks comes from the distance-to-default ratio. This key ratio compares the firm’s net worth to its volatility and thus embodies all of the key elements of default risk. Moreover, because the net worth is based on values from the equity market, it is both a timely and superior estimate of the firm’s value. Moody’s KMV transforms the distance-to-default into an expected default frequency using an empirical default distribution. In fact the EDF^{TM} is fitted non-parametrically to the distance to default, and is therefore not especially sensitive, at least on average, to model mis-specification.³ Because EDF^{TM} credit measures are based on market prices they are forward looking and reflect the current position in the credit cycle. They are a timely and reliable measure of credit quality. Another important factor that we consider in choosing this measure as our conditional default probabilities is the ability to quickly access to a large coverage of firms; in this respect, there is no other default measure that is readily available for essentially all public US companies like the Moody’s KMV EDF^{TM} does.⁴

³As reported by Berndt, Douglas, Duffie, Ferguson and Schranz (2005) it should be noted that while the measured distance to default is itself based on a theoretical option-pricing model, the function that maps DD to EDF^{TM} is consistently estimated in a stationary setting, even if the underlying theoretical relationship between DD and default probability does not apply. That is, conditional on only the DD, the measured EDF^{TM} is equal to the “true” DD-conditional default probability as the number of observations goes to infinity, under typical mixing and other technical conditions for non-parametric qualitative-response estimation.

⁴The Moody’s KMV EDF measure is also extensively used in the financial services industry. Indeed, Moody’s KMV is the most widely used name-specific major source of conditional default probability

3 Decomposing Instantaneous Spreads

If investors can conditionally diversify default and recovery risk, then the physical and risk-neutral instantaneous default loss rates will be identical; otherwise, the default event itself will be priced by the market. Even if default event risk can be (approximately) hedged, there still may exist principal-agent frictions that would lead to this risk being priced in equilibrium.⁵ Using data on spreads at several maturities, as we did in the term structure model presented in Chapter 2, we can only identify the risk-neutral instantaneous spread and the prices of systematic risk, but not the price of default event risk. In this section we illustrate one approach for decomposing instantaneous spreads into their various components utilising additional information on physical default probabilities.

As can be seen by recalling the expression $s_{j,t}^Q = h_{j,t}^Q \cdot L_{j,t}^Q$, risk-neutral instantaneous spreads embody information on the risk-neutral intensity and risk-neutral loss given default. Moreover, the risk-neutral intensity $h_{j,t}^Q$ can be split into the physical intensity $h_{j,t}^P$ and the market price of default event risk $\Gamma_{j,t}$.⁶

$$h_{j,t}^Q = h_{j,t}^P \cdot [1 + \Gamma_{j,t}] \quad (1)$$

The prices of default event risk, which may be bond-specific, differ from the prices of systematic risk, though they could be determined by the same underlying risk factors. Our approach is to derive estimates of the market price of default event risk from estimates of physical and risk-neutral intensities.

3.1 Risk-Neutral Intensities

Whereas time-variation in default probabilities is almost always taken into account when calculating loss distributions or pricing credit-risk sensitive instruments, it is often as-

estimates of which we are aware, covering over 26,000 publicly traded firms. In 2007, Moody's KMV released EDF8.0, which refines the mapping of the DD to the EDF credit measure using a much larger database observed over a longer time period. Details on this models enhancement can be found in Dwyer and Qu (2007). The results presented in our study refer to data released in Moody's KMV EDF7.0.

⁵See Berndt, Douglas, Duffie, Ferguson and Schranz (2005) for further discussion.

⁶Further discussion, including a derivation of this relationship, is provided in Piazzesi (2003).

sumed that recovery rates are either constant, or that recovery rates are independent of default probabilities. To obtain estimates of risk-neutral intensities from our estimates of instantaneous spreads, we make an assumption about the risk-neutral rate of loss given default, $L_{j,t}^Q$. We follow common practice in industry and the academic literature by assuming that $L_{j,t}^Q$ is constant over time and equal to the historical loss rate on defaulted debt. The average recovery rate on US senior unsecured corporate bonds is about 40% based on data from Moody's (see Figure 1). While it is $L_{j,t}^Q$, not $L_{j,t}^P$, that is relevant for pricing (so a constant $L_{j,t}^Q$ is not logically inconsistent with the evidence in Figure 1), fixing $L_{j,t}^Q$ based on historical experience requires the assumption that there is no risk premium on recovery, $L_{j,t}^Q = L_{j,t}^P$. By setting $L_{j,t}^Q = 0.6$, we can construct time series estimates of $h_{BBB,t}^Q$ and $h_{B,t}^Q$, which are plotted in Figure 2 (dashed lines). The risk-neutral intensities vary widely across the sample period, suggesting that a constant intensity assumption would likely fit the data quite poorly. (More formally, we can reject the null hypothesis of constant intensities based on the estimates presented in Chapter 2, Table 6.) It is also evident that risk-neutral intensities of different ratings generally move together and reach their highs and lows at similar times.⁷

4 Estimation of Physical Intensities Using EDFsTM

Obtaining estimates of physical intensities requires using an additional source of data on real-world default risk. We utilise data on physical default probabilities provided by Moody's KMV.

Our data consists of monthly time series of aggregated one-year EDFsTM on firms rated BBB and B by Standard and Poor's over the sample period 1993:10-2004:04. These are shown as the solid lines in Figure 3. For both BBB and B-rated bonds, EDFsTM were low and stable until late 1998 and then began to rise prior to the recession. Whereas the EDFsTM suggest that one-year default probabilities on BBB-rated bonds began to fall from late-2001 onwards, they indicate that conditional default probabilities on B-rated bonds rose sharply once again thereafter, following the collapse of Worldcom.

⁷The minimum and maximum values of $h_{BBB,t}^Q$ are 171 basis points (November 1997) and 324 basis points (December 2001), respectively; for $h_{B,t}^Q$, these values are 387 basis points (February 1998) and 1091 basis points (October 2001).

Preliminary evidence on the relationship of EDFsTM to our factors is given in Table 1. For both ratings, column 1 shows that the macro factors alone explain a significant portion of the variation in EDFsTM. Most of the coefficients are significant, although, somewhat unexpectedly, those on real activity have positive signs. The coefficients on real activity change sign in univariate regressions (not reported), which implies the existence of complex conditional relationships between default probabilities, economic activity and financial conditions. Column 2 shows that the latent factors from the term structure model have significant marginal explanatory power for EDFsTM. The six factors together explain 95% of the variation in aggregate EDFsTM for BBB-rated industrials. Columns 3 and 4 add total monthly corporate bond issuance and the within-month default rate (based on data from Moody's). Conditioning on the six term structure model factors used in Chapter 2, neither of these variables are statistically significant.

We assume that the physical default intensity is a function of the observable macro factors and filtered estimates of the latent factors from the term structure model estimated in Chapter 2. Preliminary results based on an affine functional form produced negative values of the physical intensity in one or more months in our sample.⁸ To avoid this undesirable feature, we specify a proportional-hazard model for physical intensities:

$$h_{j,t}^P = \exp(\omega_0 + \omega_f X_{f,t} + \omega_y X_{y,t} + \omega_\pi X_{\pi,t} + \omega_1 X_{1,t|t} + \omega_2 X_{2,t|t} + \omega_3 X_{3,t|t}) \quad (2)$$

where $X_{j,t|t}$ is the filtered estimate of $X_{j,t}$ from the term structure model (as shown in Chapter 2, Figure 6). One aspect of the way Moody's KMV constructs EDFsTM is worth highlighting in the context of set of state variables in (2). EDFsTM are based on a non-parametric mapping of distance-to-default to historical default rates of issuers within the same rating-sector category. Since, in effect, our objective here is to model EDFsTM, (2) can be seen as one way of approximating the Moody's KMV methodology, and where our indicator of financial activity, in particular, is used as a source of information on distance-to-default.

Our approach to estimating physical default intensities is closely related to two other recent studies, although there are several important differences in implementation. Berndt, Douglas, Duffie, Ferguson and Schranz (2005) estimate a latent factor Black-

⁸As noted above, this problem was not encountered in estimation of the term structure model. Spreads were generally much higher than EDFs throughout most of our sample.

Karasinski model of physical intensities using firm-level data on EDFsTM across three sectors. Duffie, Saita and Wang (2005) estimate proportional-hazard models using a panel data set based on actual survival/default histories of firms. Apart from the feature that we estimate (2) by rating category instead of by firm, the marginal contribution of this part of our study is to model physical intensities as functions of observable macroeconomic variables and the other (latent) factors found to be important for driving corporate spreads curves.

In a doubly-stochastic intensity-based model, the m -period ahead conditional physical default probability is given by:

$$P(\tau_j < t + m|t) = 1 - E_t^P \left[\exp \left(- \int_{s=t}^{t+m} h_{j,s}^P ds \right) \right] \quad (3)$$

We use maximum likelihood estimation to estimate the parameters $\{\omega_i\}$ in (2) by assuming that EDFsTM are noisy observations of the model-based one-year default probabilities given in (3):

$$EDF(t + 12|t) = P(\tau_j < t + 12|t) + e_t \quad (4)$$

where e_t is distributed i.i.d. $N(0, \sigma_e^2)$. Since $h_{j,t}^P$ is a nonlinear function of the factors, the solution of $P(\tau_j < t + m|t)$ is not known in closed form. We compute the likelihood function numerically by solving (3) using Monte Carlo simulation.

Parameter estimates are given in Table 2. For both rating categories, all of the coefficients are statistically significant except for real output in the intensity for B-rated spreads. The signs of the coefficients reflect the results in Table 1. The values of ω_f imply that a one-standard-deviation increase in the financial activity indicator raises one-year real-world default probabilities by 12 bps and 109 bps in BBB and B-rated bonds, respectively. The estimates also indicate that a positive shock to Latent 1, which was shown to raise the level of the Treasury curve, acts to reduce physical default intensities, whereas a positive shock to Latent 3 leads to an increase in physical default intensities and spreads.

Our model appears to capture much of the variation in EDFsTM, as indicated in Figure 3, which plots model-based one-year default probabilities with EDFsTM. It is evident from the graph that the estimated residuals \hat{e}_t exhibit some serial correlation. However, our model has the ability to match most of the sharp moves in EDFsTM in

the latter half of the sample, while capturing the relatively sanguine period in the mid-1990s. This is one virtue of using a proportional-hazard model in comparison to an affine specification for physical intensities.⁹

Looking back at Figure 2, the estimated sample paths of the physical intensities (dashed lines) are plotted with the risk-neutral intensities. The physical intensity for BBB-rated bonds peaks in November 2000, just prior to the recession, whereas the physical intensity for B-rated bonds, reaching a high in November 2002, appears to have been significantly influenced by changes in perceptions of default risk in lower-rated firms following the accounting scandals at Enron and Worldcom. The physical intensities are evidently much more volatile in the latter half of the sample.¹⁰ They are also more volatile than the risk-neutral intensity of the same rating. This and other features of the relationship between physical and risk-neutral intensities are examined in the next section.

4.1 Market Prices of Default Event Risk

Given our estimates of physical and risk-neutral intensities, we construct estimates of the market price of default event risk from the relationship $\Gamma_{j,t} = h_{j,t}^Q/h_{j,t}^P - 1$ (see (1)). In Figure 4 we plot monthly time series estimates of $\Gamma_{BBB,t}$ and $\Gamma_{B,t}$. It is evident that the price of default event risk on BBB-rated bonds is higher on average and more volatile than that for B-rated bonds. The average values of $\Gamma_{BBB,t}$ and $\Gamma_{B,t}$ are estimated to be 8.5 and 1.5, respectively, and their in-sample standard deviations are 4.9 and 1.2. These values are somewhat larger than found elsewhere in the literature (Driessen (2005), Berndt, Douglas, Duffie, Ferguson and Schranz (2005), Amato and Remolona (2005)), and is further evidence that investors require more compensation per unit of default event risk for bonds of higher credit quality.

However, as with our estimates of systematic risk premia, measured prices of default event risk may implicitly incorporate the effects of taxes and liquidity. For example, Driessen (2005) estimates that a liquidity premium and taxes account for 13 basis points

⁹An even better alternative would seem to be a regime-switching model, a subject for future research.

¹⁰The full-sample standard deviations of $h_{BBB,t}^P$ and $h_{B,t}^P$ are 29 basis points and 368 basis points, respectively.

and 33 basis points, respectively, of the expected excess return in a 10-year BBB-rated corporate bond. This compares to a default event risk premium of 31 basis points. Moreover, previous studies indicate that the sizes of liquidity premia are roughly equal across rating categories, which might explain, at least partly, higher measured values of the price of default event risk in higher-rated bonds (i.e. liquidity premia represent a larger fraction of investment grade spreads).

What drives variation in the price of default event risk? By construction, the price of default event risk is a nonlinear function of the factors in our model. To gauge sensitivities to these factors, in Table 3 we report estimates of regressions on levels of the factors and squares of the factors. For both rating categories, the estimates under column 1 show that the levels of the macro factors alone account for 75% and 65% of the variation in $\Gamma_{BBB,t}$ and $\Gamma_{B,t}$, respectively. Adding the latent factors to the regressions helps explain almost all of the variation in the prices of default event risk (column 2). Consequently, adding in squares of the factors adds little in terms of improving the fit of the regressions (and the coefficients on the linear terms remain statistically significant).

An examination of the regression coefficients reveals several insights. First, the price of default event risk is countercyclical: a one-standard deviation increase in real economic activity leads to a decline of 0.67 in $\Gamma_{BBB,t}$ (based on results in column 2). Real activity has a smaller, but still negative impact on the price of default event risk on B-rated bonds. Second, the price of default event risk falls with a marginal increase in the financial activity factor. Third, there appears to be a strong link between the prices of default event risk and the filtered latent factors from the term structure model. In particular, compensation for default event risk rises with Latent 1 and falls with Latent 3. Recall from Chapter 2 above that positive innovations to these factors lead to a narrowing and widening of spreads, respectively. Thus, even though spreads widen with an increase in Latent 3, the associated increase in real-world default probabilities tends to be larger in percentage terms.

Berndt, Douglas, Duffie, Ferguson and Schranz (2005) discuss several possible reasons for obtaining large and variable measured prices of default event risk. These include mismeasurement of real-world default probabilities (or, for that matter, risk-neutral default probabilities) or erroneous assumptions about risk-neutral loss given default (in particular, that it may be time-varying), both of which imply that estimates of default event risk premia are flawed. A third and more fundamental explanation concerns the

impact of changes in the relative demand and supply for risk bearing in credit markets. We have little to add on mismeasurement. If loss given default is countercyclical, then by assuming it is constant means we are likely overestimating both the level and degree of variation in the prices of default event risk.

Descriptive studies of credit spreads often focus on the nature of the default process when interpreting results, although recovery (defined as the average amount recovered on the bonds of a defaulting firm) may also play a central role. In fact, there is substantial evidence that recovery varies over the business cycle, with recovery rates being lower during recessions. Moreover, default rates also vary with the business cycle, being higher during recessions. Together, these patterns imply a quite strong negative correlation between default rates and recovery rates, at least in the aggregate and for corporate bonds. As found in Altman, Brady, Resti and Sironi (2005) and other academic research, annual corporate default rates are negatively correlated with annual average recovery rates as measured by post-default trading prices. As shown in Figure 6, a linear regression of annual average senior unsecured bond recovery rates on annual corporate speculative-grade default rates yields an R-square value of 50%. Both variables also seem to be driven by the same common factor that is persistent over time and clearly related to the business cycle: in recessions or industry downturns, default rates are high and recovery rates are low (Figure 1). While this pattern in $L_{j,t}^P$ is now widely recognized, this correlation is rarely accommodated in econometric specifications of defaultable bond pricing models. Typically, for pricing, $L_{j,t}^Q$ is assumed to be constant. Two drawbacks in our approach to estimating $h_{j,t}^Q$ are that risk-neutral rates of loss given default may differ from real-world loss rates and they may vary systematically over time. In regard to the former, unfortunately, as noted by Pan and Singleton (2005), there is as yet little compelling evidence on how risk-adjusted expected recovery rates differ from real-world recovery rates. In regard to the latter, as mentioned before, recent empirical evidence on the time series properties of recovery rates suggests that they are negatively correlated with default rates. The time variation in recovery rate distributions should also have an effect on bond prices and spreads; since risks are larger in a world where recovery rate distributions vary over time than in one where they are static, spreads should be higher. If so, this would mean that our estimates of $h_{j,t}^Q$ in Figure 2 are too high when default rates are highest (e.g. just prior to and during the recession in 2001) and too low at other times. However, experimental results

in Berndt, Douglas, Duffie, Ferguson and Schranz (2005) suggest that the upward bias is probably a small fraction of the default event risk premium. Although, as they noted, there is little conclusive evidence on the properties of *risk-neutral* loss given default, which is the relevant quantity for pricing purposes.

In a recent contribution, Bruche and González-Aguado (2006) assess the importance of allowing for systematic time-variation in recovery rates in the context of a consumption-based asset pricing model. To gauge what proportion of the spread is a risk premium and what proportion compensates for expected losses, they calculate the spreads on senior unsecured bonds that would result if agents were risk neutral for a world in which recovery rate distributions are static, and for one in which recovery rate distributions vary. They found that spreads are much larger in downturns than in upturns in both models and regardless of the assumptions about risk aversion, indicating that the fluctuations in the spread over the cycle are caused by differences in expected losses in downturns and upturns, which in turn are driven mainly by variation in default probabilities. Letting recovery rate distributions vary increases the amplitude of the swings in spreads over the cycle only slightly, since time-variation in recovery rate distributions amplify the effect of time variation in default probabilities. They show that the difference in spreads between upturns and downturns in a model in which recovery rates vary is $149-64=85\text{bp}$. In a model in which recovery rates do not vary, it is slightly lower, $147-70=77\text{bp}$. However, the average spread (either the simple average over the two states, or weighting by unconditional probabilities) changes at most by a few basis points in moving from a model in which recovery rate distributions are static to a model in which recovery rate distributions vary with the cycle. These results suggest that allowing for time variation of recovery rate distributions in a pricing model could allow a slightly better matching of spreads over the cycle, but is unlikely to alter the ability of the model to match the level of spreads. Based on these observations, we can state that the time-variation in recovery rate distributions does amplify risk, but this effect is much smaller than the contribution of the time variation in default probabilities.

An alternative possibility is that our measures of default event risk premia really reflect uncertainty premia. Compensation for uncertainty aversion may vary over time, for example, as investors become more or less confident about the reliability of public information on corporate balance sheets. To assess this hypothesis, consider the autumn of 2002, when the measured prices of default event risk rose sharply. We have already

noted that this is the period following the accounting scandals at Enron and Worldcom. Even though credit fundamentals seemed to be on firmer ground at that time, amidst corporate deleveraging and improved growth prospects (and as captured by our measures of real-world default probabilities), if investors are uncertainty averse, they may have required a premium to hold corporate bonds in the wake of the accounting scandals. However, uncertainty aversion would seem to have difficulty in explaining the large drop in the price of default event risk in BBB-rated bonds in September-October 1998. After several sanguine years for corporate bond investors, corporates started to amass debt, Russia defaulted, and the events surrounding LTCM shook financial markets. If anything, one would have expected uncertainty premia to be rising, not falling, during this period.

Turning to more fundamental explanations, if the amount of risk-bearing capital available in the corporate bond market is roughly fixed over short intervals of time, then an increase in risk (e.g. volatility) could also lead to an increase in the price of risk. However, our finding of a negative marginal relationship between the prices of default event risk and the financial activity factor would seem to contradict this: implied volatility has a positive loading in this factor (see Chapter 2, Appendix A). In fact, our regression results may be revealing reverse causation if firms increase leverage when the price of default event risk, and hence borrowing costs, are relatively low.

To further investigate the risk-taking capacity hypothesis, we add the trailing three-month moving average of corporate bond issuance and total number of defaults to regressions of the prices of default event risk on the macro factors. Following Berndt, Douglas, Duffie, Ferguson and Schranz (2005), these variables are meant to be proxies for the relative amount of risk capital available in the corporate bond market. However, as reported in columns 4 and 5 in Table 11, we find that neither of these variables is statistically significant conditional on the presence of the macro factors in the regression.

4.2 Survival Probabilities

In this and in the previous chapter we have considered two types of risk in corporate bonds that may be priced by the market: systematic risk in the state variables that determine risk-free rates and instantaneous spreads, and default event risk. To get a sense of the relative importance of these two types of risk premia in spreads, in Figure

5 we plot model-based five-year survival probabilities constructed under three different probability measures.

The solid lines (PP) in the figure show real-world survival probabilities, which are calculated using the physical intensity and the process for the factors under the physical measure: $E_t^P \left[\exp \left(- \int_{s=t}^{t+60} h_{j,s}^P ds \right) \right]$. The dashed lines (PQ) are survival probabilities calculated using the risk-neutral intensity but under the physical measure: $E_t^P \left[\exp \left(- \int_{s=t}^{t+60} h_{j,s}^Q ds \right) \right]$. Compared to real-world probabilities, this second measure involves an adjustment (typically downwards) to survival probabilities that takes account of investors' aversion to default event risk. Finally, the dash-dotted lines (QQ) are survival probabilities calculated using the risk-neutral intensity under the risk-neutral measure: $E_t^Q \left[\exp \left(- \int_{s=t}^{t+60} h_{j,s}^Q ds \right) \right]$. This measure, which is the most relevant one for pricing corporate bonds, adjusts probabilities for both default event risk and systematic risk.

It is evident from the figure that, even at the relatively long horizon of five-years, most of the difference between real-world probabilities and those used in pricing (PP vs. QQ) can be attributed to risk adjustments for default event risk. The proportion of the risk adjustment in QQ-probabilities attributed to systematic risk appears to increase when default risk rises, for example, during the recession in 2001 and, in the case of high-yield bonds, in the autumn of 2002. Nonetheless, it is always smaller in magnitude than the adjustment due to default event risk aversion.

5 Conclusion

In this study, in addition to estimating risk-neutral default intensities, we provided estimates of physical default intensities using data on Moody's KMV EDFsTM as a forward-looking proxy for default risk. We estimated the market price of default event risk across the business cycle and, in particular, we assessed how it is related to observable measures of macroeconomic activity. Our estimates of physical and risk-neutral default intensities obtained using data on EDFsTM and spreads, respectively, provide new evidence on the size and evolution of the price of default *event* risk. We found that the real and financial activity indicators, along with filtered estimates of the latent factors from our term structure model, explain a large portion of the variation in EDFsTM

across time. Furthermore, measures of the price of default event risk implied by estimates of physical and risk-neutral intensities indicate that compensation for default event risk is countercyclical, varies widely across the cycle, and is higher on average and more variable for higher-rated bonds.

Further research into the empirical consequences of relaxing the assumption of constant *risk-neutral* loss given default seems warranted. Given the negative relationship between default probabilities and recovery rates, a question that should be raised for future research is by how much we overestimate the prices of default event risk if we ignore this negative relationship. In fact, from the point of view of a holder of a diversified portfolio of corporate bonds the fact that recovery rates are low precisely in situations in which many companies default is important because the negative relationship between recoveries and default probabilities amplifies the systematic risk of the portfolio. A way to address this issue would be to specify an econometric model in which the joint time-variation in default rates and recovery rate distributions is directly taken into account.

References

- [1] Allen, L and A Saunders (2003): “A survey of cyclical effects in credit risk measurement models”, BIS Working Paper no. 126.
- [2] Altman, E I (1968): ”Financial ratios, discriminant analysis, and the prediction of corporate bankruptcy”, *Journal of Finance*, 23, pp 589–609.
- [3] Altman, E I, B Brady, A Resti and A Sironi (2005): “The link between default and recovery rates: theory, empirical evidence and implications”, *Journal of Business*, 78(6), pp 2203–2227.
- [4] Altman, E I , R G Haldeman, and P Narayanan (1977):.”ZETATM analysis: a new model to identify bankruptcy risk of corporations”, *Journal of Banking and Finance*, 1, pp 29-54.
- [5] Amato, J and E Remolona (2005): “The pricing of unexpected credit losses”, BIS Working Paper no. 190.
- [6] Ang, A and M Piazzesi (2003): “A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables”, *Journal of Monetary Economics*, 50, pp 745–787.
- [7] Ang, A, A Dong and M Piazzesi (2005): “No-arbitrage Taylor rules”, working paper, Columbia University.
- [8] Bakshi, G, D Madan and F Zhang (2001): “Understanding the role of recovery in default risk models: empirical comparisons of implied recovery rates”, working paper, University of Maryland.
- [9] Bakshi, G, D Madan and F Zhang (2004): “Investigating the role of systematic and firm-specific factors in default risk: lessons from empirically evaluating credit risk models”, working paper, University of Maryland.
- [10] Bangia, A, F X Diebold, A Kronimus, C Schagen, and T Schuermann (2002): ”Ratings migration and the business cycle, with application to credit portfolio stress testing”, *Journal of Banking & Finance*, 26, pp 445–474.

- [11] Behar, R, and K Nagpal (2001): "Dynamics of rating transition", *Algo Research Quarterly*, 4 , pp 71–92.
- [12] Bernanke, B S, M Gertler and S Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework", in J B Taylor and M Woodford (eds), *Handbook of Macroeconomics*, vol. 1, Amsterdam: North-Holland, pp 1341-93.
- [13] Berndt, A, R Douglas, D Duffie, M Ferguson and D Schranz (2005): "Measuring default risk premia from default swap rates and EDFs", working paper, Stanford University.
- [14] Bharath, S and T Shumway (2004): "Forecasting default with the KMV-merton model", working paper, University of Michigan.
- [15] Black, F and M Scholes (1973): "The pricing of options and corporate liabilities", *Journal of Political Economy*, 81, pp 637-654.
- [16] Bhon, J, N Arora and I Korablev (2005): "Power and level validation of the Moody's KMV EDF credit measure in the U.S. market", Moody's KMV White paper.
- [17] Bruche, M and C González-Aguado (2008): "Recovery rates, default probabilities and the credit cycle", working paper, CEMFI.
- [18] Cantor, R and C Mann (2003): "Measuring the performance of corporate bond ratings", Moody's Special Comment, April.
- [19] Chava, S and R Jarrow (2004): "Bankruptcy prediction with industry effects", *Review of Finance*, 8, pp 537–569.
- [20] Collin-Dufresne, P, R Goldstein and J Spencer Martin (2001): "The determinants of credit spread changes", *Journal of Finance*, 56, pp 2177–207.
- [21] Crosbie, P J and J R Bohn (2002): "Modeling default risk", Technical Report, KMV, LLC.
- [22] Crosbie, P J and A Kocagil (2003): "Modeling default risk", *Modeling Methodology*, Moody's KMV.

- [23] Dai, Q and K Singleton (2000): “Specification analysis of affine term structure models”, *Journal of Finance* 55, pp 1943–1978.
- [24] Delianedis, G and R Geske (2001): “The components of corporate credit spreads: default, recovery, tax, jumps, liquidity and market factors”, working paper 22–01, Anderson Graduate School of Management, UCLA.
- [25] Driessen, J (2005): “Is default event risk priced in corporate bonds?”, *Review of Financial Studies*, 18, pp 165–195.
- [26] Dwyer D W and S Qu (2007): ”EDF 8.0 model enhancements”, working paper, Modeling Methodology, Moody’s KMV.
- [27] Dwyer, D W and I Korablev (2007): ”Power and level validation of Moody’s KMV EDFTM credit measures in North America, Europe, and Asia”, working paper, Modeling Methodology, Moody’s KMV.
- [28] Duffee, G (1999): “Estimating the price of default risk”, *Review of Financial Studies*, 12, pp 197–226.
- [29] Duffie, D., and K Singleton (1999): “Modeling term structures of defaultable bonds”, *Review of Financial Studies*, 12, pp 687–720.
- [30] Duffie, D and K Singleton (2003): *Credit Risk: Pricing, Measurement and Management*, Princeton University Press.
- [31] Duffie, D, L Saita and K Wang (2007): “Multi-period corporate default prediction with stochastic covariates”, *Journal of Financial Economics*, 83, pp 635–665.
- [32] Elton, E, M Gruber, D Agrawal and C Mann (2001): “Explaining the rate spread on corporate bonds”, *Journal of Finance*, 56, pp 247–77.
- [33] Eom, Y, J Helwege and J Huang (2004): “Structural models of corporate bond pricing: an empirical analysis”, *Review of Financial Studies*, 17, pp 499–544.
- [34] Fisher, E , R Heinkel and J Zechner (1989): ”Dynamic capital structure choice: theory and tests”, *Journal of Finance*, 44, pp 19–40.

- [35] Gertler, M and C Lown (1999): “The information in the high-yield bond spread for the business cycle: evidence and some implications”, *Oxford Review of Economic Policy*, 15, pp 132–150.
- [36] Hackbarth, D, J Miao and E Morellec (2006): ”Capital structure, credit risk, and macroeconomic conditions”, *Journal of Financial Economics*, 82, pp 519-550.
- [37] Huang, J and M Huang (2003): “How much of the corporate-treasury yield spread is due to credit risk?: a new calibration approach”, working paper, Penn State University.
- [38] Kavvathas, D (2001): ”Estimating credit rating transition probabilities for corporate bonds”, working paper, University of Chicago.
- [39] Kealhofer, S (2003): “Quantifying credit risk I: default prediction”, *Financial Analysts Journal*, January–February, pp 30–44.
- [40] Janosi, T, R Jarrow and Y Yildirim (2001): “Estimating expected losses and liquidity discounts implicit in debt prices”, working paper, Cornell University.
- [41] Jarrow, R, D Lando and F Yu (2005): “Default risk and diversification: theory and applications”, *Mathematical Finance*, 15, pp 1–26.
- [42] Jonsson, J and M Fridson (1996): “Forecasting default rates on high-yield bonds”, *Journal of Fixed Income*, June, pp 69–77.
- [43] Lando, D (1998): “Cox processes and credit-risky securities”, *Review of Derivatives Research*, 2, pp 99-120.
- [44] Lando, D and T Skødeberg (2002): ”Analyzing rating transitions and rating drift with continuous observations”, *Journal of Banking and Finance*, 26, pp 423–444.
- [45] Leland, H (1994): ”Corporate debt value, bond covenants, and optimal capital structure”, *Journal of Finance*, 49, pp 1213–1252.
- [46] Longstaff, F, S Mithal and E Neis (2005): “Corporate yield spreads: default risk or liquidity? New evidence from the credit default swap market”, *Journal of Finance*, 60, pp 2213–2253.

- [47] Merton, R C (1974): "On the pricing of corporate debt: the risk structure of interest rates", *Journal of Finance*, 29, pp 449-470.
- [48] Morris, C, R Neal, and D Rolph (2001): "Interest rates and credit spreads", working paper, Indiana University.
- [49] Nickell, P, W Perraudin, and S Varotto (2000): "Stability of Rating Transitions", *Journal of Banking & Finance*, 24, pp 203–227.
- [50] Pan, J and K Singleton (2005): "Default and recovery implicit in the term structure of sovereign CDS spreads", working paper, Stanford University.
- [51] Piazzesi, M (2003): "Affine term structure models", working paper, UCLA.
- [52] Schultz, P (2001): "Corporate bond trading costs: a peek behind the curtain", *Journal of Finance*, 56, pp 677–98.
- [53] Singleton, J K (2006): *Empirical Dynamic Asset Pricing*, Princeton University Press.
- [54] Tang, D and H Yan (2006): "Macroeconomic conditions, firm characteristics, and credit spreads", *Journal of Financial Services Research*, 29, pp 177-210.
- [55] Vasicek, O (1977): "An equilibrium characterization of the term structure", *Journal of Financial Economics*, 5, pp 177–188.
- [56] Vassalou, M and Y Xing (2004): "Default risk in equity returns", *Journal of Finance*, 59, pp 831–868.
- [57] Yang, J (2003): "An empirical investigation of the effects of macroeconomic variables on default risk", working paper, University of Toronto.

Table 1
Regressions of EDFsTM on Macro Factors

	Dependent Variable:							
	BBB-rated Industrials EDF TM				B-rated Industrials EDF TM			
	1	2	3	4	1	2	3	4
Fin	2.11 (0.10)	1.20 (0.09)	1.18 (0.10)	1.23 (0.10)	18.82 (1.44)	11.13 (1.52)	10.89 (1.56)	10.68 (1.67)
Real	0.82 (0.10)	0.65 (0.06)	0.65 (0.06)	0.66 (0.06)	1.35 (1.45)	2.61 (0.94)	2.55 (0.94)	2.56 (0.94)
Infl	0.56 (0.08)	0.34 (0.04)	0.35 (0.05)	0.35 (0.05)	2.71 (1.11)	-1.33 (0.72)	-1.24 (0.74)	-1.38 (0.73)
Latent 1		-0.50 (0.08)	-0.48 (0.08)	-0.50 (0.08)		-6.20 (1.23)	-5.95 (1.27)	-6.14 (1.23)
Latent 2		0.63 (0.06)	0.62 (0.06)	0.64 (0.07)		0.09 (1.04)	0.02 (1.05)	-0.06 (1.07)
Latent 3		0.63 (0.05)	0.64 (0.05)	0.63 (0.05)		11.10 (0.78)	11.21 (0.79)	11.09 (0.78)
Bond iss			0.00 (0.00)				0.00 (0.00)	
Def rate				-0.02 (0.04)				0.40 (0.60)
R^2	0.82	0.95	0.95	0.95	0.69	0.89	0.89	0.89

Table 2
Estimates of Physical Intensity Parameters

	BBB-rated Spreads	B-rated Spreads
ω_0	-5.93 (0.02)	-3.49 (0.03)
ω_f	0.35 (0.04)	0.26 (0.04)
ω_y	0.13 (0.03)	0.00 (0.03)
ω_π	0.09 (0.02)	-0.05 (0.02)
ω_1	-0.27 (0.03)	-0.31 (0.03)
ω_2	0.30 (0.03)	0.09 (0.03)
ω_3	0.27 (0.03)	0.44 (0.03)
σ_e	0.056 (0.006)	0.685 (0.045)

Table 3
Regressions of Price of Default Event Risk on Macro Factors

	Dep. Var.: Γ_t^{BBB}					Dep. Var.: Γ_t^B				
	1	2	3	4	5	1	2	3	4	5
Fin	-4.87 (0.29)	-1.05 (0.19)	-2.14 (0.27)	-5.52 (0.35)	-4.25 (0.52)	-1.13 (0.09)	-0.25 (0.05)	-0.45 (0.07)	-1.22 (0.10)	-1.02 (0.15)
Real	-1.14 (0.30)	-0.67 (0.12)	-1.10 (0.12)	-1.28 (0.27)	-1.01 (0.29)	-0.26 (0.09)	-0.28 (0.03)	-0.35 (0.03)	-0.28 (0.08)	-0.24 (0.08)
Infl	-1.11 (0.23)	-0.33 (0.09)	-0.05 (0.09)	-1.05 (0.22)	-1.12 (0.21)	0.02 (0.07)	0.30 (0.03)	0.33 (0.03)	0.00 (0.06)	0.00 (0.06)
Latent 1		2.92 (0.15)	2.48 (0.20)				0.74 (0.04)	0.67 (0.05)		
Latent 2		-2.43 (0.13)	-2.05 (0.12)				-0.32 (0.04)	-0.17 (0.03)		
Latent 3		-1.93 (0.10)	-2.06 (0.09)				-0.71 (0.03)	-0.79 (0.02)		
Fin ²			0.25 (0.09)					0.05 (0.02)		
Real ²			-0.13 (0.12)					-0.01 (0.03)		
Infl ²			0.11 (0.08)					0.02 (0.02)		
(Latent 1) ²			-0.47 (0.09)					0.00 (0.02)		
(Latent 2) ²			0.09 (0.08)					-0.01 (0.02)		
(Latent 3) ²			0.44 (0.06)					0.16 (0.02)		
Issuance				0.05 (0.03)					0.00 (0.01)	
Defaults					-0.23 (0.13)					-0.04 (0.04)
R^2	0.75	0.97	0.98	0.81	0.80	0.65	0.96	0.98	0.73	0.73

Figure 1
Moody's Annual Average Recovery Rates

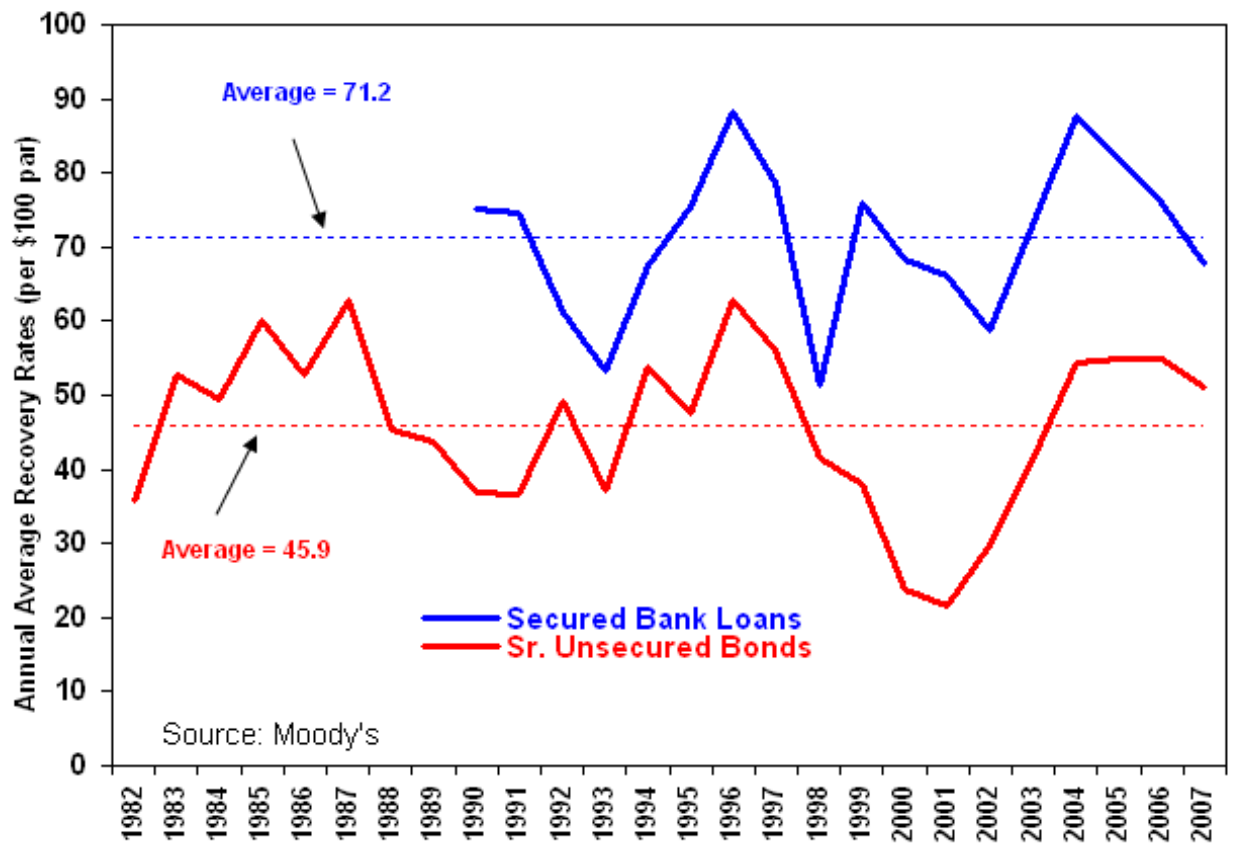


Figure 2
Risk-Neutral and Physical Default Intensities

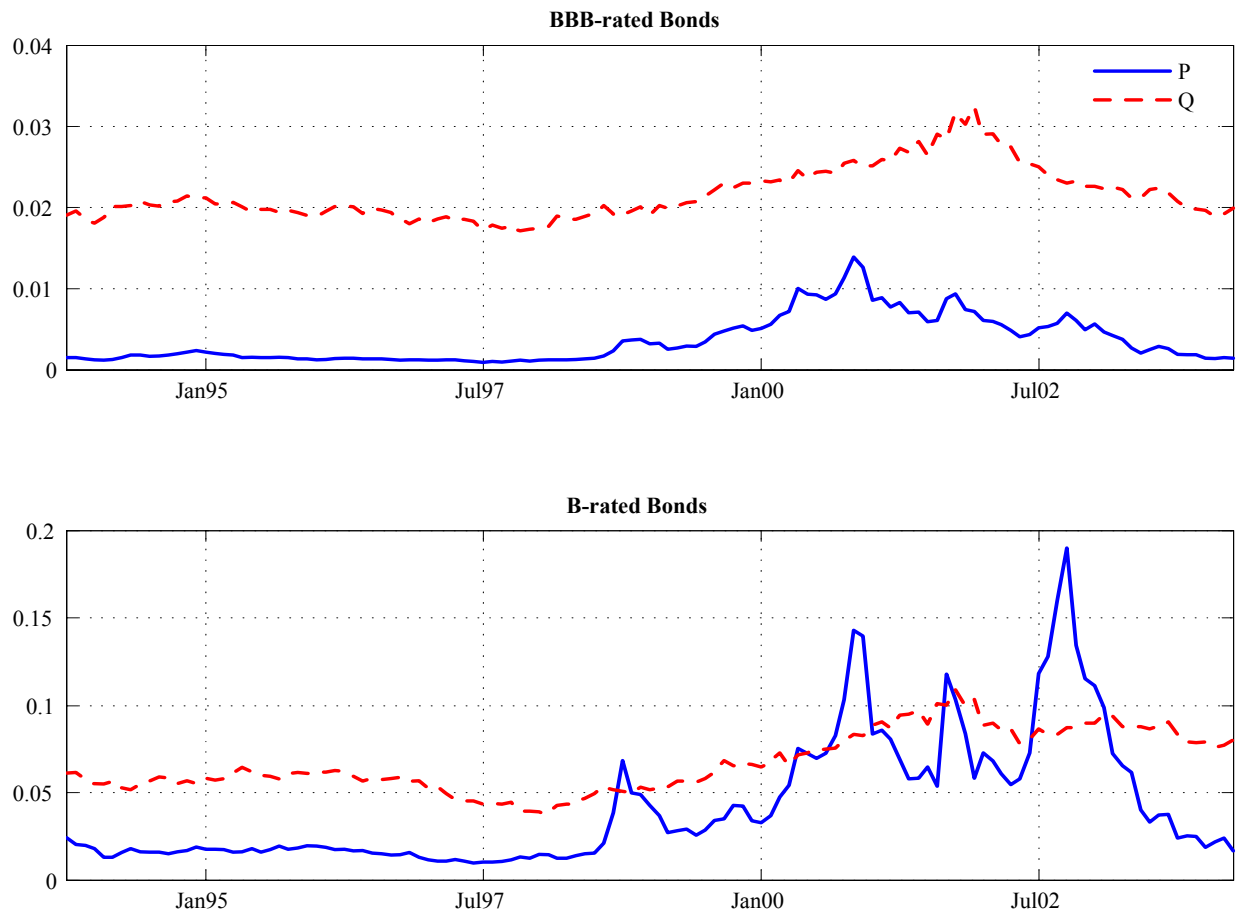


Figure 3
EDFsTM and Model-Based Physical Default Probabilities:
One-Year Horizon

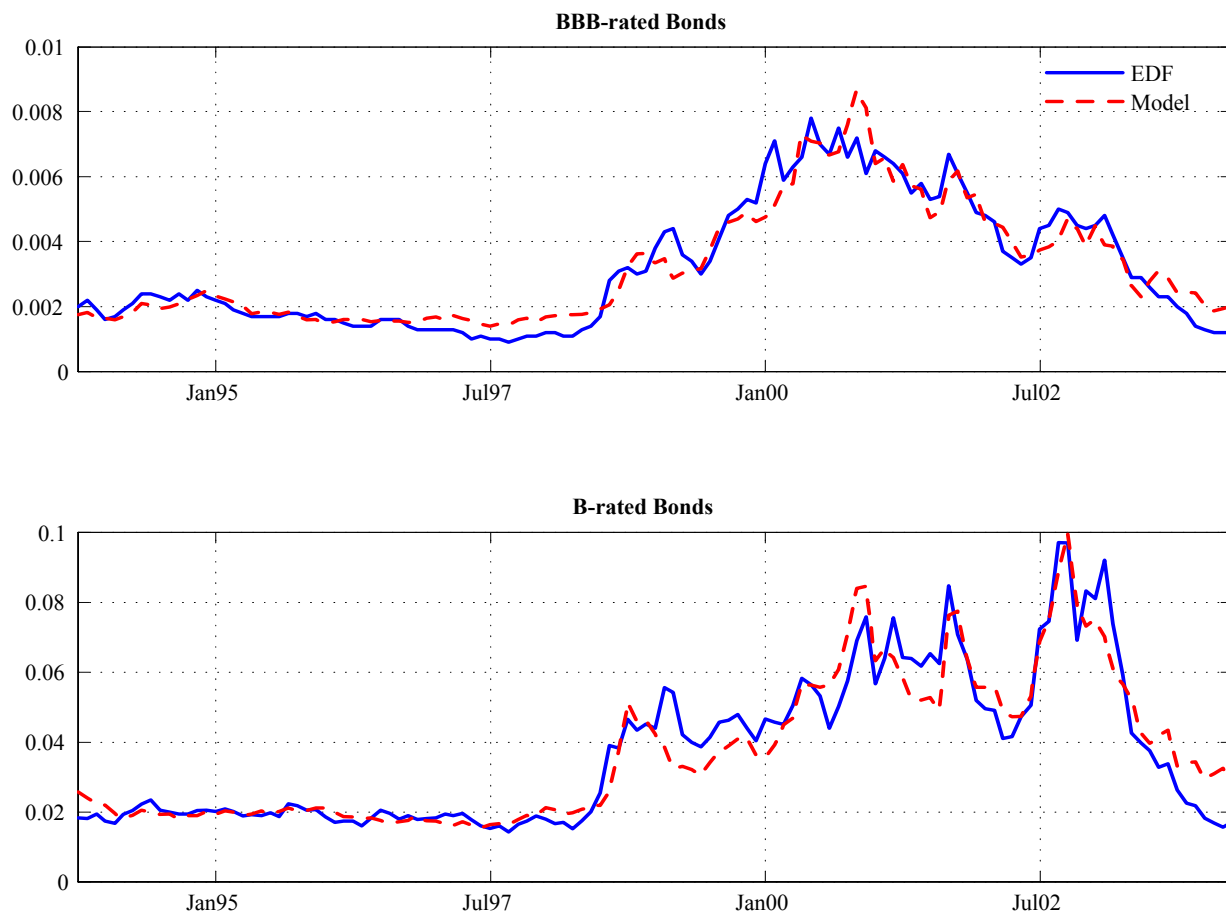


Figure 4
Market Prices of Default Event Risk

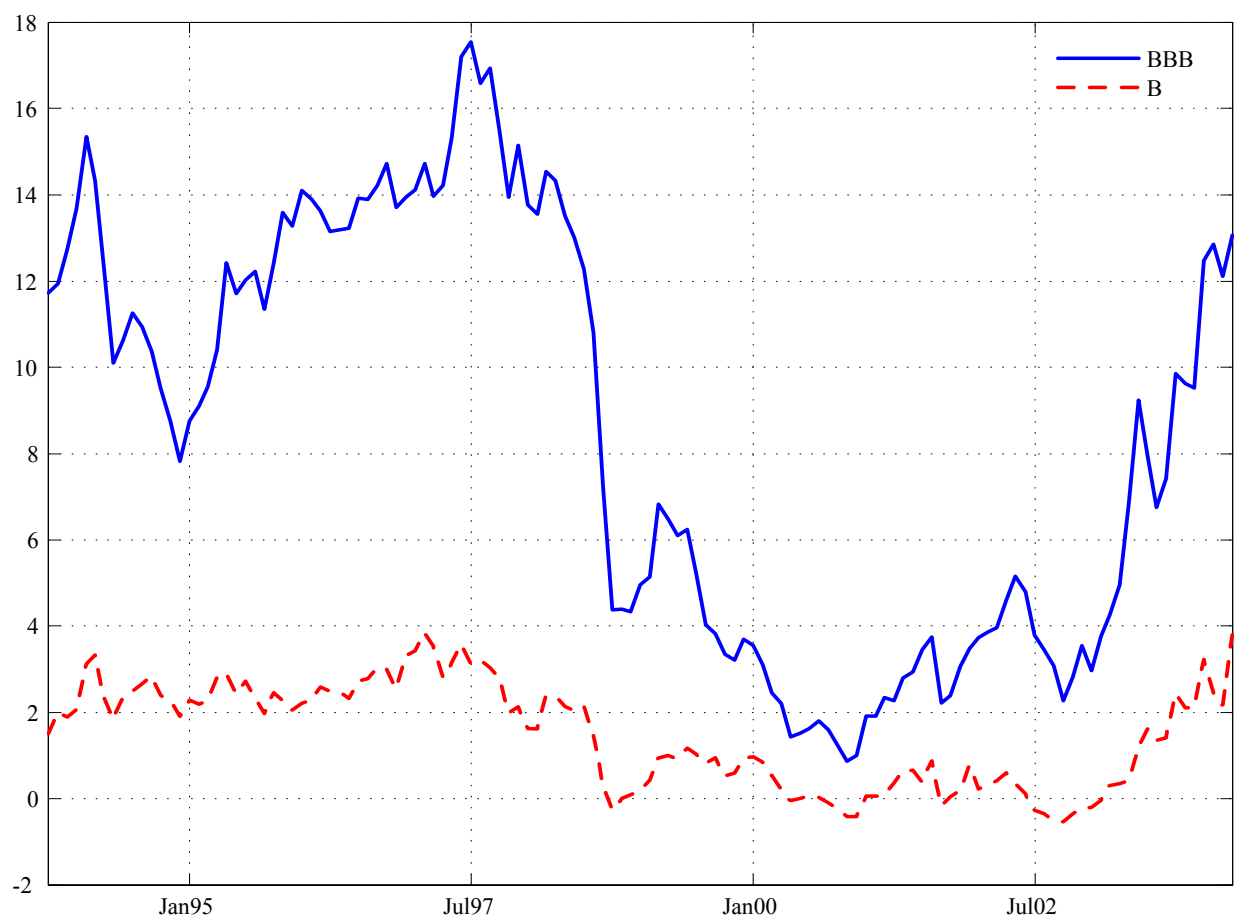


Figure 5
Model-Based Risk-Neutral and Physical Survival Probabilities:
Five-Year Horizon

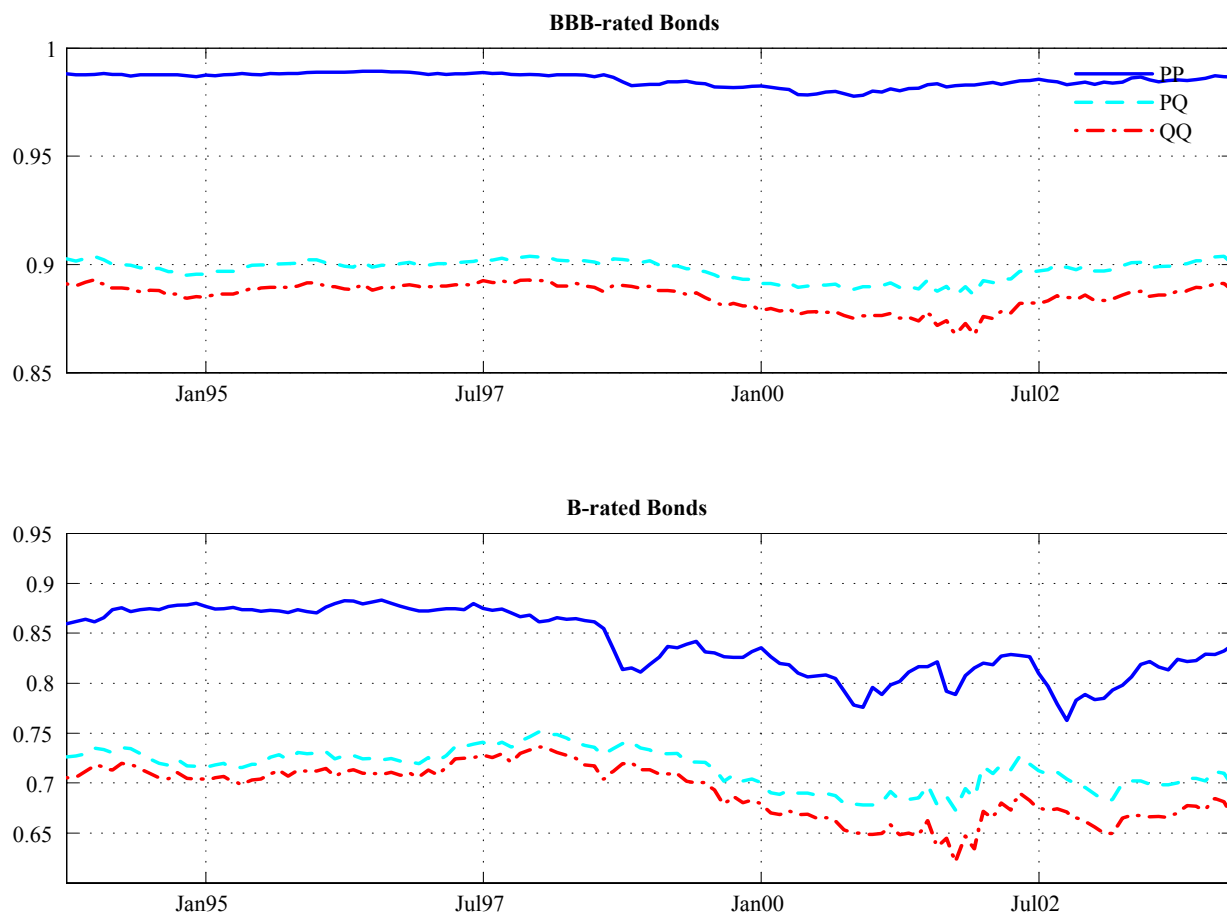


Figure 6
Correlation between Default and Recovery Rates, 1982-2007

