Modelling the Implied Volatility Surface: Does Market Efficiency Matter?

An Application to MIB30 Index Options

Gianluca Cassese
Bocconi University, Milan and
University of Lugano

Massimo Guidolin*
University of Virginia

JEL codes: G12, G13.

Abstract

We analyze the volatility surface vs. moneyness and time to expiration implied by MIBO options written on the MIB30, the most important Italian stock index. We specify and fit a number of models of the implied volatility surface and find that it has a rich and interesting structure that strongly departs from a constant volatility, Black-Scholes benchmark. This result is robust to alternative econometric approaches, including generalized least squares approaches that take into account both the panel structure of the data and the likely presence of heteroskedasticity and serial correlation in the random disturbances. Finally we show that the degree of pricing efficiency of this options market can strongly condition the results of the econometric analysis and therefore our understanding of the pricing mechanism underlying observed MIBO option prices. Applications to value-at-risk and portfolio choice calculations illustrate the importance of using arbitrage-free data only.

1. Introduction

In recent years, we have witnessed many attempts at investigating the mechanism by which markets price stock options by modeling the structure and dynamics of the implied volatility surface (IVS).[1] This paper focuses on two novel aspects of statistical models of the IVS: the existence of complicated patterns of correlation and heteroskedasticity across heterogeneous strikes and maturities (see

*Correspondence to: Massimo Guidolin, Department of Economics, 118 Rouss Hall, University of Virginia, Charlottesville, VA - 22903, USA. Tel: 434-924-7654; Fax: 434-982-2904; e-mail: mg8g@virginia.edu.

[1] In Black and Scholes' (1973) model the price c_t^{BS} of a European call at time t is a function of a number of observable parameters and one unknown parameter, the volatility σ. Given the market price c_t of the option, the implied volatility σ^{IV} is the solution to the equation $c_t = c_t^{BS}(\sigma)$. Similar definition applies to put implied volatilities. It is easy to show that the relationship between the option price and the level of volatility is strictly increasing, so that one may interpret implied volatility as a transformation of the original price *independently of the actual applicability* (correct specification) *of the Black-Scholes model.*

Rénault, 1997) for a theoretical treatment); careful filtering of the data to eliminate records that reflect mispricings and that are incompatible with the absence of arbitrage opportunities (hence equilibrium).

Since Rubinstein (1985), it is well known that option markets are characterized by systematic deviations from the constant volatility benchmark of Black and Scholes (1973), a fact that has become even more evident after the world market crash of October 1987. These anomalies have been described either in terms of a volatility smile (or smirk, see Rubinstein, 1994 and Dumas et al., 1998) vs. moneyness or as the presence of a term structure in implied volatilities (Campa and Chang (1995)). Furthermore, the IVS is now understood to dynamically evolve over time, in response to news affecting investors' beliefs. Although much literature has focused on the IVS of CBOE index options (written on the S&P 100 and S&P 500 indices), these results are not specific to North American markets only. Similar patterns have been documented for European markets (see Gemmill, 1996, Peña et al., 1999, Tompkins, 1999, and Cavallo and Mammola, 2000).

Departures from the traditional Black-Scholes benchmark have spurred interest in the econometric modeling of the IVS. Despite recent advances in the theory of option pricing under stochastic volatility and/or jumps, on the empirical side very few studies have tried to exploit the full *panel* nature of options data sets, i.e. the fact that researchers have available both long time series of prices and rich cross-sections along the strike price and the maturity date dimensions. For instance, Dumas et al. (1998) propose a model in which implied volatilities are a quadratic function of the strike price and also depend on time to maturity. However they simply estimate it on a sequence of weekly cross-sections of S&P 500 option prices and observe that there is strong time-variation in the estimated coefficients. Therefore the time series structure of the data is completely lost and the cross-sectional estimation repeated at each point in time. To our knowledge, the only exception is Ncube (1996). In his application to daily FTSE 100 index options, he fits and compares to standard OLS both fixed and random effects panel models. Although these strategies are likely to accommodate some of the features of option pricing errors, like non-zero serial and cross-correlations, the assumptions on the random disturbances underlying the IVS are the classical ones, i.e. homoskedastic, serially uncorrelated errors with zero (simultaneous) cross correlations. On methodological grounds, our paper tries to overcome the shortcomings of Ncube's work by applying techniques now quite common in empirical economics and fully consistent with the presence of both heteroskedasticity and of non zero correlations, i.e. Parks' (1967) two stages feasible GLS method.

A second issue that naturally arises when modeling the IVS is the impact of market inefficiency. Indeed, contracts deep-in (and sometimes out-of) the-money tend to be less liquid and are therefore often mispriced. Similarly, very-short term and long-term option contracts sometimes command low trading volumes and are thus prone to mispricings. These issues are all the more important when using data from stock index option markets with an overall degree of efficiency and depth substantially inferior to their U.S. counterparts. The most common solutions to this problem — such as discarding the observations violating a limited number of no-arbitrage conditions (typically the lower bound condition only) or restricting the sample to narrow ranges of moneyness and time to expiration — do not appear to be entirely satisfactory. Our paper also documents the impact of pricing inefficiencies on the econometrics of the IVS for a non-CBOE index options market.

We pursue these objectives by modeling and estimating the IVS characterizing the market for options on the Italian MIB30 index, the so-called MIBO market — one of the most important segments of the Italian Derivatives Market (IDEM). In particular, we use 9 months of transaction data, sampled at a half-an-hour frequency during each of the business days in the sample.[2] Such a relatively young derivatives market offers in fact the best chances to study the potential links between mispricings/inefficiencies and the perception of the IVS dynamics an econometrician would derive from the estimation of a statistical model. Capelle-Blancard and Chaudury (2001), Mittnik and Rieken (2000), Nikkinen (2003), Peña et al. (1999), and Puttonen (1993) have recently stressed the importance of studying the efficiency and pricing mechanisms of such relatively less developed stock index option markets. A related paper is Cassese and Guidolin (2004) who study MIB30 index options, but does not consider the importance of market efficiency for both the econometrics of the MIBO IVS and for practical financial decisions.

Our results for the Italian options market are only partially consistent with previous findings concerning North American or other European markets. In fact, no arbitrage restrictions fail to hold rather often and (ruling out sheer irrationality) this suggests that on the MIBO market frictions play a role that is much more relevant than on benchmark CBOE markets. Assuming the presence of frictions in the form of bid/ask spreads, we proceed to filter out all the observations violating the most elementary no-arbitrage restrictions. Using alternative data sets in terms of quality of the prices included we then estimate the MIBO IVS: we document that the structure of the IVS perceived by an econometrician does considerably depend on the "pricing quality" of the underlying data. Arbitrage-ridden data offer a picture of the surface quite different from (relatively) arbitrage-free data. This is worrisome, as the presence of niches of pricing inefficiency seems to be so important to radically change our ability to quantify the risk/return trade-off perceived by market participants.

We then offer two examples of how frictions and inefficiencies can substantially affect financial decisions based on parameters (related to the risk/return trade-off) commonly implied out of option prices. In the first application, we show that standard Value-at-Risk measures implied by our IVS models crucially depend on the quality of the options data. In the second example, a simple multi-period portfolio choice problem is solved under the assumption that index options provide informative and efficient forecasts of future volatility. We show once more that the preliminary treatment of the data impacts the resulting estimates in ways that have dramatic consequences for a simple asset allocation problem.

The paper is organized as follows. In Section 2 we briefly describe the data and some of the institutional characteristics of the MIBO market. In Section 3 we document a number stylized facts concerning the MIBO IVS, thus motivating the sections to follow. In section 4 we systematically apply no-arbitrage tests showing that a striking percentage of the data does reflect significant

[2]Apart from the seminal paper by Barone and Cuoco (1989), Cavallo and Mammola (2000) contains only a brief treatment of one of the dimensions of the MIBO IVS, the relationship between implied volatilities and moneyness. No specific effort is directed at investigating the stochastic process of the overall IVS.

mispricings. We then introduce a simple structure for transaction costs and eliminate all the observations that still violate at least one of the no-arbitrage restrictions. By purging the original data set of mispricings under alternative levels of frictions we obtain several data sets of varying quality. Section 5 takes up the task of formally modeling the IVS. We document the importance of arbitrage violations in the estimation. Section 6 describes the results of two applications to financial decisions that depend on estimated, dynamic models of the MIBO IVS. Section 7 concludes.

2. **The Data**

We analyze a high-frequency data set of European options written on the MIB30, the most important Italian stock index.[3] The MIB30 index is a capital-weighted average of the price of 30 Italian blue chips, which represent approximately 80% of the whole Italian stock market. Data are sampled at a frequency of 30 minutes from 9 a.m. to 6 p.m. each day starting on April 6, 1999 and ending on January 31, 2000, for a total of 300 calendar days and approximately 15 observations a day.[4] Each observation record reports the contemporaneous (as 'stamped' by the exchange) value of the index, the risk-free interest rate, the cross-section of transaction option prices (over alternative strikes and maturities) and the bid and ask volumes.[5] The interest rate is computed as an average of the bid and ask three month LIBOR rates. Summary statistics are reported in Table 1.

According to IDEM market rules enforced during our sample period, prices are quoted for the strike nearest to the index, two strikes above and two below it. Strike prices differ by 500 index points. Prices are quoted for contracts with the three shortest monthly maturities and the three shortest quarterly maturities. Since the longest monthly maturity coincides with the shortest quarterly maturity, we have a total of five different maturities for each strike. Therefore at each point in time (day/time of the day), we have a vector of approximately 25 prices for call and put contracts. After filtering the data for obvious misrecordings (e.g. negative prices or missing data), we proceed to drop prices with no trade volume: we are then left with a total of $75,900$ prices ($37,920$ calls and $37,980$ puts).

By distinguishing contracts on the basis of moneyness and the length of their residual life we obtain a detailed description of the composition of the sample. In the following we will consider an option as being at the money (ATM) if the strike price is within 2% of the index; if it is within 5% (but apart for more than 2%) the option will be considered in the money (ITM) or out of the money (OTM), respectively (depending on its intrinsic value); an option will be considered to be

[3] The MIBO, established in November 1995, is a fully automated *quote-driven* market. Market makers have the obligation to quote prices for a specified set of contracts, expressly indicated in the market rules. Contracts are settled in cash. During 1999 the volume of exchanges (in milions of Euros) has been equal to $399,031$ and the number of traded contracts $2,236,241$.

[4] All prices in our data set are the last available *transaction* prices in the preceding half-an-hour interval. When no transactions took place for a given contract, the price is reported as missing. Missing observations are dropped from the anlysis. Option prices are expressed in "index points", with a value of 2.5 euro each.

[5] Because of standing IDEM rules, bid/ask quotes are not released and therefore unavailable; only bid/ask volumes are released to the public. Unfortunately, this is not uncommon with derivatives markets in continental Europe. For instance, Mittnik and Rieken (2000) face a similar constraint on German-DAX data.

deep in-the-money (DITM) or deep out-of-the-money (DOTM) if its strike price differs from the value of the underlying by more than 5%. We also define the following maturity classes: a contract has *very short* time to expiration, τ (measured in calendar days), if $\tau \in (0, 7]$, *short* if $\tau \in (7, 25]$, *medium* if $\tau \in (25, 50]$, and *long* if $\tau \in (50, \infty)$. The most important class in the sample is that of ATM options with short residual life (16%). More generally, ATM options represent more than one third of the data set, while short- and medium-term contract account for almost 80%.

3. The MIBO Implied Volatility Surface: Stylized Facts

Starting with the moneyness dimension of the IVS, Figure 1 plots full-sample as well as sub-period averages and medians of implied volatility when classified in 21 moneyness intervals of 1% size, starting at 0.89 and up to 1.10. Overall, the IVS describes an asymmetric smile when plotted against moneyness. Medians and means are not very different, confirming that DITM options command an implied volatility which is 5-8% higher than ATM options; while OTM options have roughly the same mean implied volatility as ATM contracts, DOTM options imply again volatilities which are above the ATM levels. Although the meaning of averaging (or calculating the median of) the implied volatilities of contracts with different time-to-maturity is uncertain, it is undeniable that the top panel of Figure 1 is striking evidence that one of the basic assumptions of Black-Scholes (1973) — constant volatility, independent of the underlying spot price — hardly applies to the Italian stock index options market.[6]

Figure 1 about here

The bottom panel of Figure 1 plots average volatilities vs. moneyness for three sub-periods of equal length: 04/06/1999 - 07/15/1999, 07/15/1999 - 10/25/1999, and 10/26/1999 - 01/31/2000. While the first and last periods produce jagged smiling shapes, the second is an asymmetric smile similar to the one obtained for the full sample. The variety of shapes obtained through a simple decomposition into three sub-samples makes us suspect the presence of remarkable instability in the MIBO IVS.

Next, we go beyond simple measures of location and examine the IVS for a few alternative days. For instance consider April 16, 1999. Figure 2 plots four IV curves as a function of moneyness for three consecutive trading times for which we have information (11:49 am, 12:19 p.m., and 12:49 p.m.), besides the closest moment to market closing in our data set, 5:19 p.m. Reading the plots in a clockwise direction, we have an initial example of stability of the IVS (between 11:49 am and 12:19 p.m., when it describes an almost perfectly skewed shape, an asymmetric smile) followed by a sudden shift to (an almost equally perfect) smile. However, by the end of the day (5:19 p.m.) the IVS have once more changed, taking a shape in which DOTM options have much higher implied volatility than all other moneyness classes.

Figure 2 about here

[6] Such a pattern is consistent with the preliminary findings of Cavallo and Mammola (2000) who, using daily data for the period Dec. 1996 - Sept. 1997, report an asymmetric smile.

Figure 2 suggests that on the MIBO the IVS can take (even in an interval of a few hours) many alternative shapes and be subject to sudden breaks. Figure 3, first panel gives an idea on the tremendous instability of the IVS with respect to moneyness by plotting for each of the four data sets volatilities as a function of moneyness over the entire sample period. The range of variation of implied volatility for each level of moneyness is striking, going from [5%, 35%] for ATM contracts to roughly [10%, 50%] for DITM and DOTM options. All this suggests that in the aggregate the MIBO IVS is likely to display non-flat shapes vs. moneyness, although a plot including all the daily IV curves is in fact consistent with the presence of smiles, skews, as well as other shapes. The lower panel does stress this point. Another implication is that — ruling out the unlikely hypothesis that perfect volatility smiles dominate all the time — the IV shapes are changing over time in response to the MIB30 index swings.

Figure 3 about here

Figure 4 shows that similar remarks apply to the other dimension of the IVS, the term structure. Focusing on the afternoon of Sept. 7 1999,[7] we can see that not only a variety of shapes of the IVS vs. time-to-maturity are possible — at first hump-shaped, then upward sloping, then 'smiling', and finally downward sloping — but also that dramatic changes can occur in half-an-hour only. For instance, on that day the term structure evolved from hump-shaped to upward sloping between 1:05 p.m. and 2:35 p.m., with two further breaks between 2:35 p.m. and 3:35 p.m.. At market close, the IVS was decreasing vs. time-to-maturity, another possible structure never appeared while the MIBO market had been open during the day. Also in this case, sudden breaks seem to occur and on the whole the IV shapes are highly unstable.

Figure 4 about here

We omit a series of plots of IVs vs. time-to-maturity similar in spirit to Figure 3 as they would iterate the point that for short, medium, and long times-to-maturity the range of observed IVs is rather large, [10%, 40%]. Once more, such wide ranges of variations are fully consistent with both a number of shapes for the term structure of MIBO IVs and the presence of strong time heterogeneity.

4. **Pricing Efficiency**

A crucial issue in options markets is the existence of arbitrage opportunities.[8] Since in a companion paper (Cassese and Guidolin, 2004) we investigate this aspect in detail, for the current purposes we limit ourselves to a small set of key points.

For frictionless options markets the absence of arbitrage is equivalent to the following pricing

[7] This choice is not totally random, as in order to be able to draw term-structure plots we require at least three different maturities being simultaneously traded.

[8] This issue has been addressed, among many others, by Ackert and Tian (2001), George and Longstaff (1993), Kamara and Miller (1995), Nisbet (1992), Ronn and Ronn (1989).

rules:

$$c_t(K,\tau) = E_{Q,t}\left[e^{-r\tau}\max(S_T - K, 0)\right] \qquad (1)$$
$$p_t(K,\tau) = E_{Q,t}\left[e^{-r\tau}\max(K - S_T, 0)\right], \qquad (2)$$

where c_t and p_t indicate the time t price of a call and a put with strike K and time to maturity $\tau \equiv T - t$.[9] Frictions and other market imperfections are responsible for frequent failure of (1)-(2) to hold in the data, although a tractable model of option prices in the presence of frictions is not yet available. In the absence of a satisfactory model, our choice is to treat frictions as affecting the difference between selling and buying net prices (in analogy with the bid/ask spread). Discarding fixed costs — the effective burden of which depends on the volume of the transaction and is therefore hard to assess — we model frictions as a fixed proportion of the asset price, i.e. as an additional component to the bid/ask spread. This choice allows us to restrict attention to bid/ask spread for options, the bid/ask spread for the underlying and the cost of taking a short position in the underlying.[10]

Let the superscript a denote ask prices and b the bid prices (inclusive of frictions). Let T_t^S represent the cost of taking a short position in the stock index.[11] In particular, defining α as the spread on the option, β as the spread on the MIB30, γ as the proportional transaction cost on sales of the underlying, we write

$$c_t^b = (1-\alpha)\,c_t, \quad c_t^a = (1+\alpha)\,c_t, \quad S_t^b = (1-\beta)\,S_t, \quad S_t^a = (1+\beta)\,S_t \qquad (3)$$

(for a call), and $T_t^S = \gamma S_t$. Then the bid/ask spread is 2α and 2β for options and the MIB30, respectively. Under these assumptions, Cassese and Guidolin (2004) derive a number of no arbitrage conditions to be tested. These conditions are listed below:[12]

(a). Lower Bound;

[9] For the sake of simplicity, we will assume throughout that the interest rate r is constant.

[10] Cassese and Guidolin (2003) also discuss the potential role of other types of frictions — microstructural features of the IDEM, cash dividends, and taxation — as possible determinants of arbitrage violations. They conclude that either these additional frictions are difficult to quantify and use for empirical purposes (e.g. the special inventory position of market makers, the individual tax position of arbitrageurs, etc.) or that they are hardly relevant (cash dividends).

[11] An approximate replication of an index sale can only be obtained via the corresponding futures market, the FIBO30, i.e. by selling the future on the MIB30. This is an ordinary sale transaction so that no particular costs apply apart from the corresponding bid/ask spread. Unfortunately, this strategy is not always available in the Italian stock market since the expiration dates of futures and options market match only imperfectly. Potential alternatives to a short position in the matched futures have the disadvantage to display highly imperfect correlation with the underlying and therefore imply costs.

[12] See Cassese and Guidolin (2003) for further details on how these condions are checked. For most of them, textbook treatments are available, see e.g. Epps (2000). Box spread conditions have been recently introduced by Ronn and Ronn (1989). Spread maturity conditions appear to be novel and are derived in Cassese and Guidolin (2003); in practice they boil down to the following pair of strict inequalities:

(short) $[p_t(\tau_2) + c_t(\tau_1)](1-\alpha) - [p_t(\tau_1) + c_t(\tau_2)](1+\alpha) + K\left[e^{-r\tau_1} - e^{-r\tau_2}\right] < 0$

(long) $[p_t(\tau_1) + c_t(\tau_2)](1-\alpha) - [p_t(\tau_2) + c_t(\tau_1)](1+\alpha) + K\left[e^{-r\tau_2} - e^{-r\tau_1}\right] < 0.$

(b). Strike Monotonicity;

(c). Maturity Monotonicity;

(d). Butterfly;

(e). Put/Call Parity;

(f). Reverse Strike Monotonicity;

(g). Box Spreads;

(h). Maturity Spreads.

For instance, in the presence of frictions the standard lower bound condition is represented by the following inequalities

$$\begin{array}{ll} \text{(put)} & K\exp(-r\tau) - \left[\frac{S_t}{1-\beta}\,(1+\beta) + p_t\,(K)\,(1+\alpha)\right] < 0 \\ \text{(call)} & -K\exp(-r\tau) + \left[\frac{S_t}{1+\beta}\,(1-\beta)\,(1-\gamma) - c_t\,(1+\alpha)\right] < 0 \end{array}$$

When either of the conditions are violated, their left-hand side represents the arbitrage profit that could be made on the MIBO. For empirical purposes, we set $\alpha = \beta$ and $\gamma = 0$.[13] One can easily check that all the profits derived from violation of conditions (a)-(h) are decreasing functions of the parameter $\alpha = \beta$. In particular, when $\alpha = 0$, inequalities like the ones listed above represent potential arbitrage profits violating the pricing rules (1)-(2). For positive values of α, violations of the no arbitrage conditions are far less attractive, thus explaining how profit opportunities that exist in theory may not be available in practice. One can easily check that all the computed profits are decreasing functions of the parameter α. In particular, when $\alpha = 0$ conditions similar to the put-call parity bounds also give expressions for the potential arbitrage profits violating the pricing rules (1)-(2). For positive values of α, the profits deriving from violations of the no-arbitrage conditions are far less attractive, thus explaining how profit opportunities that exist in theory may not available in practice.

For the case $\alpha = 0$, the finding is striking: the number of arbitrage violations is outstanding — approximately 50% of the sample. The distribution of this total number across moneyness, time to maturity and type of condition violated is also interesting, see Table 2. It is clear that the number of arbitrage opportunities detected increases the shorter the time to maturity and the higher the moneyness. Short-term, ITM and DITM options are normally considered not very liquid: Those investors who own them have good prospects of receiving a positive final payoff if they hold their contracts rather than trade them in the attempt of reaping immediate arbitrage profits. We thus find some correlation between mispricings and liquidity. Nevertheless, even the more liquid segments of the market tend to display ratios above 30%, i.e. inefficiency is maximum when coupled with low liquidity, but it characterizes in general the MIBO market. As regards the different conditions tested, it clearly emerges that the most often violated ones are the Box spreads, the put/call parity and the Butterfly spreads. The two Box conditions are violated equally often,

[13] Results in Cassese and Guidolin (2003) show that γ has little impact on the number and size of the existing arbitrage opportunities.

but the parity condition is highly asymmetrical and it is the short hedge the one that the MIBO has the hardest time to satisfy. This makes intuitive sense as the trading strategy that would allow arbitrageurs to exploit the corresponding mispricing requires to take a short position in the index, which is difficult and costly.[14]

Figure 5 about here

Figure 5 plots the percentage number of violations of at least one of the above conditions as a function of α. In panel A we plot the ratio of arbitrage violations over the sample size for conditions implying a low number of violations. Percentage ratios in this case rapidly converge to zero as α increases. Panel B refers to the two sides of the put-call parity: although it is still clear that violations of the short end occur more frequently than those of the long side, it is remarkable that the high ratios of these violations quickly drop to zero as soon as α reaches about 2%. Panel C stresses the existence of other conditions that are both frequently violated in the absence of frictions (box and maturity spreads imply about 20% violations for $\alpha = 0$) and that remain important throughout the range of all the values for α we consider. For instance, at $\alpha = 4\%$ the two box spread conditions still originate a 5% each of violations, while the maturity spread condition is still at a very persistent 15%. The plot documents that some kinds of pricing inefficiencies are very persistent and that in particular, the maturity spread restriction is scarcely influenced by the value of the transaction costs. Obviously, as α increases the overall number of violations declines to zero.

We conclude that the MIBO market is characterized by remarkable niches of inefficiency even after considering the role of transaction costs and other frictions. Therefore any econometric analysis of the structure and dynamics of the IVS should consider the possibility that the results depend on the presence of observations that violate some of the rational pricing bounds established by (a)-(h). To this end we discard from our sample those prices involving at least one type of arbitrage opportunity for different values of α so to obtain four distinct data sets:

1. The original data set (73,529 observations). This amounts to a sample free of arbitrage only for $\alpha = +\infty$.[15]

2. A medium-frictions data set (67,962 observations) corresponding to $\alpha = 5\%$.

3. A low-friction data set (57,379 observations) for $\alpha = 2\%$. Since this value of α is the most plausible in the light of market regulations, this data set is probably the most realistic one.

4. A frictionless data set (36,623 observations) derived by setting $\alpha = 0\%$. Obviously, this corresponds to dropping about 51% of the original observations and makes this fourth data set the highest quality one. We will often refer to this fourth data set as the arbitrage-free one.

[14] Mittnik and Rieken (2000) make a similar remark with reference to options on the German DAX stock index.

[15] In practice, this is true only as a first approximation as we anyway purge the data set of observations violating the lower bound conditions, i.e. implying a negative implied volatility. Elimination of lower bound violations explains the loss of 2,371 observation from the original 75,900.

In Section 5 we study whether an econometrician's perception of the stochastic process of the IVS may be influenced by the pricing quality of the data.

5. Structural models of the IVS

Given our finding that the MIBO IVS has a rich structure that departs from the Black-Scholes (1973) benchmark, in this section we try to refine our understanding of its determinants by fitting a host of alternative 'reduced form' models of implied volatility. Defining z_t as $K/\left[e^{r_t \tau_t} S_t\right]$,[16]

1. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \epsilon(z_t, \tau_t)$
2. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \epsilon(z_t, \tau_t)$
3. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \beta_2 (\ln z_t)^2 + \epsilon(z_t, \tau_t)$
4. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 OTM_t + \beta_2 (ITM_t)^2 + \epsilon(z_t, \tau_t)$
5. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 OTM_t + \beta_2 (\ln z_t)^2 + \epsilon(z_t, \tau_t)$
6. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 OTM_t + \beta_2 (\ln z_t)^2 + \beta_3 ITM_t + \epsilon(z_t, \tau_t)$
7. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \beta_2 (\ln z_t)^2 + \gamma_1 \tau_t + \gamma_2 (\ln z_t) \tau_t + \epsilon(z_t, \tau_t)$
8. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \beta_2 (\ln z_t)^2 + \gamma_1 \tau_t + \gamma_2 (\ln z_t) \tau_t + \gamma_3 \tau_t^2 + \epsilon(z_t, \tau_t)$
9. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 NS_t + \beta_2 NS_t^2 + \gamma_1 \tau_t + \gamma_2 NS_t \tau_t + \epsilon(z_t, \tau_t)$

Models 1-6 follow Peña et al. (1999, pp. 1159-1160), apart for the fact that the regressand is specified as the logarithm of the implied volatility (see Ncube, 1996). By construction (after getting rid of violations of the lower bound condition) $\sigma^{IV}(z_t, \tau_t) > 0$, so the left-hand side is always well defined. The advantage of this choice is to make the random regressands consistent with the errors $\epsilon(z_t, \tau_t)$, commonly interpreted as (possibly normal) random draws from a distribution symmetric around zero. Model 1 corresponds to the assumption of constant volatility (e^{β_0}) of Black-Scholes. It is a useful benchmark as it allows to measure what is the additional percentage of variability in the IVS (over time and contracts) that the use of additional regressors allows to capture. Models 2 and 3 correspond to the case of an IVS which is either a linear or a quadratic function of moneyness, although the IVS does not depend on time-to-expiration. As for models 4-6, define the following piecewise functions:

$$OTM_t = \begin{cases} \ln z_t & \text{if } z_t < 1 \\ 0 & \text{if } z_t \geq 1 \end{cases}, \qquad ITM_t = \begin{cases} 0 & \text{if } z_t < 1 \\ \ln z_t & \text{if } z_t \geq 1 \end{cases}. \tag{4}$$

The former indicator measures moneyness when the contract is OTM and is zero otherwise, while the latter measures moneyness when the contract is ITM. Clearly, $OTM_t + ITM_t = \ln z_t \; \forall z_t$. Consequently, model 4 captures an asymmetric smile, linear for $z_t < 1$ and quadratic for $z_t \geq 1$. Model 5 still represents an asymmetric smile, since for $z_t < 1$ the IVS is described by a polynomial of second degree, while for $z_t \geq 1$ the IVS reduces to the upward sloping branch of a quadratic function. Model 6 is yet another variation, in which for $z_t < 1$ the IVS is a polynomial of second

[16] z_t is a notion of moneyness that employs the forward price at the denominator. In what follows, τ_t is expressed in number of calendar days as a fraction of a year.

degree with coefficients β_1 and β_2, while for $z_t \geq 1$ a different polynomial of second degree is fitted, this time with coefficients β_2 and β_3. Notice that since $\ln z_t$ is employed and the piecewise functions of moneyness in (4) depend on the logarithm of moneyness only, e^{β_0} always measures ATM implied volatility. Models 7 and 8 are inspired instead by Dumas et al.'s (1998, p. 2068) 'ad hoc strawman'. Model 7 allows the IVS to change as a function of time-to-expiration too. τ_t also appears in an interaction term, $\tau_t z_t$. The interaction term might be crucial in capturing the infra-daily variation in the term structure of implied volatility detected in Section 2. Model 8 differs from 7 as also a quadratic term in τ_t is used as a regressor. Finally, model 9 follows Gross and Waltner (1995) in using an alternative to the variable z_t, the normalized strike:

$$NS_t = \frac{\ln\left(\frac{S_t e^{r\tau_t}}{K}\right)}{\sqrt{\tau_t}},$$

where τ_t is expressed as a fraction of a 365 days' year. Model 9 is otherwise identical to model 7.

Unfortunately, it is not possible to simply run OLS regressions of a vector of implied volatilities corresponding to different days/time of the day, moneyness, and time-to maturity on vectors of regressors according to each of the models 1 - 9. The problem is that since the observations come from a panel data set — along several dimensions, time, moneyness, and time-to-expiration — the random disturbances $\epsilon(z_t, \tau_t)$ are unlikely to be *spherical*, i.e. to have identical variance and to be uncorrelated. For instance, it is plausible that, because of the lower liquidity, certain regions (DITM and DOTM) of the IVS be characterized by more volatile random shocks, a source of heteroskedasticity. Similarly, it is likely that in a high frequency data set certain times of the day (like opening, lunch time, etc.) be characterized by more volatile random influences than others. Finally, depending on the dynamics of the market's (risk neutral) beliefs underlying the pricing of derivative securities, it is plausible that shocks to the IVS be correlated across moneyness classes (for options with different maturities) and/or across maturities (for given moneyness).

Therefore our estimation strategy takes into explicitly account the panel nature of the data and consists of an application of Parks' (1967) method after implementing suitable procedures of transformation of the original data set(s).[17] In the following we provide a brief account of the estimation strategy and report on the resulting perception of the MIBO IVS as a function of the pricing efficiency of the data used in the analysis.[18]

[17] We also resort to a second empirical strategy, OLS regressions on pooled time series / cross section data supplemented by calculation of heteroskedasticity-autocorrelation consistent estimates of the covariance matrix of the estimated regression coefficients as in Newey and West (1987). Since HAC estimators have now entered the regular toolkit of all empirical economists, also this approach is quite common in the literature (see for instance Peña et al. (1999)). However with our data, the results obtained were spurious due to the high level of serial correlation in implied volatilities (as shown by the Durbin-Watson statistics for regression residuals) and are therefore omitted.

[18] Clearly, the first-best is a strategy that models the factors causing the mispricings (e.g. lack of liquidity, missing markets like in the case of the FIBO30, etc.) and thus imposes structural restrictions on the resulting estimates. However, such models as well as the techniques of detection and measurement of the inefficiencies implicit in reported options prices are still in their infancy. Hence the second-best, 'data- filtering' approach followed by our paper.

5.1. *Feasible GLS estimation on panel data*

We approach the estimation problem trying to exploit both the cross section and the time series dimensions of the data and apply a method that explicitly takes care of the non-spherical nature of the random disturbances, Parks' (1967) iterative GLS approach. While extremely common in many fields of empirical economics, we are not aware of any other applications to modeling the IVS options data.[19] The application of Parks' method requires first that a two transformations be applied to the data. In particular we take two steps:

a. We subject the data to a reduction process by which, for each recorded trading time, we extract only 20 observations, corresponding to all the possible combinations (the order does not matter) of the five categories of moneyness — {DOTM, OTM, ATM, ITM, DITM} — and the four categories of time-to-maturity — {very short, short, medium, long}. The classes of moneyness and time-to-expiration are defined in Section 2. It often happens that a given moneyness class contains multiple observations. In these cases we extract the observation with the lowest (highest) moneyness in the case of DOTM (DITM) options, and use the mid-point observation based on a moneyness ranking for the remaining three classes. This transformation inevitably induces some loss of data. For instance, since the unfiltered ($\alpha \to \infty$) high frequency data provide us with 3,434 observations over time (at half-an-hour intervals), the resulting sample is in principle composed of 68,680 observations, implying a minimal loss of information. In practice, it happens that a few classes of moneyness may not be represented; especially in the case of time-to-maturity, at most three classes are simultaneously present throughout the sample. It turns out that the 'reduced' unfiltered sample consists of 21,240 observations, between 1/3 and 1/4 of the original number. Similar selection procedures are applied for lower levels of α. On the other hand, the resulting data sets have the structure of balanced panels in which the cross-sectional identifiers are now the 20 moneyness/time-to-maturity classes.

b. We allow the covariance matrix of the random errors affecting the IVS to have arbitrary patterns of heteroskedasticity, *serial*, and *cross-sectional* correlation, as synthesized by a full matrix rank covariance matrix Ω.

Write the generic model for time t (defined by day/hour of the day) as

$$\begin{aligned} y_{it} &= \beta_0 + \mathbf{x}'_{it}\beta_1 + \epsilon_{it} \qquad i = 1, ..., 20 \quad \text{or} \\ \mathbf{y}_t &= \beta_0 \iota_{20} + X_t \beta_1 + \epsilon_t \end{aligned}$$

where $E[\epsilon_t] = 0$ and $E[\epsilon_t \epsilon_t'] = \Sigma_t$. y_{it} collects the log implied volatility at time t for class i, while the row vector $\mathbf{x}'_{it}$ contains the regressors characterizing models 1 - 9. Let's now stack the T observations

[19] Ncube (1996) estimates reduced form models of the FTSE100 IVS using panel methods, fitting both a dummy variable model and a random effects model. Although the specification of strike-specific intercepts or random terms might help capturing the heteroskedasticy otherwise present in the data, these techniques still assume perfectly spherical disturbances and are unlikely to accommodate for the presence of serial correlation.

(for instance 3,434 for the unfiltered data) on the different times and write the model in compact fashion as:

$$Y = \beta_0 + X\beta_1 + \epsilon.$$

Following Parks (1967), we initially assume Σ_t is constant over time and that no *serial* correlation patterns be present, so that the overall covariance matrix of the IV errors can be effectively described by $\Omega = \Sigma \otimes I_T$. At this point it is well known that the GLS estimator

$$\hat{\beta}_1^{GLS} = (X'\Omega X)^{-1} X'\Omega Y$$

is consistent and efficient, and also yields consistent estimates of the covariance matrix of the estimated coefficients, $(X'\Omega X)^{-1}$. Unfortunately, Ω (more precisely, Σ) is unknown and must be first replaced by a consistent estimate, such as

$$\widehat{\Omega}^{OLS} = \hat{\Sigma}^{OLS} \otimes I_T = \left[T^{-1} \sum_{t=1}^{T} (\hat{\epsilon}_t^{OLS})(\hat{\epsilon}_t^{OLS})' \right] \otimes I_T$$

where $\hat{\epsilon}_t^{OLS} = \hat{\mathbf{y}}_t - \hat{\beta}_0^{OLS} \iota_{20} - \mathbf{X}_t' \hat{\beta}_1^{OLS}$ and $\hat{\beta}_1^{OLS} = (X'X)^{-1} X'Y$. The resulting estimator

$$\hat{\beta}_1^{FGLS} = \left(X' \widehat{\Omega}^{OLS} X \right)^{-1} X' \widehat{\Omega}^{OLS} Y$$

is called the feasible GLS.[20] Under a variety of conditions (see Parks (1967)) it has been shown to be consistent and unbiased. Asymptotically, it is also equivalent to MLE and therefore it is fully efficient. Even in the absence of normality, it can be interpreted as a pseudo-maximum likelihood estimator that retains all the asymptotic properties of MLE estimators (see Gouriéreux and Monfort, 1984). Notice however that the assumption of Σ_t constant over time is easily rejected by most data sets. In our case, it is likely (at least within a given class of contracts) that pricing errors might be long-lived and hence serially correlated. Therefore we resort to a further step. We regress (by OLS) the panel residuals on their lagged values and estimate the matrix R in the multivariate model

$$\hat{\epsilon}_t^{FGLS} = R \hat{\epsilon}_{t-1}^{FGLS} + \mathbf{u}_t \tag{5}$$

where $\mathbf{u}_t$ is spherical. Finally, we apply OLS to the (so called Prais-Winsten) transformed model

$$y_t - \hat{R} y_{t-1} = \beta_0 (I - \hat{R}) + (X_t - X_{t-1} \hat{R}) \beta_1 + \mathbf{u}_t,$$

which yields consistent and efficient estimates of $\hat{\beta}_0^{Parks}$ and $\hat{\beta}_1^{Parks}$**,** along with an unbiased estimate of their covariance matrix.

[20] In practice we iterate over the two steps of finding a consistent estimator for Ω based on the residuals obtained in step $i-1$, estimating $\hat{\beta}_{(i)}^{FGLS}(\hat{\Omega}_{(i-1)})$ and then calculating the corresponding residuals for step i until convergence of the estimates of β is obtained. Although our data set is relatively large, convergence is fast.

5.2. *Empirical Results*

For both the unfiltered and the arbitrage-free samples, Table 3 reports descriptive statistics (mean, median, and standard deviation) for each of the 20 classes defined above. Most of the contract classes are represented in the sample in a balanced way, although (as it is to be expected) long-term, DITM and DOTM contracts are underrepresented (less than 1,000 observations each). Given maturity, means and medians describe smiles for short maturities, and smirks for medium and long term contracts. Given moneyness, the term structure of implied volatilities is generally downward sloping, which is consistent with our previous remarks. Decreasing α has mainly the effect of expelling extreme IVs from the sample, and this is reflected in the smaller values of panel B, especially for short term contracts. Since all these impressions coincide with our comments in Section 3, we surmise that the reduced sample is highly representative of the original data. Therefore we apply the estimation procedure outlined above.

For the unfiltered data, Panel A of Table 4 reports on the output of the first step of the estimation procedure, i.e. $\hat{\beta}^{FGLS}$ and the p-values obtained from the covariance matrix $\left(X'\widehat{\Omega}X\right)^{-1}$. If we were really convinced that IVS errors are serially uncorrelated, these would be our panel estimates. All the parameter estimates are statistically significant at p-values indistinguishable from zero, and the resulting $\bar{R}^2$ are of the same order of magnitude. Model 8 returns the highest $\bar{R}^2$ (0.11) while model 9 provides a poor fit. However serial correlation of the IV disturbances is troublesome, as stressed by very low and highly significant Durbin-Watson statistics in the last column of the Table. Therefore we apply the second stage of Parks's method. We estimate by OLS the model:

$$\hat{\epsilon}_t^{FGLS} = \rho I_{20} \hat{\epsilon}_{t-1}^{FGLS} + \mathbf{u}_t,$$

a simplification of (5) to the case in which serial correlation is common in intensity to all classes of option contracts. Since we do not have any theoretical reason to assume that IV shocks have a different persistence as a function of moneyness and/or time-to maturity, and this assumption remarkably simplifies the task, we proceed to derive our final (Parks) estimates from the Pras-Winsten modified regression:[21]

$$y_t - \widehat{\rho} y_{t-1} = \alpha(1 - \widehat{\rho}) + (X_t - \widehat{\rho} X_{t-1})\beta + \mathbf{u}_t.$$

Panel B of Table 4 reports the results. As suspected from panel A, serial correlation is the principal problem plaguing the unfiltered data. Adopting Parks' GLS correction changes some of the estimates and in general increases the standard errors by several orders of magnitudes. In particular, the variable capturing the interaction effects between moneyness and time-to-maturity is always insignificant. However, the correction is quite successful, in the sense that now all the D-W statistics (not reported) fall in the range [2, 2.5]. Comparing models 3 and 7, and models 7 and 8, it appears

[21] The estimates of ρ are reported in the last column of panel B of table IV. In general they are in the range 0.89-0.90 and all of them are highly significant. Davidson and MacKinnon (1993, pp. 371-372) give also technical reasons for why it might be wise to specify R as a scalar matrix.

that in order to obtain a good fit incorporating time-to-maturity is crucial, while also squared time-to-maturity helps.[22] Figure 6 (right graph) plots the IVS implied by the Parks' coefficient estimates under model 8, the one guaranteeing the highest $\bar{R}^2$ in panel A of Table 4. Notice how the model captures the transition from perfectly symmetric smiles for short-term options to flatter and more asymmetric smiles (smirks) for medium- and long-term contracts. The fitted term structure of implied volatility is instead downward sloping for ATM and ITM contracts and describes another smile for DOTM options.

Figure 6 about here

Table 5 applies Parks' estimation procedure to the arbitrage-free data set. Table VI inspects what lies between the two extremes of $\alpha \to \infty$ and $\alpha = 0$. In the case of the arbitrage-free sample, the same process under (a) above, gives a data set of 20,356 observations, more than 50% of the original sample size. Panel A of Table 5 reports the output of the first step of the Parks' estimation procedure. There is only one remarkable difference with respect to Table 4: on arbitrage-free data, model 9 outperforms model 8, displaying significant estimates only and (in panel A) a striking $\bar{R}^2 = 0.31$. Apparently, data sets plagued by violations of basic no-arbitrage conditions display a law of motion for the IVS which is sensibly different from the one characterizing arbitrage-free option prices. Comparing Tables 4 and 5 it is also apparent that a few estimated coefficients in the best fitting models do switch signs. A last interesting finding is that while an econometrician using the unfiltered data would be probably led to infer that the interactions effects between maturity and moneyness are hardly important and certainly insignificant under a statistical viewpoint, the estimated coefficients associated with interactions become on the contrary quite significant when data of better quality are used. This is comforting, offering an explanation for the term structure shifts uncovered in Section 3.

Table VI further stresses these points by comparing the rankings of the best four models in terms of fit (as measured by their $\bar{R}^2$). Apart from the drastic change of status of model 9, we also observe that model 3 — implying a very simple smiling structure of the IVS without any term structure or interaction effects — looses importance and finally disappears from the rankings as the pricing efficiency underlying the data is increased throughout our experiments. Although the differences are not alarming, as α declines many coefficients become smaller albeit estimated with higher precision. Figure 6 completes the picture by plotting the estimated MIBO IVS under the best fitting model when $\alpha = 0$. Obviously the left-hand panel strongly differs from the right-hand one. Not only the MIBO IVS is rich and therefore worthy of empirical analysis, but what can we learn about it seems to be related in quite a precise way to the degree of efficiency we impute to the Italian options market.[23]

[22]Notice that the $\bar{R}^2$ in panel B of Table IV cannot have the same interpretation as the ones in panel B. Another disadvantage of this procedure that should be pointed out is that — because of the use of lagged variables in the regressions — many less observations are actually available for estimation purposes.

[23]We have also applied a shortcut approach that has proven rather popular in the literature (e.g. Ncube (1996), Dumas et al. (1998), and Peña et al. (1999)): OLS regressions on a sequence of cross sections, one at each point in time. This strategy completely disregards the panel nature of the data and implies a remarkable loss of efficiency in

5.3. *Analysis on first-differenced data*

There is at least one unresolved issue left open by the analysis so far: how restrictive is the assumption that $R = \rho I_{20}$, i.e. that the serial correlation coefficient of the random shocks must be identical across moneyness and maturity classes? Instead of generalizing Parks' method to the case of R full matrix, we take a shortcut that maintains the flavor of Parks' procedure but that appears to more closely correspond to a number of papers that have formally modeled implied volatilities (e.g. Christensen and Prabhala, 1998): we apply FGLS on a panel in which the regressands are defined as the first difference of log-implied volatility. For instance, in the case of model 8:

$$\begin{aligned} 8''.\ \Delta \ln \sigma^{IV}(z,\tau) &= \beta_0 + \beta_1 \Delta \ln z_t + \beta_2 (\Delta \ln z_t)^2 + \gamma_1 \Delta\tau_t + \gamma_2 (\Delta \ln z_t) \Delta\tau_t + \\ &\quad + \gamma_3 (\Delta\tau_t)^2 + \epsilon(z,\tau), \end{aligned}$$

where $\Delta \ln \sigma^{IV}$ is defined as the change in log-implied volatility within the same moneyness and maturity class over two consecutive trading days. Similar transformations apply to all variables and models investigated in the paper. Clearly the transformed model has a different meaning relative to the original one, as now changes in log-moneyness and time-to-maturity explain not the log-level of the IV, but its change over time. It is similarly obvious that all problems of excessive serial correlation in ϵ_t caused by the persistence in log-implied volatilities, ought to disappear once the transformed model is embraced. However, such type of models are on equal footing as 1.-9. if the objective is not only to explain how the IVS looks like, but instead whether our ability to empirically pin down its properties do in fact depend on the quality of the underlying data.

Panels C of Tables 4 and 5 show results for the models in first differences. Results are qualitatively similar at least in two ways. First, once more the ranking across models provided by the $\bar{R}^2$ statistic strongly depends on the quality of the data employed in the estimation.[24] Therefore while the unfiltered data ($\alpha \to \infty$) reveal that the best fit is unequivocally provided by model 6, the arbitrage-free data set ($\alpha = 0$) shows that once more the superior model is 8. Also in this case, as the quality of the data set improves, the $\bar{R}^2$s increase from 1-2% to almost 3%. Second while Table 4, panel C shows some discontinuity vs. the Parks' estimates in panel B, this does not occur in Table 5: for instance, out of 23 estimated slope coefficients, 20 keep their signs unchanged going from panel B to C, and in at least half of the cases the magnitude of the estimates are practically

the estimates. Moreover, its output consists of a time series of estimated coefficients, a different vector for each point in time covered by the sample. Although reporting means or medians of the estimates over time is common practice, the logical background for this operation is unclear. It turns out that model 8 is consistently the best, with median $\bar{R}^2$s ranging from 0.67 to 0.87, followed by model 9. Although the ranking over models is not affected by the efficiency of the data used in the estimation, we have other indications that the presence of misspriced options does matter for an econometrician's perception of the IVS. First, average and median $\bar{R}^2$s systematically increase as α declines; second, for large αs, it is common to find large differences between average and median estimates, a sign of instability over the sequence of cross sections. Detailed results are available upon request.

[24] Notice that FGLS for first-differenced data allow us to interpret the $\bar{R}^2$ in standard fashion. One additional advantage is that the number of useful observations for estimation purposes is close to the original sample size, roughly 20,000 observations.

identical; in particular, there are clear indications that implied volatilities decrease in time to maturity and increase in moneyness, although significant interactions between maturity and moneyness emerge in models 7 and 8. The only structural change that can be observed involves the squared moneyness regressor, that fails to be significant when first-differenced models are entertained.

5.4. *Misspecification tests*

In Section 5.2 we have established provisional rankings across models that fail to lie on firm grounds by relying on FGLS $\bar{R}^2$, i.e. referred to estimates that do not correct for serial correlation in the residuals. It is therefore important to develop ways to formally assess if any of the models might be considered correctly specified in the light of the information carried by opponent models. In this final subsection, we take care of this point.

With reference to the formal ranking of the models under analysis, some of the tests are easily implemented because of their nested nature: given a pair of models that differ only by the fact that one model employs additional regressors relative to the opponent, it is well known (e.g. Davidson and MacKinnon, 1992, pp. 193-194) that a specification check consists simply of testing (using F or likelihood ratio statistics) whether the additional regressors significantly improve the model's fit. In this respect, a number of nesting relationships are clear from the models' list initially provided: 1. is nested within 2.; 2. within 3.; 3. within 7., and 7. within 8.; 5. is nested within 6. Exploiting these relationships and the fact that when two models differ only by one regressor the F-statistic is just the squared t-statistic, some of the nested misspecification tests can simply be 'eye-balled' from Tables 4 and 5. For instance, looking at panel C (when data are first-differenced) of Table 5, it turns out that while the null of correct specification of 1. (no explanatory variable) is clearly rejected, there is no evidence in favor of 3. when 2. is the null model; also, 5. appears to be misspecified in the light of 6., and 7. in the light of 8. Therefore the residual nested test involves models 2. and 8., and clearly supports model 8.[25] Similar sequences of nested tests reveal that the correct specification of 2. and 7. cannot be rejected when working with unfiltered data (i.e. panel C of Table 4), with 7. supported over 2. by a LR test.[26]

However, some interesting comparisons cannot be simply performed using t- or LR-tests as the corresponding models are non-nested. In particular, with high quality data, it remains to be tested whether 6. is misspecified in the light of 8., and whether 8. is supported given the fit provided by 9.; with unfiltered data, non-nested tests ought to involve 6. and 7., and 7. and 9. We briefly recall the tools required by non-nested model testing and then proceed to comment on the results (see Davidson and MacKinnon, 1992 and 1993)). For concreteness let's examine the case of models 6.

[25] 2. and 8. differ by as many as 4 regressors. The implied LR statistic is 1,439.4, which is obviously highly significant under a $\chi^2_{(4)}$.

[26] 2. and 7. differ by 3 regressors. The implied LR statistic is 176.0, which is highly significant under a $\chi^2_{(3)}$.

and 8., although the principle easily generalizes. Consider the two models

$$H_1 : \Delta \ln \sigma_t^{IV} = \beta_0 + \beta_1 \Delta OTM_t + \beta_2 (\Delta \ln z_t)^2 + \beta_3 \Delta ITM_t + \epsilon_t = \mathbf{x}_t' \theta_1 + \epsilon_t$$
$$H_2 : \Delta \ln \sigma_t^{IV} = \beta_0 + \beta_1 \Delta \ln z_t + \beta_2 (\Delta \ln z_t)^2 + \gamma_1 \Delta \tau_t + \gamma_2 (\Delta \ln z_t) \tau_t + \\ + \gamma_3 (\Delta \tau_t)^2 + \epsilon_t = \mathbf{z}_t' \theta_2 + \epsilon_t$$

where $\mathbf{x}_t \equiv [1 \ \Delta OTM_t \ (\Delta \ln z_t)^2 \ \Delta ITM_t]'$, $\mathbf{z}_t \equiv [1 \ \Delta \ln z_t \ (\Delta \ln z_t)^2 \ \Delta \tau_t \ (\Delta \ln z_t) \tau_t \ (\Delta \tau_t)^2]'$ which clearly contains 4 columns that cannot be written as linear combinations of columns of $\mathbf{x}_t$. Two tests can be now performed: First, a t- test of $\zeta = 0$ in the artificial (compound) regression

$$H_c : \Delta \ln \sigma_t^{IV} = (1-\zeta) \mathbf{x}_t' \theta_1 + \varsigma \mathbf{z}_t' \hat{\theta}_2 + u_t = \mathbf{x}_t' \theta_1 + \zeta \left(\mathbf{z}_t' \hat{\theta}_2 - \mathbf{x}_t' \theta_1 \right) + u_t,$$

where $\hat{\theta}_2$ is the FGLS estimate of model H_2. Clearly, the null of $\zeta = 0$ implies that no significant additional fit is provided by model H_2 and hence that there is no evidence of misspecification of H_1. This test is commonly called a J (joint) test.[27] Importantly, the panel nature of our data set that advises using GLS method poses no problem to the validity of this artificial regression approach, see Davidson and MacKinnon (1992, p. 127). Second, one can use a t- test of $\phi = 0$ in the artificial (compound) regression

$$H_c' : \Delta \ln \sigma_t^{IV} = \phi \mathbf{x}_t' \hat{\theta}_1 + (1-\phi) \mathbf{z}_t' \theta_2 + u_t = \mathbf{z}_t' \theta_2 + \phi \left(\mathbf{x}_t' \hat{\theta}_1 - \mathbf{z}_t' \theta_2 \right) + u_t,$$

where $\hat{\theta}_1$ is the FGLS estimate of model H_1. Clearly, the null of $\phi = 0$ implies that no significant additional fit is provided by model H_1 and hence that there is no evidence of misspecification of H_2. Interestingly, it is possible that either both models be rejected as misspecified (i.e. $\zeta \neq 0$ and $\phi \neq 0$) or that both models pass the misspecification test ($\zeta = 0$ and $\phi = 0$).

We report results in Table 7 for models 6, 7, and 9 when $\alpha \to \infty$, and 6, 8, 9 when $\alpha = 0$. As explained, tests must be performed for each pair of models by allowing each of them to play the role of H_1. With reference to the unfiltered data set, there is clear evidence in favor of model 7: while this model does not appear to be misspecified when it is artificially compounded with models 6 and 9, all other models are misspecified. Once more, switching to a high quality data set changes the conclusions on which model is less likely to be misspecified, as model 9 emerges as the only framework that does not admit 'integration' with fit information provided by either models 6 or 8.

Although our analysis just scratches the surface of a systematic investigation of structural models of the IVS dynamics, two closing considerations are in order. First, the casual observation that the IVS is very complicated and capable of displaying many heterogenous patterns is confirmed by Figure 6, where it is obvious that the relationship between volatility and moneyness (and hence the

[27] Alternatively, when the artificial compound regression takes the form

$$\Delta \ln \sigma_t^{IV} - \mathbf{x}_t' \hat{\theta}_1 = \mathbf{x}_t' \mathbf{b} + \varsigma \left(\mathbf{z}_t' \hat{\theta}_2 - \mathbf{x}_t' \hat{\theta}_1 \right) + u_t,$$

where $\hat{\theta}_1$ is the FGLS estimate of the parameters in model H_1, the test is called a P-test. We have also tried this variant of the non-nested methodology obtaining qualititatively identical results. In practice, the J test is known to reject too much in small samples. This does not seem to be too problematic given our sample size.

MIB30 index) is also a function of the time to expiration of the contracts considered, and that the term structure of IVs depends on moneyness. The MIBO IVS is truly a tridimensional concept. Second, we have indirectly documented a massive impact of the possible presence of observations containing arbitrage opportunities in a data set and the choice of reduced form model of the IVS. This is likely to be crucial both for understanding the process of price formation in derivative markets and for forecasting and operational purposes.

6. **Applications**

6.1. *Value-at-Risk Estimation*

The value-at-risk (VaR) of a given portfolio summarizes the expected maximum loss over a target horizon within a given confidence interval.[28] In particular, the time t *relative* VaR at a confidence level η over a horizon T is defined as

$$VaR(\eta, T) = E_t[V_{t+T}] - V^{\eta}_{t,t+T},$$

where $V^{\eta}_{t,t+T}$ is such that $\Pr_t\{V_{t+T} \leq V^{\eta}_{t+T}\} = \eta$ (the η-th percentile) and both $E_t[\cdot]$ and $\Pr_t\{\cdot\}$ are conditional to the information available at time t (see Jorion, 2001, p. 109).

The MIB30 options IVS models estimated in Section 5 offer an opportunity to perform VaR calculations for many portfolios containing assets whose payoffs/returns depend on the MIB30 index. Consider for instance the fitted values produced by model 9 on some data set of quality determined by the parameter α and using some estimation technique:[29]

$$\begin{aligned} \ln \widehat{\sigma^{IV}(z,\tau)} &= \hat{\beta}_0 + \hat{\beta}_1 \frac{\ln S_t + (r_t - \delta_t)\tau - \ln K}{\sqrt{\tau}} + \hat{\beta}_2 \frac{[\ln S_t + (r_t - \delta_t)\tau - \ln K]^2}{\tau} + \\ &\quad + \widehat{\gamma}_1 \tau + \widehat{\gamma}_2 [\ln S_t + (r_t - \delta_t)\tau - \ln K] \sqrt{\tau}. \end{aligned} \tag{6}$$

Once the fitted values are available, calculation of $\hat{\sigma}^{IV}(z,\tau)$ is straightforward. Given the current stock price S_t and the interest rate r_t, assume that the implied volatility $\hat{\sigma}^{IV}(z,\tau)$ represents an unbiased and efficient forecast of the future, average volatility realized over the interval $[t, t+\tau]$.[30] Therefore $\hat{\sigma}^{IV}(z,\tau)$ represents the (annualized) forecast of MIB30 volatility for a period of length τ. Obviously, such a volatility is also a function of moneyness and therefore changes (for a given contract) as S_t changes. This opens interesting possibilities to exploit our (parametric) knowledge of the predictable component in implied volatility, which is essential for VaR applications.[31] Suppose

[28] Jorion (2001) is a readable introduction to the theory and practice of VaR. Batten and Fetherson (2002) collects papers documenting recent advances and open issues in risk management.

[29] Similar remarks hold for all IVS models entartained in this paper.

[30] For Black-Scholes implied volatilities, this is approximately correct only for ATM contracts (see Poteshman (2000)): $\sigma^{IV}(z,\tau) = E\left[\tau^{-1}\int_t^{t+\tau} \sigma_s ds\right]$. However, many Authors (e.g. Canina and Fliglewski (1993)) have justified the expectation that implied volatilities should be unbiased predictors of subsequently realized volatilities using a projection argument for *all* levels of moneyness.

[31] For instance, Jorion (2001, p. 184) argues that "(...) time series models [of predictable variation in volatility] are inherently inferior to forecasts of risk contained in option prices.". Options implied volatilities are in fact inherently

one needs to simulate a path for the MIB30 index over the interval $[t, T]$. Observe that by setting $\hat{\beta}_1' \equiv \hat{\beta}_1 \ln S_t$, $\hat{\beta}_1'' \equiv \hat{\beta}_1(r_t - \delta_t)$, $\hat{\beta}_2' \equiv \hat{\beta}_2 (\ln S_t)^2$, $\hat{\beta}_2'' \equiv \hat{\beta}_2(r_t - \delta_t)^2$, $\hat{\beta}_2''' \equiv \hat{\beta}_2(\ln S_t)(r_t - \delta_t)$, $\hat{\beta}_2^{iv} \equiv \hat{\beta}_2(\ln S_t)$, $\hat{\beta}_2^{v} \equiv \hat{\beta}_2(r_t - \delta_t)$, $\widehat{\gamma}_2' \equiv \widehat{\gamma}_2 \ln S_t$, and $\widehat{\gamma}_2'' \equiv \widehat{\gamma}_2(r_t - \delta_t)$, one can re-write (6) as:

$$\begin{aligned}\ln \widehat{\sigma^{IV}(K,\tau)} \;&=\; \hat{\beta}_0 + \hat{\beta}_1' \frac{1}{\sqrt{\tau}} + \hat{\beta}_1'' \sqrt{\tau} - \hat{\beta}_1 \frac{\ln K}{\sqrt{\tau}} + \hat{\beta}_2' \frac{1}{\tau} + \hat{\beta}_2'' \tau + \hat{\beta}_2 \frac{(\ln K)^2}{\tau} + 2\hat{\beta}_2''' + \\ &\quad -2\hat{\beta}_2^{iv} \frac{\ln K}{\tau} - 2\hat{\beta}_2^{v} \ln K + \widehat{\gamma}_1 \tau + \widehat{\gamma}_2' \sqrt{\tau} + \widehat{\gamma}_2'' \tau^{3/2} - \widehat{\gamma}_2 (\ln K)\sqrt{\tau}.\end{aligned}$$

This shows that conditional to asset prices at time t, the implied volatility function may be written in the arguments τ and K only, with S_t, r_t, and δ_t absorbed in the values of the estimated coefficients. At this point, assuming the MIB30 changes at points $t+\tau$, $t+2\tau$, ..., $t+T$, it is possible to generate the return $r_{t,\tau}$ for the period $[t, t+\tau]$ from a conditional density with volatility $\hat{\sigma}^{IV}(S_t, \tau)$, the return for the period $[t+\tau, t+2\tau]$ from a conditional density with volatility $\hat{\sigma}^{IV}(S_{t+\tau}, \tau)$ (where $S_{t+\tau} = S_t(1 + r_{t,\tau})$), and so on; the final $t+T$ MIB30 index will be given by

$$S_{t+T} = S_{t+T-\tau}(1 + r_{t+T-\tau,\tau}) = S_t \times (1 + r_{t,\tau}) \times (1 + r_{t+\tau,\tau}) \times ... \times (1 + r_{t+T-\tau,\tau}),$$

where $r_{t+T-\tau,\tau}$ has been generated from a density with volatility $\hat{\sigma}^{IV}(S_{t+T-\tau}, \tau)$.

In the following, we focus on the simplest possible portfolio, the MIB30 itself, although extensions to more interesting cases (for instance, to combinations of the MIB30 and protective – i.e. long – puts) in which multiple risk factors are relevant and the portfolio value is a nonlinear function of these factors are logically straightforward. We apply two alternative and popular VaR methods: the delta-normal and a Monte Carlo simulation approach. Table 8 shows results for both methods for two confidence levels ($\eta = 5$ and 1 percent) and two alternative horizons (1 and 12 months). Calculations are performed at the end of our sample period, at the closing of January 31, 2000 when the MIB30 index was 42,130 and the yield curve approximately spanned the interval 3-6% at various maturities.

The delta normal is a local valuation method that makes a parametric assumption for the distribution of the risk factors, in our case MIB30 index returns. The most common among the parametric assumptions, conditional normality, leads to the following closed-form expression for $VaR(\eta, T)$:[32]

$$VaR(\eta, T) = c(\eta) S_t \hat{\sigma}^{IV}(S_t, T),$$

where $c(\eta)$ is the value of a standard normal Z such that $\Pr\{Z \leq c(\eta)\} = \eta$ (i.e. 1.645 if $\eta = 0.05$ and 2.326 if $\eta = 0.01$), and $\hat{\sigma}^{IV}(S_t, T)$ is the ATM forecast of volatility over the period $[t, t+T]$. Similarly to Table 6, Table 8 (panel A) reports VaR estimates for the best IVS model under alternative assumptions on the amount of frictions (α) characterizing the IDEM, in the range $\alpha = 0\%$ to

forward looking, for instance discounting possible structural breaks that are necessarily reflected by historical data only a long time after the occurrence of the break.

[32] The assumption of normality over $[t, T]$ is standard in some VaR literature but does not need to hold in practice. Panel B of Table VIII is an implicit assessment of the quality of this assumption as predictable, time-varying volatility delivers an unconditional $[t, T]$ distribution that is potentially highly non-normal.

$\alpha \to \infty$. Clearly, since different values for α impose the choice of different IVS models and lead to heterogeneous parameter estimates (even within the same model), we can assess the effects of using data sets of poor quality by comparing the resulting VaR measures. The last row in the panel also reports results for the level of α (2%) we consider most plausible when the presence of serial correlation and heteroskedasticity is ignored altogether and simple OLS estimation on pooled cross section - time series data is performed. Such a strategy is hardly correct given the features of the data set, but may represent a tempting short-cut in practice. The results show a clear dichotomy between the first-order effects due to the selection of the IVS model, and second-order effects caused by heterogeneous estimates obtained from data sets of different quality in terms of ruling our arbitrage opportunities. Model 8 $-$ that would be selected on the basis of the 'unfiltered' data set $-$ leads to grossly inflated VaR estimates for all levels of η and the horizons T. The error is potentially large, easily in the order of a few thousand points of the MIB30 index. When model 9 is selected as the best fitting one (which happens for all finite αs), the precise estimation techniques and hence parameter values obtained have relevant effects, but in the order of a few hundreds index points only. Interestingly, using OLS methods that ignore the presence of non-spherical structure in implied volatility shocks, seems to lead to underestimation of the risk effectively implied by the MIB30 index, especially over short time intervals.

The Monte Carlo approach is a full valuation method that consists of repeating successive draws of MIB30 returns from a parametric conditional distribution to create a set of Q independent simulated paths, $\{r^q_{t+\zeta\tau}\}_{\zeta=0}^{(T-\tau)/\tau}$, $q = 1, 2, ..., Q$. The VaR is then calculated from the simulated T-periods ahead distribution of 'final' values of the MIB30 index, $\{S^q_{t+T}\}_{q=1}^{Q}$.[33] In our example, we take $Q = 20,000$ and assume that at each time $t + \zeta\tau$ the MIB30 index returns density is conditionally $N(\mu, \hat{\sigma}^{IV}(S_{t+\zeta\tau}, \tau))$, where $\hat{\sigma}^{IV}(S_{t+\zeta\tau}, \tau)$ is predicted from some IVS model. τ is set to match a bi-weekly frequency (i.e. 0.00521) and $\mu = 0.0059\%$, the bi-weekly mean return on the index.[34] Table 8 (panel B) reports VaR estimates for the best IVS model under alternative assumptions on the amount of frictions. Once more, since different values of α imply the choice of different IVS models and parameter estimates, we assess the effects of using data sets of poor quality by comparing the resulting VaR measures. The last row shows results for $\alpha = 2\%$ when simple OLS estimation on pooled cross section - time series data is performed. The results show first of all how imprecise the delta-normal method can be for the MIB30 index: for all ηs, the 1-month VaR seems to be overestimated by local methods while the one year VaR is on the opposite underestimated. In particular, the 12-month VaR characterizing the MIB30 seemed to be substantial at the end of January 2000, in excess of 50% of the value of the index then prevailing. The observed difference

[33] Chapter 12 of Jorion (2001) gives further details.

[34] The frequency to be used in simulations for VaR purposes is discussed in Jorion (2001, p. 293). $\tau = 1$ day (i.e. $\tau = 1/365$) may appear to be the other natural choice $-$ i.e. MIB30 index returns could be simulated at a daily frequency. We also use this different parameterization and notice minor differences relative to the results reported in Table VIII. However, $\tau = 1$ is also an akward choice in terms of our models of predictable variation in implied volatility, as very few contracts with only 1 calendar day to maturity were actually actively traded in our sample. On the opposite, $\tau =$ half-a-month is much more typical and actually close to the average option residual life of 26 days reported in Table I.

between first-order effects – related to model selection – and second order effects – related to parameter estimation – emerge: when the unfiltered data lead to selecting model 8, the VaR is systematically overestimated by several thousand index points. Incorrectly, employing simple OLS in place of the two-stage Parks' estimation method does not matter much for short horizons, but is important over the 12-month horizon.

6.2. *Portfolio Choice*

Another class of crucial financial decisions for which our results on the MIBO IVS dynamics are crucial is asset allocation. Among the others, Barberis (2000) and Campbell and Viceira (1999) have recently focused on optimal portfolio choice when excess stock returns follow realistic stochastic processes that imply the presence of predictability. Ferson and Siegel (2001) examine instead the asset allocation effects of the presence of stochastic volatility (heteroskedasticity) in excess stock returns. In this sub-section we propose a similar exercise for the simple case in which the riskless rate is set to be constant (at the Jan. 31, 2000 short-term, LIBOR level of 3.505 percent) and the process of MIB30 index returns is described by:

$$r_{t+\zeta\tau} \sim N(\mu, \hat{\sigma}^{IV}(S_{t+\zeta\tau}, \tau)),$$

i.e. index returns are conditionally normal with constant mean and time-varying volatility given by the predicted values of some (best-fitting) IVS model. Notice that the existence of correlation between volatility and the MIB30 level $S_{t+\zeta\tau}$ creates dependence through the second moment and predictability that can be exploited for asset allocation purposes.

The remaining details of the asset allocation exercise are standard in the literature. We consider a finite horizon investor, who maximizes expected utility from the consumption of final wealth, and who chooses between allocating funds to either the MIB30 index or the riskless asset (cash) that pays a constant rate of return:

$$\max_{\omega} \quad E_t\left[\frac{W_{t+T}^{1-\gamma}}{1-\gamma}\right]$$
$$s.t.\ W_{t+1} = W_t\left\{(1-\omega)\exp\left(Tr^f\right) + \omega\exp\left(\sum_{\zeta=0}^{(T-\tau)/\tau} r_{t+\zeta\tau}\right)\right\}, \tag{7}$$

where ω is the percentage weight to stocks, and $\sum_{\zeta=0}^{(T-\tau)/\tau} r_{t+\zeta\tau}$ represents the overall stock return over the period $[t, T]$. For simplicity, we further impose no-short sale restrictions, $\omega \in [0, 1]$. Clearly, this representation of the problem implies that the portfolio strategy is of a simple buy-and-hold type and that the investor has standard power, time-separable utility function with constant relative risk aversion γ. Needless to say, extensions to more realistic set-ups are possible (for instance, admitting the presence of rebalancing as in Lynch, 2001, or more realistic preferences as in Campbell and Viceira, 1999) but beyond the scope of this paper.

We solve the above portfolio choice problem employing Monte-Carlo methods as in Barberis (2000):

$$\max_{\omega} Q^{-1} \sum_{q=1}^{Q} \left\{ \frac{\left[(1-\omega)\exp\left(Tr^{f}\right) + \omega \exp\left(\sum_{\zeta=0}^{(T-\tau)/\tau} r_{t+\zeta\tau}^{q}\right)\right]^{1-\gamma}}{1-\gamma} \right\}.$$

The simulation methods are the same described in Section 6.1. We consider again $Q = 20,000$ and set τ to match a bi-weekly frequency (i.e. 0.00521) and $\mu = 0.0759 \times 0.00521$, the bi-weekly mean return on the index.[35] Optimal portfolio weights to the MIB30 index are shown in .Table 1X for three alternative investment horizons ($T = 1$, 12, and 60 months), and two different levels of the coefficient γ typical in the literature (4, 10, and 20). The calculation is also repeated for multiple values of α that imply the choice of different IVS models and parameter estimates in the $\hat{\sigma}^{IV}(S_{t+\zeta\tau}, \tau)$ functions. Therefore, also in this case we take interest in the effects of 'data quality' on financial decisions. Similarly to Table 8, the last row shows results for $\alpha = 2\%$ for the best IVS model under OLS estimation on pooled cross section - time series data. Also in this case, all asset allocation choices are calculated as of Jan. 31, 2000.

Interestingly, the patterns of dependence of optimal portfolio choices on the quality of the data and the methods of estimation/values of the parameters are completely consistent with the results obtained for VaR. This makes intuitive sense as optimal asset allocation is after all a function of the predictive density of MIB30 returns, while VaR simply takes interest in the extreme percentiles of this density. Once more, the real difference seems to be between model 8 and model 9, and we have reason to be suspicious of decisions supported by a model that is selected on the basis of data that contain an impressive percentage of arbitrage violations. In general, portfolio weights implied by model 8 are biased against equity holdings. Otherwise, all the parameter estimates obtained by data sets supporting the selection of model 9 are similar, although some puzzling deviation (for $T = 12$ months) can be observed in the case in which model 9 is estimated using (incorrect) OLS methods on pooled cross section/time series. In general, the stocks vs. cash (domestic) allocation of an Italian investor moves away from stocks towards the riskless asset as the investment horizon grows, which is consistent with the increasing VaR uncovered in Table 8: the weight of the tails of the MIB30 returns density grows with T at a speed possibly higher than the mean, thus tilting the optimal risk/return trade-off away from equities.[36]

[35] We also experiment with $\tau =$ one day (i.e. $\tau = 1/365$) and notice insignificant differences relative to the results reported in Table IX. Notice that the mean is set to match the sample mean return on the MIB30 over a longer time interval, 01/01/1995-01/31/2000, to avoid performing rather unrealistic long-term (5 year) asset allocation with a negative equity premium. Using $\mu = 0.141\%$ would have given very modest values for $\hat{\omega}$, always below 0.05.

[36] As expected, the allocation to stocks declines as risk aversion γ increases. The no-short sale constraint binds only for $T = 1$ month and $\gamma = 4$ and thus is unlikely to be responsible for the results on the importance of appropriate selection and estimation of an IVS model.

7. Conclusion

This paper has analyzed the structure of the implied volatility surface — the trivariate relationship between stock index return volatilities implied by option prices, moneyness, and time to maturity — characterizing the Italian stock index options market, the MIBO. A first contribution of the paper is to perform an exploratory analysis of the determinants of the MIBO IVS. Since in a companion paper (see Cassese and Guidolin, 2004) we have found that the MIBO is characterized by resilient niches of pricing inefficiency that can hardly be explained by sensible levels of transaction costs, we try to map the quality of the data in terms of incidence of violations of a number of no-arbitrage restrictions into the perception of the MIBO IVS an econometrician would develop by estimating a reduced form model. A second contribution of the paper is technical, as models of the IVS are estimated using GLS panel techniques that fully accommodate for the presence of heteroskedasticity and serial correlation in option pricing errors.

We find that the MIBO IVS possesses a number of features of its own that have not been previously documented. More importantly, these feature seem to strongly depend on the quality of the data employed in the estimation. As a rough approximation, smiles seem to better describe low quality data sets containing high percentages of arbitrage opportunities. The structure of the IVS is on the other hand better characterized by a combination of implied volatility smirks and downward sloping term structures as observations causing mispricing are progressively eliminated. Finally, the applications in Section 6 illustrate that such incorrect perceptions of the IVS may have important (at times devastating) effects on financial decisions/assessments (such as VaR) that more and more are predicated as optimal when based on implied, derivative-driven parameters. We are left wondering about the potential impact that reduced pricing efficiency might have had already on our perception of the pricing mechanisms at work in many other derivatives market which are younger and probably not as liquid as the North American benchmarks.

Acknowledgment: We wish to thank Alberto di Stefano from *BSI* (Banca della Svizzera Italiana) who has made available the dataset on which this paper is based, Damiano Brigo and all seminar participants at the Fifth Workshop of Quantitative Finance, in Siena, Italy. One anonymous referee provided stimulating comments and helped improving the paper.

References

[1] Ackert, L., Tian, Y., 2001. Efficiency in Index Options Markets and Trading in Stocks Baskets. Journal of Banking and Finance, 25, 1607-1634.

[2] Barberis, N., 2000. Investing for the Long Run When Returns Are Predictable. Journal of Finance, 55, 225-264.

[3] Barone, E., Cuoco, D., 1989. The Italian Market for 'Premium' Contracts: An Application of Option Pricing Theory. Journal of Banking and Finance, 13, 709-745.

[4] Batten, J., Fetherston, T. 2002, Financial Risk and Financial Risk Management, Elsevier Science, JAI.

[5] Black, F., Scholes, M. 1973. The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 81, 637-654.

[6] Campa, J., Chang, K. 1995. Testing the Expectations Hypothesis on the Term Structure of Volatilities. Journal of Finance, 50, 529-547.

[7] Campbell, J.Y., Viceira, L., 1999. Consumption and Portfolio Decisions when Expected Returns are Time Varying. Quarterly Journal of Economics, 114, 433-495.

[8] Capelle-Blancard G., Chaudury, M. 2001. Efficiency Tests of the French (CAC40) Options Market. SSRN Working Paper.

[9] Cassese, G., Guidolin, M., 2004. Pricing and Informational Efficiency of the MIB30 Index Options Market. An Analysis with High Frequency Data. Economic *Notes*, forthcoming.

[10] Cavallo, L., Mammola, P. 2000. Empirical Tests of Efficiency of the Italian Index Options Market. Journal of Empirical Finance, 7, 173-193.

[11] Christensen, B., Prabhala, N., 1998. The Relation between Implied and Realized Volatility. Journal of Financial Economics, 50, 125-150.

[12] Davidson, R., MacKinnon, J., 1992. Model Specification Tests and Artificial Regressions. Journal of Economic Literature, 30, 102-146.

[13] Davidson, R., MacKinnon, J., 1993. Estimation and Inference in Econometrics. Oxford University Press, Oxford.

[14] Dumas, B., Fleming, J. Whaley, R., 1998. Implied Volatility Functions: Empirical Tests. Journal of Finance, 53, 2059-2106.

[15] Ferson, W., Siegel, A., 2001. The Efficient Use of Conditioning Information in Portfolios. Journal of Finance, 56, 967-982.

[16] George, T., Longstaff, F., 1993. Bid-Ask Spreads and Trading Activity in the S&P 100 Index Options Market. Journal of Financial and Quantitative Analysis, 28, 381-397.

[17] Gemmill, G., 1996. Did Option Traders Anticipate the Crash? Evidence from Volatility Smiles in U.K. with U.S. Comparisons. Journal of Futures Markets, 16, 881-897.

[18] Gouriéreux, C., Monfort, A., 1984. Pseudo-Maximum Likelihood Methods: Theory. Econometrica, 52, 681-700.

[19] Gross, L., Waltners, N., 1995. S&P 500 Options: Put Volatility Smile and Risk Aversion. Salomon Brothers, mimeo.

[20] Jorion, P., 2001, Value at Risk. McGraw-Hill, New York.

[21] Kamara, A., Miller, T., 1995. Daily and Intradaily Tests of European Put-Call Parity. Journal of Financial and Quantitative Analysis, 30, 519-539.

[22] Lynch, A., 2001. Portfolio Choice and Equity Characteristics: Characterizing the Hedging Demands Induced by Return Predictability. Journal of Financial Economics, 62, 67-130.

[23] Mittnik, S., Rieken, S., 2000. Put-Call Parity and the Informational Efficiency of the German DAX-Index Options Market. International Review of Financial Analysis, 9, 259-279.

[24] Ncube, M., 1996. Modelling Implied Volatility with OLS and Panel Data Models. Journal of Banking and Finance, 20, 71-84.

[25] Newey, W., West, K., 1987. A Simple, Positive Semi-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. Econometrica, 55, 703-708.

[26] Nikkinen, J., 2003. Normality Tests of Option-Implied Risk Neutral Densities: Evidence from the Small Finnish Market. International Review of Financial Analysis, 12, 99-116.

[27] Nisbet, M., 1992. Put-Call Parity Theory and an Empirical Test of the Efficiency of the London Traded Options Market. Journal of Banking and Finance, 16, 381-403.

[28] Parks, R., 1967. Efficient Estimation of a System of Regression Equations When Disturbances are Both Serially and Contemporaneously Correlated . Journal of the American Statistical Association, 62, 500-509.

[29] Peña, I., Rubio, G., Serna, G., 1999. Why Do We Smile? On the Determinants of the Implied Volatility Function. Journal of Banking and Finance, 23, 1151-1179.

[30] Poteshman, A., 2000. Forecasting Future Volatility from Option Prices. mimeo, University of Illinois.

[31] Puttonen, V., 1993. Boundary Conditions for Index Options: Evidence from the Finnish Market. Journal of Futures Markets, 13, 545-563.

[32] Rénault, E., 1997. Econometric M*odels of Option Pricing Errors. in: Advances in Economics and Econometrics:* Theory and Applications: Seventh World Congress, volume 3, 223-78

[33] Ronn, A., Ronn, E., 1989. The Box Spread Arbitrage Conditions: Theory, Tests and Investment Strategies. Review of Financial Studies, 2, 91-108.

[34] Rubinstein, M., 1985. Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 Through August 31, 1978. Journal of Finance, 40, 455-480.

[35] Rubinstein, M., 1994. Implied Binomial Trees. Journal of Finance, 49, 781-818.

[36] Tompkins, R., 1999. Implied Volatility Surfaces: Uncovering Regularities for Options on Financial Futures. working paper No. 49, University of Vienna.