Università della Svizzera italiana

Faculty of Economics

Hakim Dall'O

Essays on the Valuation and Hedging of Derivative Securities

Submitted for the degree of Ph.D. in Finance at
Faculty of Economics
University of Lugano

Thesis Committee:

Prof. G. Barone-Adesi, advisor, University of Lugano and Swiss Finance Institute
Prof. A. Berardi, University of Verona
Prof. N. Carcano, University of Lugano and MVC & Partners
Prof. A. Plazzi, University of Lugano and Swiss Finance Institute

24th June 2011

swiss:finance:institute

To my family

ACKNOWLEDGEMENTS

I would like to thank my supervisor, Professor Giovanni Barone-Adesi for the opportunity he offered me and for all the advice he gave me in the last 4 years. Another special thanks to Professor Nicola Carcano for all the precious time he dedicated to me. Then, a general but really deep thank you to all the professors of the department, because “their doors were never closed…”. But, among all the professors a special thanks is for Professor Francesco Franzoni because he was not only a professor, but also, a great friend in this journey.
A deep thanks to my family to whom this work is dedicate because of their constant support and encouragement.. “all can be parents…but few are really father and mother…”. Special thanks to my brother and my sister for their love and their belief in me.

A special thanks to my lifelong friends: Ragno, Simone, Raffa, Melissa and to the new friends who made these 6 years in Lugano full of joy and full of fun: Fuxio, Graziano, Davide, Ilaria, Sofy, Fudo, Cota, Dima, Dimitr, Roberta, Maria, Masha, Mihaela, Natalie, Fleur, Alex, Gianpaolo, Jacopo, Stefano P., Tamara, Diego, and Jost.

Contents

Presentation of the work

The following work is submitted for the degree of Ph.D. in Finance at the Faculty of Economics, University of Lugano. During my Ph.D. I wrote three articles dealing with two main subjects: the first one is the so called “Kernel Puzzle” while the second one is the problem of immunization of portfolio of treasury and corporate bonds.

For the first topic, I wrote an article with professor Giovanni Barone Adesi: “Is the Pricing Kernel Monotone?”. In this article we provide a new method to derive the state price density per unit probability based on option prices and GARCH model. We derive the risk neutral distribution using the result in Breeden and Litzenberger (1978) and the historical density adapting the GARCH model of Barone-Adesi, Engle, and Mancini (2008). We take the ratio of these two probabilities in order to describe the shape of the state price density and to evaluate its consistency with economic theory. We find that using a large dataset and introducing non-Gaussian innovations, the pricing kernel puzzle that arises in Jackwerth (2000) disappears both in a single day and over an average of different days with options expiring at the same maturity. We also evaluate the price kernel at the onset of the recent crisis.

For the second topic I wrote two articles. The first one is a joint work with professor Nicola Carcano which deals with the immunization of a portfolio of treasury bonds against interest rate risk. This article has been concluded in 2011 and it is forthcoming on the “*Journal of Banking and Finance*”. In particular in this article we test alternative models of yield curve risk by hedging US Treasury bond portfolios through note/bond futures. We show that traditional implementations of models based on principal component analysis, duration vectors and key rate duration lead to high exposure to model errors and to sizable transaction costs, thus lowering the hedging quality. Also, this quality randomly varies from one model and hedging problem to the other. We show that accounting for the variance of modeling errors substantially reduces both hedging errors and transaction costs for all considered models. Additionally, it leads to much more stable weights in the hedging portfolios and as a result to more homogeneous hedging quality. On this basis, error-adjusted principal component analysis is found to systematically and significantly outperform alternative models.

The last article is a joint work with professors Giovanni Barone-Adesi and Nicola Carcano and deals with the problem of immunization of a portfolio of corporate bonds. In this article, we test alternative strategies for hedging a portfolio composed from BBB-rated corporate bonds. Our results highlight a change of regime. From 2000 to 2007, a hedging strategy based only on T-bond futures would have reduced the variance of the portfolio by circa 83.5%. This compares well to the maximum variance reduction of 50% reported by previous studies hedging corporate bonds through T-bond and S&P500 futures. We attribute this improvement to the use of four futures contracts with different maturity and to the consideration of modeling errors. On the contrary, in 2008 and 2009 T-bond futures would have been insufficient to successfully hedge our bond portfolio. The use of the 5-year CDX contract would have only marginally improved the quality of hedging. We attribute the disappointing hedging performance of CDX to counterparty risk and show that credit derivatives free of default risk could have led to a variance reduction over 64% even during 2008 and 2009.

Chapter 1
The “Pricing Kernel Puzzle”

Is the Price Kernel Monotone?

Giovanni Barone-Adesi
University of Lugano and Swiss Finance Institute
Lugano, Switzerland
giovanni.barone-adesi@lu.unisi.ch

Hakim Dall'O
University of Lugano and Swiss Finance Institute
Lugano, Switzerland
hakim.dallo@lu.unisi.ch

January, 2010

Abstract

We provide a new method to derive the state price density per unit probability based on option prices and GARCH model. We derive the risk neutral distribution using the result in Breeden and Litzenberger (1978) and the historical density adapting the GARCH model of Barone-Adesi et al. (2008). We take the ratio of these two probabilities in order to describe the shape of the state price density and to evaluate its consistency with economic theory. We find that using a large dataset and introducing non-Gaussian innovations, the pricing kernel puzzle that arises in Jackwerth (2000) disappears both in a single day and over an average of different days with options expiring at the same maturity. We also evaluate the price kernel before and during the recent crisis and we look at the change in the shape in order to evaluate the difference.

Keywords: Pricing kernel, State price density per unit probability, Risk neutral, Historical distribution.

JEL Classification: G12, G13, G14.

1 Introduction

According to economic theory, the shape of the kernel price or the state price density (SPD) per unit probability (also known as the asset pricing kernel (Rosenberg and Engle (2002)) or stochastic discount factor (SDF) (Campbell et al. (1997))) is a decreasing function in wealth.

In their paper, Jackwerth (2000) find a kernel price before the crash of 1987 in agreement with economic theory, but a discordant result for the post-crash period. After their work, a number of papers have been written on this topic explaining the reason for this puzzle. Rosenberg and Engle (2002), Detlefsen et al. (2007) and Jackwerth (2004) are among the most interesting papers on this topic. Unfortunately, none of them found an answer to this puzzle. In all of their papers they found problems in the methodology previous papers presented and tried to improve them, but the result was the same: the puzzle remained.

An answer to this puzzle has been given in Chabi-Yo et al. (2005) where they argue that the main problem in this puzzle is the regime shifts in fundamentals: when volatility changes, the kernel price is no longer monotonically decrease. In each regime they prove that the kernel price is consistent with economic theory, but when there is a shift in regime the kernel price changes in its shape and it is no longer consistent with economic theory.

In a recent paper, Barone-Adesi et al. (2008) compute again the kernel price in a parametric method and they find kernel prices consistent with economic theory. In particular they find kernel price consistency for fixed maturities. They do not use different maturities as Aït-Sahalia and Lo (1998) and therefore they avoid the problem that arises when the maturity is different, but they do not consider the change in fundamental as a relevant aspect of the computation. This result can be explained if we consider that the sample they use is very short in time (3 years) and in that period (2002 - 2004) the volatility does not make big change.

In this paper we use real data to describe the shape of the kernel price as presented in the market. We compute the kernel price both in a single day and as an average of kernel prices in a period of time (considering a fixed maturity). We want to understand the implication of the changing regime using two measures of moneyness: in the first case we consider the kernel price as a function of only two parameter (we do not take into consideration the changing regime) and then as a function of underlying, interest rate and volatility. In order to evaluate the kernel price we need to take a broad index which attempts to cover the entire economy. We decide to use options index on the S&P500 with a time series of 12 years (from the 2nd of January 1996 to the 31st December 2007).

Evaluating the kernel price in a period of time without taking into consideration the change in volatility should lead to a kernel not anymore consisted with economic theory. Surprising, when we compute the kernel price considering only two parameters (the underlying and the interest rate), the average of kernel price is consistent with economic theory (with the exceptions of some points). The main reason for this result is the methodology we use to compute the two probabilities.

In order to estimate the risk neutral distribution, we use the well known result in Breeden and Litzenberger (1978). The difference with previous works is in the options we use. Instead of creating option prices through nonparametric or parametric models (all the previous research use artificial price of options and this could introduce a bias in the methodology), we use only the options available on the market. We then construct the historical density using the GJR GARCH model with Filter Historical Simulation already presented in Barone-Adesi et al. (2008).

As discussed in Barone-Adesi et al. (2008), among the several GARCH models, the GJR GARCH with FHS has the flexibility to capture the leverage effect and has the ability to fit daily S&P500 index returns better. Then, the set of estimated scaled innovations gives an empirical density function that incorporates excess skewness, kurtosis, and other extreme return behavior that is not captured in a normal density function. These features avoid several problems in the estimation of the kernel price. For example, using a simple GARCH model where the innovations are standard normal $(0, 1)$ leads to a mispecification of the return of the underlying.

Once we have the two probabilities, we take the ratio between the two densities, discounted by the risk-free rate, in a particular day, and we get the kernel price for a fixed maturity. We repeat the same procedure for all the days in the time series which have options with this maturity and then we take the average of the kernel price through the sample.

We also evaluate how the shape of the price kernel changes before and during a crisis (the 2008 crisis). We notice that the three periods before the crisis (2005, 2006 and 2007) exhibit fairly monotonically decreasing paths, while during the crisis, the kernel price remains monotonically decrease but has higher values. (consistency with the idea that during a crisis an investor increase the risk aversion).

In order to evaluate the impact of the shifting regime, we repeat the computation of the differen kernel prices considering the volatility as a parameter of the kernel function. Surprising, the results have an improvement, but thanks to the methodology we used in the case of kernel price without volatility, the results are quite similar, supporting our first intuition that the changing regime is an important aspect, but also the methodology has a strong impact on the final result.

The remainder of this paper is organized as follows. In section 2, we present a review of the literature and we define the "pricing kernel puzzle". In section 3, we define our method to estimate the kernel price. We explain the application of the result of Breeden and Litzenberger (1978) for a discrete case and we derive the risk neutral distribution. We then estimate the historical density using a GJR GARCH method with FHS and we take the kernel price from a particular day as well as the kernel price over the time series of our sample. In section 4, we provide other evidence of our results. First we plot kernel price with different maturities to prove the robustness of our methodology, then we take the average of these different kernel prices and we show that the average of SPD per unit probabilities with closing maturities have a monotonically decreasing path. In section 5, we present the change in the kernel price shape before and during the recent crisis. In section 6, we extend our model, using a kernel price with 3 parameters (underlying, volatility and risk-free), and in section 7 we offer some conclusions.

2 Review of the Literature

In this section we derive the price kernel as in microeconomic theory and also as in probability theory. We then present some methods, parametric and non parametric to derive the kernel price.

2.1 Price kernel and investor preference

The ratio between the risk neutral density and the historical density is known as the price kernel or state price density per unit probability. In order to explain the relationship between the risk-neutral distribution and the historical distribution we need to introduce some basic concepts from macroeconomic theory. In particular, we use a representative agent with a utility function $U(\cdot)$. According to economic theory (the classical von Neumann and Morgenstern economic theory), we have three types of investors: risk averse, risk neutral and risk lover. The utility function $U(\cdot)$ of these investors is a twice-differentiable function of consumption c: U(c). The common property for the three investors is the non-satiation property: the utility increase with wealth e.g. more wealth is preferred to less wealth, and the investor is never satisfied - he never has so much wealth that getting more would not be at least a little bit desirable. This condition means that the first derivative of the utility function is always positive. On the other hand, the second derivative changes according to the attitude the investor has towards the risk.

If the investor is risk averse, his utility function is an increasing concave utility function which displays a strictly negative second derivative. The risk neutral investor has a second derivative equal to zero while the risk seeker has a second derivative strictly positive, which means a convex utility function.

Defining u($\cdot$) as the single period utility function and β as the subjective discount factor, we can write the intertemporal two-period utility function as

$$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1}.)$$

We introduce ξ as the amount of an asset the agent chooses to buy at time t, e as the original endowment of the agent, P_t as the price of the asset at time t and x_{t+1} as the future payoff of the asset. The optimization problem is:

$$\max_{\xi} u(c_t) + E_t\left[\beta c_{t+1}\right],$$

subject to

$$c_t = e_t - P_t\xi,$$

$$c_{t+1} = e_{t+1} + x_{t+1}\xi.$$

The first constraint is the budget constraint at time 1, while the second constraint is the Walrasian property e.g. the agent will consume all of his endowment and asset's payoff at the last period. Substituting the constrains into the objective and setting the derivative with respect to ξ equal to zero we get:

$$P_t = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right].$$

We define

$$\beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\right] = m_{t,t+1} = MRS,$$

as the Marginal rate of Substitution at time t. The MRS is also known as the Stochastic Discount Factor (SDF) or Price Kernel. Therefore the price of any asset can be expressed as

$$P_t = E_t \left[m_{t,t+1} x_{t+1} \right]. \tag{1}$$

In a continuous case, the price of any asset can be written as

$$P_t^p = \int_0^\infty m_{t,T}(S_T) x_T(S_T) p_{t,T}(S_T) dS_T, \tag{2}$$

where $p_{t,T}(S_T)$ is the probability of state S_T (for the rest of the paper we refer to this probability as the historical probability).

To define the price of an asset at time t, under the risk neutral measure, we can write equation 2 as:

$$P_t^q = e^{-rt} \int_0^\infty x_T(S_T) q_{t,T}(S_T) dS_T, \tag{3}$$

where $q_{t,T}(S_T)$ is the state price density (for the rest of the paper we refer to this probability as the risk neutral probability). At this point, combining equation 2 and 3 we can derive the SDF as:

$$m_{t,T}(S_T) = e^{-rt} \frac{q_t(S_T)}{p_t(S_T)} \tag{4}$$

In this case we consider a two period model where the price kernel is a function only of the underlying and of the risk-free rate. In the following part we will see how to have a kernel price with more parameters.

In their papers Arrow (1964) and Pratt (1964) find a connection between the kernel price and the measure of risk aversion of a representative agent. We can define the agent's coefficient of relative risk aversion (RRA) as:

$$\rho_t(S_T) = -\frac{S_T u''(S_T)}{u'(S_T)}. \tag{5}$$

A decreasing relative risk aversion indicates that the percentage of wealth one is willing to expose to risk increase with wealth. Constant relative risk aversion implies that the percentage of wealth one is willing to expose to risk remains unchanged as wealth increases or decreases. Increasing relative risk aversion means that the percentage of wealth one is willing to expose to risk falls as wealth increases.

The absolute risk aversion is the absolute amount of wealth an individual is willing to expose to risk as a function of changes in wealth. The absolute risk aversion can be decreasing, constant or increasing in wealth. Decreasing absolute risk aversion means that the amount of wealth someone is willing to expose to risk increases as wealth increase. Constant absolute risk aversion means that the amount of wealth one is willing to expose to risk remains unchanged as wealth increase or decrease. Increasing absolute risk aversion means that one's tolerance for absolute risk exposure falls as wealth increases.

The pricing kernel can be written as function of the marginal utility as:

$$m_{t,T}(S_T) = \beta \frac{u'(S_T)}{u'(S_t)}, \tag{6}$$

and the first derivative is:

$$m'_{t,T}(S_T) = \beta \frac{u''(S_T)}{u'(S_t)}, \tag{7}$$

Using the first and the second derivatives of the utility function from equation 6 and equation 7 we can write the RRA as:

$$\rho_t(S_T) = -\frac{S_T \beta m'_{t,T}(S_T) u'(S_t)}{\beta m_{t,T}(S_T) u'(S_t)} = -\frac{S_T m'_{t,T}(S_T)}{m_{t,T}(S_T)}. \tag{8}$$

Using the definition of MRS, we can write the RRA as:

$$\begin{aligned}
\rho_t(S_T) &= -\frac{S_T \left[\beta \frac{q_t(S_T)}{p_t(S_T)}\right]'}{\left[\beta \frac{q_t(S_T)}{p_t(S_T)}\right]}, & (9)\\
&= -S_T \frac{\left[\frac{q'_t(S_T)p_t(S_T) - q_t(S_T)p'_t(S_T)}{p_t^2(S_T)}\right]}{[q_t(S_T)/p_t(S_T)]}, & (10)\\
&= S_T \left[\frac{p'_t(S_T)}{p_t(S_T)} - \frac{q'_t(S_T)}{q_t(S_T)}\right]. & (11)
\end{aligned}$$

2.2 Nonparametric and parametric estimation

There are several methods to derive the kernel price. There are both parametric models and nonparametric models. In this section we give a review of the most well-known method used in literature. We focus particularly on the nonparametric model because they do not assume any particular form for the risk neutral and historical density and also for the kernel price.

One of the first papers to recover the price kernel in a nonparametric way is Aït-Sahalia and Lo (1998). In their work they derive the option price function by nonparametric kernel regression and then, throughout the result in Breeden and Litzenberger (1978), they compute the risk neutral distribution. Their result is not consistent with economic theory, but, because they look at the time continuity of $m_{t,T}$ across time, we may understand their results as estimates of the average kernel price over the sample period, rather than as conditional estimates.

Additional problems in their article are discussed in Rosenberg and Engle (2002). In particular they suggest that the non specification of the investors beliefs about future return probabilities could be a problem in the evaluation of the kernel price. Also their use of a very short period of time, 4 years, to estimate the state probabilities may be problematic. Moreover, they depart from the literature on stochastic volatility, which suggests that future state probabilities depend more on the recent events than past events. In fact, past events remain useful for prediction of future state probabilities. In order to avoid this problem we use a dataset of 12 years of option prices.

A work close in spirit to Aït-Sahalia and Lo (1998) is Jackwerth (2000). His article is one of the most interesting pertaining to this literature. Beyond the estimation technique he used, his paper also opened up the well-known "pricing-kernel puzzle". In his nonparametric estimation of the kernel price, Jackwerth finds that the shape of this function is in accordance with economic theory before the crash of 1987, but not for the period after the crash. He concludes that the reason is the mispricing of options after the crash.

Both articles could incur some problems that cause the kernel price and the relative risk aversion function (RRA) to be not consistent with economic theory. In Aït-Sahalia and Lo (1998), we see that, if the bandwidth changes, the RRA changes as well and this means that the methodology used influences the shape of the RRA; on the other hand, in Jackwerth (2000), the use of option prices after the crisis period could influence the shape of the kernel price if volatility is misspecified.

Another nonparametric estimation model for the kernel price is given by Barone-Adesi et al. (2008), where they relax the normality assumption in Rosenberg and Engle (2002) and provide a nonparametric estimation of the ratio $q_{t,T}/p_{t,T}$. While in the papers by Aït-Sahalia and Lo (2000) and

Jackwerth (2000) results are in contrast with the economic theory, Barone-Adesi et al. (2008) find a kernel price which exhibits a fairly monotonically decreasing shape.

Parametric methods to estimate the kernel price are often used in literature. Jackwerth (2004) provides a general review on this topic, but for the purpose of our work we do not go into detail on parametric estimation. As pointed out by Birke and Pilz (2009) there are no generally accepted parametric forms for asset price dynamics, for volatility surfaces or for call and put functions and therefore the use of parametric methods may introduce systematic errors.

Our goal is to test whether a different nonparametric method, starting with option pricing observed in the market, respects the conditions of no-arbitrage present in Birke and Pilz (2009). In particular, we test if the first derivative of the call price function is decreasing in the strike and the second derivative is a positive function. These conditions should guarantee a kernel price monotonically decreasing in wealth.

It is important to underlying that our kernel price is a function of three variables: the underlying price, the risk-free rate and the volatility. In the first part, we use only the first two of them: the underlying asset and the risk-free rate. At the end of the article we extend our methodology introducing the volatility.

3 Empirical kernel price

In this section we compute the kernel price as the ratio of the risk-neutral density and the historical one, discounted by the risk-free interest rate. In the first part we describe how we compute the risk-neutral density. In the second part, we explain our computation of the historical density. In each part we describe the dataset we use and our filter for cleaning it.

For the risk-free we use the Unsmoothed Fama-Bliss zero-coupon rate. The methodology followed for the estimation of these rates has been described in Bliss (1997).

3.1 Risk-neutral density

Breeden and Litzenberger (1978) shows how to derive the risk-neutral density from a set of call options with fixed maturity. They start from a portfolio with two short call options with strike K and long two call with strikes K - ϵ and K + ϵ and they consider $\frac{1}{2\epsilon}$ shares of this portfolio. The result is a butterfly spread which pays nothing outside the interval [K - ϵ ; K + ϵ]. Letting ϵ tend to zero, the payoff function of the butterfly tends to a Dirac delta function with mass at K, i.e. this is simply an Arrow-Debreu security paying $1 if $S_T = K$ and nothing otherwise (see Arrow (1964)). In this case, define K as the strike price, S_t the value of the underlying today, r as the interest rate, and τ as the maturity time, the butterfly price is given by

$$P_{butterfly}(S_T) = \frac{1}{2\epsilon}\left[2C(S_t, K, T, r) - C(S_t, K-\epsilon, T, r) - C(S_t, K+\epsilon, T, r)\right] \tag{12}$$

for $\epsilon \to 0$ and substituting equation 8 in equation 3 we get that the price of the butterfly is:

$$\lim_{\varepsilon \to 0} P_{butterfly} = \lim_{\varepsilon \to 0} e^{-rT} \int_{k+\varepsilon}^{k+\varepsilon} x_{butterfly}(S_T) q_t(S_T) dS_T. \tag{13}$$

If we solve this equation, we get that

$$\lim_{\varepsilon \to 0} P_{butterfly} = e^{-rT} q_t(S_T). \tag{14}$$

Rearranging equations 9 and 10 we can have that

$$\lim_{\varepsilon \to 0} P_{butterfly} = e^{-rT} q_t(S_T) = e^{-rT} \frac{\partial^2 C(S_t, K, T)}{\partial K^2}\bigg|_{S_T = K} \tag{15}$$

This result suggests that the second derivative of a call price (we will see that it is also true for a put price) with respect to the strike price gives the risk neutral distribution. In the next part of this section we see how to apply this result in the discrete case.

In the following, we consider three call options with strikes K_i, K_{i-1}, K_{i+1}, where $K_{i+1} > K_i > K_{i-1}$. We have seen that the price of a call option can be written as:

$$C(S_t, K, T) = \int_K^{\infty} e^{-rT}(S_T - K) f(S_T) dS_T. \tag{16}$$

We define $F(x)$ as the cumulative distribution function, $f(x)$ as the probability density, $C(S_t, K, T)$ as the price of a European call option, $P(S_t, K, T)$ as the price of a European put option, and K as the strike price of the reference option. According to the result in Breeden and Litzenberger (1978) taking the first derivative with respect to the strike price, we get:

$$\frac{\partial C(S_t, K, T)}{\partial K} = \frac{\partial}{\partial K}\left[\int_K^\infty e^{-rT}(S_T - K)f(S_T)dS_T\right] =$$

$$= e^{-rT}[-(K-K)f(K) + \int_K^\infty -f(S_T)dS_T] =$$

$$= e^{-rT}\int_K^\infty -f(S_T)dS_T = -e^{-rT}[1 - F(K)]$$

$$F(K) = e^{rT}\frac{\partial C(S_t, K, T)}{\partial K} + 1 \tag{17}$$

Which gives us the value for F(K) which is the cumulative distribution function. In order to find F(x) in the discrete case, we can use the following approximation:

$$F(K_i) \approx e^{rT}\left[\frac{C_{i+1}(S_t, K, T) - C_{i-1}(S_t, K, T)}{K_{i+1} - K_{i-1}}\right] + 1 \tag{18}$$

Now, take the second derivative in a continuous case, we have:

$$f(K) = e^{rT}\frac{\partial^2 C(S_t, K, T)}{\partial K^2}\bigg|_{S_T = K}, \tag{19}$$

that is the result in equation 15. We can approximate this result in the discrete case as:

$$f(K_i) \approx e^{rT}\frac{C_{i+1}(S_t, K, T) - 2C_i(S_t, K, T) + C_{i-1}(S_t, K, T)}{(K_{i+1} - K_i)^2}\bigg|_{S_T = K}. \tag{20}$$

The same is also true for the put options. In that case the two approximate distributions (cumulative and density) are given as:

$$F(K_i) \approx e^{rT}\left[\frac{P_{i+1}(S_t, K, T) - P_{i-1}(S_t, K, T)}{K_{i+1} - K_{i-1}}\right], \tag{21}$$

and

$$f(K_i) \approx e^{rT}\frac{P_{i+1}(S_t, K, T) - 2P_i(S_t, K, T) + P_{i-1}(S_t, K, T)}{(K_{i+1} - K_i)^2}\bigg|_{S_T = K}. \tag{22}$$

A recent paper by Figlewski (2008) is very close in spirit to our work. In his paper he derives the risk neutral distribution using the same result in Breeden and Litzenberger (1978). We differ from him in some aspects. First, we use the bid and ask prices that are given on the market to construct butterfly spreads. e.g. for the long position the ask price is used and for the short position the bid price. This way we avoid negative values in the risk-neutral distribution and we respect the no-arbitrage condition described in Birke and Pilz (2009). Second, we do not need to convert the bid, ask, or mid-prices into implied volatility to smooth the transition from call to put because we take the average of butterfly prices from several days with equal maturities and this improves the precision of our result (see Figure 2). Other similar works are discuss in Bahra (1997), Pirkner et al. (1999) and Jackwerth (2004).

In Bahra (1997), the author proposes several techniques to estimate the risk neutral density. For every method he explains the pros and the cons. He then assumes that the risk neutral density can be derived either in a parametric method, by solving a least squares problem, or in a nonparametric method, using kernel regression. In our work, using a time series of options over a sample of 12 years and taking the average of them, we avoid the parametric or nonparametric pricing step and therefore we rely only on market prices.

In Pirkner et al. (1999), they use a combination approach to derive the risk neutral distribution. They combine the implied binomial tree and the mixture distributions to get the approach called "Mixture Binomial Tree". The main difference with this work is in the use of the options types. In their work, they use American options and therefore they could have the problem of early exercise. In our sample, we consider only European options to be sure to have the risk neutral density for the appropriate expiration time.

Jackwerth (2004) may be considered as a general review of different methods and problems. He concentrates in particular on nonparametric estimation, but he gives a general overview also on parametric works, dividing every parametric work in a particular class and explaining the positive and negative aspects.

In the next section we explain the estimation method we use for the risk neutral distribution.

3.2 Risk-neutral estimation

We use European options on the S&P 500 index (symbol: SPX) to implement our model. We consider the closing prices of the out-of-the-money (OTM) put and call SPX options from 2^{nd} January 1996 to 29^{th} December 2007. It is known that OTM options are more actively traded than in-the-money options and by using only OTM options one can mitigate the potential issues associated with liquidity problems.

Option data and all the other necessary data are downloaded from OptionMetrics. We compute the risk-neutral density at two different maturities: 37 days and 72 days. Our choice of maturities is random and the same procedure can be applied for other maturities. We download all the options from our dataset with the same maturities (e.g. 37 days and 72 days) and we discard the options with an implied volatility larger than 70%, an average price lower than 0.05 or a volume equal to 0. In table 1 we summarized the number of options that we have for each maturity.

Now, we construct butterfly spreads using the bid-ask prices of the options. The butterfly spread is formed by two short call options with strike K_i and long two call with strikes K_{i+1} and K_{i-1}. We divide the dataset and we construct a butterfly spread for everyday (clearly the butterfly spread must be symmetric around K_i and the issue time and the maturity time must be the same for all options). By dividing the dataset for each day we obtain that the issue and maturity times are the same, we only have to ensure that there is symmetry around the middle strike. We try to use the smallest distance possible in the strikes to construct the butterfly spread. Following the quotation for the SPX we use a difference of 5 points. However, for the deep-out-of-the-money options we need to take into consideration a larger distance because there are less options traded. In that case, we arrive to have spreads of 10 to 50 points. Having a spread of 5 to 50 points does not impact our probability computation because we adjust it by the spread.

Our methodology to construct butterflies is very simple. We download the option prices, order by strike, from smallest to largest, we check if the first three options are symmetric around the second one. If so, we construct the butterfly, then we take into consideration the second to fourth values and we check again if they are symmetric around the third, repeating this process for every value. In this way, we have butterfly spreads which overlap some options. We repeat the construction of the butterfly spread with the same maturity for each day available in our dataset.

At the end of this procedure we get a number of butterfly prices summarized in table 1. In figure 1, we take a day at random from our sample and we apply equations (21) and (22) to compute the risk-neutral distribution, the historical distribution and their ratio as the SPD per unit probability. We take as an example the 11 August 2005, and we look at options with a maturity equal to 37 days. We see that for one day, the kernel price shows a monotonically decreasing path in S_T with some jumps, because we do not smooth the results.

[Figure 1 about here]

At this point, we take into consideration the moneyness of each butterfly. As reference moneyness of the butterfly spread, we use the moneyness of the middle strike. We round all the butterfly moneyness to the second digit after the decimal point and we take the average of all the butterfly prices with equal moneyness.[1] In table 1 we summarize the results after this procedure. At this point we are ready to plot the risk-neutral distribution as an average of the butterfly prices for a fixed maturity over a twelve year period.

3.3 Historical density

In order to construct the historical density we use a GARCH approach. Specifically, we use the asymmetric GJR GARCH model. As discussed in Barone-Adesi et al. (2008), among the several GARCH models, the GJR GARCH has the flexibility to capture the leverage effect and has the ability to fit daily S&P500 index returns. Under the historical measure, the asymmetric GJR GARCH model is

$$log\frac{S_t}{S_{t-1}} = \mu + \epsilon_t, \tag{23}$$

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2 + \gamma I_{t-1}\epsilon_{t-1}^2, \tag{24}$$

where $\epsilon_t = \sigma_t z_t$, and $z_t \sim f(0,1)$, and $I_{t-1} = 1$ when $\epsilon_{t-1} < 0$, and $I_{t-1} = 0$ otherwise. The scaled return innovation, z_t, is drawn from the empirical density function $f(\cdot)$, which is obtained by dividing each estimated return

[1]In order to find an equal moneyness it is necessary to round the moneyness values, to the second decimal digit. At this point we are left with a lower number of prices that we average.

innovation, $\hat{\epsilon}_t$, by its estimated conditional volatility $\hat{\sigma}_t$. This set of estimated scaled innovations gives an empirical density function that incorporates excess skewness, kurtosis, and other extreme return behavior that is not captured in a normal density function.

The methodology we use to estimate the historical density is as follows. We have butterfly prices for each day in the time series of twelve years, with equal maturity. For each day, we estimate the parameters of the GJR GARCH using a time series of 3500 returns from the S&P500. Once we get the parameters for each day, we simulate 35000 paths and we look at the probability that at maturity we exercise the butterfly. e.g. we count the number of paths that at maturity are in the range $[K_i$ - K_{i-1}, $K_i + K_{i+1}] =$ l (the length of the interval)[2]. To compute the probability density under the historical measure, we apply the following relation:

$$p = \frac{\frac{\#of\ paths\ in\ the\ interval\ l}{total\ number\ of\ paths}}{l}. \tag{25}$$

Once we have computed the probability for each day, we can apply the same methodology we use for the risk-neutral distribution. We round the butterfly moneyness to the second digit after the decimal point and we take the average over the sample period.

[Figure 2 about here]

We use the mid-strike for the butterfly and we round the moneyness to two decimal places. We take the average throughout the time series and we plot the distribution in respect to the moneyness.

3.4 Kernel price

We apply the definition given in equation 4 in order to get the kernel price. From previous calculations we obtain the average risk-neutral distribution for the fixed maturity and also the average historical density. We then discount this ratio by the risk-free rate. To estimate the average kernel price we take the kernel price of each day and then we compute the average from all the

[2]In our sample we use intervals with different lengths: most of them are intervals with a length of 10 points, but we also have some intervals with 25 or 50 points, and these intervals are in some cases overlapping

days in our time series. We compute the average across time because of the low number of data at each time.

We must keep in mind, throughout the process, that the risk-neutral distribution and the historical distribution we plotted in figure 2, are not the same ones we used to derive the kernel price. The kernel price is the average of the kernel price of each day. The distributions are the average of each day and therefore have a different kernel price.

[Figure 3 about here]

For all different maturities we get a monotonically decreasing path for the kernel price and all of these are in accordance with economic theory[3].

4 Averaging price kernel over time

In this section we show the robustness of our methodology and we try to find a smoothness criteria for smoothing our price kernels. First of all, we show different price kernels with maturity close to the one we showed before. According to economic theory, closing maturity price kernels should have similar shape and therefore, averaging closing maturity, could be a good criteria for smoothing.

The robustness of the methodology comes from the fact the the different price kernels should be similar. In order to verify this, we create to samples: the first one has maturities equal to 36, 37, 38 and 39 days. The second one, 71, 72, 73 and 74 days.

We use the approach explained in previous section to derive the price kernel for a fixed maturity. By this method we derive the price kernels for the maturities in the two samples and in the following figures we plot the results of the two samples.

[Figure 4 about here]

The kernel prices show a clearly monotonically decreasing path except in some points likely to be due to the discretization of the data. In order to verify that the price kernels are monotonous over time, we plot the kernel

[3]The presence of some jumps is due to the empirical analysis we compute. In fact, we do not introduce any smoothing criteria.

price as the average of different days. In particular, referring to the two samples (the first one is for maturities equal to 36, 37, 38 and 39 days and the second sample for maturities equal to 71, 72, 73 and 74 days), we take the average over the 4 different maturities. We expect to find a kernel price that is monotonically decreasing in wealth, because of the close maturities in our sample.

[Figure 5 about here]

As we see in figure 4, the kernel prices close in maturity, have similar path. This result supports the robustness of our methodology.

In figure 5, we plot the average for the two samples and we get decreasing kernel prices. In this way we were able to have a sort of smoothing criteria without using methods which bias our dataset.

5 Price Kernel around a crisis

In this section we evaluate the change of kernel price during a crisis. In particular, we look at kernel prices before and during a crisis. We divide our sample in 4 periods. Every period is from 9 to 12 months and we take periods which show a similar range in volatility according to the VIX index.

5.1 Estimate pricing kernels in different periods

In this subsection we explain how we choose different samples for the period before and during a crisis. In particular we look at the VIX index, given by the CBOE.

[Figure 6 about here]

From the VIX index, we can identify four different periods between August 2004 and August 2008. The first period is from the 12^{th} of August 2004 to the 15^{th} of September 2005. In this period we can see that the volatility is between 10 and 20 points. The second period is between 10^{th} of November 2005 to 10^{th} of October 2006. In this second period the volatility is again in a limited range between 10 to 20 points. The third period, which is before the crisis period, is between 14^{th} of June 2006 to 14^{th} of June 2007. Even here the volatility is in a range of 10 to 20 points. The last period, the period of the beginning of the crisis is between 11^{th} of October 2007 and 14^{th} of August 2008. In this period the volatility is much higher and it is in a range between 10 to 30 points.

[Figure 7 about here]

For each period, we compute the price kernel by the methodology presented in sections 3 and 4. We fix a maturity (in this case we look at maturity of 37 days) and we plot the kernel price of each period.

As expected, for the three periods before the crisis we get price kernels monotonically decreasing and very similar in shape to each other.

[Figure 8 about here]

For the kernel price of the crisis period, we have higher values. This is exactly what we expected to obtain. The probability of negative outcomes is more higher and therefore we give more weight to negative outcomes. This result is quite interesting. It proves how, using a robust method, the problem arising in Jackwerth (2000) disappear and we still have results consistent with economic theory.

6 Kernel price as a function of volatility

In this section we would like to extend our model and consider the kernel price as a function of more variables. In fact, as explained in Chabi-Yo et al. (2005), one way to overcome the problem of non monotonicity of the price kernel is consider another factor: the volatility.

In a previous section we compute the price kernel as a function of two variables: the underlying and the risk-free rate. We know from Pliska (1986), Karatzas et al. (1987), and Cox and Huang (1989) that the kernel price is characterized by at least two factors: the risk-free rate and the market price of risk. In our analysis we would like to consider the kernel price as a function of three different factors: the risk-free rate, the underlying price and the volatility. We have already introduce the underlying price and the risk free-rate. Now we want to introduce also the volatility: $m_{t,T}(S_T, r_f, \sigma)$. Because we look at a price kernel in a two-period model, we are not interested in looking at the dividends, but an extension with multi-period kernel price could be interesting. In that case we should consider another factor: the dividends.

We have already introduced the underlying price in our kernel price when we use, as strike price, the moneyness. The moneyness is nothing else then the k/S_t. In order to introduce the volatility we take as a reference the idea by Carr and Wu (2003). They use a moneyness defines as:

$$moneyness = \frac{log(K/F)}{\sigma\sqrt{T}},$$

where F is the futures price, T is the maturity time and σ is the average volatility of the index.

For our propose, we can change this formula in:

$$moneyness = \frac{K}{S_t * \sigma},$$

Time to maturity is constant over the sample we consider because we fix the maturity at the beginning. Therefore, we do not include the square of the time in the analysis. Furthermore, the volatility is not anymore the average volatility, but the implied volatility of each option.

The procedure for derive the kernel price is again the same we have seen in the previous sections [4] and therefore our result for maturities equal 36, 37, 38 and 39 as well as 71, 72, 73 and 74 are:

[Figure 9 about here]

Also in this case the result are consistent with economic theory. In particular we can see how the kernel prices for close maturities are very similar in agreement with what we expected.

7 Conclusion

This paper proposed a method to evaluate the kernel price in a specific day for a fixed maturity as well as the average of different kernel prices in a time series of 12 years for a fixed maturity. Using option prices on the S&P 500, we

[4] There is only a small difference when we round the new moneyness in order to average different periods. We do not take the second digit after the point, but only the first one.

derive the risk-neutral distribution through the well-known result in Breeden and Litzenberger (1978). We compute the risk neutral distribution in each day where we have options with a fixed maturity.

Then, we compute the historical density, for the same maturity, each day, using a GARCH method, based on the filter historical simulation technique. We then compute the ratio between the two probabilities discounted by the the risk-free rate, in order to derive the kernel price for that given day. We show that in a fixed day (chosen at random in our sample) the risk-neutral distribution taken from the option prices respects the no-arbitrage condition proven in Birke and Pilz (2009).

Therefore, we show that the ratio between the two probabilities, in that particular day, is monotonically decreasing, in agreement with economic theory (see figure 1). We also show how the average of the different kernel prices across 12 years display the same monotonically decreasing path (see figure 3).

We also prove that average price kernels over time, if we take closing maturities, exhibit a monotonically decreasing path in agreement with economic theory.

In the extension of our model, considering the volatility in the kernel price, the average across several years results more smooth and still coherent with economic theory.

In the last part, we show the changing in shape of different price kernels before and during the recent crisis. We see that before the crisis the price kernels are monotonically decreasing with a similar value while during the crisis the pricing kernel reaches higher values. This is exactly what we expected to obtain. The probability of negative outcomes is higher and therefore the market gives more weight of negative outcomes.

References

Aït-Sahalia, Y. and Lo, A. W. (1998). Nonparametric estimation of state-price densities implicit in financial assets prices. *Journal of Finance*, 53:499–548.

Aït-Sahalia, Y. and Lo, A. W. (2000). Nonparametric risk management and implied risk aversion. *Journal of Econometrics*, 94:9–51.

Arrow, k. J. (1964). The role of securities in the optimal allocation of risk-bearing. *The review of Economic Studies*, 31:91–96.

Bahra, B. (1997). Implied risk-neutral probability density functions from options prices: Theory and application. *Working Paper, Bank of England.*

Barone-Adesi, G., Engle, R. F., and Mancini, L. (2008). A garch option pricing model with filtered historical simulation. *Review of financial studies*, 21:1223–1258.

Birke, M. and Pilz, K. F. (2009). Nonparametric option pricing with no-arbitrage constrains. *Journal of Financial Econometrics*, 7:53–76.

Bliss, R. (1997). Testing term structure estimation methods. *Advanced in Futures and Options Research*, 9:197–231.

Breeden, D. T. and Litzenberger, R. H. (1978). Prices of state-contigent claims implicit in option prices. *Journal of Business*, 51:621–651.

Campbell, J. Y., Lo, A. W., and C., M. (1997). The econometrics of financial markets. *Princeton University Press.*

Carr, P. and Wu, L. (2003). The finite moment log stable process and option pricing. *Journal of Finance*, LVIII:753–777.

Chabi-Yo, F., Garcia, R., and Renault, E. (2005). State dependence in fundamentals and preferences explains risk-aversion puzzle. *Working Paper.*

Cox, J. C. and Huang, C. F. (1989). Optimal consumption and portfolio policies when asset prices follow a diffusion process. *Journal of Economic Theory*, 49:33–83.

Detlefsen, K., Härdle, W. K., and Moro, R. A. (2007). Empirical pricing kernels and investor preferences. *Working Paper, Humboldt University of Berlin.*

Figlewski, S. (2008). The risk neutral probability distribution for the u.s. stock market. *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle.*

Jackwerth, J. (2000). Recovering risk aversion from option prices and realized returns. *Review of financial studies*, 13:433–451.

Jackwerth, J. C. (2004). Option-implied risk-neutral distributions and risk aversion. *Charlotteville: Research Foundation of AIMR.*

Karatzas, I., Lehoczky, J. P., and Shreve, S. E. (1987). Optimal portfolio and consumption decisions for a small investor on a finite horizon. *Journal of Control and Optimization*, 25:1557–1586.

Pirkner, C. D., Weigend, A. S., and Heinz, Z. (1999). Extracting risk-neutral density from option prices using mixture binomial trees. *Proc. IEEE/IAFE 1999 Conf. Comput Intell. Financial Eng.*, pages 135–158.

Pliska, S. (1986). A stochastic calculus model of continuous trading: Optimal portfolios. *Mathematics of Operations Research*, 11:371–382.

Pratt, J. (1964). Risk aversion in the small and in the large. *Econometrica.*

Rosenberg, J. V. and Engle, R. F. (2002). Empirical pricing kernels. *Journal of Financial Economics*, 64:341–372.

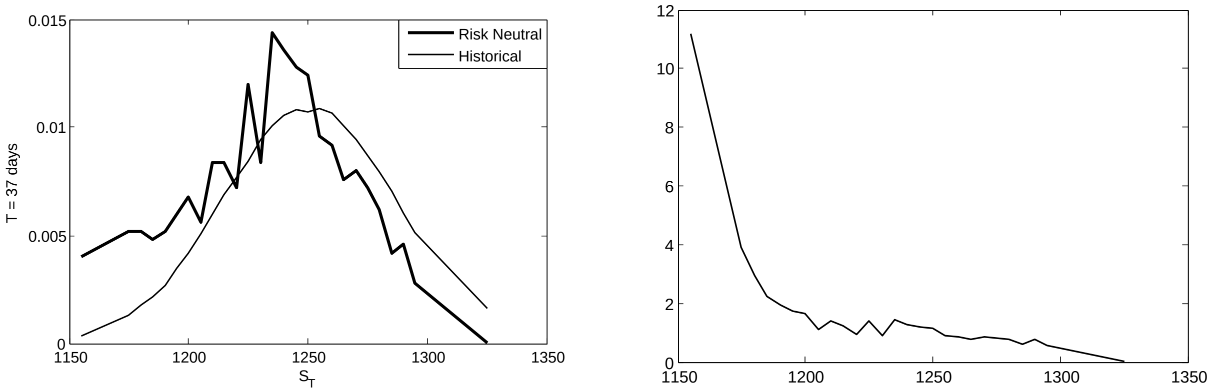


Figure 1: Left: Risk-neutral distribution (black line) and the historical distribution (gray line). We take one day at random from our sample (11 August 2005) with maturity equal to 37 days. Right: SPD per unit probability for this particular day (11 August 2005).

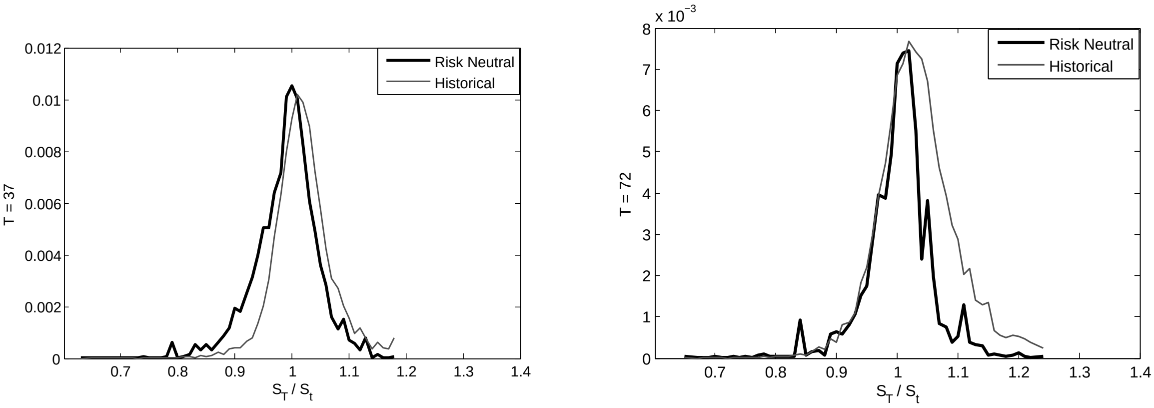


Figure 2: Risk neutral and historical distribution as the average of 12 years risk neutral and historical distribution for a fixed maturity. Left: maturity equal to 37 days, right maturity equal to 72 days.

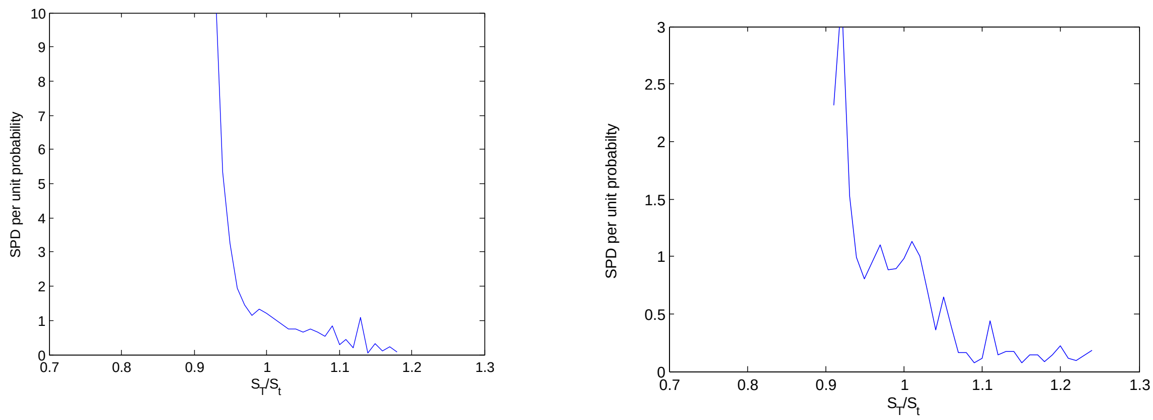


Figure 3: SPD per unit probability as the average of the SPD per unit probability throughout the time series of 12 years and with equal maturity. Left: 37 days, right 72 days. It is important to bear in mind that this SPD per unit probability is not derived from the two distributions given in figure 2.

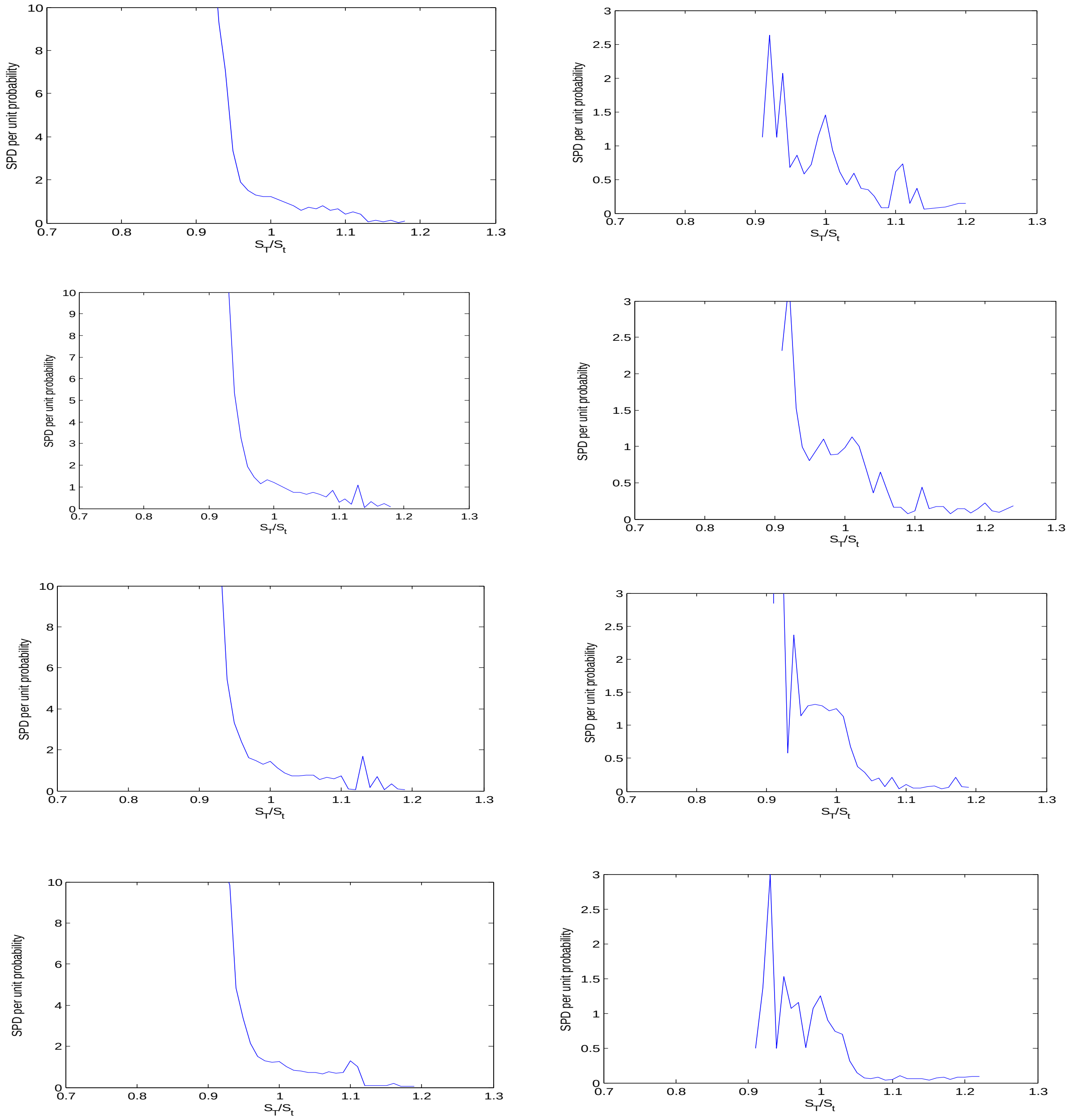


Figure 4: SPD per unit probability for different maturities. Left: from top to bottom, SPD per unit probability for maturity equal to 36, 37, 38 and 39. Right: from top to bottom, SPD per unit probability for maturity equal to 71, 72, 73 and 74.

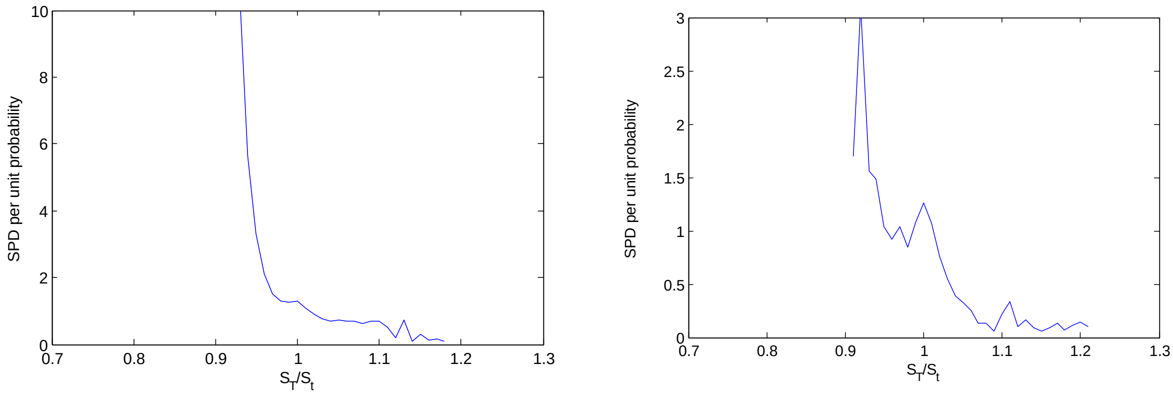


Figure 5: SPD per unit probability over time. Left: SPD per unit probability for the first sample (from 36 to 39 days). Right: SPD per unit probability for the second sample (from 71 to 74 days).

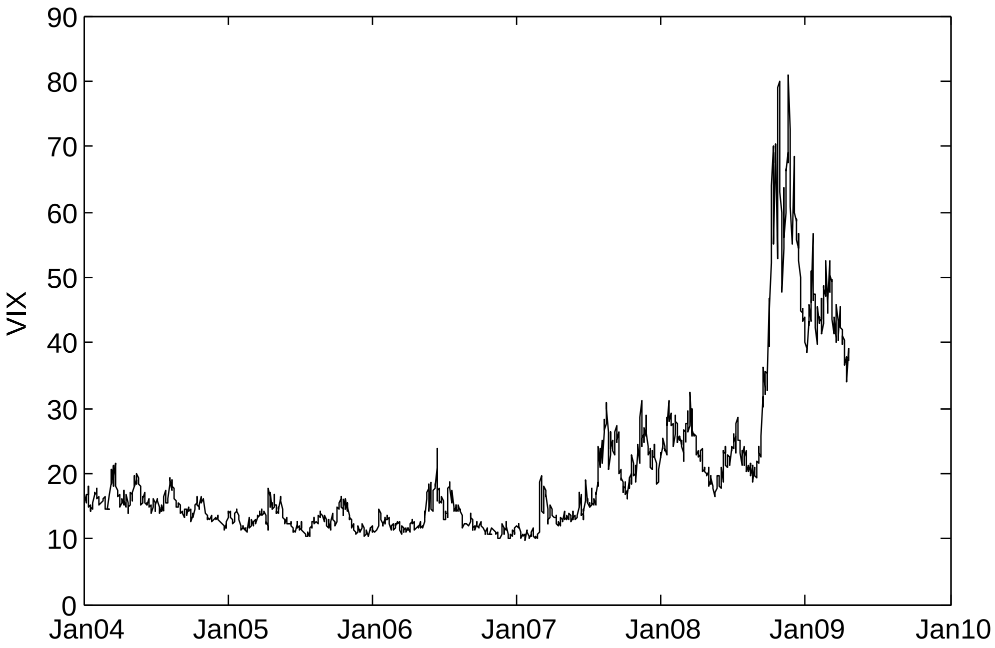


Figure 6: The VIX index between 2^{nd} of January 2004 and 21^{st} of April 2009

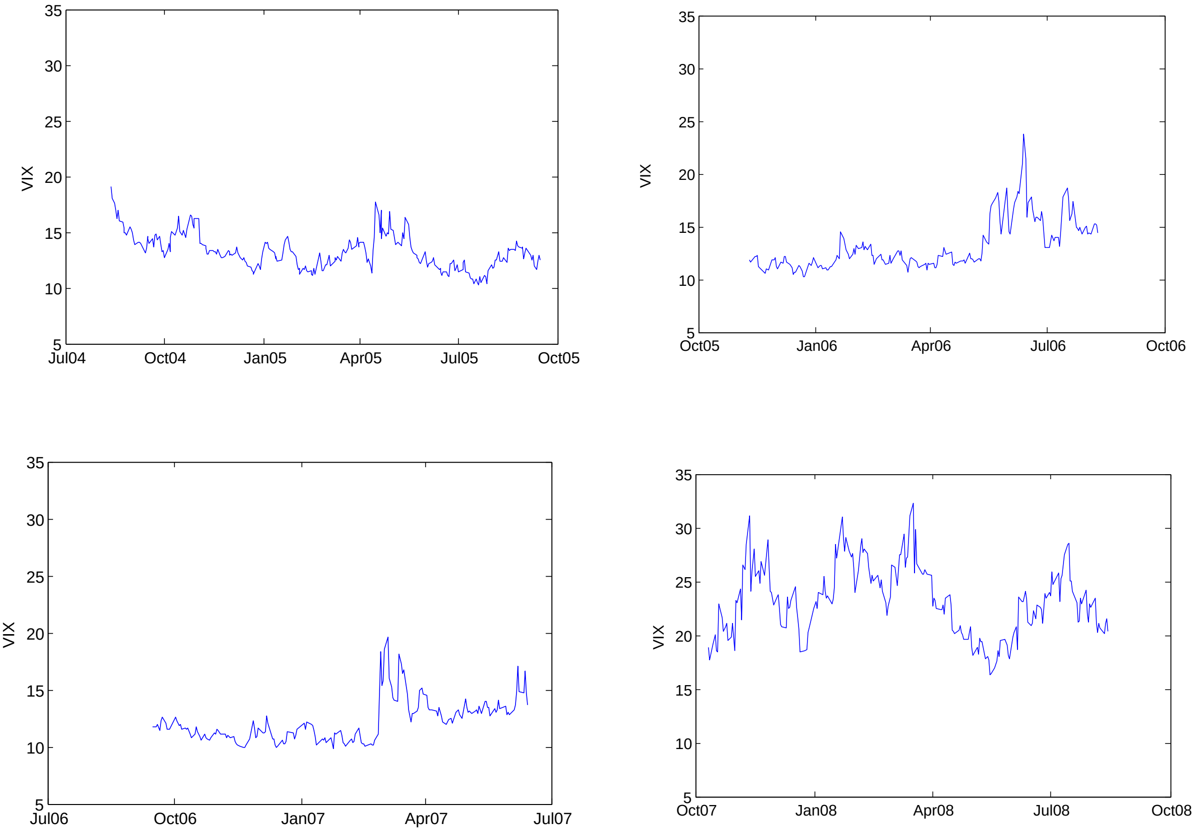


Figure 7: The VIX samples we use to compute the different SPD per unit probabilities over different years.

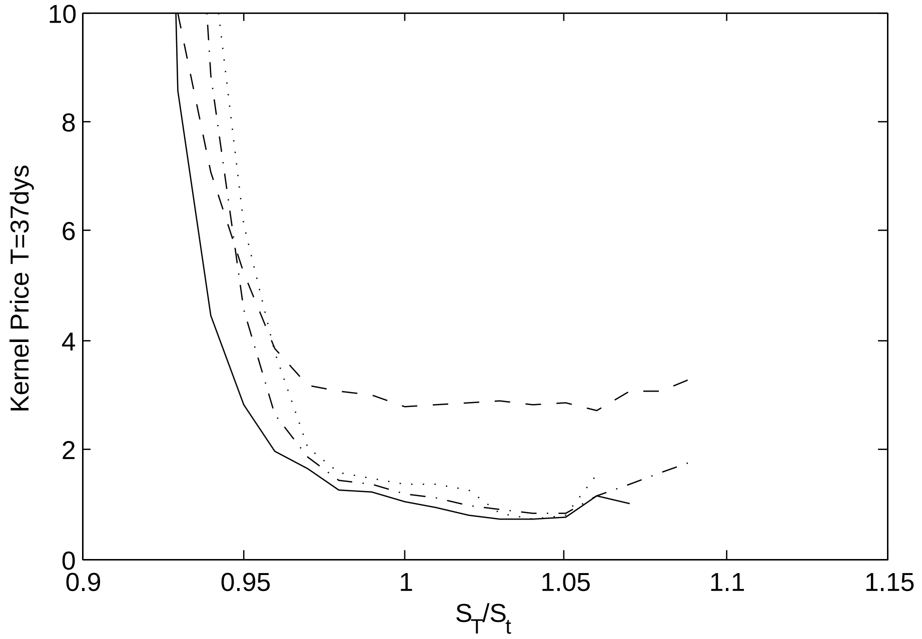


Figure 8: The kernel prices for the four samples we create looking at different levels of volatility index. The solid line, dot line and the dot-dash line are the pre-crash kernel prices, while the dash line is the kernel price in the crisis period.

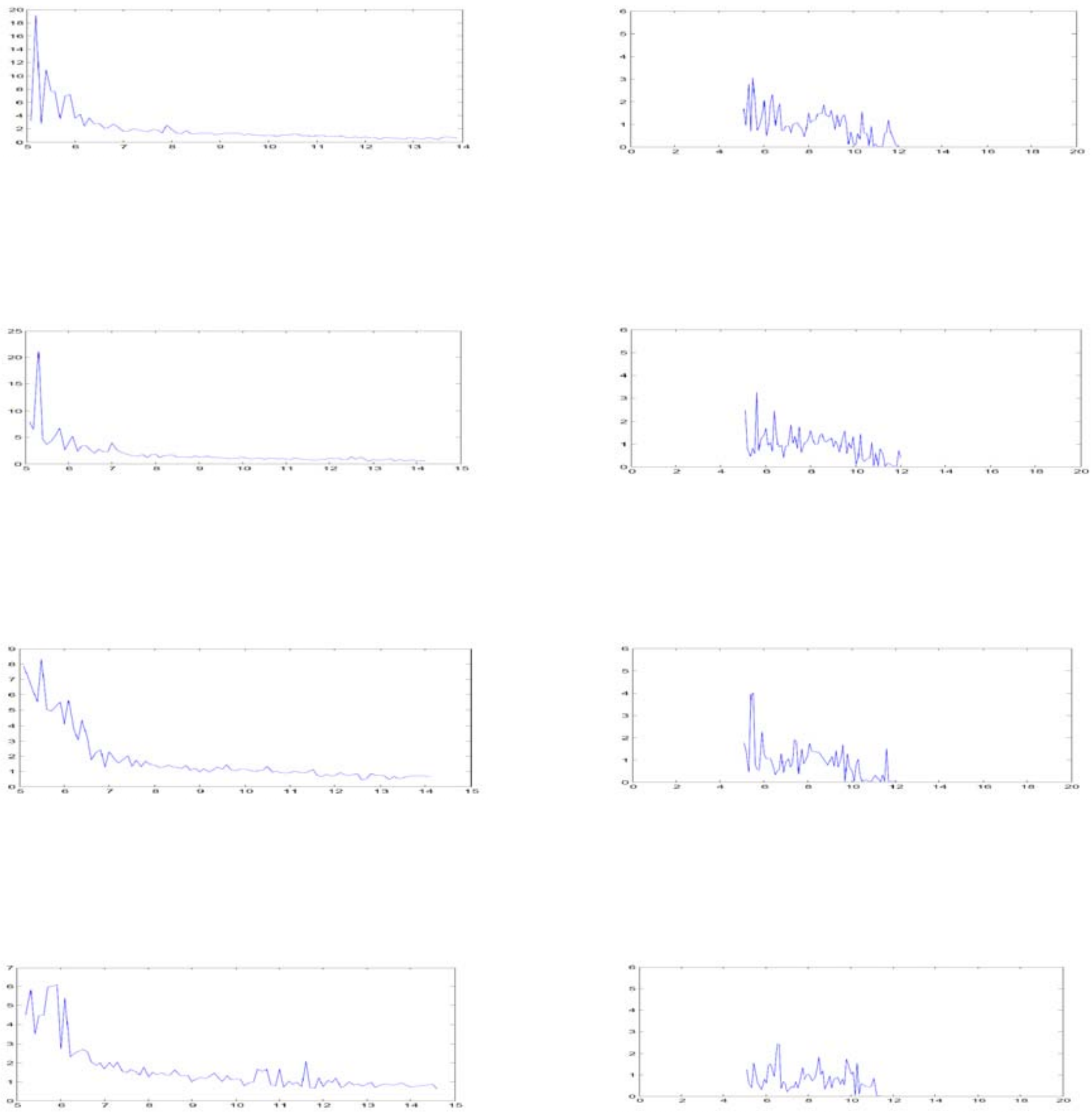


Figure 9: SPD per unit probability for different maturities. Left: from top to bottom, SPD per unit probability for maturity equal to 36, 37, 38 and 39. Right: from top to bottom, SPD per unit probability for maturity equal to 71, 72, 73 and 74.

Maturity and type of option	Sample	Number of butterfly spreads	Number of butterfly spreads after rounding
37 days call	2903	2590	54
37 days put	3544		
72 days call	1497	1196	58
72 days put	2025		

Table 1: Summary of the number of option data used for our method. The second column is the number of options from our dataset once the options with implied volatility larger than 70%, average price lower than 0.05 and a volume equal to 0 have been discarded. The third column is the number of butterfly spreads we can construct. The fourth column is the number of butterfly spreads after we round the moneyness to two decimal places. The first two rows refer to the call and put options for maturity equal to 37 days and the third and fourth for maturity equal to 72.

Chapter 2
Hedging Treasury Bond Portfolios

ALTERNATIVE MODELS FOR HEDGING YIELD CURVE RISK: AN EMPIRICAL COMPARISON

This version: 2nd of January, 2011

ABSTRACT

We test alternative models of yield curve risk by hedging US Treasury bond portfolios through note/bond futures. We show that traditional implementations of models based on principal component analysis, duration vectors and key rate duration lead to high exposure to model errors and to sizable transaction costs, thus lowering the hedging quality. Also, this quality randomly varies from one model and hedging problem to the other. We show that accounting for the variance of modeling errors substantially reduces both hedging errors and transaction costs for all considered models. Additionally, it leads to much more stable weights in the hedging portfolios and – as a result – to more homogeneous hedging quality. On this basis, error-adjusted principal component analysis is found to systematically and significantly outperform alternative models.

Keywords: Yield curve risk, interest rate risk, immunization, hedging.

JEL codes: G11; E43

Introduction

We define yield curve risk as the risk that the value of a financial asset might change due to shifts in one or more points of the relevant yield curve. As such, it represents one of the most widely spread financial risks: each institution having to match future streams of assets and liabilities is exposed - up to a certain extent - to it.

A simple, but effective way to cope with yield curve risk is to match positive with negative cash-flows. Unfortunately, the dates and the amounts of future cash-flows are often subject to constraints, so that implementing an accurate matching might either not be possible or be very expensive. In these cases, *immunization* techniques are employed to manage yield curve risk. These techniques make the sensitivity of the assets and the liabilities to yield curve changes similar to each other, so that the overall balance sheet will not be largely affected by these changes.

Initially, academicians and practitioners focused on the concept of *duration* - introduced by Macaulay (1938) - for implementing immunization techniques. Duration represents the first derivative of the price-yield relationship of a bond and was shown to lead to adequate immunization for parallel yield curve shifts[1].

The assumption of parallel yield curve shifts could be released thanks to the concept of *convexity* which was initially related to the second derivative of the price-yield relationship (Klotz (1985)). However, the impact of interest rate changes over a few weeks is normally well-approximated by duration. Bierwag et al. (1987) and Hodges and Parekh (2006) showed that the usefulness of convexity is generally not related to better approximating the price-yield relationship, but rather to the fact that hedging strategies relying on duration- and convexity-matching are consistent with plausible two-factor processes describing non-parallel yield curve shifts.

Extensions of these strategies were based on *M-square* and *M-vector* models introduced by Fong and Fabozzi (1985), Chambers et al. (1988), and Nawalka and Chambers (1997). Similarly as for convexity, most of these models relied on the observation that further-order approximations of the price-yield relationship lead to immunization strategies which are consistent with multi-factor processes accurately describing actual yield curve shifts. Nawalka et al. (2003) reviewed these *duration vector* (DV) models and developed a *generalized duration vector* (GDV).

A second class of immunization models relied on a statistical technique known as *principal component analysis* (PCA) which identifies orthogonal factors explaining the largest possible proportion of the variance of interest rate changes. Litterman and Scheinkman (1988) showed that a 3-factor PCA allows to capture the most important characteristics displayed by yield curve shapes: level, slope, and curvature. Accordingly, models matching the sensitivity of assets and liabilities to these three components should lead to high-quality hedging.

A third approach relied on the concept of *key rate duration* (KRD) introduced by Ho (1992). According to this approach, changes in all rates along the yield curve can be represented as linear interpolations of the changes in a limited number of rates, the key rates. A significant extension of this approach in the presence of restricting constraints was developed by Reitano (1996).

In practice, yield curve hedging techniques mostly rely on one of these three classes of models. However, we are not aware of a conclusive evidence on their relative performance[2]. On the contrary, studies conducting empirical tests of these models reported puzzling results: models capable to better capture the dynamics of the yield curve were not always shown to lead to better hedging. This was the case of the volatility- and covariance-adjusted models tested by Carcano and Foresi (1997) and of the 2-factor PCA tested by Falkenstein and Hanweck (1997) which was found to lead to better immunization than the corresponding 3-factor PCA.

Carcano (2009) tested a model of PCA-hedging which controls the exposure to model errors. He found that – by introducing this adjustment - 3-component PCA does lead to better hedging than 2-component PCA, as theory would suggest. On this basis, he claimed that random changes in the exposure to model errors might have led previous empirical tests of alternative hedging models to inconclusive results.

The goal of this paper is to identify the model capable of minimizing yield curve risk based on a sound empirical evidence. We expected the exposure to model errors to play a crucial role in determining the performance of alternative models. Once appropriate consideration is given to this exposure, the success of the tested models should mainly depend on how well the underlying stochastic process catches the actual dynamics of the yield curve. Accordingly, we extended all three mainstream immunization approaches in order to account for model errors and compared them among themselves and with their traditional implementations.

We relied on previous evidence that three factors are sufficient to explain the vast majority of the yield curve dynamics and tested only three-factor models. Accordingly, we expected the quality of the resulting error-adjusted hedging strategies to be comparable. The PCA model is constructed in a way to explain the largest possible part of the variance of yield curve shifts based on three orthogonal factors. Accordingly, we suspected that - once we account for model error exposure - this model would slightly outperform the alternative models.

We tested the three models by hedging portfolios of US T-bonds and T-notes through T-bonds and T-notes futures. The results confirmed our expectations: even though we could not clearly rank the models based on their traditional version, hedging based on PCA is consistently the best choice when the error-adjustment is introduced. This adjustment is also shown to improve the performance of all three models and to lead to substantially more stable hedging equations.

The remainder of the paper is organized as follows: section 1 presents the hedging models we are going to test and their theoretical justification. Section 2 describes our dataset and testing approach. Section 3 reports our results, both on the full sample as well as on three sub-samples, while Section 4 concludes and indicates some possible directions for future research.

1. The Hedging Methodology

1.1. *The sensitivity of bond and future prices*

We consider the problem of immunizing a risk-free bond portfolio which at time t has a value V_t by identifying the optimal underlying value ϕ_y to be invested in each of the four US T-note/T-bond futures (the 2-year, the 5-year, the 10-year, and the 30-year contracts). We group the cash flows of the bond portfolio and of the cheapest-to-deliver (CTD) bonds underlying the futures in n time buckets. Following the most common approach to this immunization problem as in Martellini and Priaulet (2001), we impose the so-called *self-financing constraint*:

$$\sum_{y=1}^{4} \phi_{y,t} \equiv H_t = V_t \tag{1.}$$

In our context, the latter constraint implies that the market value of the portfolio to be hedged must be equal to the market value of the underlying CTD bonds of the hedging portfolio. In practice, the amounts to be invested in the hedging portfolio are often constrained, even though the form of these constraints can differ from the last equation. Accordingly, we felt that including a constraint would make our empirical tests more realistic.

We decided to analyze the quality of alternative hedging models on a relatively short hedging horizon, which was set equal to one month. This choice was motivated by the fact that many institutional investors and portfolio managers do have a time horizon of 1 to 3 months, when they set up their hedging strategies. After this period, they mostly reconsider the whole hedging problem and determine a new strategy.

The market risk for the portfolio to be hedged comes from unexpected shifts in the corresponding continuously compounded zero-coupon risk-free rates $R(t,D_k)$, where D_k indicates the duration and maturity of the corresponding time bucket. We assume, for simplicity, that all rates are martingales: that is, $E[dR(t,D_k)]=0$ for every k and t. Accordingly, we computed the unexpected return of a bond as the return in excess of what would have been obtained if the promised yield had remained constant throughout the hedging period[3].

Approximating the dynamics of the term structure through a limited number of factors results in a difference between the modeled and actual dynamics of interest rates, the *model error*. For a generic 3-factor model of the term structure of interest rates, we can describe the dynamics of the zero-coupon risk-free rate $R(t,D_k)$ of maturity D_k as:

$$dR(t,D_k) \equiv \sum_{l=1}^{3} c_{lk} F_t^l + \varepsilon(t,D_k) \tag{2.}$$

where F^l_t represents the change in the *l-th* factor between time t and $t+1$, c_{lk} represents the sensitivity of the zero-coupon rate of maturity D_k to this change, and ε represents the model error.

As reported in several papers, like Hodges and Parekh (2006), the impact of monthly rate changes on the price of a zero-coupon bond can be well-approximated by its duration. Accordingly, we will follow this simplifying approach.

Estimating the sensitivity of future prices to changes in zero rates is more complex. In the past, researchers implementing hedging strategies through note and bond futures attempted to simplify the problem. One approach has been to calculate the sensitivity of futures through standard regression analysis (an example of this approach can be found in Kuberek and Norman (1983)). Such an approach implicitly assumes that the sensitivity of the future price is constant over time, whereas practitioners know well that this sensitivity varies significantly with the underlying

CTD bond. Since we intended to test realistic hedging strategies, we decided not to follow this approach.

The approach we decided to follow in the estimation of bond future sensitivity can be better described by splitting the market price of these futures in two components: the theoretical price excluding the value of the embedded options and the basis.

The theoretical price of a bond future excluding the value of the embedded options *FP* can be represented by the following expression:

$$FP_t = \frac{1}{CF_{CTD}} \left[\sum_{k=s+1}^{n} \frac{cf_{CTD,k}}{e^{R(t,D_k)D_k}} e^{R(t,D_s)D_s} - AI_{CTD,s} \right] \quad (3.)$$

where *CF* indicates the Conversion Factor, $cf_{CTD,k}$ indicates the cash-flow paid by the cheapest-to-deliver bond at time *k*, and $AI_{CTD,s}$ represents the accrued interests of the cheapest-to-deliver bond on the expiration date *s* of the future contract. The only cash-flows of this bond which are relevant for the valuation of the future contract are the ones maturing after the expiration date *s*.

Approximating the effect of rate changes on the price of a zero bond by its duration, the percentage sensitivity of the future price to these changes can be expressed as:

$$\frac{\frac{\partial FP_t}{FP_t}}{\partial R(t,D_k)} \approx -\frac{D_k}{FP_t} \frac{e^{R(t,D_s)D_s}}{CF_{CTD}} \frac{cf_{CTD,k}}{e^{R(t,D_k)D_k}} \equiv -D_k \omega_{CTD,k,t} \quad (4.)$$

for all rates maturing after the future contract (i.e.: $k > s$) and

$$\frac{\frac{\partial FP_t}{FP_t}}{\partial R(t,D_s)} \approx \frac{D_s}{FP_t}\frac{e^{R(t,D_s)D_s}}{CF_{CTD}}\left[\sum_{k=s+1}^{n}\frac{cf_{CTD,k}}{e^{R(t,D_k)D_k}}\right] = D_s\sum_{k=s+1}^{n}\omega_{CTD,k,t} \equiv -D_s\omega_{CTD,s,t} \qquad (5.)$$

for the zero rate with maturity equal to the expiration of the future contract, where $\omega_{CTD,k,}$ represents the percentage of the CTD future price related to the CTD cash-flow with maturity *k* and is defined based on the last two equations. The sensitivity of the future price to changes in zero rates maturing before the future contract is zero, which implies: $\omega_{CTD,k,t} = 0$ for all $k < s$.

The second component of the market price of a future is represented by the basis. Within the basis, we can identify three further components:

1. The carry, which we estimated as the difference between the yield of the CTD bond and the 1-month risk-free rate applied from the starting date of the hedging period to the expiration of the future contract. The basis net of carry – the so-called *net basis* – is the sum of the two following components 2 and 3.
2. A possible mispricing between the cash and the future market and/or data quality issues, like the difference in the time at which spot and future prices are observed (5 pm for bonds, 2 pm for futures) and in their meaning (mid price for bonds, closing price for futures).
3. The value of the embedded options. As illustrated by Fleming and Whaley (1994), future contracts embed 4 types of options. The first option is a quality option that permits the short position to deliver the CTD bond to the long position. The other three options are defined time options[4].

For all components of the basis, we need to distinguish expected from unexpected changes. If the latter changes display a dependency on yield curve shifts, this would represent a further source of future price sensitivity to such shifts and would influence the optimal hedging strategy.

For the carry, given our focus on the next expiring future, actual changes during the hedging period are dominated by its time-decay. Accordingly, we estimated this time-decay as a component of futures expected return and neglected unexpected changes due to modifications in the yield curve shape.

For the net basis, we followed Grieves et al. (2010) in the assumption that this value should be expected to be linearly amortized in order to get to zero by the contract expiration. An analysis we performed on the average absolute value of the net basis confirmed that the hypothesis of a linear amortization is fully consistent with empirical evidence. Accordingly, we estimated the unexpected change as the difference between the actual value of the net basis at the end of the hedging period and its expected value. In the expiring months of future contracts (therefore, in one-third of our test sample), we imposed this difference to be zero because of the non-arbitrage condition. We then performed a regression of unexpected changes in the net basis on changes in the 1-year risk-free rate for each of the future contracts without finding any evidence of a statistically significant dependency[5]. This result allows us to estimate the sensitivity of bond futures to unexpected changes in the yield curve simply based on equations (4.) and (5.).

In order to assess if our results are robust to a possible underestimation of the sensitivity of embedded options, we will perform a sub-sample analysis. In fact, Rendleman (2004) and Grieves et al. (2010) highlighted that the value of the delivery option has a low impact on hedging strategies based on the next-expiring future contract, when yields are not *too close* to the notional coupon of the future contract. Accordingly, we will assess if and how our results are sensitive to the difference between market yields and the notional coupon.

1.2. *The development of the hedging equations*

Given the approach to estimate the sensitivity of bond and future prices to zero rate changes described in the previous sub-section, we approximate the total unexpected return ψ provided by the combination of the two portfolios *V* and *H* as follows:

$$\psi_t \approx -\sum_{y=1}^{4} \phi_{y,t} \sum_{k=1}^{n} dR(t, D_k) D_k \omega_{y,k,t} + \sum_{k=1}^{n} dR(t, D_k) D_k A_{k,t} \qquad (6.)$$

where A_i indicates the present value of the bond portfolio cash-flows included in the *i-th* time bucket. As in Carcano (2009), we assume that the error terms ε of two zero rates of different maturity are independent from each other. Additionally, we assume that the error term of the zero rate of maturity D_k is independent from the fitted values $\sum_{l=1}^{3} c_{l\xi} F_t^l$ of all considered zero rates, including the zero rate of maturity D_k [6]. On this basis and relying on the definition of $dR(t,D_k)$ given in (2.), the expected squared value of the unexpected return can be approximated by:

$$E_t[\psi_t]^2 \approx \sum_{k=1}^{n}\sum_{v=1}^{n}\left(D_k A_{k,t} - \sum_{y=1}^{4}\phi_{y,t} D_k \omega_{y,k,t}\right)\left(D_v A_{v,t} - \sum_{y=1}^{4}\phi_{y,t} D_v \omega_{y,v,t}\right) E_t\left[\sum_{l=1}^{3} c_{lk} F_t^l \sum_{h=1}^{3} c_{hv} F_t^h\right] + \\ + \sum_{k=1}^{n}\sigma_\varepsilon(t,D_k)^2\left(D_k A_{k,t} - \sum_{y=1}^{4}\phi_{y,t} D_k \omega_{y,k,t}\right)^2 \tag{7.}$$

If we construct the Lagrangian function as:

$$L(\varphi_t, \mu_t) = E(\psi_t^2) - \mu_t\left(\sum_{y=1}^{M+1}\phi_{y,t} - H_t\right) \tag{8.}$$

and we set its first derivatives equal to zero, we obtain the self-financing constraint (1.) and the following 4 equations for each future *j* included in the hedging portfolio:

$$2\sum_{k=1}^{n}\left(\left[\sum_{v=1}^{n}\omega_{j,v,t} D_v D_k E_t\left[\sum_{l=1}^{3} c_{lk} F_t^l \sum_{h=1}^{3} c_{hv} F_t^h\right] + \sigma_\varepsilon(t,D_k)^2 D_k^2 \omega_{j,k,t}\right]\left[\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t} - A_{k,t}\right]\right) = \mu_t \tag{9.}$$

The proof of the second order condition of the minimization can be obtained analogously as in Carcano (2009). For each error-adjusted hedging strategy, the optimal weights ϕ_y to be invested in each future have been calculated based on the last set of equations.

The key advantage of the error-adjustment can be identified by analyzing the two terms of equation (7.). Traditional hedging methods are nested within this equation since they simply ignore its second term. In fact, it can be easily seen that if we set the model error volatility σ_ε equal to zero, the whole equation (7.) reduces to zero when the following equation is true for each risk factor *l*:

$$\sum_{k=1}^{n}\left(A_{k,t} c_{lk} D_k - \sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t} c_{lk} D_k\right) = 0 \tag{10.}$$

The last equation summarizes the common idea behind all traditional hedging equations, which is that the sensitivity of the portfolio to be hedged to the three risk factors must be exactly replicated by the sensitivity of the hedging portfolio. However, this idea has the disadvantage of ignoring the second term of equation (7.) and particularly the exposure to the model error for each zero rate of maturity *k*, which is represented by the following expression:

$$\left(D_k A_{k,t} - \sum_{y=1}^{4} \phi_{y,t} D_k \omega_{y,k,t} \right)^2 \tag{11.}$$

The unconstrained set of equations (10.) can lead to high values for expression (11.). This weakness of traditional hedging approaches can be substantially reduced by the error-adjustment, which leads to a hedging portfolio minimizing the whole equation (7.) and not only its first term. In the following paragraphs, we explain how we estimated the PCA, DV, and KRD models and recall the specific form of equations (10.) for their traditional implementation.

In the case of the PCA model, the factors included in equation (2.) are the three principal components. The factors, factor sensitivities, and error terms have been directly obtained by the application of the PCA methodology. The weights ϕ_y to be invested in each future according to the traditional implementation of PCA must satisfy the self-financing constraint and equations (10.), which in this case take the following form:

$$\begin{aligned}
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}c_{1k}D_k &= \sum_{k=1}^{n}A_{k,t}c_{1k}D_k \\
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}c_{2k}D_k &= \sum_{k=1}^{n}A_{k,t}c_{2k}D_k \\
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}c_{3k}D_k &= \sum_{k=1}^{n}A_{k,t}c_{3k}D_k
\end{aligned} \tag{12.}$$

For the DV model, we refer to Chambers et al. (1988). The process underlying this model can be considered as a special case of the generic process (2.), where the three sensitivity parameters c_{lk} have been set equal to - respectively - 1, D_k, and D_k^2. In addition to the self-financing constraint, the traditional version of the DV model leads to the following system of hedging equations:

$$\begin{aligned}
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}D_k &= \sum_{k=1}^{n}A_{k,t}D_k \\
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}D_k^2 &= \sum_{k=1}^{n}A_{k,t}D_k^2 \\
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}D_k^3 &= \sum_{k=1}^{n}A_{k,t}D_k^3
\end{aligned} \tag{13.}$$

The linear 3-factor process of zero rate changes which is consistent with this model and minimizes the model error is represented by:

$$dR(t, D_k) \equiv F_t^1 + F_t^2 D_k + F_t^3 D_k^2 + \varepsilon(t, D_k) \tag{14.}$$

where the factors and the error terms have been estimated applying the ordinary least square technique to the changes in all considered zero rates between time t and $t+1$. It is visible that if we multiply each term of the last equation by D_K in order to estimate the sensitivity of the bond price to the zero rate change, we obtain the overall sensitivity to the factor changes on which the equations in (13.) are based.

A review of the DV methodology is given in Nawalkha et al. (2003) who propose and test a generalization of it. They found out that – for short immunization horizons like the one we are going to assume – a GDV model leading to lower exponents for D_k than in (14.) leads to better immunization. They suggest that the reason for this result might be that lower exponents are consistent with mean reverting processes leading to higher volatility for short-term rates than for long-term rates (a characteristic consistently displayed by yield curve shifts).

Particularly, they suggest a model which results in setting the three sensitivity parameters of expression (2.) equal to - respectively - $D_k^{-0.75}$, $D_k^{-0.5}$, and $D_k^{-0.25}$. This leads to the following system of hedging equations:

$$\begin{aligned}
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}D_k^{0.25} &= \sum_{k=1}^{n}A_{k,t}D_k^{0.25}\\
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}D_k^{0.5} &= \sum_{k=1}^{n}A_{k,t}D_k^{0.5}\\
\sum_{k=1}^{n}\sum_{y=1}^{4}\phi_{y,t}\omega_{y,k,t}D_k^{0.75} &= \sum_{k=1}^{n}A_{k,t}D_k^{0.75}
\end{aligned} \tag{15.}$$

Following the same reasoning described above, the linear 3-factor process of zero rate changes which is consistent with the last set of equations and minimizes the model error is represented by:

$$dR(t,D_k) \equiv \frac{F_t^1}{D_k^{0.75}} + \frac{F_t^2}{D_k^{0.5}} + \frac{F_t^3}{D_k^{0.25}} + \varepsilon(t,D_k) \tag{16.}$$

where the factors and the error terms have been estimated like in the DV model.

It should be highlighted that Nawalkha et al. (2003) tested a version of DV and GDV models including a minimization of the squared values of the weights ϕ_y.

This was motivated by the fact that the number of the hedging instruments exceeded the hedging constraints. This does not apply to our case, since we have four hedging constraints (e.g.: for the GDV model, the three constraints reported under (15.) and the self-financing constraint) and four hedging instruments (the four bond/note future contracts included in our dataset).

For the KRD model, we refer to Ho (1992). The resulting process of zero rate changes can be described as:

$$dR(t, D_k) \equiv \sum_{l=1}^{3} c_{lk}^{KRD} F_t^{\,l} + \varepsilon(t, D_k) \tag{17.}$$

where in this case the factor F^l represents the l-th key zero rate change and c_{lk} represents the sensitivity of the zero-coupon rate of maturity D_k to this change which has been defined following Nawalkha et al. (2005).

In addition to the self-financing constraint, the resulting system of hedging equations for the tradition KRD model is:

$$\begin{aligned}
\sum_{k=1}^{n} \sum_{y=1}^{4} \phi_{y,t} \omega_{y,k,t} c_{1,k}^{KRD} D_k &= \sum_{k=1}^{n} A_{k,t} c_{1,k}^{KRD} D_k \\
\sum_{k=1}^{n} \sum_{y=1}^{4} \phi_{y,t} \omega_{y,k,t} c_{2,k}^{KRD} D_k &= \sum_{k=1}^{n} A_{k,t} c_{2,k}^{KRD} D_k \\
\sum_{k=1}^{n} \sum_{y=1}^{4} \phi_{y,t} \omega_{y,k,t} c_{3,k}^{KRD} D_k &= \sum_{k=1}^{n} A_{k,t} c_{3,k}^{KRD} D_k
\end{aligned} \tag{18.}$$

Also in this case, the error terms have been estimated applying the ordinary least square technique to the changes in all considered zero rates between time *t* and *t+1*, where the 2-year, 12-year, and 22-year zero rates have been used as key rates and have been assumed to be also exposed to model errors.

2. The dataset and the testing approach

We tested the alternative hedging strategies on 144 monthly periods from December 1996 to December 2008. The portfolio to be hedged is formed by 8 US Treasury bonds and notes. We defined 8 time buckets with maturity equal to – respectively – 2, 4, 6, 8, 10, 16, 20, and 26 years. In order to select the securities included in the portfolio to be hedged, we impose three conditions: the bonds or notes must have a publicly held face value outstanding of at least 5 billion US$, the first coupon must already have been paid and the maturity date must be as close as possible to the one of the corresponding time bucket[7].

The hedging portfolio was formed by the four US T-bond and T-note future contracts with denomination of – respectively – 2, 5, 10, and 30 years. We always referred to the next expiring future contract.

For each contract and each month, we identified the cheapest-to-deliver bond following the net basis method. As pointed out by Choundhry (2006), there is no consensus about the best way to identify the CTD. The two most common methods rely either on the net basis or on the implied repo rate (IRR). In academia, the second method is the most widely used, while practitioners often argue that the net basis approach should be used since - as pointed out by Chance (1989) - it measures the actual profit and loss for a cash-and-carry trade. The cheapest-to-deliver bonds have been identified relying on the monthly baskets of deliverable bonds and conversion factors (CF) kindly provided to us by the Chicago Mercantile Exchange (CME).

We extracted all information related to US Treasury bonds and notes (both for the securities included in the portfolio to be hedged as well as for the cheapest-to-deliver bonds of the future contracts) from the CRSP database. This included both mid prices and reference data. The closing price of the future contracts has been provided by Datastream. From both databases, we only downloaded end-of-month data.

In order to estimate the sensitivity of each financial instrument to the three selected factors, we calculated the present value of each individual cash-flow. For the future contracts, this calculation was based on the cheapest-to-deliver bonds. The discount rate we used for this calculation relied on the Unsmoothed Fama-Bliss zero-coupon rates. The methodology followed for the estimation of these rates has been described in Bliss (1997). We used the same set of zero rates between May 1975 and December 1996 to estimate the parameters of all tested hedging models.

We test our hedging strategies by varying the weights invested in the 8 bonds of the portfolio to be hedged. The first 3 portfolios are identified as *bullet* portfolios, because the vast majority of the bond positions matures in the same period. For the *short bullet*, this period is within 5 years; for the *medium bullet*, it is between 8 and 16 years, and for the *long bullet* it is over 20 years. The other three portfolios replicate typical bond portfolio structures: *ladders* (evenly distributed bond maturity), *barbells* (most bonds mature either in the short term or in the long term), and *butterflies* (long positions in bonds maturing either in the short term or in the long term and short positions in bonds maturing in the medium term). The set of equations (9.) is solved at the end of each month for each hedging strategy and each of the 6 bond portfolios; the hedging portfolio for the following month is based on the resulting weights ϕ_y for each future contract.

In order to assess the quality of a certain immunization strategy, we analyze the Standard Error of Immunization (SEI), that is, the average absolute value of the *hedging error*. The hedging error is the difference between the unexpected return of the bond portfolio to be hedged and the unexpected return of the futures portfolio. Lower SEI indicates higher quality of the immunization strategy. The unexpected return of the bond portfolio is based on the excess return provided by the CRSP database for the individual bonds. For the future contracts, the actual return has been calculated in two different ways depending if the contract expired during the hedging period or not. In the case of no-expiration, the actual return has been simply calculated as the percentage change in the quoted future price. In the case of a contract expiration, the actual return has been calculated assuming an opening of the future position at the end of the previous month and a delivery of the cheapest-to-deliver bond at the end of the expiration month. The cheapest-to-deliver bond has been identified as the bond with the highest delivery volume based on the actual delivery statistics provided by the CME. The unexpected return of each future contract has been calculated as the difference between the actual return and the expected return for the long position which – as explained in 1.1 - was set equal to the sum of the time-decay expected for the carry and the linear amortization of the net basis.

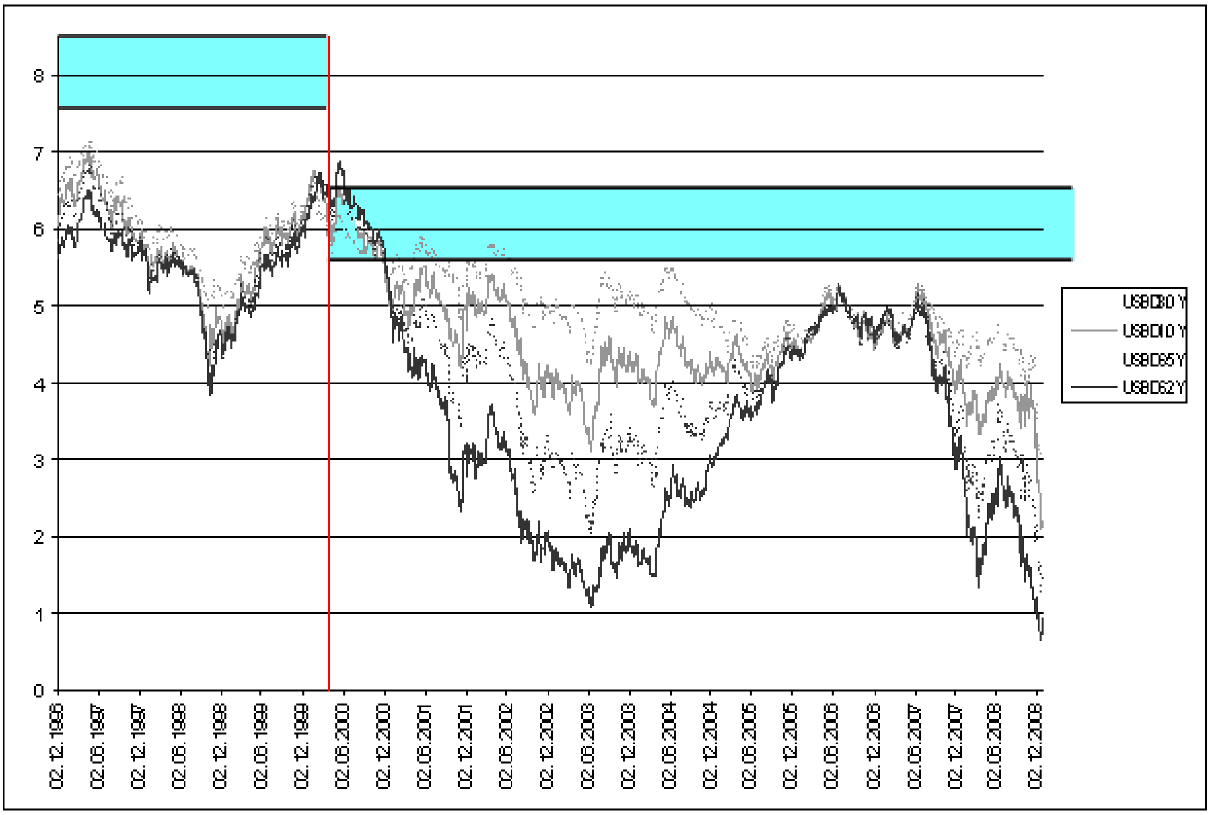
Given the dependency of different hedging strategies on the same case and time, we estimate statistical significance following an approach of matched pairs experiment. In other words, we calculated the difference between the absolute value of the hedging errors generated by two strategies on the same case and holding period. Our inference refers to the mean value of this difference. As a benchmark model, we use the error-adjusted PCA, which was expected to be the best performer.

For each hedging problem, we also estimate the square root of the average sum of the squared weights ϕ_y expressed as percentages of the bond portfolio value. This estimate is a useful proxy of the level of transaction costs implied by each hedging strategy. In fact, these costs are normally proportional to the sum of the absolute value of all long and short future positions.

Finally, we analyzed our dataset in order to assess when market yields should be considered too close to the notional coupon of the future contracts. As explained in section 1.1, we intend to base our sub-sample analysis on this assessment. This will allow us to isolate the observations for which the impact of the delivery option is likely to be tangible from the rest of the sample.

We followed Grieves et al. (2010) in defining *too close* as an absolute distance not greater than 0.5%. Figure 1 highlights the period during which market yields were within this distance from the notional coupon. It was the period starting from March 2000 (when the notional coupon was lowered from 8% to 6%) and ending in June 2004 (when the market yield of the 30-year bond briefly touched the lower limit of our range). Accordingly, we will use these two dates to limit our sub-samples.

Fig. 1. Assessing the distance between the yields of the 2-year, 5-year, 10-year and 30-year Treasury bonds and the future notional coupon (Source: Datastream)



Note: the chart represents the continuous time series of the par yields of the US Treasury bonds with maturity equal – respectively – to 2, 5, 10, and 30 years. The marked area represents a distance of +/- 0.5% from the notional coupon of the future contracts. When bond yields are within this area, the value of the embedded options is expected to be particularly relevant also for the next-expiring future contracts.

3. **The results**

After estimating the parameters of the tested models between May 1975 and December 1996, we analyzed the size of the model errors on the same sample. As expected, all models explain a high proportion of the variance of interest rate changes, but this proportion is slightly higher for the PCA model (circa 95%) than for the DV model (circa 93%) and the KRD model (circa 92%). The main reason for the worse performance of the latter models is their inability to correctly account for the term structure of volatility (i.e.: the higher volatility of short-term rates). The GDV model shares this strength of the PCA model and leads to similar model errors.

The results of the strategies based on the PCA, DV, and KRD models are reported in Exhibits 1 to 3. For the sake of brevity, we have not reported the results provided by the GDV model which led to significantly worse hedging than the simpler DV model. This outcome is not consistent with the abovementioned findings of Nawalkha et al. (2003). We believe that the reason for this inconsistency is the relatively high sensitivity of the futures to changes in the zero rate of maturity *s* (the expiration date of the contract), which affects the full cost-of-carry[8]. This leads to a high exposure to model errors which overwhelms the relatively good quality of the underlying process of interest rate changes.

Exhibit 1 shows a comparison of the results of the three methods in their traditional forms. As expected, these results are puzzling. Even though we know the interest rate processes underlying these models to be of a comparable quality, their hedging performance is quite different: for the PCA model, the average hedging error represents less than 10% of the unexpected return volatility we intended to hedge, whereas this ratio is substantially higher for the KRD and the DV models. Following Carcano (2009), we believe this outcome to be due to a widely different exposure to model errors; the substantially lower Squared Weights statistic of the PCA relatively to the other two models suggests that this might indeed be the case. However, this observation raises a further question: why is the Squared Weights statistic of similar models estimated on the same data set so different? We will come back to this question at the end of this section.

Exhibit 1

Testing the most common hedging techniques in their traditional form. (December 1996 – December 2008, 144 monthly observations)

Case Description	Portfolio to be hedged	Traditional PCA			Traditional KRD			Traditional DV		
	Standard deviation of unexpected return (σ_V)	SEI over σ_V (1)	(2)	Squared weights(3)	SEI over σ_V (1)	(2)	Squared weights	SEI over σ_V (1)	(2)	Squared weights
Short Bullet	1.44%	9.12%	**	1.11	10.85%	***	1.18	12.09%	***	1.38
Medium Bullet	1.84%	7.85%	**	1.10	9.84%	***	1.48	11.04%	***	1.68
Long Bullet	2.18%	8.47%	***	1.26	18.69%	***	4.61	20.26%	***	4.85
Ladder	1.82%	8.59%	***	1.14	15.36%	***	2.92	16.88%	***	3.16
Barbell	1.81%	9.66%	***	1.24	21.04%	***	4.41	22.69%	***	4.58
Butterfly	1.81%	11.58%	***	1.47	29.27%	***	6.57	30.94%	***	6.62
Average	1.81%	9.18%		1.22	17.75%		3.53	19.24%		3.71

Note: (1) SEI (Standard Error of Immunization) represents the average absolute value of the hedging error; the hedging error is the difference between the unexpected return of the bond portfolio to be hedged and the unexpected return of the future portfolio. (2) Statistical significance is related to the average difference between the absolute value of the hedging errors for the tested strategy and the error-adjusted PCA: "*" indicates 10% significance, "**" indicates 5% significance, and "***" indicates 1% significance. (3) It indicates the square root of the average sum of the squared weights ϕ_y expressed as percentages of the value of the bond portfolio.

Our second step is to analyze the performance of the three methods in their corresponding error-adjusted versions. In Exhibit 2, we compare these results. They support our initial hypothesis that controlling the exposure to the model errors significantly improves the hedging quality. In particular, the error adjustment leads to an average reduction in the SEI of 17% for the PCA model, whereas this reduction equals 39% for the KRD and 53% for the DV models. This reduction is statistically significant for each model and each of the 6 tested bond portfolios. The reduction in the squared weights statistics obtained for the error-adjusted models is also very substantial, thus highlighting a second important advantage of this adjustment: the cut in transaction costs. If the costs of setting up the hedging strategy are indeed proportional to our squared weights statistics, then the reduction in these costs would be around 50% for the PCA and 80% for the other two models.

Also, the differences among the hedging performances of the three models are significantly lower in Exhibit 2 than in Exhibit 1. Since the quality of the underlying interest rate processes is comparable, this makes sense and confirms that the results reported in Exhibit 1 were influenced by widely different model error exposures. However, the slightly superior quality of the process of interest rate changes underlying the PCA model systematically leads to better hedging: on each of the 6 tested bond portfolios, the PCA model outperforms both alternative models and this outperformance is always statistically significant.

Exhibit 2

Testing the most common hedging techniques in their error-adjusted form. (December 1996 – December 2008, 144 monthly observations)

Case Description	Error-Adjusted PCA			Error-Adjusted KRD			Error-Adjusted DV		
	SEI over σ_V (1)	(2)	Squared weights(3)	SEI over σ_V (1)	(2)	Squared weights	SEI over σ_V (1)	(2)	Squared weights
Short Bullet	8.00%		0.53	9.01%	***	0.54	8.96%	**	0.54
Medium Bullet	7.14%		0.62	7.99%	**	0.62	7.83%	*	0.59
Long Bullet	6.99%		0.75	10.44%	***	0.91	7.73%	***	0.73
Ladder	7.25%		0.60	9.69%	***	0.67	7.62%	*	0.59
Barbell	7.74%		0.64	12.08%	***	0.78	8.67%	***	0.60
Butterfly	8.92%		0.80	15.97%	***	0.98	13.48%	***	0.62
Average	7.64%		0.66	10.90%		0.75	9.00%		0.61

Note: (1) SEI (Standard Error of Immunization) represents the average absolute value of the hedging error. The hedging error is the difference between the unexpected return of the bond portfolio to be hedged and the unexpected return of the future portfolio. (2) Statistical significance is related to the average difference between the absolute value of the hedging errors for the tested strategy and the error-adjusted PCA: "*" indicates 10% significance, "**" indicates 5% significance, and "***" indicates 1% significance. (3) It indicates the square root of the average sum of the squared weights ϕ_y expressed as percentages of the value of the bond portfolio.

In Exhibit 3, we report the hedging quality statistics we would have obtained if the performance of the hedging portfolio would have been calculated on the initial cheapest-to-deliver bonds, instead of on the future contracts. The purpose of this exhibit is to provide us with an attribution of the hedging error. In fact, the difference between the SEI reported in Exhibit 1 (Exhibit 2) and the one reported in Exhibit 3 for the traditional (error-adjusted) form of the tested models is an estimate of the impact on the hedging errors of elements which are specific to the future contracts and do not influence the prices of the cheapest-to-deliver bonds.

Exhibit 3

Calculating the performance of hedging models based on the initial cheapest-to-deliver bonds (December 1996 – December 2008, 144 monthly observations).

Case Description	PCA		KRD		DV	
	Traditional(1)	Error-Adjusted(1)	Traditional(1)	Error-Adjusted(1)	Traditional(1)	Error-Adjusted(1)
Short Bullet	7.55%	4.50%	7.44%	5.40%	9.61%	5.59%
Medium Bullet	6.29%	4.57%	7.05%	5.64%	8.58%	5.25%
Long Bullet	6.73%	4.74%	16.69%	7.67%	18.95%	4.89%
Ladder	6.85%	4.47%	13.06%	6.54%	15.36%	4.35%
Barbell	7.79%	5.01%	19.16%	8.67%	21.65%	5.53%
Butterfly	9.50%	6.64%	27.71%	12.67%	30.58%	8.61%
Average	7.42%	5.00%	15.47%	7.83%	17.75%	5.68%

Note: (1) SEI (Standard Error of Immunization) as a percentage of the standard deviation of the unexpected return from the bond portfolio to be hedged. SEI represents the average absolute value of the hedging error. The hedging error is the difference between the unexpected return of the bond portfolio to be hedged and the unexpected return of the future portfolio. The latter unexpected return has been calculated based on the cheapest-to-deliver bonds identified at the beginning of the hedging month and not – like in the previous exhibits - on the quoted future prices.

This comparison highlights that the future-specific hedging error varies for all tested strategies between 1.5% and 3.3% of the total risk we intended to hedge. For most practitioners, such numbers are likely to give support to the standard practice of hedging yield curve risk through these future contracts.

However, if a very high accuracy of the hedging strategy is required, these future-specific discrepancies deserve further attention. In fact, when we calculate the ratios between the hedging errors reported in Exhibit 2 and in Exhibit 3, we see that the use of futures leads to an increase in hedging errors of 40-60% relatively to error-adjusted strategies based on bonds.

Accordingly, it makes sense to analyze the possible sources of these future-specific hedging errors more in detail. A first possible source is represented by the abovementioned data quality issues, which are specific to our testing dataset and would not affect a real-life hedging problem. Accordingly, this source of hedging errors makes our strategies based on bond futures looking worse than they really are. Unfortunately, it is impossible to estimate how much of the future-specific hedging error is due to data quality issues.

Further sources of future-specific hedging errors are represented by a temporary mispricing between the spot and future bond markets and by actual changes in the cheapest-to-deliver bonds and/or in the value of the embedded options. The latter changes explain why "*…the future price not only does not behave like any one bond or note, but behaves instead like a complex hybrid of the bonds and notes in the deliverable set..*" (Burghardt et al. (2005))

Our sub-sample analysis can help us to get a feeling for the relative importance of these sources of future-specific hedging errors. Its results are summarized within Exhibit 4.

Exhibit 4

Alternative hedging models based on bond futures: sub-sample analysis.

Case Description	PCA		KRD		DV	
	Traditional(1)	Error-Adjusted(1)	Traditional(1)	Error-Adjusted(1)	Traditional(1)	Error-Adjusted(1)
Sub-Sample 1	**Dec. 1996 to Feb. 2000 (Average Net Basis: -3.73%)**					
Short Bullet	14.49%	12.95%	16.66%	15.11%	19.33%	13.28%
Medium Bullet	11.93%	10.85%	15.45%	13.77%	18.26%	11.59%
Long Bullet	12.66%	11.14%	33.76%	18.04%	40.21%	12.42%
Ladder	13.20%	11.70%	26.50%	16.95%	31.61%	12.26%
Barbell	14.50%	12.97%	37.45%	19.89%	44.15%	14.22%
Butterfly	16.24%	14.70%	51.71%	24.30%	61.15%	17.21%
Average	13.76%	12.31%	30.62%	18.05%	36.24%	13.44%
Sub-Sample 2	**March 2000 to May 2004 (Average Net Basis: 3.23%)**					
Short Bullet	7.07%	6.70%	8.42%	6.50%	9.13%	7.30%
Medium Bullet	6.49%	6.09%	8.00%	5.82%	8.35%	6.41%
Long Bullet	6.63%	5.56%	7.68%	6.30%	9.26%	6.86%
Ladder	6.64%	5.90%	7.88%	6.14%	9.06%	6.53%
Barbell	7.12%	6.05%	8.50%	7.55%	10.51%	7.53%
Butterfly	8.40%	6.52%	10.18%	10.01%	12.81%	15.33%
Average	7.03%	6.10%	8.41%	7.02%	9.83%	8.26%
Sub-Sample 3	**June 2004 to Dec. 2008 (Average Net Basis: 2.23%)**					
Short Bullet	8.03%	6.35%	9.90%	7.98%	10.84%	8.11%
Medium Bullet	6.80%	5.99%	8.38%	6.77%	9.50%	7.07%
Long Bullet	7.91%	6.05%	21.06%	10.22%	19.81%	5.91%
Ladder	7.88%	6.00%	16.49%	9.07%	16.23%	6.01%
Barbell	9.41%	6.44%	24.17%	12.12%	22.54%	6.64%
Butterfly	12.04%	8.04%	35.07%	17.07%	31.55%	9.70%
Average	8.71%	6.48%	19.75%	10.68%	18.88%	7.17%

Note: (1) SEI (Standard Error of Immunization) as a percentage of the standard deviation of the unexpected return from the bond portfolio to be hedged. SEI represents the average absolute value of the hedging error. The hedging error is the difference between the unexpected return of the bond portfolio to be hedged and the unexpected return of the future portfolio.

For each sub-sample, we reported in Exhibit 4 the average value of the net basis across all future contracts and test cases. It is visible that the quality of the hedging strategies relatively to the level of risk we intended to hedge was significantly poorer in the first sub-sample. The negative average net basis suggests that a combination of data quality issues and/or mispricing might be responsible for this observation. On the contrary, we notice that the relative quality of the hedging strategies in the second sub-sample is very good: the highest average level of net basis supports our expectation of a higher value of the delivery options for this sub-sample. We also observed that changes in the cheapest-to-deliver bonds during the hedging month have been almost 5 times more frequent in this sub-sample than in the other sub-samples. Nevertheless, this does not seem to have negatively affected the hedging quality. As a result, we suspect that data quality issues and/or mispricing are responsible for the largest part of future-specific hedging errors.

Moreover, our sub-sample analysis highlights the robustness of the error adjustment and the superiority of the error-adjusted PCA model: only in one case (the DV hedging of the butterfly portfolio in the second sub-sample) the traditional model performs better than the corresponding error-adjusted one, whereas the error-adjusted PCA consistently outperforms the alternative models on average and on the vast majority of the portfolios.

Finally, Exhibit 4 highlights a problem related to the traditional implementation of the three hedging strategies we already remarked on Exhibit 1: their quality appears to be extremely volatile across different models and time periods. We suspected this to be due to an instability of the solutions to their hedging equations. Exhibit 5 provides clear support to this hypothesis for the PCA model.

Exhibit 5

Sensitivity of PCA hedging models to small changes in the coefficients.

Mean Weights Sensitivity						
	c_1		c_2		c_3	
Case Description	Traditional	Error-Adjusted	Traditional	Error-Adjusted	Traditional	Error-Adjusted
Short Bullet	2.0	0.2	1.7	0.1	1.4	0.1
Medium Bullet	1.8	0.3	1.5	0.2	1.2	0.1
Long Bullet	2.2	0.8	1.9	0.3	1.5	0.2
Ladder	2.1	0.4	1.8	0.2	1.4	0.2
Barbell	2.4	0.5	2.0	0.3	1.6	0.4
Butterfly	2.8	0.6	2.4	0.3	2.3	0.3
Average	2.2	0.5	1.9	0.2	1.6	0.2
Volatility Weights Sensitivity						
	c_1		c_2		c_3	
Case Description	Traditional	Error-Adjusted	Traditional	Error-Adjusted	Traditional	Error-Adjusted
Short Bullet	4.8	0.1	4.9	0.1	4.9	0.0
Medium Bullet	4.3	0.1	4.5	0.1	4.5	0.1
Long Bullet	4.6	1.1	4.7	0.2	4.8	0.1
Ladder	4.6	0.1	4.7	0.1	4.8	0.1
Barbell	5.0	0.2	5.1	0.1	5.2	0.8
Butterfly	5.7	0.2	5.9	0.2	6.2	0.1
Average	4.8	0.3	5.0	0.1	5.1	0.2

Note: Mean Weights Sensitivity indicates how much the weighting of the four future contracts changes on average for a unit change in each of the three PCA coefficients – c_1, c_2, and c_3. Volatility Weights Sensitivity indicates the standard deviation of these average changes in the weighting of the four future contracts across our test sample. Please, refer to the main text for a more formal definition of these variables.

In order to estimate the stability of the optimal hedging weights for the PCA model, we shifted the full term structure of the PCA coefficients c_l in a parallel way and recalculated the optimal weights for each of the four future contracts. On this basis, we estimated the absolute value of the change in the weights for each contract - $\partial\phi_y$ - and the average of these values for the four contracts. We defined the mean value of this average across our data set as "Mean Weights Sensitivity":

$$\text{Mean Weights Sensitivity} \equiv \frac{1}{T}\sum_{t=1}^{T}\frac{\frac{1}{4}\sum_{y=1}^{4}\left|\partial\phi_{y,t}\right|}{\partial c_{lk}}$$

whereas the standard deviation of this average across our data set was defined as "Volatility Weights Sensitivity":

$$\text{Volatility Weights Sensitivity} \equiv \sqrt{\frac{1}{T}\sum_{\tau=1}^{T}\left[\frac{\frac{1}{4}\sum_{y=1}^{4}\left|\partial\phi_{y,\tau}\right|}{\partial c_{lk}}-\frac{1}{T}\sum_{t=1}^{T}\frac{\frac{1}{4}\sum_{y=1}^{4}\left|\partial\phi_{y,t}\right|}{\partial c_{lk}}\right]^2}$$

The stability statistics reported in Exhibit 5 have been estimated by shifting the term structure of the PCA coefficients up and down by 0.02 and taking the average value of the two resulting statistics.

The results reported in Exhibit 5 provide us with an overwhelming evidence of the higher stability of the error-adjusted model with respect to the traditional model. The mean sensitivity across the six bond portfolios for the traditional strategy is between 4.8 (for c_1) and 8.2 (for c_2) times larger than for the error-adjusted strategy. The volatility of the sensitivity across the six bond portfolios for the traditional strategy is even between 15 (for c_1) and 39 (for c_2) times larger than for the error-adjusted strategy, thus highlighting that in some test cases the changes in the optimal weightings of the traditional strategy have been extremely large.

Stability could very well be the greatest advantage of the error-adjusted hedging strategies. In fact, unstable values of the optimal weights imply higher transaction costs for resetting a traditional strategy and inconsistent quality across different models and time periods, the evidence of which emerges very clearly from the sub-sample analysis reported in Exhibit 4.

4. **Conclusions**

Our results highlight that traditional implementations of the most common models used for hedging yield curve risk often lead to high exposure to model errors and to sizable transaction costs, thus lowering the hedging efficiency. The exposure to model errors generated by these implementations varies quite randomly across hedging problems, so that the resulting hedging quality is rather heterogeneous.

As a consequence, including some mechanisms to control the exposure to model errors is of paramount importance for a sound implementation of these models. We presented the results of explicitly accounting for the variance of the model errors displayed by each zero rate. We found out that the reduction in both the hedging errors and the transaction costs is substantial: the errors are reduced on average by 17% for the PCA model, by 39% for the KRD model and by 53% for the DV model. If the costs of setting up the hedging strategy are proportional to our squared weights statistics, then the reduction in these costs would be around 50% for the PCA and 80% for the other two models.

What is perhaps more important is that the error adjustment makes the optimal weights of the hedging strategies far more stable: on average, traditional hedging models are between 5 and 8 times more sensitive to changes in the coefficients than error-adjusted models. This leads the latter models to deliver a much more homogeneous hedging quality across different time periods and bond portfolios.

Nevertheless, we do find that the error-adjusted PCA model systematically outperforms all alternative models. To the best of our knowledge, this result is new. We attribute it to the better quality of the interest rate process underlying the PCA model, which explains the largest possible part of the variance of yield curve shifts based on three orthogonal factors.

Finally, our study shows that bond futures can effectively be used to hedge the yield curve risk of a bond portfolio. When error-adjusted models are applied, only 7.5%-10% of the risk to be hedged is left as a hedging error (gross of the effect of the 3-hour difference between spot and future end-of-day prices). This is still circa 3% more than what we would obtain by using bonds to hedge other bonds. However, futures present other advantages, such as strongly reduced need of cash, higher liquidity, and lower transaction costs.

All abovementioned results have been found to be robust to sub-sample analysis and to 6 different structures of bond portfolios. Our sub-sample analysis seems to suggest that future-specific errors are more due to data quality issues and/or mispricing than to issues related to the value of the embedded options and/or changes in the CTD bond.

Acknowledgements

We are grateful to Robert R. Bliss for having allowed us to use his yield curve estimates and to Ray Jireh and Daniel Grombacher from the CME for having provided us with the relevant data underlying the bond future contracts. We would also like to thank Giovanni Barone-Adesi and participants to the Interest Rate Risk Modeling conference in London, April 2010, for helpful comments. Finally, the constructive criticism of an anonymous referee was essential to significantly improve the first draft of this paper. All remaining errors or omissions should only be charged to the authors.

References

Bierwag, G.O., Kaufman, G.G., Latta, C.M., 1987. Bond portfolio immunization: Test of maturity, one- and two-factor duration matching strategies. Financial Review May, 203-219.

Bierwag, G.O., 1987. Duration analysis: Managing interest rate risk. Ballinger: Cambridge.

Bliss, R.R., 1997. Testing term structure estimation methods. Advances in Futures and Options Research 9, 197-231.

Burghardt, G.D., Belton, T.M., Lane, M., Papa, J., 2005. The Treasury Bond Basis. Library of investment and finance. McGraw-Hill.

Carcano, N., 2009. Yield curve risk management: adjusting principal component analysis for model errors. Journal of Risk 12, n.1, 3-16.

Carcano, N., Foresi, S., 1997. Hedging against interest rate risk: reconsidering volatility-adjustment immunization. Journal of Banking and Finance 21, 127-143.

Chambers, D. R., Carleton, W. T., McEnally, R. M., 1988. Immunizing Default-Free Bond Portfolios with a Duration Vector. Journal of Financial and Quantitative Analysis 23, 89-104.

Chance, D. M., 1989. An introduction to Derivatives. Dryden: Virginia.

Choudhry, M., 2006. The Futures Bond Basis, John Wiley and Sons, LTD. West Sussex: England.

Fisher, L., Weil, R.L., 1971. Coping with the risk of interest rate fluctuations: Returns to bondholders from naive and optimal strategies. Journal of Business 4, 408-431 .

Fleming, J., Whaley, R.E., 1994. The value of wildcard options. Journal of Finance 49, 215–236.

Fong, H.G., Fabozzi, F.J., 1985. Derivation of risk immunization measures. Fixed Income Portfolio Management. Dow Jones-Irwin, Homewood, IL, 291-294.

Grieves, R., Marcus, A., 2005. Delivery options and Treasury bond futures hedge ratios. Journal of Derivatives 13, 70−76.

Grieves, R., Marcus, A., Woodhams, A., 2010. Delivery options and convexity in Treasury bond and note futures. Review of Financial Economics 19, 1–7.

Henrard, M., 2006. Bond Futures and Their Options: More than the Cheapest-to-Deliver; Quality Option and Margining. Journal of Fixed Income 2, 62-75.

Ho, T.S.Y., 1992. Key rate durations: Measures of interest rate risks. Journal of Fixed Income 2, 29–44.

Hodges, S., Parekh N., 2006, Term-structure slope risk: convexity revisited. Journal of Fixed Income 3, 54-60.

Klotz, R.G., 1985. Convexity of fixed-income securities. Salomon Brothers, New York.

Litterman, R., Scheinkman, J., 1988. Common factors affecting bond returns. Financial Strategies Group Publications, September Goldman, Sachs & Co, New York.

Macaulay, F.R., 1938. The movements of interest rates, bond yields and stock prices in the United States since 1856. National Bureau of Economic Research, New York.

Martellini, L., Priaulet, P. 2001. *Fixed-income securities: dynamic methods for interest rate risk pricing and hedging*. John Wiley & Sons Ltd, Baffins Lane, Chichester: England.

Nawalkha, S.K., Soto, G.M., Beliaeva, N. A., 2005. Interest Rate Risk Modeling. John Wiley & Sons, Inc. Hoboken, New Jersey.

Nawalkha, S.K., Soto, G.M., Zhang, J., 2003. Generalized M-Vector Models for Hedging Interest Rate Risk. Journal of Banking & Finance 27, 1581–1604.

Nawalka, S.K., Chambers, D.R., 1997. The M-Vector model: Derivation and testing of extensions to M-square. Journal of Portfolio Management 23, 92-98.

Kuberek, R.C., Norman G.P., 1983. Hedging Corporate Debt with U.S. Treasury Bond Futures. Journal of Futures Markets 4, 345-353.

Reitano, R.R., 1996. Non-parallel Yield Curve Shifts and Stochastic Immunization. Journal of Portfolio Management, Winter, pp. 71-78.

Rendleman, R.J., 2004. Delivery Options in the Pricing and Hedging of Treasury Bond and Note Futures. Journal of Fixed Income 2, pp. 20-31.

Endnotes

[1] The original formulation of duration relied on flat yield curves, but this restriction was overcome thanks to the formulation proposed by Fisher and Weil (1971). For an extensive review of how the concept of duration was developed during the last century, see Bierwag (1987).

[2] Nawalkha et al. (2005) affirm that the DV model must be considered more robust and suitable for hedging purposes when time series of interest rate changes are non-stationary, since in this case the estimates of PCA models are highly instable. They also highlight that the KRD model leads to an arbitrary selection of the number and maturity of the key rates and to implausible shapes for the yield curve shifts.

[3] Given the short time horizon of our hedging strategies, the impact of time decay (which would lead to move down the yield curve in order to identify the expected yield at the end of the hedging period) can be neglected.

[4] The first time option consists in the possibility for the short position to deliver at any time during the expiration month (generally speaking, early delivery is preferable if the cost of financing exceeds the CTD coupon and vice versa). The second time option – the so-called *end-of-the-month option* - consists in the possibility for the short position to deliver during the final business days of the deliverable month after the invoice price has been locked in. The third time option is the so-called *wild-card option*. It consists in the possibility for the short position to lock in the invoice price at 3 pm during the delivery month and make the delivery if the spot price falls below the established invoice price between 3 pm and 5 pm.

[5] In theory, the sensitivity of the value of the embedded options to interest rate changes could be calculated analytically or by simulations relying on a pricing model for these options. However, such a calculation is challenging. Past attempts to simplify it have been found to be unreliable: for example, Grieves and Marcus (2005) assumed that the embedded quality option can be represented as a switching option between only two bonds. Latest research has shown that this simplification is too crude to accurately describe future price sensitivity (see, for example, Henrard (2006) and Grieves et al. (2010)). As a result, numerical procedures based on arbitrage-free term structure models are recommended when an accurate evaluation of the quality option is needed. For 3-factor models like the ones we are testing, these procedures are not trivial and would probably lead our results to be highly dependent on the specific modeling assumptions.

[6] For the PCA model, this assumption is fulfilled by construction. Considering the way how we estimated the error terms $\varepsilon(t, D_K)$ for the other models, this assumption is also fulfilled by construction as far as the independency between the error term and the fitted value of the same zero rate is concerned. For models other than PCA, the independency between the error term $\varepsilon(t, D_K)$ and the fitted value of zero rates with maturity other than D_K is a simplifying assumption.

[7] The first condition ensures a good level of liquidity for the considered securities, while the second one allows us to avoid the complexity linked to the potential irregularity of the first coupon payment and the third one leads to spread the securities as evenly as possible within the selected range of maturities.

[8] Given our testing approach, the maturity s is very short (ranging from one to three months). Now, the GDV method leads to much higher values of the sensitivity parameters c_l for very short-term rates than for any other rate. Since the portfolio to be hedged displays a much lower sensitivity to changes in very short-term rates than the future contracts, the GDV minimization procedure leads to wide long-short future positions having the goal of offsetting the high sensitivity of the zero rate of maturity s to the three risk factors.

Chapter 3

Hedging Corporate Bond Portfolios

Managing Corporate Bond Risk: New Evidence in the Light of the Sub-prime Crisis

Giovanni Barone-Adesi[a], Nicola Carcano[a,b], Hakim Dall'O[a,c]

[a] *University of Lugano, Via Buffi 13, 6900 Lugano, Switzerland*
[b] *MVC & Partners SA, Via della Posta 4, 6901 Lugano, Switzerland*
[c] *UBS SA, 8001 Zurigo, Switzerland*

This version: May 2011

ABSTRACT

We test alternative strategies for hedging a portfolio composed from BBB-rated corporate bonds. Our results highlight a change of regime. From 2000 to 2007, a hedging strategy based only on T-bond futures would have reduced the variance of the portfolio by circa 83.5%. This compares well to the maximum variance reduction of 50% reported by previous studies hedging corporate bonds through T-bond and S&P500 futures. We attribute this improvement to the use of four futures contracts with different maturity and to the consideration of modeling errors. On the contrary, in 2008 and 2009 T-bond futures would have been insufficient to successfully hedge our bond portfolio. The use of the 5-year CDX contract would have only marginally improved the quality of hedging. We attribute the disappointing hedging performance of CDX to counterparty risk and show that credit derivatives free of default risk could have led to a variance reduction over 64% even during 2008 and 2009.

Most institutional and private investors have to face the issue of managing portfolios of corporate bonds and their risks. Empirical tests of the techniques available for managing these risks have not been numerous so far; relatively more work has been done on government bonds than on corporate bonds. The sub-prime crisis of 2008 offered us at the same time a strong motivation to improve these risk management techniques and a new set of data on which these techniques can be tested. These data are particularly interesting because they relate to exceptional market conditions and represent a realistic stress-scenario. Starting from this outset, our research had the goal of extending to investment-grade corporate bonds the most promising risk management techniques developed for government bonds and to test their application in the years preceding and following the sub-prime crisis based on a range of different financial instruments. Since real-life bond portfolios normally display a high level of issuer diversification, we will focus our analysis on systematic, rather than on idiosyncratic risk.

Hedging corporate bond portfolios implies hedging two main sources of risk: interest rate risk and credit risk[1]. The first source of risk is related to the dynamics of the risk-free term structure of interest rates, whereas the second source is related to the dynamics of the credit spreads implicit in the individual bond prices and to the risk of a

[1] Depending on their characteristics, corporate bonds can be exposed to a number of other risks, of which liquidity is often the most serious one. However, given our focus on large-size issues of plain-vanilla investment-grade bonds, we assumed the role of these additional risk factors to be subordinated to interest rate and credit risk. Our results confirmed this assumption.

possible default. In the next paragraphs, we will summarize some key findings of previous research work performed around these two sources of risk.

Different techniques to hedge the risk-free term structure of interest rates have been developed over the past forty years. The techniques most commonly used in practice rely on one of the following approaches: principal component analysis (PCA), duration vector (DV), or key rate duration (KRD). Ample literature exists on each of these approaches. Selected reference papers presenting and motivating these techniques are the ones by Litterman and Scheinkman (1988) for PCA, Nawalka et al. (2003) for DV, and Ho (1992) for KRD. Nawalka et al. (2005) provide an up-to-date and accurate review of all three approaches. Carcano (2009) as well as Carcano and Dall'O (2011) tested a generalized version of these three approaches taking into account modeling errors; they show that this generalization leads to important advantages both in terms of hedging quality and of transaction costs, while allowing a clear ranking of the three approaches: the 3-factor PCA model consistently outperforms the other two approaches. Accordingly, we based our analysis on PCA relying on at least three principal components.

The focus of this paper is on short-term immunization strategies for investment-grade bonds. Since the risk of a default in the next few weeks is negligible for an investment-grade bond, we can focus our discussion of the second main source of risk on the dynamics of credit spreads. For hedging purposes, the key questions related to this risk are: how large is its systematic – as opposed to the idiosyncratic – component and how many factors are important to explain the dynamics of this component? These questions have motivated extensive efforts both of academicians and practitioners, but no consensus has been reached on their answers.

With respect to the first question, a group of scholars – including Elton et al. (2001) as well as Collin-Dufresne et al. (2001) - supported the view that a wide proportion of the credit spread can be explained by systematic factors. However, Longstaff et al. (2005) found out that the component of spreads which is not due to expected default losses is strongly related to measures of bond-specific illiquidity such as the bid-ask spread and the outstanding principal amount. Also, a more recent analysis conducted by Ahn et al. (2009) relying on canonical correlations concludes that the spread is driven for circa 50% by elements related to the individual corporate bonds. Opinions are even more diversified with respect to the second question, that is, the systematic factors underlying the dynamics of credit spreads. Some of the earlier studies – like Bodie et al. (1993) and Cumby and Evans (1995) – concluded that this spread has been mainly driven by two factors: expected default loss and tax premium. Later on, Elton et al. (2001) estimated that expected default loss and tax premium can explain 53.9% of total credit spread. They found out that using traditional Fama-French factors, 85% of the spread that is not accounted for by taxes and expected default can be explained as a reward for bearing systematic risk. Since expected default loss and tax premium are relatively static, this risk premium is responsible for most of the dynamics of credit spreads. At the same time, Collin-Dufresne et al. (2001) found out that credit spread is explained for 25% by expected default and recovery rate, while the remaining 75% is explained by a single factor of their PCA. This factor did not seem to be clearly related to variables traditionally used as proxies for systematic risk (spot rate, slope of the yield curve, leverage, volatility, probability or magnitude of a downward jump in firm value, and business climate). Collin-Dufresne et al. (2001) concluded that this factor could be somehow linked to liquidity and that the non-default component of spreads is weakly related to the tax component. More recently, Longstaff et al. (2005) found out that expected default loss represents the majority of corporate spread and that the non-default component is time-varying, mean-reverting and does not seem to be due to taxes. Even though performing a formalized test of these opposing theses is beyond the scope of this paper, we will comment on the evidence with respect to these questions which seems to emerge from our results.

Most empirical tests of strategies to hedge corporate bond portfolios relied either only on T-bond futures or on T-bond futures and S&P500 futures. Grieves (1986) explained the rational for hedging a portfolio of corporate bonds using T-bond futures and S&P500 futures on a data sample of industrial bonds[2]. He shows that the variance of a portfolio of industrials corporate bonds rated as *Baa* by Moody's can be reduced by circa 38% using only T-bond futures, whereas this reduction increases to 51% if we also use the S&P500 futures. Marcus and Ors (1996) found out that returns of lower-rated bonds are more

[2] Grieves (1986) decided not to focus on bonds issued by utility companies because return on equity, revenues and risk of default of these companies are strongly influenced by regulators. A similar approach has been more recently suggested by Eom et al. (2004) and has been followed for our own empirical tests.

related to the equity market than those of higher-rated bonds and that this relationship is much stronger during periods of low consumer confidence. Accordingly, the calculation of a constant hedge ratio across the full business cycle leads to almost no variance reduction. The inconsistency with the findings of Grieves (1986) is due to the significant difference in the analyzed time periods: 1982 – 1994 for Marcus and Ors (1996), 1982 – 1985 for Grieves (1986). However, if a Consumer Confidence Index below its historic mean is used to identify periods of pessimism and if the strategy including the S&P500 futures is calibrated and used only during these periods, the variance reduction for portfolios of Baa-rated bonds increases to 49%. These results highlight that the S&P500 futures can – in combination with T-bond futures – lead to a significant variance reduction. However, Ioannides and Skinner (1999) showed that the S&P500 futures alone cannot lead to variance reductions higher than 23% for BBB-rated bonds, whereas T-bond futures alone can lead to reductions up to 38.6% for long-term BBB-rated bonds.

Over the last years, credit derivatives – and in particular Credit Default Swaps (CDS) contracts – have been increasingly used to hedge credit spread risk. Accordingly, we considered necessary for our research to include CDS in the framework of the considered hedging strategies. To the best of our knowledge, this is the first paper to attempt an empirical test of hedging strategies combining T-bond futures and CDS contracts. Given that CDS total return indices are mostly still in development, the use of CDS for hedging purposes implies facing the complexity of converting CDS spreads into CDS returns. An additional complexity linked to the use of CDS for hedging a portfolio of corporate bonds has to do with the pricing of these contracts. As highlighted by Longstaff et al. (2005) and Blanco et al. (2005), in normal conditions CDS and bond spreads should converge to each other. Unfortunately, the recent crisis showed that this convergence is not guaranteed during exceptional market conditions: the *basis*, that is the difference between CDS spreads and bond spreads with equal maturity and underlying entity, is close to zero during regular market conditions, but became strongly negative during the sub-prime crisis. Fontana (2010) presents empirical evidence suggesting that counterparty risk and overall shrinkages in capital market liquidity appear to have been the key drivers of this phenomenon, whereas specific illiquidity in the corporate bond market – identified as an important driver of the basis by Acharya and Pedersen (2005) and by Brunnermeier (2009) – seems to have played a limited role in this occasion. The specific dynamics of the CDS basis are likely to negatively influence the quality of hedging strategies including these contracts. Accordingly, every analysis of such strategies must cope with this complexity.

The rest of this article is organized as follows: Section I presents the dataset and our methodology for calculating unexpected returns, Section II formally derives the algorithms for the tested hedging strategies, Section III presents the results obtained for each strategy and our explanations of them, whereas Section IV summarizes our conclusions.

I. Data

A. Corporate Bonds and Corporate Yield Curves

For corporate bonds, we relied on the database which was kindly made available to us by Wilshire Associates. Time series of market prices were available for a sufficient number of bonds starting from September 2000. Accordingly, we decided to work on 15-day hedging periods. While still a realistic representation of the frequency of rebalancing adopted by many practitioners, this length allowed us to rely on a number of observations – 219 – which is sufficient for adequate sub-sample analyses. In fact, we tested the alternative hedging strategies on three different sub-samples: the first sub-sample goes from September 2000 to December 2004, the second sub-sample goes from January 2005 to December 2007, whereas the last sub-sample goes from January 2008 to December 2009.

Carcano (2009) as well as Carcano and Dall'O (2011) recently analyzed the performance of alternative strategies for hedging government bonds. In order to provide empirical evidence of general relevance for investment-grade corporate bonds, we decided to analyze alternative strategies for hedging bonds rated as *BBB* by S&P. Given the relatively high correlation of credit spread changes among investment-grade bonds, successfully hedging portfolios of government bonds and BBB-rated bonds implies with very high probability successfully hedging any portfolio of investment-grade bonds.

In order to limit the relevance of idiosyncratic risk and to extend our tests over the full term structure of interest rates, we built corporate bond portfolios composed from eight BBB-rated bonds issued in US$ by different companies. Following Eom et al. (2004), we did not consider issuers belonging to the financial, utility and energy sectors. We defined eight time buckets with maturity equal to – respectively – 2, 4, 6, 8, 10, 16, 20, and 26 years. At the beginning of each year, we selected the bonds to be included in the portfolio based on three conditions: a publicly held face value outstanding of at least 100 million US$, an already-paid first coupon and a maturity as close as possible to the one of the corresponding time bucket[3]. Furthermore, we selected only fixed-rate coupon bonds which did not include any embedded options and paid a regular coupon every 6 months. Finally, we selected bonds which - during the year - were not subject to any rating upgrade or downgrade, so that their market benchmark could still be represented by one yield curve: the term structure of BBB-rated bonds. The results reported by Carcano (2009) as well as by Carcano and Dall'O (2011) highlight that hedging quality measured on *ladder* (i.e.: equally-weighted) bond portfolios is representative of average hedging quality measured on a number of different bond portfolio structures. Also, these results highlight that ranking of error-adjusted hedging strategies does not depend on individual bond weightings. Accordingly, we focused on equally-weighted portfolios of the eight bonds selected at the beginning of each year.

[3] The first condition ensures a good level of liquidity for the considered securities, while the second one allows us to avoid the complexity linked to the potential irregularity of the first coupon payment and the third one leads to spread the securities as evenly as possible within the selected range of maturities.

For all financial instruments included in our tests, we need to estimate the unexpected returns. In general, these estimates rely on the assumption that all considered interest rates are martingales. Past studies have reported that the overall effect on hedging errors of this simplifying assumption is very small (see, for example, Carcano and Foresi (1997)). Accordingly, we compute the unexpected return of a corporate bond as the return in excess of what would have been obtained if the yield-to-maturity had remained constant throughout the hedging period[4].

Finally, we need to estimate the sensitivity of each bond to the considered principal components. This estimate cannot simply be based on historical prices since the sensitivity of each bond varies over time with its time to maturity. In order to overcome this difficulty, we calculated for each bond the present value of its cash-flows at the beginning of a certain hedging period. This was done by selecting the zero-coupon rate matching the cash-flow maturity from the appropriate yield curve (i.e.: the risk-free curve for Treasury bonds and the BBB curve for corporate bonds). This present value represents the portion of a bond price whose systematic risk is related to the dynamics of the selected zero-coupon rate. For calculating the bond sensitivity to the principal components, we still need to estimate the sensitivity of the zero-coupon rates to these components, which is performed via the PCA.

Zero-coupon as well as par yield curves for BBB-rated bonds issued by industrial companies have been provided by Bloomberg. The 3-month, 1-year, 5-year, 10-year, 20-

[4] Given the short time horizon of our hedging strategies, the impact of time decay (which would lead to move down the yield curve in order to identify the expected yield at the end of the hedging period) can be neglected.

year, and 30-year zero rates have been used to estimate the PCA parameters; linear interpolations of these rates have been used to discount the bond cash-flows to their present value. The 5-year par yield has been used to estimate the theoretical credit spread forwards.

B. T-Bond Futures and US Treasury Yield Curves

The first type of hedging instruments we tested is represented by the four US T-bond and T-note futures contracts with denomination of – respectively – 2, 5, 10, and 30 years. We always referred to the next expiring futures contract. For each contract and each month, we identified the cheapest-to-deliver bond following the net basis method[5] and relying on the monthly baskets of deliverable bonds and conversion factors (CF) kindly provided to us by the Chicago Mercantile Exchange (CME). The closing price of the futures contracts has been provided by Datastream.

For these futures contracts, we calculate the actual return in two different ways depending if the contract expires during the hedging period or not. In the case of no-expiration, we calculate the actual return simply as the percentage change in the quoted futures price. In the case of a contract expiration, we calculate the actual return assuming an opening of the futures position at the end of the previous month and a delivery of the cheapest-to-deliver bond at the end of the expiration month. We identify the cheapest-to-

[5] As pointed out by Choundhry (2006), there is no consensus about the best way to identify the CTD. The two most common methods rely either on the net basis or on the implied repo rate (IRR). In academia, the second method is the most widely used, while practitioners often argue that the net basis approach should be used since - as pointed out by Chance (1989) - it measures the actual profit and loss for a cash-and-carry trade.

deliver bond as the bond with the highest delivery volume based on the actual delivery statistics provided by the CME. Following Carcano and Dall'O (2011), we calculate the unexpected return of each futures contract as the difference between the actual return and the expected return for the long position, which was set equal to the sum of the time-decay expected for the carry and the linear amortization of the net basis.

As we will explain more formally in the next section, the estimation of the sensitivity of each T-bond futures to the considered risk factors relies on the underlying cheapest-to-deliver bonds. We extracted all information related to US Treasury bonds and notes from the CRSP database. This included both mid prices and reference data. On this basis, we calculated the present value of each individual cash-flow using the US Treasury zero-coupon rates estimated through the Unsmoothed Fama-Bliss methodology described in Bliss (1997). The 3-month, 1-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year, and 30-year zero rates have been used to estimate the PCA parameters; linear interpolations of these rates have been used to discount the bond cash-flows to their present value. The 5-year US Treasury par yield provided by Bloomberg has been used to estimate the theoretical credit spread forwards.

C. S&P500 Futures

Prices for the continuous series of the S&P500 futures have been downloaded from Bloomberg. We calculate the actual return simply as the percentage change in the quoted futures price. The unexpected return is computed by subtracting the expected return from the actual return. The expected return is computed as the annualized equity risk premium multiplied by the year fraction corresponding to each hedging period. We assumed the equity risk premium to be constant and estimated it over a long time horizon (from July 1982 to April 2010) as the difference between the annualized average monthly return provided by the S&P500 Index and the average 1-month US Treasury rate.

D. North-American Investment Grade CDX Contracts

The 5-year CDX.NA.IG series from the 20th September 2004 to the 20th March 2010 (series number 3 to number 13) have been downloaded from Bloomberg. In order to be able to use the CDX index to hedge our corporate bond portfolio, we needed to convert the CDX spread (which is how the CDX is traded in the market) into a return. For this conversion, we relied on the ISDA CDX standard model, whose theoretical derivation is described in O'Kane and Turnbull (2003). According to this model, the *Cash Settlement Amount (CSA)* is the amount paid by the protection buyer to the protection seller on an assumed cash settlement date of trade-date plus 3 business days. In the CSA calculation, the premium that has accrued from accrual initial date to trade-date (where both dates are inclusive) enters as a nonnegative number with a negative sign. This leads to the fact that the carry for the protection buyer is negative, which is consistent with the insurance-like nature of a CDS. The parameter which represents the market value of the CDX position net of the effect of the premium accrual is defined as *upfront* and is calculated as the sum of the CSA and the accrued premium divided by the notional.

Since the premium for each series is reset to reflect current market conditions, we consider its accrual as a proxy for the expected return of the CDX contract. Accordingly, we estimated the unexpected return provided by the CDX contract to the protection buyer during a certain period as the change in the upfront. Therefore, the time series of unexpected returns for the 5-year CDX contract has been constructed as the change in the upfront calculated on the on-the-run series[6]. Within our data sample, no credit events took place on the on-the-run series for the 5-year CDX.NA.IG. Accordingly, we did not have to consider the additional complexity linked to the existence of multiple versions of the same series[7]. For the same reason, we did not have to include in the performance calculation any *protection leg*, that is, any payment by the protection seller as a compensation to the protection buyer for the loss related to the credit event.

II. Methodology

A. General Framework

We intend to immunize a portfolio composed from eight BBB-rated bonds denominated in USD against market risk. We define market risk as the risk of an

[6] Obviously, when the on-the-run series changes, one must pay attention to calculate the change in the upfront on the same series. Accordingly, the first hedging period considering the new on-the-run series must be the one starting on - or immediately after - the issue date of the new series.

[7] In fact, if there is a default in one of the basket components, the series is updated to a new version. For example, series number 4, version 1 was issued on the 20th of March 2005. The basket for this series consisted of 125 entities. If, during the period this series was on the run (from the 20th of March 2005 to the 20th of September 2005, when series number 5 was issued) one of the entity in the index had defaulted, the series would have become series number 4, version 2 and its basket would have consisted only of 124 components.

unexpected shift of the term structure of interest rates for BBB-rated bonds. As explained above, we assume that all interest rates are martingales; accordingly, all changes in the term structure are considered to be unexpected. Moreover, as reported among others by Hodges and Parekh (2006), the impact of monthly rate changes on the price of a zero-coupon bond can be well-approximated by its duration. Since our hedging periods normally extend over 15 days, we will follow this simplifying approach.

In order to estimate the sensitivity of a certain bond to yield curve changes, we decompose the present value of the bond into the present value of its cash-flows calculated based on the appropriate zero-coupon yields: the symbol $\omega_i(t,D_k)$ indicates the present value of bond i cash-flow with maturity D_k as a percentage of its total present value at time t. Within this context, we approximate the unexpected return provided by a certain corporate bond i from time t to time $t+\Delta t$ as:

$$\frac{P_{i,t+\Delta t}}{P_{i,t}} - E_t\left[\frac{P_{i,t+\Delta t}}{P_{i,t}}\right] \approx -\sum_{k=1}^{n} dR_{BBB}(t,D_k)D_k\omega_i(t,D_k) + \varepsilon_i(t) \tag{1}$$

where P indicates the bond dirty price, $R_{BBB}(t,D_k)$ indicates the zero-coupon market yield for BBB-rated bonds with duration D_k at time t, and $\varepsilon_i(t)$ indicates the idiosyncratic return provided by bond i from time t to time $t+\Delta t$. The summation goes from *1* to n because we assume to observe changes in n points of the term structure. Next, we define the stochastic process driving the zero-coupon yield curve for BBB-rated bonds based on the dynamics of the L risk factors F as:

$$dR_{BBB}(t, D_k,) \equiv \sum_{l=1}^{L} c_{l,BBB,k} F_t^l + \varepsilon_{BBB}(t, D_k) \tag{2}$$

where $c_{l,BBB,k}$ indicates the sensitivity of the zero-coupon yield for BBB-rated bonds with duration D_k to the l-th risk factor and $\varepsilon_{BBB}(t,D_k)$ indicates the residual modeling error (that is, the part of the dynamics of the yield curve which cannot be explained by the considered risk factors).

In order to define the unexpected return provided by the *y-th* T-bond futures contract we relied on the approach presented by Carcano and Dall'O (2011), who claim that unexpected changes in the futures basis can – for practical purposes - be considered independent from yield curve changes. Accordingly, the unexpected return can be simply represented based on the theoretical price of a bond futures *FP* excluding the value of the embedded options, like for corporate bonds:

$$\frac{FP_{y,t+\Delta t}}{FP_{y,t}} - E_t\left(\frac{FP_{y,t+\Delta t}}{FP_{y,t}}\right) \approx -\sum_{k=1}^{n} dR_{RF}(t, D_k) D_k \omega_{CTD(y),k,t} \tag{3}$$

with $R_{RF}(t,D_k)$ indicating the zero-coupon yield for risk-free bonds with duration D_k at time t and $CTD(y)$ the cheapest-to-deliver bond of the futures contract y. In this case, we do not need to take into account any idiosyncratic return because the methodology we adopted for calculating the risk-free zero-coupon rates ensures that T-bond market prices are perfectly explained by the dynamics of these rates. For T-bond futures expiring on date *s*, the term $\omega_{CTD(y)}$ is defined as follows:

$$\frac{\frac{\partial FP_t}{FP_t}}{\partial R(t,D_k)} \approx -\frac{D_k}{FP_t}\frac{e^{R(t,D_s)D_s}}{CF_{CTD}}\frac{cf_{CTD,k}}{e^{R(t,D_k)D_k}} \equiv -D_k\omega_{CTD,k,t} \tag{4}$$

for all rates maturing after the futures contract (i.e.: $k > s$) and

$$\frac{\frac{\partial FP_t}{FP_t}}{\partial R(t,D_s)} \approx \frac{D_s}{FP_t}\frac{e^{R(t,D_s)D_s}}{CF_{CTD}}\left[\sum_{k=s+1}^{n}\frac{cf_{CTD,k}}{e^{R(t,D_k)D_k}}\right] = D_s\sum_{k=s+1}^{n}\omega_{CTD,k,t} \equiv -D_s\omega_{CTD,s,t} \tag{5}$$

for the zero rate with maturity equal to the expiration of the futures contract (i.e.: the zero rate affecting the cost-of-carry). In the last two equations, *CF* indicates the Conversion Factor and $cf_{CTD,k}$ indicates the cash-flow paid by the cheapest-to-deliver bond at time *k*. Because of the definition of T-bond futures contracts, the sensitivity of the futures price to changes in zero rates maturing before the futures contract is zero, which implies: $\omega_{CTD,k,t} = 0$ for all $k < s$.

Similarly to corporate bond yields, the stochastic process driving risk-free zero-coupon yields can be represented as follows:

$$dR_{RF}(t,D_k,) \equiv \sum_{l=1}^{L} c_{l,RF,k}F_t^l + \varepsilon_{RF}(t,D_k) \tag{6}$$

where the symbols have the same meaning as in equation (2), but now refer to risk-free zero-coupon yields.

Finally, we need to define stochastic process driving the unexpected returns provided by the S&P500 futures based on the same L risk factors used for the BBB and risk-free yields:

$$\frac{S\,\&\,PFut_{t+\Delta t}}{S\,\&\,PFut_{t}} - E_t\left[\frac{S\,\&\,PFut_{t+\Delta t}}{S\,\&\,PFut_{t}}\right] \equiv \sum_{l=1}^{L} c_{l,S\&P} F_t^l + \varepsilon_{S\&P}(t) \tag{7}$$

where the symbols have the same meaning as in equation (2), but now refer to the S&P500 futures contract. In the next sub-sections, we will show how we calculated the optimal weights for each strategy starting from this set of sensitivity equations and stochastic processes.

B. Hedging through T-Bond Futures

In the case of this hedging strategy, the hedging portfolio is exclusively composed from T-bond futures. Accordingly, we set the number of the considered risk factors L also equal to four (the need for this setting will become clear as soon as we will present the resulting system of equations). Also, we assume that the dynamics of these risk factors are independent from each other; since we are going to identify these dynamics through PCA, this assumption is true by construction. Relying on equations (1) to (6), the total unexpected return ψ provided by the combination of the bond and the hedging portfolio is:

$$\psi_t \approx -\sum_{i=1}^{8} A_{i,t}\left[\sum_{k=1}^{n} D_k \omega_i(t,D_k)\left[\sum_{l=1}^{L} c_{l,BBB,k} F_t^{\,l} + \varepsilon_{BBB}(t,D_k)\right] + \varepsilon_i(t)\right] - \\ -\sum_{y=1}^{4} \phi_{y,t}\left[\sum_{k=1}^{n} D_k \omega_{CTD(y),k,t}\left[\sum_{l=1}^{L} c_{l,RF,k} F_t^{\,l} + \varepsilon_{RF}(t,D_k)\right]\right] \tag{8}$$

where A_i represents the amount invested in bond *i* and ϕ_y represents the optimal weight to be invested in the *y-th* futures contract. The latter weights shall be interpreted as the market value of the underlying CTD bonds. Now, if we assume that the modeling errors ε_{BBB} and ε_{RF} as well as the idiosyncratic bond returns ε_i are independent from each other and from the risk factors F, the partial derivatives of the variance of the unexpected return with respect to the optimal weight ϕ_y can be approximated by the following expression:

$$\frac{\partial E[\psi_t]^2}{\partial \phi_{y,t}} \approx 2\sum_{l=1}^{L} E\left[F_t^{l\,2}\right]\left[\sum_{k=1}^{n} c_{l,BBB,k} D_k \sum_{i=1}^{8} A_{i,t}\omega_i(t,D_k) + \sum_{j=1}^{4}\phi_{j,t}\left[\sum_{k=1}^{n} c_{l,RF,k} D_k \omega_{CTD(j),k,t}\right]\right]\sum_{k=1}^{n} c_{l,RF,k} D_k \omega_{CTD(y),k,t} + \\ + 2\sum_{k=1}^{n}\left[D_k \sum_{j=1}^{4}\phi_{j,t}\omega_{CTD(j),k,t}\right] E\left[\varepsilon_{RF}(t,D_k)^2\right] D_k \omega_{CTD(y),k,t} \tag{9}$$

The optimal hedging strategy is obtained when the last equation is simultaneously set equal to zero for each of the four considered T-bond futures. For simplicity, we will assume that not only the sensitivity coefficients *c*, but also the variances of the risk factors and of the modeling errors are constant over time. In order to solve this system of equations more efficiently, we express the last equation using matrices and vectors:

$$\frac{\partial E\left[\psi_t^{\,2}\right]}{\partial \boldsymbol{\varphi}_\mathbf{t}} \approx 2\,\boldsymbol{\omega}_t\,'\mathbf{c}'\Omega_F \mathbf{C}_{BBB}\mathbf{A}_{BBB,t}\,' + 2\,\boldsymbol{\omega}_t\,'\mathbf{c}'\Omega_F \mathbf{c}\boldsymbol{\omega}_t\;\boldsymbol{\varphi}_\mathbf{t}\mathbf{'} + 2\boldsymbol{\omega}_t\,'\,\Omega_\varepsilon \boldsymbol{\omega}_t \boldsymbol{\varphi}_\mathbf{t}\mathbf{'} = 0 \tag{10}$$

where we have defined the involved vectors and matrices in the following way:

$$\boldsymbol{\omega}_t \equiv \begin{pmatrix} D_1\omega_{CTD(1),1,t} & D_1\omega_{CTD(2),1,t} & \dots & D_1\omega_{CTD(4),1,t} \\ D_2\omega_{CTD(1),2,t} & D_2\omega_{CTD(2),2,t} & \dots & D_2\omega_{CTD(4),2,t} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ D_n\omega_{CTD(1),n,t} & D_n\omega_{CTD(2),n,t} & \dots & D_n\omega_{CTD(4),n,t} \end{pmatrix}$$

$$\mathbf{c} \equiv \begin{pmatrix} c_{1,RF,1} & c_{1,RF,2} & \dots & \dots & c_{1,RF,n} \\ c_{2,RF,1} & c_{2,RF,2} & \dots & \dots & c_{2,RF,n} \\ \dots & \dots & \dots & \dots & \dots \\ c_{4,RF,1} & c_{4,RF,2} & \dots & \dots & c_{4,RF,n} \end{pmatrix}$$

$$\boldsymbol{\Omega}_\mathbf{F} \equiv \begin{pmatrix} E\left[F_1^{\ 2}\right] & 0 & 0 & 0 \\ 0 & E\left[F_2^{\ 2}\right] & 0 & 0 \\ 0 & 0 & E\left[F_3^{\ 2}\right] & 0 \\ 0 & 0 & 0 & E\left[F_{4,}^{\ 2}\right] \end{pmatrix}$$

$$\mathbf{C}_{BBB} \equiv \begin{pmatrix} c_{1,BBB,1}D_1 & c_{1,BBB,2}D_2 & \dots & \dots & \dots & c_{1,BBB,n}D_n \\ c_{2,BBB,1}D_1 & c_{2,BBB,2}D_2 & \dots & \dots & \dots & c_{2,BBB,n}D_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ c_{4,BBB,1}D_1 & c_{4,BBB,2}D_2 & \dots & \dots & \dots & c_{4,BBB,n}D_n \end{pmatrix}$$

$$\mathbf{A}_{BBB,t} \equiv \left(\sum_{i=1}^{8} A_i\omega_i(t, D_1) \quad \sum_{i=1}^{8} A_i\omega_i(t, D_2) \quad \dots \quad \dots \quad \sum_{i=1}^{8} A_i\omega_i(t, D_n) \right)$$

$$\boldsymbol{\varphi}_\mathbf{t} \equiv \left(\phi_{FUT(1),t} \quad \phi_{FUT(2),t} \quad \phi_{FUT(3),t} \quad \phi_{FUT(4),t} \right)$$

$$\mathbf{\Omega}_{\varepsilon} \equiv \begin{pmatrix} E\left[\varepsilon_{RF}(D_1)^2\right] & 0 & 0 & 0 & 0 \\ 0 & E\left[\varepsilon_{RF}(D_2)^2\right] & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 0 & E\left[\varepsilon_{RF}(D_n)^2\right] \end{pmatrix}$$

On this basis, the system of equations (10) can immediately be solved obtaining[8]:

$$\boldsymbol{\varphi}_{\mathbf{t}}' = -\left[\boldsymbol{\omega}_t'\left(\mathbf{c}'\Omega_F\mathbf{c} + \Omega_\varepsilon\right)\boldsymbol{\omega}_t\right]^{-1}\boldsymbol{\omega}_t'\mathbf{c}'\Omega_F\mathbf{C}_{BBB}\mathbf{A}_{BBB,t}' \tag{11}$$

In order to implement this hedging strategy, we estimated the sensitivity coefficients *c*, the variances of the risk factors and of the modeling errors through a PCA performed jointly on the BBB zero rates listed in I.A and on the risk-free zero rates listed in I.B. Firstly, we estimate these parameters on data from 1995 to 2000 and we apply them between 2000 and 2005. Secondly, we re-estimate these parameters on data from 1995 to 2005 and we apply them between 2006 and 2009. At the beginning of each hedging period, we solve the system of equations (11) based on current values of the risk-free and BBB zero-coupon yield curves. The resulting optimal weights are applied for the hedging period and then re-calculated in the same way at the beginning of the following period. This approach ensures a rigid out-of-sample testing framework. For each period, we then calculate the hedging error as the sum of the unexpected return provided by the bond

[8] The proof of the second order condition of the minimization can be obtained analogously as in Carcano (2009).

portfolio and the unexpected return provided by the hedging portfolio. The hedging error, which can also be conceived as the unexpected return of the hedged portfolio, is the key indicator on which we will base the analysis of our results in the next section.

For each period, we also estimated the optimal weights of the traditional PCA hedging strategy simply by setting the matrix $\mathbf{\Omega}_{\boldsymbol{\varepsilon}}$ equal to a matrix of zeros. In fact, Carcano (2009) highlighted that traditional PCA hedging is a special case of error-adjusted PCA hedging when the modeling errors are zero.

C. Hedging through T-Bond Futures and S&P500 Futures

This hedging strategy implies that the hedging portfolio includes the four T-bond futures as well as the S&P500 futures. Accordingly, the equation of the overall unexpected return now becomes:

$$\begin{aligned}\psi_t \approx & -\sum_{i=1}^{8} A_{i,t}\left[\sum_{k=1}^{n} D_k \omega_i(t, D_k)\left[\sum_{l=1}^{L} c_{l,BBB,k} F_t^l + \varepsilon_{BBB}(t, D_k)\right] + \varepsilon_i(t)\right] - \\ & -\sum_{y=1}^{4} \phi_{y,t}\left[\sum_{k=1}^{n} D_k \omega_{CTD(y),k,t}\left[\sum_{l=1}^{L} c_{l,RF,k} F_t^l + \varepsilon_{RF}(t, D_k)\right]\right] + \phi_{S\&P,t}\left[\sum_{l=1}^{L} c_{l,S\&P} F_t^l + \varepsilon_{S\&P}(t)\right] \quad (12)\end{aligned}$$

We followed step-by-step the same approach described for the previous hedging strategy. Obviously, in this case the number of considered principal components increases to five. Also, the time series of the unexpected returns provided by the S&P500 futures must be added to the data set underlying the PCA described for the previous hedging strategy. Some of the matrices we defined above need to be adjusted to reflect this new framework. In particular, we now have:

$$
\boldsymbol{\omega}_t \equiv \begin{pmatrix} D_1\omega_{CTD(1),1,t} & D_1\omega_{CTD(2),1,t} & D_1\omega_{CTD(4),1,t} & 0 \\ D_2\omega_{CTD(1),2,t} & D_2\omega_{CTD(2),2,t} & D_2\omega_{CTD(4),2,t} & 0 \\ & & & \\ & & & \\ D_n\omega_{CTD(1),n,t} & D_n\omega_{CTD(2),n,t} & D_n\omega_{CTD(4),n,t} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
\mathbf{c} \equiv \begin{pmatrix} c_{1,RF,1} & c_{1,RF,2} & \cdots & \cdots & c_{1,RF,n} & c_{1,S\&P} \\ c_{2,RF,1} & c_{2,RF,2} & \cdots & \cdots & c_{2,RF,n} & c_{2,S\&P} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{5,RF,1} & c_{5,RF,2} & \cdots & \cdots & c_{5,RF,n} & c_{5,S\&P} \end{pmatrix}
$$

$$
\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}} \equiv \begin{pmatrix} E\left[\varepsilon_{RF}(D_1)^2\right] & 0 & 0 & 0 & 0 & 0 \\ 0 & E\left[\varepsilon_{RF}(D_2)^2\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & E\left[\varepsilon_{RF}(D_n)^2\right] & 0 \\ 0 & 0 & 0 & 0 & 0 & E\left[\varepsilon_{S\&P500}{}^2\right] \end{pmatrix}
$$

Moreover, the matrix $\boldsymbol{\Omega}_{\mathbf{F}}$ now becomes a (5x5) matrix in order to include the variance of the fifth principal component and the matrix $\mathbf{C}_{\mathbf{BBB}}$ becomes a (5xn) matrix in order to include the sensitivity coefficients to the fifth component. On this basis, the optimal hedging weights are still represented by equation (11).

Relying on previous research cited in the introduction as well as on our own findings, we decided to apply this hedging strategy only when the level of consumer confidence is below its historic mean. Accordingly, we estimated the coefficients for the years 2000-2005 (2005-2009) on all periods from 1995 to 2000 as well from 2005 to 2009 (2000-2005) at the beginning of which the consumer confidence was below its historic mean. We applied these coefficients to the hedging periods from 2000 to 2005 (2005 to 2009) at the beginning of which the consumer confidence was below its historic mean. For all other hedging periods, we applied the previous hedging strategy relying only on T-bond futures. The remaining steps in testing the strategy including the S&P500 futures have been performed analogously to the strategy relying only on T-bond futures.

D. *Hedging through T-Bond Futures and Credit Default Swaps*

In theory, there are two possible approaches to include Credit Default Swaps (CDS) within the hedging strategy described under sub-section II.B. The first approach relies on a one-stage optimization: the CDS should be included in alternative to the S&P500 within equation (12), its coefficients should be estimated in a joint PCA with all other coefficients and used in the identical way described in sub-section II.C. for the S&P500 futures. In practice, available historical data are still insufficient to implement this approach. There are two reasons for that. Firstly, daily CDX data are only available starting from 2005. Secondly, we will highlight in section III that the inclusion of CDS seems to improve the quality of hedging only under very special market conditions. After 2005, these conditions materialized only during 2008 and 2009. This makes a robust estimate of the PCA coefficients virtually impossible.

The second approach relies on a two-stage optimization: in the first stage, the optimal weights for the T-bond futures are calculated as in II.B. Thereafter, the hedging errors are linearly regressed on the return of the CDX. Formally, we have:

$$e_{FUT,t} = \alpha + \beta R_{CDX,t} + e_{CDX,t} \tag{13}$$

where e_{FUT} represents the hedging error produced by the strategy including only the T-bond futures, R_{CDX} represents the return to the protection buyer of the CDX contract, and e_{CDX} represents the hedging error remaining after the inclusion of the CDX in the hedging portfolio.

Since the hedging errors are expressed as percentage of the market value of the bond portfolio, the optimal notional of the CDX contracts resulting from the last regression is equal to $-\beta$ times the market value of the bond portfolio. Since we always obtained negative estimates for β, this implies that we always buy protection in order to hedge the corporate bond portfolio, which is consistent with economic intuition.

A linear regression is parsimonious in terms of parameter estimation. Basically, only two parameters (α and β) need to be estimated, whereas sixteen parameters need to be estimated for our PCA model. This makes the regression-based approach better suited to our reality of a small sample of CDX data which can be used for parameter estimation.

Finally, in order to adhere to our principle of out-of-sample testing, we created ten different data samples by starting from all the observations available for 2008 and 2009 and taking one-tenth of these observations out of each sample[9]. We estimated β on each data sample and applied this estimate to the periods which were excluded from the same data sample.

E. Hedging through T-Bond Futures and Credit Spread Forwards

The last hedging strategy we intend to test is based on T-Bond futures and on credit spread forwards. The rational for the test of this strategy is to have a benchmark for the performance of the previous strategy based on the CDX contract. Since we know from previous academic research and empirical evidence that the dynamics of CDS do not always coincide with the dynamics of bond spreads, we intend to verify the impact of these discrepancies on the hedging quality. Credit spread forwards are derivative contracts whose underlying is the bond credit spread itself. Accordingly, they represent the ideal benchmark for our analysis.

A drawback of using credit spread forwards for empirical analyses is that these instruments are rather illiquid and reliable time series of their prices are not available. However, if we assume adequate collateralization of these contracts so that counterparty risk is not an issue, their prices can be calculated based on available market data and non-

[9] We excluded one every ten subsequent observations. For example, in the first sub-sample, we excluded the first observation, the eleventh observation, the twenty-first observation and so on. In the second sub-sample, we excluded the second observation, the twelfth observation, the twenty-second observation and so on.

arbitrage conditions. Under these conditions, if we identify with $Y_{BBB}(t,D_k)$ the yearly-compounded par market yield for BBB-rated bonds with duration D_k at time t and with $Y_{RF}(t,D_k)$ the same yield for US Treasury bonds, the credit spread forward expiring on date s between these two rates - $CSF_{BBB}(t,t+s,D_k)$ – can be estimated as follows:

$$CSF_{BBB}(t,t+s,D_k)=\left[\frac{[1+Y_{BBB}(t,D_k)]^{D_k}}{[1+Y_{BBB}(t,D_s)]^{D_s}}\right]^{\frac{1}{D_k-D_s}}-\left[\frac{[1+Y_{RF}(t,D_k)]^{D_k}}{[1+Y_{RF}(t,D_s)]^{D_s}}\right]^{\frac{1}{D_k-D_s}} \tag{14}$$

which is simply the difference between the two forward rates. Of course, in the analysis of the results, we need to consider that these represent theoretical – rather than actually traded - forward spreads. However, we believe that this is a reasonable approximation for benchmarking the strategy involving CDX contracts.

Accordingly, we estimated 5-year credit spread forwards for contracts expiring at the end of our 2-week hedging period as described in the last equation. Even though historical data to perform a one-stage optimization of this strategy are available, this would represent a substantial modeling difference compared with our implementation of the strategy involving CDX contracts. Since we intend to compare the results of these two strategies, we decided to implement both of them based on a similar two-stage optimization. As a result, we estimated the following linear regression:

$$e_{FUT,t}=\alpha+\beta CSD_{BBB}(t,t+\Delta t,D_k)\left[CSF_{BBB}(t,t+\Delta t,D_k)-Y_{BBB}(t+\Delta t,D_{k-\Delta t})+Y_{RF}(t+\Delta t,D_{k-\Delta t})\right]+e_{CSF,t} \tag{15}$$

where CSD_{BBB} represents the *Credit Spread Duration* of the considered spread and the other variables have been defined similarly as in regression (13).

The last regression is based on the standard cash settlement of credit spread forwards, which – for one unit of notional - is given by the multiplication of the Credit Spread Duration[10] and the difference between the traded forward spread and the spot spread observed on the forward expiration date. The estimation of β and the calculation of the optimal notional to be invested in the credit spread forwards have been performed analogously to the hedging strategy involving CDX contracts.

III. Results

Before discussing the results provided by the alternative hedging strategies, we analyze some relevant statistics related to the bonds composing the portfolio to be hedged as well as the market yield curve of BBB-rated bonds. In particular, we consider the level of correlation among the individual bonds composing the portfolio to be hedged and between this portfolio and the corporate bond market. All our hedging strategies have been calibrated on the dynamics of the corporate bond market and three of the tested strategies include a financial instrument linked to an average credit spread representing this market. Accordingly, this correlation analysis can be instructive in order to interpret the results obtained by the tested strategies.

[10] The exact definition of the Credit Spread Duration may vary from one contract to the other, but normally refers to the expiration date of the forward contract. For our purposes, we calculated this parameter as the mean value of the durations of the two 5-year bonds constructed on the corresponding BBB and risk-free par rates at the beginning of the hedging period minus the length of the hedging period.

Figure 1 depicts the average level of correlation among the eight bonds composing the portfolio to be hedged for each considered calendar year. It leaves us with the clear impression that these bonds largely moved together between 2001 and 2007, whereas this was far less the case in 2008 and 2009: the average correlation was 66% in the first seven years and only 30% in the last two years. In the introduction, we summarized some key contributions to the debate about the number of factors driving the dynamics of the credit spread. Performing a formalized test of the alternative hypotheses animating this debate goes beyond the scope of the present paper. However, since the focus of this paper is on hedging market – in opposition to idiosyncratic – risk, an estimation of the relevance of these types of risk is of paramount importance for our purposes. Some of the figures reported in Table I turn out to be helpful with this respect. First of all, we notice that the different points along the term structure of market yields for BBB-rated bonds moved more together in 2008 and 2009 than in the previous years. This was mainly the result of a very high correlation among credit spreads: from the beginning of 2008 to March 2009, credit spreads of every maturity strongly widened, whereas during the rest of 2009 all these spreads drastically tightened. Accordingly, the lower correlation in 2008 and 2009 cannot be explained by the dynamics of the market yield curve.

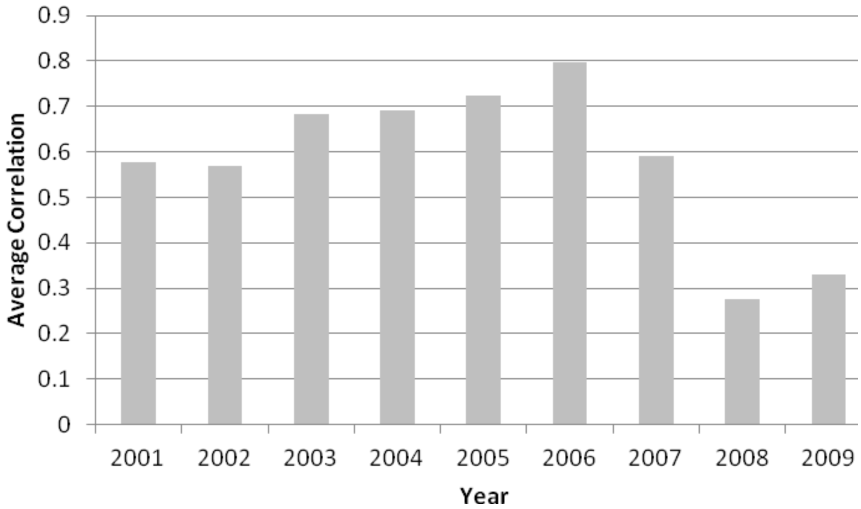


Figure 1. Average correlation of returns among the bonds composing the portfolio to be hedged. The chart represents the average level of correlation among the eight bonds composing the portfolio to be hedged. The correlation has been calculated based on the returns provided by each bond on every time period. Since the time periods are by-weekly, there are 24 observations for each of the reported years. The average correlation has been calculated as a simple arithmetic average of the correlations between each of the 28 possible pairs composed from the eight bonds.

A confirmation that the lower correlation in 2008 and 2009 is mostly due to idiosyncratic risk comes from the results of regressing the unexpected returns of the bond portfolio on the market yield curve of BBB-rated bonds. The R^2 statistic of these regressions is also reported in Table I and shows that 75% of the unexpected return variance is still explained by market risk. Even though this is circa 8% less than between 2000 and 2007, such a proportion highlights that it is sufficient to combine a few bonds in a portfolio to rapidly diversify away the dynamics of individual bonds, thus indicating that most risk during this period was indeed idiosyncratic. Since our hedging strategies assume that idiosyncratic bond returns are independent from market returns, they should not be expected to hedge the residual idiosyncratic return provided by the bond portfolio. Accordingly, the abovementioned R^2 statistics can provide us with a rough estimate of the maximum variance reduction which can be expected from the tested strategies.

Table I

Summary Statistics

The first column of this table describes the periods the statistics refer to. The second column reports the average of the correlation coefficient among the changes in the BBB zero-rates with maturity 1-year, 5-year, 10-year, and 20-year. The third column reports the average of the correlation coefficient among the changes in the BBB zero-coupon credit spread with maturity 1-year, 5-year, 10-year, and 20-year. The fourth column reports the R^2 statistic of linearly regressing the unexpected returns of the bond portfolio on the changes in the BBB zero-rates with maturity 5-year, 10-year, and 20-year. The fifth column reports the R^2 statistic of linearly regressing the average change in all considered BBB zero-rates on the changes in the risk-free zero-rates with maturity 5-year, 10-year, and 20-year. The sixth column reports the average ratio between the variance of the changes in the zero-coupon spreads and the variance of the changes in the BBB zero-rates with maturity 1-year, 5-year, 10-year, and 20-year. The three sub-periods include – respectively – 119, 191, and 48 by-weekly observations.

Period	Average Correlation BBB Rates	Average Correlation BBB Spreads	Bond Portfolio: Explanatory Power of BBB Systematic Risk	BBB Systematic Risk: Explanatory Power of Risk-Free Rates	Average Ratio Spread Variance / Total Variance
1995-Sep. 2000	75.5%	30.2%		87.8%	27.2%
Oct. 2000 - 2007	65.0%	37.0%	83.8%	83.5%	26.8%
2008-2009	76.0%	94.9%	75.5%	43.4%	57.6%

Table II reports the results obtained by the alternative hedging strategies in term of variance reduction, whereas Table III reports the first-order autocorrelation of the hedging error for the same hedging strategies. Although the variance of the returns provided by the hedged portfolio is the key statistic we use to rank the alternative hedging strategies, we believe that the first-order autocorrelation of the hedging errors should also be regarded as a relevant quality statistic. If this autocorrelation is negative, hedging errors tend to offset each other during subsequent periods. On the contrary, if this autocorrelation is positive, hedging errors tend to add up to each other during subsequent periods, thus amplifying a loss or a profit over several time periods. In most business environments, the latter situation is undesirable. Accordingly, hedging strategies leading to errors displaying a lower level of autocorrelation should – *ceteris paribus* – be preferred.

Table II

Variance Reduction Obtained by Alternative Hedging Strategies

This table reports the results obtained by the error-adjusted version of four alternative hedging strategies including different financial instruments. Variance reduction is calculated as the ratio of the variance of the unexpected returns provided by the hedged portfolio over the variance of the unexpected returns provided by the bond portfolio to be hedged less one. For the hedged portfolio relying only on T-bond futures, we also report the R^2 obtained by linearly regressing the hedging errors on the changes in the spread between the 10-year zero rate of the BBB USD yield curve and the corresponding zero rate of the US Treasury yield curve. ** indicates 2.5% statistical significance, whereas *** indicates 1% statistical significance. The statistical significance related to the variance reduction has been estimated following a matched pair experiment approach: the inference refers to the average difference between the absolute value of the hedging error for the strategy relying only on T-bond futures and the absolute value of the hedging error for the alternative strategy. The three sub-periods include – respectively – 100, 72, and 47 by-weekly observations.

Hedging Instruments		T-Bond Futures		T-bond Futures and S&P500 Futures	T-bond Futures and CDX	T-bond Futures and Credit Spread Forward
Period	Bond Portfolio Variance	Variance Reduction	R^2 of Hedging Error on Spread Changes	Variance Reduction	Variance Reduction	Variance Reduction
2000-2004	0.015%	82.3%	0.6%	76.8%		
2005-2007	0.008%	84.7%	0.0%	84.0%		
2008-2009	0.035%	23.5%	49.8% (***)	26.1%	30.2%	64.3%(**)

We implemented both the traditional version and the version including the error-adjustment for each of the four tested strategies. Consistently with Carcano and Dall'O (2011), the results we obtained for the traditional version of the strategies were systematically worse than the ones obtained for the version including the error-adjustment. However, we observed that traditional strategies often led to implausible long-short positions in the hedging instruments and – as a consequence- to far worse results than the ones reported by Carcano and Dall'O (2011). We suspect that the greater complexity of our hedging problem and the larger size of the involved hedging errors might be responsible for this outcome. Accordingly, for the sake of brevity, we report only the results provided by the error-adjusted strategies here. In the next sub-sections, we will analyze these results for each of the four tested hedging strategies.

A. *Hedging through T-Bond Futures*

The results reported in Table II highlight a clear change of regime before and during the sub-prime crises. Before the crisis, the error-adjusted version of this hedging strategy led to an average variance reduction of 83.5%. This figure almost precisely matches with the proportion of systematic risk in the bond portfolio reported in Table I (83.8%), which we considered to be a rough estimate of the maximum attainable variance reduction. This level of variance reduction is substantially higher than the one reported by previous studies on this subject and is largely consistent between the two analyzed sub-periods. Also, Table III reports a negative autocorrelation of order one for the hedging errors of this strategy before the crisis, which strengthens the evidence about the usefulness of this immunization approach.

We can explain the above result by looking at some of the figures reported in Table I. Particularly, 83.5% of the dynamics of the yield curve of BBB-rated bonds from 2000 to 2007 were explained by risk-free rates. Since we have just remarked that 83.8% of the bond portfolio variance during the same period was explained by the yield curve of BBB-rated bonds, this implies that the vast majority of the bond portfolio variance from 2000 to

2007 was explained by risk-free rates. Carcano and Dall'O (2011) reported that error-adjusted PCA strategies relying on four T-bond futures contracts can almost perfectly hedge the risk related to the risk-free yield curve, so that the very good performance of this hedging strategy between 2000 and 2007 should not be surprising. We would expect the vast majority of the residual variance of the hedged portfolio returns (16.5% of the total variance to be hedged) to be due to idiosyncratic returns of the individual bonds. This hypothesis is confirmed by the observation reported in Table II that the hedging errors produced by this strategy before the sub-prime crisis were basically uncorrelated with the dynamics of the average credit spread paid by the market on BBB-rated bonds.

The observation that - in the years preceding the sub-prime crisis – hedging errors are mainly due to idiosyncratic returns of the individual bonds has two important implications. Firstly, corporate bond portfolios which display a higher level of diversification than our equally weighted 8-bond portfolio can plausibly be hedged even more effectively. Secondly, strategies attempting to hedge against changes in the corporate bond spread are unlikely to obtain any improvements over a simpler strategy relying only on T-bond futures during the considered period, also because Table I highlighted that credit spreads of different maturity moved up to a large extent independently from each other. We will show below that this was indeed the case for all alternative strategies we tested.

Table III

Autocorrelation of the Hedging Errors Produced by Alternative Hedging Strategies

The column "Bond Portfolio" reports the coefficient of first-order autocorrelation of the unexpected return provided by the bond portfolio. The other four columns report the coefficient of first-order autocorrelation of the hedging error provided by the hedging portfolio indicated in the title row. The hedging error is computed as the sum of the unexpected return provided by the bond portfolio and the unexpected return provided by the hedging portfolio. The three sub-periods include – respectively – 100, 72, and 47 by-weekly observations.

Period	Bond Portfolio	T-Bond Futures	T-bond Futures and S&P500 Futures	T-bond Futures and CDX	T-bond Futures and Credit Spread Forward
2000-2004	1.3%	-10.3%	-13.2%		
2005-2007	1.3%	-20.0%	-20.5%		
2008-2009	27.7%	70.4%	66.0%	57.8%	21.3%

When we consider the results obtained by the same strategy in 2008 and 2009, we face a completely different picture. Not only does the variance reduction drop to 23.5%, but also the autocorrelation of the hedging error stops being negative. A positive autocorrelation of 70.4% implies that hedging errors tend to cumulate over time, so that losses tend to be followed by other losses. Considering that the unexpected returns of the unhedged bond portfolio display a substantially lower level of autocorrelation than the hedging errors and the small variance reduction, it is not at all clear that the hedged portfolio is preferable to the unhedged portfolio during this period.

Two questions arise from the observation we just made: why does hedging corporate bonds through T-bond futures stop making sense in this period and under which conditions is this likely to happen again in the future? The answer to the first question seems to be simpler: the dynamics of the credit spread which are not hedged by this strategy played a crucial role in 2008 and 2009. Clear evidence of this is provided by the R^2 statistic reported in Table II which highlights a highly statistically significant relationship between the hedging error and the credit spread during this period. The origin of this relationship can be identified in Table I: in 2008 and 2009, risk-free rates are no longer able to explain most of the dynamics of BBB yields and the main reason for this is that the variance of credit spreads as a proportion of total yield variance more than doubled. This relationship explains also the high level of autocorrelation of the hedging error. In fact, if we observe the development of the credit spread in 2008 and 2009 reported by Figure 2, we notice that positive (negative) changes have been mostly followed by other positive (negative) changes. Since these changes heavily influenced the hedging errors, a strongly positive autocorrelation of the latter errors should be expected.

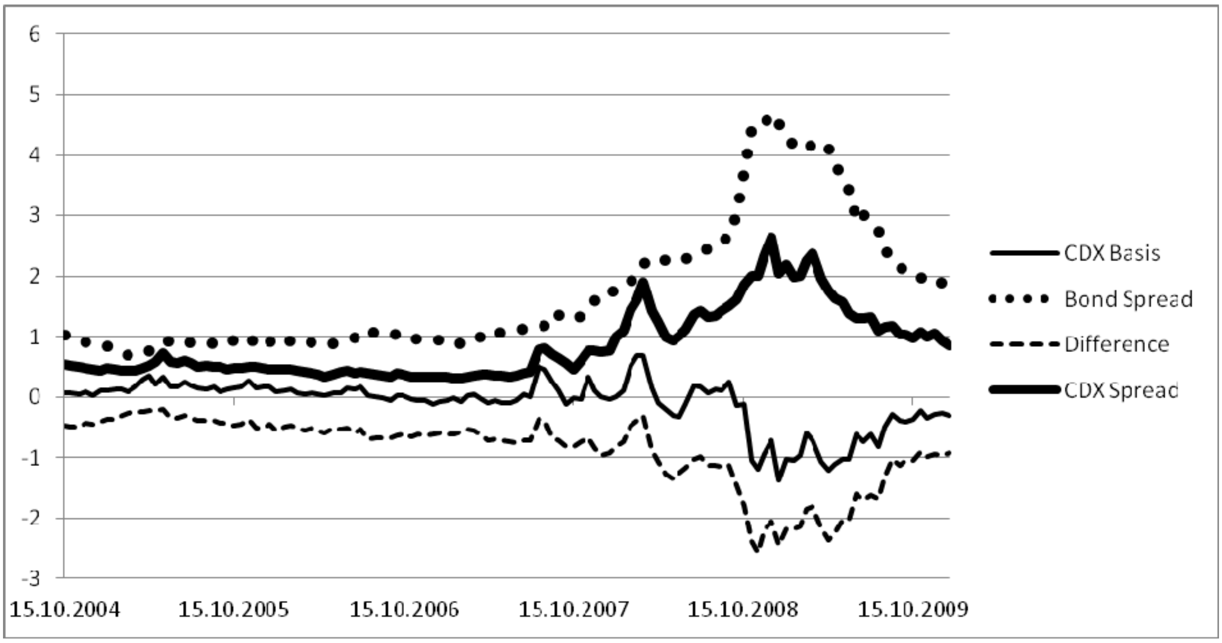


Figure 2. Level of corporate bond spread and CDX spread from 2004 to 2009. The chart represents the values of four lines from the 15th of October 2004 to the 31st of December 2009. The first line - the *Bond Spread* - has been calculated as the difference between the 5-year par rate reported by Bloomberg for BBB-rated bonds issued by industrial companies and for US Treasury bonds. The second line - the *CDX Spread* - has been obtained by Bloomberg for the 5-year Investment Grade CDX contract. The third line - the *CDX Basis* - has been calculated as the difference between the CDX Spread and the bond-to-swap spread; the latter spread is defined as the difference between the yield of the CDX basket and the 5-year swap rate. The fourth line - the *Difference* - has been calculated as the difference between the CDX Spread and the Bond Spread.

Answering the second question above is more difficult. If we analyze BBB credit spreads in the United States from 1995 to 2010, we come to the conclusion that the "normal" range for these spreads lies somewhere between 0.75% and 2%. When spreads exceed the level of 2%, an abnormal tension on the credit market seems to be in force. The latter tension can plausibly be expected to lead to sudden and remarkable shifts in the credit spreads. Accordingly, strategies hedging corporate bond risk should attempt to also hedge systematic credit spread risk during such periods. However, we realize that this answer has rather the character of anecdotic evidence than of a rigorous proof. More comprehensive empirical analyses of other countries and time periods could perhaps provide us with a sounder evidence on the conditions under which it is important to hedge systematic credit spread risk. For our purposes, we will assume that this was the case only in 2008 and 2009 and test the performance of alternative hedging strategies focusing on systematic credit spread risk during this period. Their results will be analyzed in the following paragraphs.

B. Hedging through T-Bond Futures and S&P500 Futures

The results reported in Table II and Table III highlight that the differences in variance reduction and autocorrelation of hedging errors obtained by introducing a position in the S&P500 futures within the hedging portfolio are small. Before the sub-prime crisis, our results do not support the usage of the S&P500 futures as a hedging instrument for corporate bonds; this is not surprising since in the previous sub-section we highlighted the lack of correlation between hedging errors and the credit spread paid by the market during this period. In 2008 and 2009, the S&P500 futures would have helped both in terms of variance reduction and of autocorrelation. However, the size of this improvement is so small that we should ask ourselves if the benefit is worth the effort.

There are two reasons why the impact of the S&P500 futures on the hedging quality is so small. The first reason goes back to our decision to use these futures only when the level of consumer confidence is below its historic mean, since past evidence showed that only during these phases the credit spread and the S&P500 seem to be significantly correlated to each other. This implies that in circa 40% of our test cases before the sub-prime crisis the S&P500 is not used. However, this does not apply for 2008 and 2009, a period during which the consumer confidence index was systematically below its historic mean. A second reason is responsible for the low impact of the S&P500 futures also during this period. Even if we consider only the periods with a low level of consumer confidence, the unexpected dynamics of the S&P500 explain only 8.7% of the variance of the changes in the credit spread of the 10-year zero-coupon rate for BBB-rated bonds. This proportion falls to 0.1% if we regress the full changes in the 10-year zero-coupon rate for BBB-rated bonds on the unexpected return provided by the S&P500 futures. Consistently with such a loose relationship, our PCA-based methodology leads to sell the S&P500 futures (which is consistent with our intuition), but only for an average amount of circa 2.5% of the bond portfolio value. With such a low weighting, it should not come as a surprise that the use of the S&P500 does not add significant value to the simpler hedging portfolio including solely T-bond futures.

We conclude that the relationship between the equity market and the corporate bond spread seems to be too loose to allow a meaningful use of equity-related financial instruments in hedging corporate bond risk. In the next sub-section, we will analyze if a derivative explicitly constructed on the credit risk of BBB-rated corporate bonds is of greater use for this purpose, as we would expect.

C. Hedging through T-Bond Futures and Credit Default Swaps

Considering the evidence reported above on the lack of relationship between hedging errors and credit spread changes before the sub-prime crisis, we decided to include the 5-year Investment Grade CDX contract in the hedging portfolio only during 2008 and 2009. We decided to include only one hedging instrument for the credit spread because – as reported in Table I – when the dynamics of this spread really need to be hedged, all points along the term structure of credit spread are very strongly correlated with each other. Accordingly, hedging this term structure with only one instrument should be sufficient. We decided to use the 5-year CDX contract, which is by far the most liquid one.

The results reported in Table II and in Table III suggest that this Credit Default Swap does indeed improve the quality of hedging compared to the strategy including the S&P500 futures, both in term of variance reduction and of autocorrelation dampening. However, the improvement is much smaller than one would plausibly expect. If we consider the abovementioned benchmark of 75.5% for the maximum variance reduction, we have to admit that a value of 30.2% cannot be considered too favorably. Not surprisingly, the improvement compared to the hedging portfolio including only T-bond futures is not statistically significant. Also, the problem of a significantly higher positive autocorrelation than the unhedged bond portfolio persists, so that we are still not in the position to say that this way of hedging makes more sense than just keeping the bond portfolio unhedged.

Why does a derivative explicitly designed to track the dynamics of credit risk not perform better in hedging the credit spread risk embedded in corporate bond portfolios? Figure 2 can help us to answer this question. In this figure, the line titled *Difference* represents the difference between the CDX spread and the bond spread, that is, the spread between BBB-rated corporate bond rates and risk-free rates. Since changes in the CDX spread determine the CDX performance and this performance should hedge against changes in the bond spread, if the CDX spread and the bond spread move apart from each other the hedging quality cannot be very high. The line titled *Difference* in Figure 2 highlights that during 2008 and the first part of 2009, the CDX premium reacted less than proportionally to the increase in the corporate bond spread. During the second part of 2009, the CDX premium reacted less than proportionally to the decrease in the corporate bond spread. As a result, the CDX premium tended to undershoot the actual dynamics of the corporate bond spread. Now, if this undershooting took place at a consistent rate over time, it would not represent a problem for hedging, since changes in bond spreads and CDX premiums would still be highly correlated with each other. Unfortunately, the rate of this undershooting changes over time, thus drastically reducing the level of this correlation.

Accordingly, the question shifts to the dynamics of the difference between the CDX spread and the bond spread: what can explain these dynamics ? A first explanation is that the CDX spread in reality refers to the difference between the yield of the underlying bond basket and – as suggested by Blanco et al. (2005) - the corresponding swap rate. The underlying bond basket of the Investment Grade CDX does not only consist of BBB-rated bonds issued by industrial companies: it includes also A-rated bonds and non-industrial companies. The difference between the CDX spread and the appropriate bond-to-swap spread (considering the composition of the underlying CDX basket) is the so-called *CDX basis* and represents the remaining dynamics of the line titled *Difference* in Figure 2.

In order to assess the relevance of these two explanatory factors, we estimate the CDX basis and report it in Figure 2[11]. We notice that this basis follows very similar dynamics as the line titled *Difference*, even though at a consistently lower absolute value: a linear regression analysis shows that 77.5% of the variance of changes in the difference is explained by changes in the CDX basis. Accordingly, this basis explains the vast majority of the residual hedging errors.

The natural question which arises based on these results is: were the dynamics of the CDX basis in 2008 and 2009 systematic or idiosyncratic? Were they economically justified, so that we should expect to see similar dynamics in the future? Or were they

[11] For estimating the CDX basis, we needed an estimate of the yield of the CDX bond basket. We calculated this yield as a weighted average of the 5-year BBB and A par rates, where the weights corresponded to the rating allocation displayed by each CDX series. The bond-to-swap spread was obtained by subtracting the 5-year swap rate from the yield of the CDX bond basket.

related to exceptional market inefficiencies which are unlikely to happen again? In the former case, the weakness of CDX as a hedging vehicle would be chronic, whereas in the second case there would be some more room for hope.

Since the negative CDX basis persisted for one full year, it is hard to believe that it was related to a mispricing or to a lead-and-lag relationship between the CDX and the bond markets. Consistently with the findings of Elton et al. (2001) and Collin-Dufresne et al. (2001), during the sub-prime crisis the corporate bond spread increased much more than it would have been justified simply by the expected default losses. In a frictionless market with no counterparty risk, non-arbitrage conditions would have led the CDS spread and the bond spread to move together. In fact, an arbitrageur could take advantage of a negative CDS basis by financing the acquisition of the bond at the risk-free rate and buying protection through a CDS: if the cost of the protection is lower than the bond spread, the arbitrageur would obtain a positive and riskless return. However, two problems made this kind of arbitrage cumbersome during the crisis. Firstly, finding cheap money to finance the acquisition of the bonds was not easy. This issue was more serious for single-name CDS than for our CDX contract, tough: even during the crisis, the collateral value of a well-diversified basket of investment-grade bonds was relatively high. Secondly, the dramatic increase in counterparty risk led the risk for the protection buyer to increase much more than for the protection seller: during a generalized credit crunch, the default correlation of the protection seller (normally, a bank) with the bond issuers drastically increases, thus making more likely that the protection seller will not honor her duties on the CDS. As a result of the increased counterparty risk, the negative-basis trade is far from being riskless for the arbitrageur: the negative basis simply represents a compensation for this risk. Accordinlgy, we agree with Fontana (2010) that this phenomenon was not signaling any market inefficiency. In the case of our CDX contract, we are convinced that the increase in counterparty risk was the most important reason which led the negative basis to develop and persist during the crisis and finally caused the hedging strategy under analysis to underperform.

D. Hedging through T-Bond Futures and Credit Spread Forwards

If the hypothesis described in the last sub-section is correct, we should see a very significant improvement in hedging quality by using credit derivatives which are not subject to counterparty risk. In order to test if this is indeed the case, we assumed the existence of fully collateralized credit spread forwards on the 5-year BBB par rates. The credit spreads forwards have been calculated using the 5-year par Treasury rate as a benchmark and the pricing framework described in Section II.

The results provided by this hedging strategy are reported in the last column of Table II and Table III. They show indeed a substantial improvement compared with the strategy involving the CDX contract: the variance reduction now amounts to 64.3%, thus realizing the vast majority of the potential reduction of 75.5% we estimated as a benchmark for the years 2008 and 2009. It should not be surprising that this benchmark cannot be fully matched: even though Table I highlighted that the correlation between changes in spreads of different maturity is on average very high in 2008 and 2009 (94.9%), still it is far from being perfect. Accordingly, the use of only one instrument for hedging the full term structure of credit spreads should not be expected to generate a perfect hedging of the dynamics of this term structure. Also, Carcano and Dall'O (2011) showed that hedging based on four T-bond futures is normally very effective in removing the US Treasury term structure risk. However, they reported above-average hedging errors in 2008 which are likely to be due to the exceptional market conditions experienced immediately before and after the bankruptcy of Lehman Brothers. We believe that the imperfect hedging of the full dynamics of the two involved term structures – the one of risk-free rates and the one of credit spreads – is the main reason why even the hedging strategy we analyze here cannot fully realize the potential variance reduction of 75.5%.

The improvement in variance reduction relatively to the hedging strategy based only on the T-bond futures is statistically significant at the 2.5% level. Also, the hedging errors display a first-order autocorrelation which is lower than the autocorrelation of the unhedged bond portfolio returns. Accordingly, we have a consistent evidence on the usefulness of this strategy for hedging corporate bond market risk.

As a consequence, we conclude that the tested hedging methodology is capable of reducing the variance of the hedged portfolio by almost two-thirds also in a very challenging period, like the one including the years 2008 and 2009. The difference in terms of variance reduction between the two last tested strategies – circa 34.1% of the bond portfolio variance to be hedged - represents an estimate of the hedging error due to the use of the CDX contract.

IV. Conclusions

We test alternative strategies for hedging a portfolio composed from eight BBB-rated corporate bonds of different maturity. We focus on market risk, rather than on idiosyncratic risk. Accordingly, we do not consider in our analysis hedging instruments linked to a single credit risk, like single-name CDS contracts.

All our hedging strategies result in closed-formula solutions based on a multi-factor PCA methodology. In general, we find that the traditional implementation of these hedging strategies, which ignores the modeling errors, does not lead to satisfactory results. On the contrary, an implementation considering the modeling errors can lead to satisfactory results, even though the analysis of these results leads to clearly identify two different regimes within our data sample.

In the section of our data sample preceding the sub-prime crisis (i.e.: from 2000 to 2007), a hedging strategy based only on T-bond futures would have reduced the variance of the bond portfolio by circa 83.5%. This reduction is significantly higher than the one reported by previous studies attempting to hedge corporate bonds through T-bond futures. We attribute this improvement to the use of four futures contracts with different maturity and to the consideration of modeling errors. The hedging errors tend to be negatively autocorrelated, thus reducing the cumulative error over multiple periods, and to be independent from the dynamics of the average credit spread paid on BBB-rated bonds. This suggests that the hedging errors are mainly due to the idiosyncratic returns provided by the bond portfolio to be hedged. Accordingly, it is plausible to think that increasing the number of bonds in the portfolio might lead to even higher degrees of variance reduction.

The picture we obtain for the years 2008 and 2009 is radically different: during this period, properly hedging the dynamics of the credit spreads was of paramount importance. A simple strategy relying solely on T-bond futures would have been insufficient for this purpose, so that it is not clear that such a strategy would have provided a significant added value to the bond portfolio. On the contrary, we found out that the tested hedging methodology considering modeling errors and including fully-collateralized synthetic credit derivatives could have reduced the variance by 64.3% even during these very turbulent times. Also for this sub-sample, the vast majority of the residual hedging errors is due to the idiosyncratic returns provided by the bond portfolio, so that increasing the number of bonds in this portfolio might lead to higher degrees of variance reduction.

Unfortunately, the financial instrument which is more commonly used in practice for this sort of hedging – the CDX contract – would have led only to marginal improvements with respect to the simple strategy relying solely on T-bond futures. We believe this unsatisfactory result to be due to the counterparty risk involved by this contract which led the CDX spread to undershoot the dynamics of bond spreads. Even the S&P 500 futures contract would have not helped much during this period, since its relationship with the credit spreads is too loose to lead to a significant weighting within our hedging strategies.

Also with respect to the dispute on the dynamics of credit spreads summarized in the introduction we find a strong evidence of a change in regime starting from the end of 2007. This and – possibly - other past changes in regime might contribute to explain the contradictory results obtained by scholars who analyzed this subject. Between the years 2000 and 2007, the volatility of credit spreads relatively to risk-free rates was quite low, the correlation among different points along the term structure of credit spreads was also low (suggesting the existence of more than one important systematic risk factor), and the role of idiosyncratic risk was limited. On the contrary, in 2008 and 2009 the volatility of credit spreads relatively to risk-free rates was high, the correlation among different points along the term structure of credit spreads was very high (suggesting the existence of only one important systematic risk factor), and the role of idiosyncratic risk was significant (even though it would have been sufficient to combine eight bonds issued by different companies to largely diversify this risk away).

The interpretation we give to our results is that during times of financial stability and ordinary levels of credit risk, hedging investment-grade corporate bond portfolios though T-bond futures or other instruments linked to risk-free rates can lead to very effective immunization. However, during times of generalized financial distress this is no longer the case and the use of financial instruments directly linked to credit spreads becomes essential for an accurate hedging. Derivative contracts can be very helpful for this purpose as long as they do not involve significant levels of counterparty risk which can lead to substantial divergences between their market value and their underlying value. This interpretation raises the question of how we could identify times of generalized financial distress. Barone-Adesi et al. (2011) show that the lease rate, that is the difference between the gold forward and the LIBOR rate with equal maturity, is an effective indicator of this distress. As a complement to the lease rate, we found out that credit spreads for BBB-rated bonds higher than 2% also seem to signal exceptional distress in the credit market.

Next efforts extending the work presented in this paper should in our opinion focus on collecting further empirical evidence and on the design of credit derivatives. Additional empirical evidence should analyze how far our conclusions are applicable also to other fixed-income markets and to high-yield bonds. On the other hand, our results clearly suggest that a full collateralization of contracts like CDS would substantially improve the hedging effectiveness of these contracts. We believe these findings to be reconcilable with economic intuition: since the counterparty risk involved in current CDS contracts is normally positively correlated with the underlying credit risk, market values for these contracts tend to widely deviate from their underlying values exactly when the protection buyer would wish these deviations to be as small as possible. To make this concept clearer through a non-financial analogy, we would not store fire-extinguishers within inflammable cases, since we want to be able to access the extinguishers, when we really need them. Similarly, we should design CDS and other credit derivatives in such a way that they fully represent their underlying value, when we really need them to do so. Accordingly, we would recommend to perform empirical analysis verifying that actual market prices of fully-collateralized credit derivatives track the dynamics of credit spreads better than traditional CDS contracts, as we would expect. A viable alternative to a full collateralization would be represented by the development of exchange-traded CDS contracts, the settlement of which would be guaranteed by the stock exchange, like for futures and exchange-traded options.

Acknowledgements

We are particularly indebted to Wilshire Associates, which allowed us to access its extensive data base, thus making our research possible. In particular, we thank Peter Matheos – at the time with Wilshire – for his support and encouragement and Michael John for technical assistance. We are also grateful to Robert R. Bliss for having allowed us to use his yield curve estimates and to Ray Jireh and Daniel Grombacher from the CME for having provided us with relevant data underlying the bond futures contracts. Furthermore, we would like to thank Sandipan Mullick from Markit and Matteo Borghi from the Università della Svizzera Italiana for their insights and support on calculating the performance of CDX indexes. All remaining errors or omissions should only be charged to the authors.

REFERENCES

Acharya, Viral V. Pedersen, Lasse Heje, 2005. Asset pricing with liquidity risk, *Journal of Financial Economics*, Elsevier, vol. 77(2)

Ahn, Seung, Dieckmann, Stephan, and Perez, Marcos, Fabricio, Exploring the Common Factors in the Term Structure of Credit Spreads, Arizona State University working paper, March 2008.

Barone-Adesi, Giovanni, Geman, Hélyette and Theal, John, On the Lease Rate, Convenience Yield and Speculative Effects in the Gold Futures Market (2010). Swiss Finance Institute Research Paper No. 09-07

Blanco, Roberto., Simon Brennan, and Ian W. Marsh. 2005. An Empirical Analysis of the Dynamic Relation Between Investment-Grade Bonds and Credit Default Swaps. *Journal of Finance* 60: 2255–81.

Bliss, Robert R., 1997. Testing term structure estimation methods. *Advances in Futures and Options Research* 9, 197-231.

Bodie, Zvi, Kane, Alex, and Marcus, Alan, 1993, Investments, Irwin, Homewood, Illinois.

Carcano, Nicola, and Silverio Foresi. 1997. Hedging against interest rate risk: Reconsidering volatility-adjustment immunization. *Journal of Banking and Finance* 21, 127-143.

Carcano, N., 2009. Yield curve risk management: adjusting principal component analysis for model errors. *Journal of Risk* 12, n.1, 3-16.

Carcano, Nicola, and Hakim Dall'O, 2011, Alternative models for hedging yield curve risk: An empirical Comparison*, Journal of Banking and Finance. Forthcoming.*

Choudhry, Moorad, 2006. The Futures Bond Basis, John Wiley and Sons, LTD. West Sussex: England.

Chance, Don, 1989. An introduction to Derivatives. Dryden: Virginia.

Collin-Dufresne, Pierre, Robert, .S. Goldstein, and J. Spenser, Martin, 2001 "The Determinants of Credit Spread Changes" Journal of Finance, 56, 2177-2207.

Cumby, Bob and Martin Evans, 1995, The term structure of credit risk: estimates and specification tests, Department of Economics, Georgetown University.

Elton, Edwin, Gruber, Martin, Agrawal, Deepak, and Mann, Christopher, 2001, Explaining the rate spread on corporate bonds, *Journal of Finance*, 56, 247–77.

Eom, Young, Jean Helwege, and Jing-zhi Huang. 2004. Structural Models of Corporate Bond Pricing: An Empirical Analysis. *Review of Financial Studies* 17:499–544.

Fontana, Alessandro, 2010, Essays on credit spreads, *Dissertation*. University of Venice.

Grieves, Robin, 1986, Hedging corporate bond portfolios. *Journal of Portfolio Management*, Summer, pp. 23-25.

Grieves, Robert, Marcus, Alan, Woodhams, Adrian, 2010. Delivery options and convexity in Treasury bond and note futures. *Review of Financial Economics* 19, 1–7.

Ho, T.S.Y., 1992. Key rate durations: Measures of interest rate risks. *Journal of Fixed Income* 2, 29–44.

Hodges, S., Parekh N., 2006, Term-structure slope risk: Convexity revisited. *Journal of Fixed Income* 3, 54-60.

Ioannides, M. and F. Skinner, 1999, Hedging corporate bonds. *Journal of Business Finance & Accounting* 26(7), Sept./Oct., 919-944.

Litterman, R., Scheinkman, J., 1988. Common factors affecting bond returns. Financial Strategies Group Publications, September Goldman, Sachs & Co, New York.

Longstaff, Francis, Mithal, Sanjay, and Neis, Eric, 2005, Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market, *Journal of Finance*, 60, 2213–2253.

Marcus, Alan. and Evren. Ors (1996), Hedging Corporate Bond Portfolios Across the Business Cycle, *Journal of Fixed Income*, Vol. 4, No. 4 (March), pp. 56-60.

Nawalkha, S.K., Soto, G.M., Beliaeva, N. A., 2005. Interest Rate Risk Modeling. John Wiley & Sons, Inc., Hoboken, New Jersey.

O'Kane, Dominic, and Turnbull, Stuart. Valuation of Credit Default Swaps. Fixed Income and Quantitative Credit Research. Vol. 2003-Q1/Q2, Lehman Brothers.