

# Pricing and Informational Efficiency of the MIB30 Index Options Market.

An Analysis with High Frequency Data<sup>\*</sup>

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## Abstract

We analyze the pricing and informational efficiency of the Italian market for options written on the most important stock index, the MIB30. We report that a striking percentage of the data consists of option prices violating basic no-arbitrage conditions. This percentage declines when we relax the no-arbitrage restrictions to accommodate for the presence of bid/ask spreads and other frictions, but never becomes negligible. We also investigate the informational efficiency of the MIBO and conclude that option prices are poor predictors of the volatility of MIB30 returns. This conclusion is robust to a number of statistical and sampling methods.

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## 1. Introduction

In this paper we investigate the efficiency properties of the market for options on the Italian MIB30 index, the MIBO market, one of the most important segments of the Italian Derivatives Market (IDEM). Following the creation of the index futures market (FIB30) by approximately one year, trading on the MIBO started in November 1995. The birth of the MIBO has been a crucial step towards the completion of the Italian stock market. MIB30 index options have soon become an important financial instrument, especially for institutional investors (like mutual funds) whose portfolios have typically a large share allocated to securities with a high degree of correlation with the MIB30 index.<sup>1</sup>

The importance of the Italian MIBO market parallels the prominent role of index options markets in modern financial systems. In the first place, these markets are essential for risk sharing. On the one hand, options enable portfolio managers to improve their ability to hedge the risk of unpredictable changes of financial prices. On the other hand, investors may easily take speculative positions consistent with their views on future asset price movements. In the absence of well functioning option markets both these activities — hedging and speculation — would be too costly or simply unfeasible. Second, index options markets represent the best available instrument for aggregating investors' opinions concerning the future volatility of asset returns. Therefore, an efficient options market should: (*i*) foster the implementation of hedging and speculative activities at affordable costs (*risk-sharing and pricing efficiency*), and (*ii*) accurately aggregate market beliefs concerning asset returns volatility (*informational efficiency*). The main objective of our paper is to assess these two distinct but correlated notions of efficiency of the Italian MIBO market using a high frequency data set spanning from April 1999 to January 2000. Our investigation gives us an opportunity to contribute to two distinct literatures.

First, the analysis of the risk-sharing efficiency of a financial market always raises issues on the existence of arbitrage opportunities, the sharpest antithesis to risk sharing. The assessment of the presence of arbitrage opportunities has a long tradition in empirical finance.<sup>2</sup> In fact for frictionless markets it is possible to derive constraints on options prices that, if not respected, represent arbitrage opportunities that may be exploited by explicit portfolio strategies. This approach has the advantage of being free of any theoretical assumption (save individual rationality), and provides sensible predictions based solely on first principles. The best example is the test of the put/call parity, which has been repeatedly performed over the years for different markets and samples (see Stoll (1969) for pioneer work). We will test for the presence of arbitrage opportunities in Section 3. We improve on the previous literature by proposing a wider and more complete set of no arbitrage restrictions. In

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<sup>1</sup>During 1999 (the year to which our analysis refers) the volume of trades on stocks belonging to the MIB30 basket (measured in domestic currency) accounted for approximately 76.4% of the total trading volume on the Italian stock exchange.

<sup>2</sup>Seminal papers are Stoll (1969), Gould and Galai (1974), and Klemkosky and Resnik (1979). More recently this issue has been addressed by Ackert and Tian (1999, 2000), George and Longstaff (1993), Kamara and Miller (1995), Nisbet (1992), Ronn and Ronn (1989), and Yadav and Pope (1994). However most of these papers focus on North American markets, especially on S&P 100 and S&P 500 options.

particular, we develop and test a new condition, the *maturity spread*, that is particularly useful in establishing a link between the level of efficiency of options and futures markets. Also, we carefully distinguish between tests that jointly rely on the efficiency of the markets for options and the underlying from tests exclusively measuring MIBO efficiency (see Ackert and Tian (2000)). While in some papers (see Nisbet (1992)) it has been shown that most arbitrage opportunities vanish after the introduction of frictions,<sup>3</sup> in Section 4 we take the view that the amount of market imperfections should itself be considered an indicator of market efficiency. Therefore, differently from previous contributions, we avoid conducting a simple test of market efficiency *given* transaction costs but rather treat frictions as a *parameter* that we let assume different values. We interpret efficiency as a concept with two dimensions, arbitrage opportunities *and* the size of transaction costs, and we then represent our findings via a curve that describes how arbitrage opportunities vanish when transaction costs are increased. Given the novelty of the concept of efficiency we adopt, the comparison with other papers can only be indirect: this is our stronger motivation for conducting one further study on the efficiency of financial markets.

Second, starting from Canina and Figelewski (1993), several authors have measured the informational efficiency of options markets by testing the unbiasedness of implied volatility as a predictor of ex post, realized volatility of stock returns. The common finding has been that implied volatilities are poor forecasts of future volatility and that a prediction based on option prices can easily be improved upon by using variables commonly included in the agents' information sets. These conclusions have been recently challenged by a few papers, e.g. Christensen and Prabhala (1998). In Section 5 we supplement traditional methods of investigation (such as GMM) with a number of econometric approaches, including novel, panel-oriented tools. Furthermore, we explore the relationship between pricing and informational efficiency: although the two concepts are independent one from another, one is allowed to expect evidence of correlation. We do this by probing the robustness of our results to the use of data sets exposed in different degrees to the presence of arbitrage opportunities

Our results for the Italian options market are that the no arbitrage restrictions are not satisfied for a high percentage of the data and this suggests that market frictions must be incorporated when testing arbitrage relationships. We then compute the level of frictions which would be consistent with a reasonably low ratio of arbitrage opportunities and find that implied frictions are quite substantial. The conclusion so obtained is further strengthened by investigating the informational efficiency of the market. Consistently with results concerning other markets, we report that MIBO implied volatilities are poor predictors of future volatility.

A limited number of papers have already examined the Italian index options market (see Barone and Cuoco (1989)). In particular, in a recent paper Cavallo and Mammola (2000) investigate the efficiency properties of the MIBO market. Similarly to us, they use high-frequency observations.<sup>4</sup> However they only test the put-call parity relation and apply their tests to at-the-money, short-

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<sup>3</sup>Cavallo and Mammola is an example directly pertaining to the MIBO30. However other Authors (see Ackert and Tian (2000)) establish that arbitrage violations persist despite frictions, although to a lesser extent.

<sup>4</sup>Their data set is shorter and refers to an earlier period (July 29, 1996 – February, 1997). Market rules also slightly differ. In particular, bounds on bid/ask spreads have been changed.

term contracts only, while we extend our investigations to all categories of contracts. Cavallo and Mammola partly use available data on the bid-ask spread, and partly guess the relevant transaction costs, while (as stressed above) we treat frictions parametrically and consider them as a crucial component of the notion of market efficiency.

*We have special motivation for focusing on the MIBO market. First, such a relatively young market offers a unique opportunity to apply the approach to efficiency outlined above. A number of papers (e.g. Mittnik and Rieken (2000), Peña et al. (1999), and Puttonen (1993)) have been recently devoted to the investigation of other relatively new derivatives markets. Second, given our conclusion on the quality of the prices depending on frictions, it is interesting to link the classical literature on arbitrage in options markets with tests of the informational content of prices concerning the future volatility of the underlying. For the MIBO market, we find that rejecting informational efficiency cannot be purely imputed to mispricings, as our results are robust to the exclusion of records affected by arbitrage. This outcome suggests the conjecture that informational inefficiency might lie at a deeper level than the presence of arbitrage opportunities and that it may be more difficult to explain than invoking frictions.*

The paper is organized as follows. In Section 2 we describe our data set and some of the institutional characteristics of the MIBO market. In Section 3 we report the results of no-arbitrage tests, showing in detail which conditions are more often violated, in which segment of the market this happens more frequently, and the level of the average profits associated to each type of violation. In addition to the usual no arbitrage conditions, we introduce the maturity spread and discuss its features. Given our finding of pervasive violations of the most basic no-arbitrage restrictions, in Section 4 we proceed to imply out of the data the level of frictions compatible with pricing efficiency. Section 5 documents the intrinsic informational inefficiency of the MIBO market. Section 6 concludes with policy issues and directions for future research.

## 2. The Data

We analyze a high-frequency data set of European options written on the MIB30 Italian stock index.<sup>5</sup> The MIB30 index is a capital-weighted average of the price of 30 Italian blue chips, which represent approximately 80% of the whole Italian stock market. Our sample contains data collected at a frequency of 30 minutes from 9 a.m. to 5:25 p.m. each day starting on April 6, 1999 and ending on January 31, 2000, for a total of 300 calendar days and approximately 15 observations per day. Each observation reports the value of the MIB30 index, the risk-free interest rate, the cross-section of MIBO30 option prices (over multiple strikes and maturities) and the bid and ask volumes.<sup>6</sup> The interest rate is computed as an average of the bid and ask three-month LIBOR rates. Although far

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<sup>5</sup>The MIBO, established in November 1995, is a fully automated *quote-driven* market. Market makers have the obligation to quote prices for a specified set of contracts, expressly indicated in the market rules. Contracts are settled in cash. During 1999 the volume of exchanges (in millions of Euros) has been equal to 399,031 and the number of traded contracts 2,236,241.

<sup>6</sup>Because of standing IDEM rules, bid/ask quotes are not released and therefore unavailable; only bid/ask volumes are released to the public.

being from constant over the whole period, the risk-free rate shows only two major breaks in its mean, May 5, 1999 and September 29, 1999 when the mean switches respectively from about 2.95% to 2.69% and then from 2.69% to approximately 3.37%. Table I reports basic summary statistics for the data under investigation.

The vector of option prices reports the transaction price for contracts with different strike prices and maturities. According to IDEM market rules for the period under analysis, prices were quoted for the options with strike price nearest to the index, two strikes above and two below it. Strike prices differ from one another by 500 “index points”, each of the value of 2.5 euros. Furthermore, prices were also quoted for options with the three shortest monthly maturities and the three shortest quarterly maturities. In principle this would return a vector of approximately 25 prices for call and put contracts at each point in time, i.e. about 750 prices a day. Nevertheless contracts with long maturities or with strike price far from the value of the index were not actively traded (if traded at all) and discarding them from the sample leaves us with a total 75,900 prices, of which 37,920 refer to calls and 37,980 to puts.<sup>7</sup>

Let  $S_t$  be the value of the MIB30 index at time  $t$ ,  $K$  the strike price of a given option contract, and  $\tau$  the number of days to the expiration of the contract. A major characteristic of an option is moneyness  $z_t$ , i.e. either  $S_t/K$  (for a call) or  $K/S_t$  (for a put). By distinguishing contracts on the basis of moneyness and the length of their residual life we can obtain a detailed description of the composition of our sample (see Table II). We have adopted the following definitions: an option is considered at the money (ATM) if the strike price is within 2% from the index; if it is within 5% (but apart by 2% or more) the option is in the money (ITM) or out of the money (OTM) respectively, depending on the sign (either positive or negative, respectively) of its intrinsic value  $S_t - K$ ; an option is deep in-the-money (DITM) or deep out-of-the-money (DOTM) if its strike price differs from the value of the underlying by more than 5%. We also define the following maturity classes: a contract has *very short* time to expiration if  $\tau \in (0, 7]$ , *short* if  $\tau \in (7, 25]$ , *medium* if  $\tau \in (25, 50]$ , *long* if  $\tau \in (50, 90]$ , and *very long* when  $\tau \in (90, \infty)$ . It is evident from Table II that more than half of the data set is composed of options which expire within a month, while almost no contract with residual life exceeding three months caused significant trading. This justifies merging options with maturity over 50 days and choosing a three month reference for the risk-free interest rate. The most important class in the sample is ATM options with short residual life (16%). More generally, ATM options represent more than one third of the data set, while short- and medium-term contracts account for almost 80%.<sup>8</sup>

Finally, although option prices in our sample refer to actually traded contracts (and the high frequency of the data set should bound the time lag among quotes), one should worry about the presence in the sample of index quotes possibly reporting stale prices thus making the value of the underlying non synchronous with the option. To prevent our tests from being strongly influenced by any issue of synchronicity, in what follows we focus exclusively on a class of arbitrage tests for

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<sup>7</sup>This makes the dimension of our data set quite considerable with respect to other works in this field. For instance, Cavallo and Mammola (2000) effectively use only 3,642 observations.

<sup>8</sup>These figures stress the arbitrariness of restricting the analysis to either ATM or short-term contracts only.

which synchronous reporting of option and index prices is not crucial and consider further types of arbitrage relations only for completeness.

### 3. Arbitrage Tests

For frictionless options markets absence of arbitrage is equivalent to the pricing rules

$$c_t(K, \tau) = E_{Q,t} [e^{-r\tau} \max(S_T - K, 0)] \quad (1a)$$

$$p_t(K, \tau) = E_{Q,t} [e^{-r\tau} \max(K - S_T, 0)], \quad (1b)$$

where  $c_t$  and  $p_t$  indicate the time  $t$  price of a call and a put with strike  $K$  and time to maturity  $\tau \equiv T - t$ .  $Q$  is the risk neutral probability measure. For the sake of simplicity, we will assume throughout that the interest rate  $r$  is constant.<sup>9</sup> It is then possible to derive from (1) some explicit relationships that need to hold if arbitrage is absent. Some of these conditions have been extensively studied and are described in many finance textbooks. These are: (i) the lower bound; (ii) the monotonic relationship with respect to the strike price; (iii) the “Butterfly” spread condition, and (iv) the put/call parity i.e.  $p_t(K) - c_t(K) + S_t = Ke^{-r\tau}$ . We will not spend time commenting the economic meaning of these conditions, for which the reader may refer to a number of well known papers (see, among others, Klemkosky and Resnik (1979), Figlewski (1989), Nisbet (1992), George and Longstaff (1993), and Kamara and Miller (1995)). Neither shall we be too detailed in illustrating the empirical results obtained with these tests which, although included in Table III for general reference, are of a lesser importance. Let us only mention that the put/call parity has been tested in the form of two separate inequalities, as customary, i.e.

$$p_t(K) - c_t(K) + S_t - Ke^{-r\tau} \geq 0 \quad (2a)$$

$$Ke^{-r\tau} - p_t(K) + c_t(K) - S_t \geq 0. \quad (2b)$$

Notably, the outcome of a test based on the lower bound and the put/call parity depends on the degree of efficiency in the underlying market as well as in the derivatives market. For this reason these are truly *joint* conditions and it is hard to disentangle the contributions of the two markets to possible violations. Furthermore, these tests require a synchronous recording of option prices and the underlying which is often questionable in practice. The actual implementation of the corresponding trading strategies is also rather delicate, because in order to exploit violations either of the lower bound for a call contract or of condition (2b) the investor is required to take a short position in the underlying and this is likely to be complex and costly (if possible at all). In order to partially overcome these problems and gain more details on the actual efficiency of the MIBO market *in isolation*, we also consider the following conditions:

**Maturity Monotonicity** ( $\tau_2 > \tau_1$ ):

$$c_t(\tau_2) - c_t(\tau_1) \geq 0 \quad (3a)$$

$$p_t(\tau_2) - p_t(\tau_1) - K [e^{-r\tau_1} - e^{-r\tau_2}] \geq 0 \quad (3b)$$

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<sup>9</sup>Section 4 discusses the robustness of our results to this assumption.

**Reverse Strike Monotonicity ( $K_1 > K_2$ ):**

$$(K_1 - K_2) e^{-r\tau} - c_t(K_2) + c_t(K_1) \geq 0 \quad (4a)$$

$$p_t(K_1) - p_t(K_2) - (K_1 - K_2) e^{-r\tau} \geq 0 \quad (4b)$$

**Box Spreads:**

$$[p_t(K_1) - c_t(K_1)] - [p_t(K_2) - c_t(K_2)] = (K_1 - K_2) e^{-r\tau}$$

or equivalently <sup>10</sup>

$$[p_t(K_1) - c_t(K_1)] - [p_t(K_2) - c_t(K_2)] - (K_1 - K_2) e^{-r\tau} \geq 0 \quad (5a)$$

$$(K_1 - K_2) e^{-r\tau} - [p_t(K_1) - c_t(K_1)] + [p_t(K_2) - c_t(K_2)] \geq 0 \quad (5b)$$

**Maturity Spreads:**

$$[p_t(\tau_1) - c_t(\tau_1)] - [p_t(\tau_2) - c_t(\tau_2)] = K [e^{-r\tau_1} - e^{-r\tau_2}]$$

or equivalently

$$[p_t(\tau_1) - c_t(\tau_1)] - [p_t(\tau_2) - c_t(\tau_2)] - K [e^{-r\tau_1} - e^{-r\tau_2}] \geq 0 \quad (6a)$$

$$[p_t(\tau_2) - c_t(\tau_2)] - [p_t(\tau_1) - c_t(\tau_1)] + K [e^{-r\tau_1} - e^{-r\tau_2}] \geq 0 \quad (6b)$$

It is easy to see how these were obtained. (5) and (6) directly follow from the put/call parity evaluated over a pair of different strike prices and maturities, respectively, and taking the difference between the values so obtained. (4) follows combining (5) with the monotone relationship with respect to the strike price. The advantage of (4) – (6) over the put/call parity is that the corresponding portfolios do not imply any position but in the options market. Moreover, the synchronicity issue is of minor concern. (4) and (5) have been introduced by Ronn and Ronn (1989); (6) is, to our knowledge, entirely new, and this justifies a more detailed analysis.

Let  $\tau_1 < \tau_2$  and consider selling one put and buying one call both maturing in  $\tau_1$  days and with strike price  $K$  and refinancing the position upon maturity for  $\tau_2 - \tau_1$  additional days. The resulting investment delivers at time  $t + \tau_2$  a payoff equal to  $(S_{t+\tau_1} - K) \exp\{r(\tau_2 - \tau_1)\}$ . This first trading strategy should then be confronted with the one obtained by selling one put and buying one call both maturing in  $\tau_2$  days and with strike price  $K$  while raising the amount  $K [\exp(-r\tau_1) - \exp(-r\tau_2)]$  on the bond market (the Payoffs and costs of both strategies are reported in the following table.)

$t$	$\dots$	$t + \tau_1$	$\dots$	$t + \tau_2$
$c_t(\tau_1) - p_t(\tau_1)$		$S_{t+\tau_1} - K$	→	$(S_{t+\tau_1} - K) e^{r(\tau_2 - \tau_1)}$
$c_t(\tau_2) - p_t(\tau_2) - K (e^{-r\tau_1} - e^{-r\tau_2})$				$S_{t+\tau_2} - K e^{r(\tau_2 - \tau_1)}$

This second portfolio of options has the same maturity of the previous one but a final payoff equal to  $S_{t+\tau_2} - K \exp\{r(\tau_2 - \tau_1)\}$ . Taking a short position in the former strategy and a long one in the

<sup>10</sup>Decomposing again an equation into two simultaneous inequalities.

latter leaves the investor with a portfolio yielding  $S_{t+\tau_2} - S_{t+\tau_1} \exp\{r(\tau_2 - \tau_1)\}$  at time  $t + \tau_2$  and initial cost

$$[c_t(\tau_2) - p_t(\tau_2)] - [c_t(\tau_1) - p_t(\tau_1)] - K(e^{-r\tau_1} - e^{-r\tau_2}) \quad (7)$$

Such an investment reflects the expectation that, in discounted terms, the underlying will increase over the period  $t + \tau_1, t + \tau_2$ . In fact the same payoff would be obtained buying at time  $t + \tau_1$  a future with maturity  $t + \tau_2$ , a strategy bearing no cost. It follows that whenever (7) is strictly positive – i.e. 6b is violated – the relative price of long maturities with respect to short ones is too high, given the investment opportunity offered by the future market: an arbitrage opportunity therefore arises by trading simultaneously on both markets. As a consequence, (6) is to be considered more as a test on the joint efficiency of these two markets or, more precisely, on the degree of integration between them: For this reason we preferred not to include it when computing the total number of arbitrage opportunities on the MIBO market but treat it separately in our analyses.

The results of all these tests are reported in Tables III and VI, in the columns labelled  $\alpha = 0$ : in Table III we report for each condition the percentage of contracts allowing for arbitrage<sup>11</sup>, the average level of arbitrage profits; Table VI describes the distribution of the number of violations across moneyness and time to maturity. The overall number of arbitrage violations is indeed outstanding, more than 50% of the sample. For what concerns the different conditions tested, it clearly emerges that the most often violated are Box spreads (5) and the put/call parity. The two Box conditions are violated almost equally often, but the put-call parity condition is highly asymmetrical and it is the short hedge (2b) the most relevant one. This finding reflects the higher difficulty and costs to take a short position in the index as required to exploit this violation.<sup>12</sup> Also the convexity condition (Butterfly) is often not satisfied.<sup>13</sup> The small number of violations of the monotonic relationships with respect to strike price and maturity, provides some evidence that lack of synchronicity between the MIB30 and option prices is not too serious in our data set.<sup>14</sup> The distribution of the number of violations across moneyness, time to maturity and type of condition violated is also interesting (Table VI), for two different reasons. First, it shows that arbitrage opportunities are not concentrated in the more illiquid segments of the market but rather considerably spread across moneyness and maturity. Second, the number of arbitrage opportunities detected increases the shorter the time to maturity and the higher the moneyness. Short-term, ITM and DITM options are normally considered quite illiquid: Those investors who own such options have good prospects of receiving a positive final

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<sup>11</sup>When the condition tested implies more than one contract, we arbitrarily impute violations of such conditions to only one contract.

<sup>12</sup>Cavallo and Mammola (2000) reach the same conclusion.

<sup>13</sup>This is due to the very general form in which we test it. In most studies the butterfly condition is tested by confronting each contract with just a pair of contracts, those with strike price immediately larger and smaller. For an option appearing in the  $n^{th}$  position inside a vector of 25 option prices sorted by the strike price, there are in fact  $(n - 1) \times (25 - n)$  possible hedges,  $n = 2, \dots, 24$ , for a total of 2,300.

<sup>14</sup>In fact, whenever an option contract has not been actively traded it would easily be the case that the intervening variation in the value of the index results in a vertical misalignment of option prices. Problems with the automatic quoting system made such an event extremely frequent in the first months of 1999. For this reason these additional 3 months of data were not included in our study in spite of their availability.

payoff if they hold them to maturity rather than trade for arbitrage profits. We thus find some correlation between mispricing and liquidity.

The maturity spread condition (6) is also frequently violated. The striking fact here is the deep asymmetry between the short side (6a) — which is negligible — and the long side, which is often violated. Another noteworthy feature which emerges from Table VI is that such arbitrage opportunities are almost exclusively concentrated in the segment of contracts with residual life within a month. As remarked above, (6b) indicates an excessive relative price of long maturities with respect to shorter ones and reflects the expectation of long-term increases of the index, a scenario consistent with the bull market of the last part of 1999. As previously argued, the arbitrage strategy involved implies selling a basket of options of two different maturities and buying a future with maturity equal to the difference between the two. The mispricing detected has an interesting interpretation in terms of market microstructure. IDEM rules result in an almost complete lack of maturity overlap between the options and the futures markets. On the FIB30 are traded only futures with quarterly maturities (March, June, September, and December), while Section 2 shows that 54% of traded option contracts expires within 25 days. Due to this, the arbitrage strategy mentioned turns then out to be simply not available, especially for contracts with shorter maturity. A better synchronization of the MIBO30 and the FIB30 cycles might affect the incidence of this type of mispricing and improve the overall efficiency of the IDEM.<sup>15</sup>

*In Figure 5 we draw the daily distribution of the overall incidence of arbitrage opportunities to check whether our findings may result being a simple artifact of corporate actions or other exceptional market events.<sup>16</sup> This graph suggests that this can hardly be the case: the ratio of available arbitrage opportunities is almost always above 40% and exceeds the sample average for long periods, especially November 1999 – January 2000, when the market was more bullish. Among the “low arbitrage” periods one finds also the months of June and July, those in which almost invariably dividends are paid out. All in all, there does not seem to be strong evidence either of clustering of violations around particular periods of the year or of large outliers driving the results in Table III.*

#### 4. The Role of Frictions

Whereas all the preceding conditions refer to a model of frictionless markets (such as Black and Scholes (1973)), frictions are obviously important in real markets (see the extensive treatment in Stoll (2000)). Therefore, a first way to look at the preceding results is to consider them as an indication of the distance between models of asset prices and markets.

In a market with no “real” arbitrage opportunities, arbitrage profits deduced from theory should indicate nothing but the amount paid by investors as commissions, margins, taxes, and the like. An initial assessment of the role of frictions on the MIBO may then be simply obtained by computing the

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<sup>15</sup>This characteristics of the IDEM remained unchanged through the subsequent reforms of the market structure.

<sup>16</sup>We are indebted to a referee for suggesting this possibility. Corporate events (like public offerings) might induce jumps in the price of options; in some periods trading in some components of the MIB30 index may be stopped to avoid excess price oscillations. These events are likely to induce temporary misalignments between the underlying and option prices that may appear as (apparent) arbitrage opportunities.

exact amount of the arbitrage profits detected. This information is contained in Table III, in which the average profits reported for each arbitrage condition may be simply interpreted as the average level of the frictions implied by trading in the MIBO. The whole distribution of arbitrage profits – rather than its mean – is quite interesting as well. It turns out that profits arising from conditions other than (5)-(6) are rather concentrated around low levels and that almost 70% of the sample of arbitrage profits are below 200 index points. Nevertheless, conditions (5) and (6) display much higher profits so that the maximum arbitrage profits available for each contract have an empirical distribution that is quite right-skewed, as illustrated in Figure 1.

When trying to understand the nature of such frictions, it is reasonable to restrict attention to the following aspects:

1. *Microstructural issues.* Execution delays tend to make arbitrage profits risky, although automated market circuits reduce this phenomenon. Following Gould and Galai (1974), Nisbet (1992), and Ackert and Tian (2000) we checked whether arbitrage opportunities are still available in the subsequent observation, i.e. whether arbitrage opportunities persist for at least half-an-hour. We obtain results very similar to those reported in Tables III to V.
2. *Taxation.* Tax rates tend to be non proportional, depending on the overall fiscal position of investors as, for example, whether they are home-based or not. We prefer to omit this element rather than take a totally arbitrary approach.
3. *Dividends.* Although dividends are hardly known in advance, a rather common approach consists of treating past dividends as a reliable forecast of future ones. Following this practice, we incorporated into our data the dividend yield on the MIB30 index in 1998 (equal to 1.4251%): our results remain virtually unchanged. We also investigated, in a few alternative ways, the relevance of dividends implicit in the data, obtaining almost no practical effect.<sup>17</sup> We therefore conclude that dividends can be omitted at little cost.
4. *Interest rate risk.* Changes in the interest rate imply a risk for arbitrageurs. We then computed profits out of each investment by using interest rates of a corresponding maturity, obtained by estimating the term structure of interest rates.<sup>18</sup> This approach, nevertheless, does not have noteworthy implications as the term structure of interest rates is considerably flat over this period.<sup>19</sup>
5. *Transaction costs.* Prices appearing with a positive (resp. negative) sign in the above conditions indicate that the arbitrage strategy considered is exploited by buying (resp. selling) the corresponding asset. For this reason such prices should be computed at their *ask* (resp. *bid*) value together with all commissions and fees involved in the transaction. Considering bid and

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<sup>17</sup>We tried inferring dividends from: (i) the spot-future parity; (ii) the put-call parity; (iii) the maturity spread condition. Approximately 70% of the computed yields turns out to be slightly negative while the average yield is approximately 0.02%.

<sup>18</sup>We use the cubic-splines method of estimation of the yield curve first introduced in McCulloch (1975).

<sup>19</sup>Including dividends and the term structure of interest rates produces a ratio of violations equal to 50.77%.

ask values and other transaction costs has the clear effect of raising the left hand side in all the preceding inequalities i.e. of cutting the amount of arbitrage profits.

Bid/ask spreads (inclusive of all commissions and fees) turn out to be the only kind of frictions likely to affect the preceding results in a significant way. A direct assessment of this component is, once again, less straightforward than it would appear at a first glance. This is mainly due to the well documented fact that a large number of transactions are concluded at an effective level of the spread quite far from the one officially quoted (see Stoll (2000), Vijh (1990) and George and Longstaff (1991)). But further to this, and more importantly, our aim is not that of assessing the arbitrage phenomenon at the level of costs which is likely to have prevailed. Motivated by our view on pricing efficiency discussed in the introduction, we would rather provide a characterization of arbitrage as a function of market costs as, we believe, efficiency is a concept involving both elements. For this reason we shall artificially introduce different levels of costs in the options prices included in our data set and to compute, for each such level, the incidence of arbitrage opportunities and the level of profits implied in them.

Of course there is a multiplicity of ways in which this experiment can be accomplished. The most simple and clear from an economic viewpoint is, to our judgement, the one that follows. Letting  $\alpha$  and  $\beta$  be positive parameters, define on the basis of these bid and ask prices for the options and the underlying simply by letting  $c_t^{bid} = (1 - \alpha) c_t$  and  $c_t^{ask} = (1 + \alpha) c_t$  in the case of a call and  $S_t^{bid} = (1 - \beta) S_t$  and  $S_t^{ask} = (1 + \beta) S_t$ . Then the bid/ask spread is  $2\alpha$  and  $2\beta$  for options and the MIB30, respectively, and the mid price coincides with the actual value reported in our original data set. We have found it appropriate to further impose  $\alpha = \beta$ . It can be easily viewed that in this choice is implicit the constraint that the spread be uniform across all characteristics of contracts such as moneyness, maturity and the like. Although this is in contrast with the market practice, it provides a consistent and meaningful measure of the aggregate level of trading costs implicit in a given ratio of arbitrage opportunities; some alternatives will also be discussed. The results so obtained are reported in Table III. Each column corresponds to a different level of  $\alpha$ , ranging from 0 to 10%; we then take the value  $\alpha = 5\%$  as a benchmark<sup>20</sup> and perform for this level of the spread a few further investigations, reported in Table VI and Figure 5. Eventually, in Figures 2 to 4 we plot as a function of  $\alpha$  the ratio of arbitrage violations and the level of arbitrage profits.

Table III shows that for most arbitrage conditions, the incidence of violations rapidly converges to zero as  $\alpha$  increases. As for to the two sides of the put-call parity, although it is still clear that violations of the short end occur more frequently than those of the long side, it is remarkable that the high ratios of these violations quickly drop to zero as soon as  $\alpha$  reaches about 2%. This is entirely consistent with the findings of Cavallo and Mammola (2000) – implicitly suggesting that our choice of the spread should not be inconsistent with the market data. However, other conditions, such as

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<sup>20</sup>The reason for this choice is based on the simple approach proposed by Roll (1984) to compute the bid/ask spread implicit in trade prices (for other models of the bid/ask spread see, among others, Stoll (1989), George et al. (1991) and Huang and Stoll (1997)). Assuming that the spread is constant during each trading day (although possibly different for different contracts), we obtain a time series for the bid/ask spread with average value equal to 8.95% which corresponds to  $\alpha = 4, 5\%$ .

box and maturity spreads remain important throughout the range of all the plausible values for  $\alpha$  we consider. For instance, at  $\alpha = 4\%$  the two box spread conditions still originate a 5% each of violations, while the maturity spread condition is still at a very persistent 15%. This exercise shows that some kinds of pricing inefficiencies are very persistent and that in particular condition (6b) is scarcely influenced by the value of the transaction costs.<sup>21</sup> As a result, the overall ratio of arbitrage violations is quite slowly decreasing as a function of  $\alpha$ , as illustrated in Figure 2. In Figure 4 we draw essentially the same graph as in Figure 2 (but focusing on overall violations of arbitrage conditions and violations of the maturity spread - long side, only) distinguishing between moneyness classes. This is done in order to verify that our results apply to all segments of the market. It turns out that a higher ratio of arbitrage opportunities is concentrated in the DITM segment but this difference becomes irrelevant as long as  $\alpha$  increase; furthermore, the ATM sector almost coincides with the whole sample. For what concerns condition (6b), it turns out that it is exactly ATM options those that are more seriously affected by this arbitrage opportunity, while it is the DITM the one in which such condition is least important.

Our results may be summarized as follows. First, even modest frictions can significantly reduce the number of violations, implying that most of the violations detected in Section 3 actually corresponded to very thin profit levels. For example, with  $\alpha = 2\%$  violations of the monotone spreads (with respect to either strike or maturity) drop to about 1.6% thus confirming that our data set is not affected by systematic misrecordings. Second, as  $\alpha$  increases the arbitrage conditions that remain into play are the box and maturity (long) spreads. Third, it is not possible to completely get rid of the mispricing even by raising  $\alpha$  beyond reasonable thresholds (8% or more) as mispricing persist in more than 2.5% of the sample. Although this number is not high enough to cast doubts on the meaningfulness of our tests, it witnesses the existence of niches of pricing inefficiencies that cannot be simply explained away by the existence of frictions.<sup>22</sup>

Table III also presents average level of profits which arise from the arbitrage opportunities detected. A diminishing number of arbitrage opportunities does not imply that the surviving mispricing cannot be anyway quite profitable. Figure 3 plots average arbitrage profit vs. the bid/ask spread  $\alpha$ . These plots confirm one of our previous conclusions: profits do not converge to zero as  $\alpha$  increases. On the contrary, average profits deriving from arbitrage strategies exploiting violations of conditions such as the butterfly spread seem to be increasing with  $\alpha$ . The reason for this puzzling behavior is that higher bid/ask spreads have two effects: (*i*) they discard from the sample of arbitrage oppor-

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<sup>21</sup>A strong belief that the MIB30 was bound to rise during 1999 is a reasonable explanation for some of these mispricings. For what concerns the spread conditions, observe that in these conditions the bid/ask spread is less important than in others, given our assumptions. As a pure matter of scale the spread has high impact especially when applied to the underlying (the MIB30 has a price which is several orders of magnitude higher than option contracts). Since trading in the underlying is not required to exploit misspricings evidenced by the spread conditions, increasing bid/ask spreads will reduce their percentage relevance very slowly.

<sup>22</sup>Longstaff (1995) reports that violations deriving from conditions such as monotonic or butterfly do not reach 1.5% of his sample, and the associated profits would amount to a few cents. In order to recover the same ratio in our data set we would need to impose a value of  $\alpha$  close to 6%, considerably higher than those documented by Longstaff. Moreover, residual profits would still be rather high.

tunities those associated with a lower level of profits; (*ii*) they cut down the profitability of those opportunities that remain in the sample. The combined effect may result in a positive relationship between profits and bid/ask spread whenever the number of low profit strategies is important. Figure 3 also stresses that the short edge of the put-call parity is on average, and independently of  $\alpha$ , much more profitable than the short hedge.

Table VI reports the sample distribution of the violations found by maturity and moneyness for the scenarios  $\alpha = 0$  and  $\alpha = 5\%$  focussing only on the three conditions for which the incidence of violations was the highest – i.e. butterfly, maturity, and box spreads – and the results relative to all kinds of violations ('Overall'). We find no evidence of strong patterns along the moneyness dimension: when  $\alpha = 5\%$  the percentage of violations is always between 7 and 10 percent. On the other hand, a distinct pattern exists along the maturity dimension as (again, assuming  $\alpha = 5\%$ ) the incidence of violating contracts steadily declines from 17 for very short-term options to 3 percent for long-term contracts. This is not completely unexpected as the segment with less than 7 days to expiration is universally considered illiquid and as such more prone to mispricings. However, once more just disregarding the extreme segments of the implied volatility surface – i.e. deep in- and out-of the money and short term contracts – does not change our finding that even admitting frictions in the order of  $\alpha = 5\%$  at least 5 – 6% of the available observations does violate one of the three no-arbitrage conditions reported by Table VI.

Eventually, in Figure 5 we draw the daily distribution of the incidence of arbitrage opportunities together with its average value, relatively to the value  $\alpha = 5\%$ . This chart confirms, as for the case  $\alpha = 0$  discussed in the close of the preceding section, that our findings are not affected by particular outliers, as the series oscillates quite regularly across its mean. Once again we find evidence of the fact that the concluding part of the sample would seem to be the one in which arbitrage opportunities are more pervasive.

As we stressed above, on the real market the bid/ask spread is far from being uniform: in fact the microstructure literature highlights that a uniform spread structure would be appropriate if order processing were the unique costs incurred by market makers. According to IDEM rules upper bounds for the spreads for each class of options are imposed as a function of moneyness. Nevertheless, in the definition of such constraints both moneyness and the spread are expressed in absolute values, so that the corresponding ratio of the spread to the option price may result being extremely wide<sup>23</sup> and the choice of a "realistic" structure of the spreads quite problematic. For comparison, Ackert and Tian (2000, p. 45) report that on the S&P 500 for contracts with price above (respectively, below) \$3 the bid ask spread was about 1/16 (respectively 1/32) of a point. Therefore the bid/ask spread should be at least 2% for low price (DOTM) options and can be as high as 4% for medium price options.<sup>24</sup> In George and Longstaff (1980) the percentage half spread,  $\alpha$ , on the S&P100 options

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<sup>23</sup>Philips and Smith (1980) report that for options with extremely low price the spread may even exceed 100%.

<sup>24</sup>Nisbet (1992, pp. 392-393) estimates transaction costs on the London Traded Options Market (LTOM) during 1988 finding  $0.6\% \leq \alpha \leq 3.4\%$ . Yadav and Pope (1994, pp. 925-926) investigate the market for futures on the FTSE 100 index between 1986 and 1990 estimating  $\alpha \leq 0.3\%$ , i.e. substantially lower values. Longstaff (1995) and George and Longstaff (1993) study the S&P 100 index options market (CBOE) 1988 and 1989: although they conclude that

either is *U* shaped, with minimum for ATM options, or decreases the more the option is into the money.

In order to take into account the role of moneyness – and given some of the stylized facts mentioned in the literature – we also considered a scheme in which the level  $\alpha$  of frictions is a function of monetness. We assumed that the level of the spread applying to ATM options is increased by 25% for OTM or ITM options and by 50% for DOTM and DITM options.<sup>25</sup> The outcome of this exercise is reported in Table IV. The main difference with Table III is that the incidence of arbitrage opportunities declines more rapidly as  $\alpha$  increases; on the other hand it is confirmed that some conditions display a higher incidence ratio than others and, particularly, the Maturity Spread. Observe though that the present choice of the structure of the spread implies, for each value of  $\alpha$  considered, a higher value of the average sample spread, which makes the comparison between Tables III and IV less straightforward. In the first row of Table IV we then compute the average level of the spread (“Effective  $\alpha$ ”) given the sample distribution of moneyness (as reported in Table II). If we compare each column of Table III with the column of Table IV of a corresponding level of effective  $\alpha$ , we find that indeed these two schemes are much alike, or, equivalently, that it is the mean spread which explains most of this figures, rather than its structure. For example, with a uniform spread equal to 5% we find that almost 8% of the market allows for arbitrage; when the spread is made to depend on moneyness and the resulting average spread is fixed at 4.83% (corresponding to  $\alpha = 4\%$ ) the incidence of arbitrage amounts to 8.26%. This analysis confirms that the choice of a uniform spread structure is correct as long as the aggregate level of frictions is concerned. We decided, however, to further test this conclusion.

It is recognized that, further to order processing, market makers face inventory costs arising from the risk that the option may change value during the holding period, especially as a result of a change in the underlying. This cost directly translates into the cost of hedging the corresponding risk and it can be reasonably approximated by the product of the underlying volatility times the delta of the option (see George and Longstaff, p. 386). In fact, direct interviews with market makers has confirmed that the Black and Scholes delta<sup>26</sup> is the main parameter on which the bid/ask spread is made to depend. We constructed then a further scheme for the bid/ask spread by dividing the sample into four classes depending on the absolute value of the  $\delta$ :  $|\delta| < 25\%$ ,  $25\% \leq |\delta| < 50\%$ ,  $50\% \leq |\delta| < 75\%$  and  $|\delta| \geq 75\%$ . For a given level of  $\alpha$ , we then fix the spread equal to  $\alpha$  for the first class,  $1.25\alpha$  for the second,  $1.5\alpha$  for the third and  $1.75$  for the fourth. The results obtained from this exercise are reported in Table V. The comments relative to Table IV apply here almost unchanged: the incidence is lower and decreases more rapidly as  $\alpha$  increases. Once again, confronting the level of  $\alpha$  reported for Table II with the levels of effective  $\alpha$  reported for Table V illustrates how the

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<sup>25</sup>α varies considerably, the average bid/ask spread is 2.5%.

<sup>26</sup>We also considered the case in which the spread is a function of maturity as well as of moneyness. Tha way the spread depends on the residual life is not clear nor in microstructure literature nor in the empirical invesigations consulted (see George and Longstaff (1993)).

<sup>26</sup>We recall that iIn the model of Black and Scholes the delta of a call contract is equal to  $\delta_t = \Phi \left\{ \frac{\ln \frac{S_t}{k} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \right\}$ , where  $\sigma$  is the volatility and  $\Phi$  is the cumulative normal distribution.

prevailing differences are mainly due to the different average levels of costs assumed.

*In conclusion, the MIBO market is characterized by arbitrage opportunities even after considering the role of transaction costs and other frictions. Even when plausible frictions for ATM and medium-term contracts are considered, roughly 10% of the data still implies riskless profitable strategies.* Some inefficiency may actually be caused by the loose bounds on bid/ask spreads (when expressed as ratios) mandated by the IDEM. Also, the current structure of the IDEM appears to be responsible for some of the mispricing we have detected, especially for the most persistent ones that are not much affected by the introduction of frictions. For instance, we suspect that a relevant percentage of the violations detected might be easily removed by improving the feasibility of synchronized trading (on instruments with identical maturity) on the MIBO and FIB30 markets. On the other hand, there is no doubt that the level of arbitrage profits detected after filtering for the bid/ask spread is not negligible. Finally, trading strategies triggering short positions in the index tend to exhibit higher profitability, and this is maybe a consequence of the existence of portfolio constraints, particularly for institutional investors. The latter is clearly a kind of market imperfection that cannot be accounted for by any data analysis although we may conjecture that it does have significant role.

## 5. Informational Efficiency

A debated question in the literature concerns the informational content of implied volatility: If the options market incorporates all available information through an efficient pricing mechanism, then it should not be possible to improve volatility forecasts over and above the volatility implied by option prices.<sup>27</sup> Sections 3 and 4 offered indications against the null of pricing efficiency. Although the two concepts of efficiency need not coincide,<sup>28</sup> we suspect that an imperfect pricing mechanism may prove unable to serve as an unbiased aggregator of beliefs. *For instance, when the lower bound condition is violated implied volatility takes on negative values which cannot represent sensible forecasts of future, realized volatility: informational efficiency tests have routinely purged the data of these types of mispricings. One naturally wonders whether removing records producing other types of arbitrage violations might somehow affect the results. In this section we pursue this approach in a systematic fashion: after having reviewed a few standard techniques and developed a novel set of econometric tools that appear to be particularly fit to the MIBO30 market, we proceed to test for informational efficiency using data sets of variable quality, in the sense that variable percentages of mispricings are removed from the sample before implementing the tests.*<sup>29</sup>

A first issue in implementing this type of tests is defining the *realized* volatility of returns on

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<sup>27</sup>Implied volatility is defined as the value of the volatility parameter (in annualized units) such that the observed price of an option and its theoretical Black-Scholes value coincide.

<sup>28</sup>In fact a market unable to correctly reflect available information need not display arbitrage opportunities. In principle, even the presence of mispricings violating no arbitrage conditions might be consistent with informational efficiency, although we argue in the following that this is a harder case to defend.

<sup>29</sup>Most of the previous literature does not pay any attention to the effects of arbitrage violations on the tests of informational efficiency, apart from the preventive elimination of all prices violating the lower bound conditions (i.e. of all negative IVs). Pricing efficiency is simply assumed.

the underlying asset. Given a series of high-frequency MIB30 prices  $\{S_j\}_{j=t}^{\tau}$  we follow Canina and Figlewski (1993) (CF for short) and define realized volatility between  $t$  and  $\tau$  as the annualized standard deviation of the continuously compounded MIB30 returns  $\{R_j\}_{j=t+1}^{\tau}$ :

$$s(t, \tau) = \sqrt{\frac{1}{\tau - t} \sum_{j=t+1}^{\tau} (R_j - \bar{R})^2} \quad (8)$$

Since in general we have observations on 15 half-an-hour periods per trading day, the annualized volatility of the MIB30 can be simply obtained by multiplying the high-frequency  $s(t, \tau)$  by the square root of  $15 \times 252 = 3,780$ . We call this quantity  $\sigma^*(t, \tau)$ . Analogous definitions apply to data sampled at a different frequency.

Assuming that at time  $t$  the IV on an option with maturity in  $\tau > t$  represents the MIBO market's prediction of the future volatility over the interval  $[t, \tau]$  (Day and Lewis (1988)), a test of rationality of this forecast can be obtained from the regression

$$\sigma^*(t, \tau) = \alpha + \beta IV(t, z_t, \tau) + u(t, z_t, \tau), \quad (9)$$

(where  $E[u(t, \tau) | F_t] = 0$  and  $F_t$  is the information set available to the market at time  $t$ ). The null hypothesis is  $\alpha = 0$  and  $\beta = 1$  so that  $E[\sigma^*(t, \tau) | F_t] = IV(t, \tau)$  (unbiasedness). Deviations of  $\alpha$  and/or  $\beta$  from the values 0 and 1 illustrate the presence of biases and hence irrationality of markets' forecasts. The notation  $IV(t, z_t, \tau)$  stresses that at time  $t$  several IVs corresponding to different moneyness levels  $z_t$  are available with maturity  $\tau$ . Analogously, since  $IV(t, z_t, \tau)$  is maintained to be formed by an efficient market able to incorporate all available information into prices, if  $x_t \in F_t$  is any piece of public information, then the (encompassing) regression

$$\sigma^*(t, \tau) = \alpha + \beta IV(t, z_t, \tau) + \gamma x_t + u(t, z_t, \tau) \quad (10)$$

should still give  $\alpha = 0$ ,  $\beta = 1$ , and  $\gamma = 0$ . In the following we use two alternative definitions of  $x_t$ . First, following CF (p. 671) we employ a 30 (trading) days moving average of realized daily volatility  $\sigma^*$ , appropriately annualized. CF use a 60-days average, but given our shorter sample size (in terms of days, not of overall number of observations) we opt for 30 (CF expressly point out that similar results were obtained using 30 instead of 60). Second, we set  $x_t = IV(t-1, z_t, \tau+1)$ , the lagged implied volatility for an option with identical moneyness and maturity.

Unfortunately, it is not possible to simply estimate regressions (9)-(10) by OLS using all implied volatilities corresponding to different days/time of the day, moneyness, and time-to-maturity levels. At least three problems have been discussed in the literature:

1. Since the observations come from a panel, the random disturbances  $u(t, z_t, \tau)$  are unlikely to be *spherical*, i.e. to have identical variance and to be uncorrelated. For instance, it is plausible that because of the lower liquidity, certain categories of contracts (DITM and DOTM) be characterized by more volatile random shocks, a source of heteroskedasticity. Similarly, in a high frequency data set certain periods of the day (like opening, lunch time, etc.) may display

more volatile random influences. Finally, it is possible that disturbances be correlated across moneyness classes and/or across maturities. It is well known that in these circumstances OLS estimates are inefficient and would produce a biased estimate of the covariance matrix. Notice that these issues uniquely derive from the panel nature of the data, i.e. they cannot be simply removed by careful choice of the sampling method. Of course, one solution is to break down the panel in one or more time series. For instance, CF apply tests (9)-(10) to 32 classes defined by moneyness and maturity. The problem with this approach is that, at least in principle, contradictory answers may be reached on heterogenous classes. Christensen and Prabhala (1998, CP for short) focus instead on the 1-month, ATM segment of the S&P 100 options market. There is obviously a trade-off between the quantity of information lost in the process and the clarity of the results that are achievable.

2. As stressed by both CF and CP, even in pure time series tests the disturbances  $u(t, z_t, \tau)$  will be serially correlated if the sampling method uses overlapping observations. From (8) it is clear that given  $\tau$ ,  $\sigma^*(t, \tau)$  will share  $\tau - k$  terms with the set of realized historical volatilities  $\{\sigma^*(t + k, \tau)\}_{k=1}^\tau$ . This introduces dependence in the regressands of (9)-(10). The same phenomenon affects 30 days moving averages of realized daily volatility. Standard econometric theory suggests that OLS estimators have poor finite sample properties and that their traditional standard errors are likely to be understated. CF offer a partial solution to the problem by calculating autocorrelation-consistent standard errors in a GMM framework. CP offer a ultimate solution that relies on avoiding the use of overlapping observations altogether.
3. CP and Jorion (1995) provide strong arguments that would indicate that  $IV(t, z_t, \tau)$  might be plagued by substantial measurement errors. Errors-in-variables are known to bias downwards the OLS estimates of the coefficient associated with an imprecisely measured regressor: this would easily cause (9) to reject unbiasedness. Moreover, provided  $IV(t, z_t, \tau)$  and  $\sigma^*(t, \tau)$  are positively correlated, the OLS estimate of  $\gamma$  in (10) might be also biased upwards, causing rejections of the efficiency hypothesis. As illustrated in CP, the solution consists in adopting an instrumental variable approach to the estimation of (9)-(10). This problem is equally serious in panel and pure time series regression models.

In the following we implement a number of strategies. We approach the problem of the non-sphericality of the random disturbances in three alternative ways. First, as in CF, we apply GMM estimation to high-frequency time series concerning separate contract classes defined in terms of time-to-maturity and moneyness (Section 5.1). In practice, this is equivalent to OLS estimation with corrections applied in the form of an autocorrelation-consistent estimator of the covariance matrix. The disadvantage is that multiple sets of estimates are obtained for (9)-(10). Second, after transforming the available data set, we apply feasible GLS estimation to each separate class of moneyness/maturity (Section 5.2). In their discussion of CF results, CP experiment with FGLS stressing that, similarly to the instrumental variables method, they provide consistent estimates in the presence of autocorrelated errors. Once more, multiple estimates emerge, one per contract

class. Third, we estimate the regression coefficients using Parks' (1967) two-stage panel method (Section 5.3). To our knowledge this method has never been applied. It offers the advantage of imposing structure useful to deal with the presence of serial correlation (both originating from the panel structure and from the use of overlapping observations) and heteroskedasticity in the errors. Moreover, it provides a unique estimate of the coefficients in (9)-(10). We also deal with two additional concerns. In Section 5.4 we apply CF-style GMM estimation to daily, closing observations for ATM options only. In fact, the use of high-frequency, transactions data might introduce in the analysis uncontrolled amounts of noise, possibly driving the results. A streamline exercise based on daily observations for the most liquid segment of the market helps shedding light on the issue. In Section 5.5 we experiment with CP's instrumental variable method. We apply their methods both to daily, closing, overlapping and non-overlapping observations for ATM options 15 days to maturity. All of these tests lead to quite a uniform conclusion: the MIBO market seems to couple pricing and informational inefficiencies in large amounts. Moreover, the fact that either the *original* data described in Section 2 or that the *arbitrage-free* data derived in Section 4 be used in the econometric tests turns out to be inconsequential.

### 5.1. GMM Tests

Similarly to CF, we subject our data to a reduction process by which, for each recorded trading time, we extract only 20 observations, corresponding to all possible combinations of 5 categories of moneyness (DOTM, OTM, ATM, ITM, DITM) and 4 categories of time-to-maturity (very short, short, medium, long). The classes of moneyness and time-to-expiration are defined as in Section 2. It often happens that a given moneyness class contains multiple observations. In these cases we extract the observation with the lowest (highest) moneyness in the case of DOTM (DITM) options, and simply use the mid-point observation based on a moneyness ranking for the remaining three classes. Since our high frequency sample consists of 3,434 observations over time, the resulting panel data set is in principle composed of 68,680 observations, thus implying a minimal loss of information. In practice, it turns out that a few classes of moneyness are missing; especially in the case of time-to-maturity, at most two classes are simultaneously present throughout the sample. The ‘reduced’ data set consists of 21,240 observations.<sup>30</sup>

Define  $\boldsymbol{\theta} \equiv [\alpha \ \beta]', \mathbf{x}_i \equiv [1 \ IV_i]', X = [\mathbf{x}_1 \ \mathbf{x}_2 \dots \ \mathbf{x}_{N_{m,\tau}}]'$  and let  $N_{m,\tau}$  denote the total number of observations (over time) that fall in a given class of moneyness and time-to-maturity (indexed by  $m, \tau$ ). CF estimate (9)-(10) by GMM using data for each separate class of contracts. Although the point estimates of the regressor coefficients are identical to OLS, the advantage of this estimation

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<sup>30</sup>Descriptive statistics for each of the 20 classes show that contract categories are represented in a balanced way, although (as it is to be expected) long-term, deep ITM and OTM contracts are under-represented (less than 1,000 observations each). A table is available from the Authors upon request.

method is that the resulting estimate of the covariance matrix of the estimators,

$$Cov\left(\hat{\theta}_{m,\tau}^{GMM}\right) = (X'X)^{-1} \left[ N_{m,\tau}^{-1} \sum_{i=1}^{N_{m,\tau}} \hat{u}_i^2 \mathbf{x}_i \mathbf{x}'_i + N_{m,\tau}^{-1} \sum_{i=1}^{N_{m,\tau}} \sum_{j=i+1}^{N_{m,\tau}} I_{i,j} \hat{u}_i \hat{u}_j (\mathbf{x}_i \mathbf{x}'_j + \mathbf{x}_j \mathbf{x}'_i) \right] (X'X)^{-1}, \quad (11)$$

is robust to arbitrary forms of correlation in the errors  $u(t, z_t, \tau)$  arising from the fact that contracts overlap. In particular, the indicator variable  $I_{i,j} = 1$  if and only if two observations are associated with overlapping contracts, and zero otherwise. By running a few Monte Carlo experiments, CF show that for the problem at hand even in small samples the standard errors extracted from (11) are quite accurate. Table VIII reports the results for the original data set. Although  $\hat{\alpha}^{GMM}$  is never significant,  $\hat{\beta}^{GMM}$  is also never close to 1; in 12 cases out of 20  $\hat{\beta}^{GMM}$  is not significantly different from zero, in one case it is significantly negative. The  $R^2$  coefficients are in general very small, between 0.38% for OTM, very short-term options and 15% for ATM, very short-term contracts. For the 7 classes of contracts for which  $\hat{\alpha}^{GMM}$  is statistically nil while  $\hat{\beta}^{GMM} > 0$ ,  $\hat{\beta}^{GMM}$  is always statistically less than one and in 6 cases the IVs anyway fail the encompassing tests (10). In general, a moving window volatility index calculated on high frequency MIB30 returns seems to have a good forecasting power (the  $R^2$  in the encompassing regression always doubles), although the sign of  $\hat{\gamma}^{GMM}$  is significantly negative for 13 categories of contracts, possibly an indication of mean reversion. All in all, this battery of tests reveals strong informational inefficiency of the MIBO market.

Our findings improve only slightly when GMM methods are applied to arbitrage-free data. This confirms that pricing efficiency is not sufficient for informational efficiency to result. Table IX reports on the arbitrage-free data set estimates: we obtain higher values for the  $R^2$ s (38% for OTM, short-term options). However, with the only exception of ITM short-term contracts (that anyway fail the encompassing tests), it remains true that the estimated slope coefficients are in general quite small, often not even significantly positive (occasionally significantly negative), which is again inconsistent with informational efficiency.

## 5.2. Feasible GLS Tests

Write a generic cross-sectional regression for time  $t$  as

$$\mathbf{y}_t = \beta_0 \boldsymbol{\epsilon}_{20} + X_t \boldsymbol{\beta} + \mathbf{u}_t$$

where  $E[\mathbf{u}_t] = 0$  and  $E[\mathbf{u}_t \mathbf{u}'_t] = \Sigma_t$ .  $\mathbf{y}_t$  collects the ex-post realized volatility between time  $t$  and  $t + \tau$ , while the matrix  $X_t$  contains the regressors,  $IV(t, z_t, \tau)$  in (9) and  $[IV(t, z_t, \tau) \ x_t]'$  in (10). Let's stack the  $T = 3,434$  observations on the different times in the sample and write the model in compact fashion as:

$$Y = \boldsymbol{\beta}_0 + X \boldsymbol{\beta}_1 + \mathbf{u}.$$

Following Parks (1967), we initially assume  $\Sigma_t$  is constant over time and that no *serial* correlation patterns be present, so that the overall covariance matrix of the IV errors can be effectively described

by  $\Omega = \Sigma \otimes I_T$ . It is well known that the GLS estimator

$$\hat{\beta}^{GLS} = (X' \Omega X)^{-1} X' \Omega Y$$

is consistent and efficient, and also yields consistent estimates of the covariance matrix of the estimated coefficients,  $(X' \Omega X)^{-1}$ . Unfortunately,  $\Omega$  is unknown and must be first replaced by a consistent estimate, such as

$$\hat{\Omega}^{OLS} = \hat{\Sigma}^{OLS} \otimes I_T = \left[ T^{-1} \sum_{t=1}^T (\hat{\mathbf{u}}_t^{OLS})(\hat{\mathbf{u}}_t^{OLS})' \right] \otimes I_T$$

where  $\hat{\mathbf{u}}_t^{OLS} = \hat{\mathbf{y}}_t - \hat{\beta}_0^{OLS} \mathbf{x}_{20} - \mathbf{X}'_t \hat{\beta}^{OLS}$ . The resulting estimator,  $\hat{\beta}^{FGLS} = (X' \hat{\Omega}^{OLS} X)^{-1} X' \hat{\Omega}^{OLS} Y$ , is called the feasible (F)GLS. Under a variety of conditions (see Parks (1967)) it has been shown to be consistent and unbiased.<sup>31</sup>

We estimate equations (9)-(10) by FGLS separately for each of the 20 classes of moneyness/time to maturity. This choice remarkably simplifies the econometrics, since with only one observation per day we only have to worry about serial correlation of the disturbances. Table IXI presents the results. Panel A refers to the original data: for all classes, IVs fail the rationality tests as predictors of future MIB30 volatility during the life of the contract. The intercept is significantly positive (with p-values systematically below 5%) for *all* classes of contracts. On the other hand, the slope is most of the time (12 out of 20 cases) not statistically different from zero, and when it is different from zero it is often negative (7 cases). In general the coefficients are very small, and the maximum  $R^2$  is 0.0245. Overall, also with this method our first data set seems to strongly reject the hypothesis of informational efficiency of the MIBO market. These results are similar — if not stronger — to those of CF. Panel B offers a very similar picture for the arbitrage-free data set. Improvements are minimal, in the sense that only one of the estimated  $\alpha$  fails to be statistically significant, while there are still four  $\beta$  coefficients which are significantly negative; the  $R^2$  coefficient generally increases, but the fundamental idea is still that options' IVs contain only partial and biased information on realized volatility.

### 5.3. Panel Regressions

The assumption of  $\Sigma_t$  constant over time is easily rejected by most data sets. In our case, it is likely that forecast errors might be long-lived and hence serially correlated. Therefore we follow Parks (1967) two-stage method resort to a further step *to jointly take into account the presence of heteroskedasticity and serial correlation, phenomena made extremely likely by the high frequency nature of our data*. We regress (by OLS) the panel residuals on their lagged values and estimate the matrix  $R$  in the multivariate model

$$\hat{\mathbf{u}}_t^{FGLS} = R \hat{\mathbf{u}}_{t-1}^{FGLS} + \boldsymbol{\nu}_t \quad (12)$$

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<sup>31</sup>Asymptotically, it is also equivalent to the MLE and therefore is fully efficient. Even in the absence of normality, it can be interpreted as a pseudo-maximum likelihood estimation that retains all the asymptotic properties of MLE estimators (see Gouriéroux and Monfort (1984)).

where  $\nu_t$  is spherical. Finally, we apply OLS to the (so called Prais-Winsten) transformed model

$$y_t - \hat{R}y_{t-1} = \beta_0(I - \hat{R}) + (X_t - X_t\hat{R})\beta + \nu_t,$$

which yields consistent and efficient estimates of  $\hat{\beta}_0^{Parks}$  and  $\hat{\beta}_1^{Parks}$ , along with an unbiased estimate of their covariance matrix.

In practice, for the *original data* we find that the first-stage, FGLS residual do indeed display a high degree of persistence ( $\hat{\rho} = 0.999$ ). Therefore we apply the second stage of Parks's method. We estimate by OLS the model:

$$\hat{u}_t^{FGLS} = \rho I_{20} \hat{u}_{t-1}^{FGLS} + \nu_t,$$

a simplification of (12) to the case in which serial correlation is common in intensity to all classes of option contracts. Since we do not have any reason to suspect that random disturbances have a different persistence as a function of moneyness and/or maturity, and this assumption simplifies the task, we proceed to derive our final estimates from the Pras-Winsten modified regression:

$$y_t - \hat{\rho}y_{t-1} = \alpha(1 - \hat{\rho}) + (X_t - \hat{\rho}X_t)\beta + \nu_t.$$

We thus find (p-values in parenthesis):

$$\sigma^*(t, \tau) = \frac{0.213}{(0.0000)} - \frac{0.0027}{(0.0000)} IV(t, z_t, \tau) + u(t, z_t, \tau).$$

The  $R^2$  is of 0.8% only. Using the same econometric techniques we obtain a much higher explanatory power by estimating the encompassing regressions

$$\begin{aligned} \sigma^*(t, \tau) &= \frac{0.360}{(0.0000)} - \frac{0.0021}{(0.0000)} IV(t, z_t, \tau) - \frac{0.8754}{(0.0000)} \sigma_{MA}^*(t - 30, t) + u(t, z_t, \tau) \quad (R^2 = 0.2183) \\ \sigma^*(t, \tau) &= \frac{0.214}{(0.0000)} - \frac{0.0037}{(0.0000)} IV(t, z_t, \tau) - \frac{0.0025}{(0.0000)} IV(t - 1, z_t, \tau) + u(t, z_t, \tau) \quad (R^2 = 0.0105). \end{aligned}$$

Particularly in the first case, the  $R^2$  is quite high, and simple 30-days rolling window standard deviations forecasts future MIB30 volatility much better than IVs. As expected, the MIBO market simultaneously displays pricing and informational inefficiencies. In the case of the *arbitrage-free data*, the results are very similar. Through the same steps above, the forecasting regression is estimated to be:

$$\sigma^*(t, \tau) = \frac{0.237}{(0.0000)} - \frac{0.0003}{(0.1169)} IV(t, z_t, \tau) + u(t, z_t, \tau).$$

Although the panel estimate of the common  $\beta$  stops being significantly negative, this is meaningless as IV forecasts remain largely biased and the  $R^2$  negligible. We find confirmation that the clues of informational inefficiency are strong and probably unrelated to pricing efficiency.<sup>32</sup>

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<sup>32</sup>Results from encompassing regressions are similar to those from original data and therefore not reported. Interestingly, rejections at the GMM or FGLS level for most option classes need not imply an overall rejection when panel methods are used. The reason is twofold: first, only using panel methods one effectively takes into account of the presence of heteroskedasticity; second, there is no general result concerning the ranking of standard errors when corrections are applied using a HAC-type blanket approach vs. modeling the possible patterns of heteroskedasticity and autocorrelation.

#### 5.4. Tests on Daily, ATM Prices

We extract from our high-frequency data set a smaller sub-sample of closest-ATM, closing prices. For each day in the sample and all traded maturities, we take the 5:25 p.m. observations as representative of closing prices. We then select the closest-ATM contract, either put or call. This procedure returns a sample of 356 observations. Maturities are between a minimum of 1 day to a maximum of 108 days. ATM contracts, especially of short- to medium-term are the most actively traded ones; equivalently, their prices are unlikely to be stale. Replication of the tests of Sections 5.1-5.3 based on liquid contracts only ought to shield us from the possibility that high-frequency data might contain too much noise to allow options implied volatilities to emerge as unbiased and efficient forecasts of future volatility.<sup>33</sup>

In the following we limit ourselves to report results for the GMM-based tests first proposed by CF and for the case  $x_t \equiv \sigma_{MA}^*(t-30, t)$  only. We obtain the following estimates of (9)-(10) (p-values in parenthesis):

$$\sigma^*(t, \tau) = 0.106 + 0.275 IV(t, \tau) + u(t, \tau) \quad (R^2 = 0.0269) \quad (13)$$

$$\sigma^*(t, \tau) = 0.141 + 0.400 IV(t, \tau) - 0.359 \sigma_{MA}^*(t-30, t) + u(t, \tau) \quad (R^2 = 0.0762). \quad (14)$$

Notably, the rationality regression provides indications that are only mildly unfavorable to the hypothesis of efficiency of the MIBO market:  $\hat{\alpha}$  is no longer significant, while  $\hat{\beta}$  becomes now significantly positive. However, the null that  $\hat{\beta} = 1$  can be still rejected at all significance levels. The encompassing regression still gives negative results: although IVs become now significant in explaining future realized volatility, past realized volatility remains significant and displays a coefficient of roughly the same magnitude as IVs. As in Table VIII, the estimated coefficient for 30-days rolling window standard deviation is negative. The  $R^2$  obtained by including past historical volatility increases, from 2.7% to 7.6%.<sup>34</sup>

#### 5.5. Measurement Error Issues. Tests on Non-Overlapping Data.

CP stress that the estimates in CF might be plagued by substantial measurement error problems concerning the IVs, due for instance to misspecification of the Black-Scholes model from which options volatilities are implied. Therefore they present an instrumental variables procedure that ought to help avoid biases in the OLS estimates of (9)-(10) and hence incorrect conclusions from tests of market efficiency. In the following we apply CP's algorithms to the same sample of daily, closing

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<sup>33</sup>As an added payoff, using ATM options only gets around the issue of the existence of a ‘smile’ of implied volatility in MIBO prices. Chernov (2001) shows that even in the presence of stochastic volatility (the most plausible reason for the existence of the smile), ATM Black-Scholes implied volatility contains a negligible bias in forecasting realized volatility over the remaining life of a contract.

<sup>34</sup>We further break down the closing ATM sample based on time-to-maturity of the included options. To this purpose we use the same maturity classes defined in Section 2. No material differences along the maturity dimension emerge. We also perform tests based on arbitrage-free, daily closing prices. However, since the two data sets differ by a modest number of observations, the test results are similar and therefore omitted.

ATM prices used in Section 5.4. This choice simplifies calculations, makes the econometric problem standard (instrumental variable estimation in panel environments is all but straightforward), and has logical foundation in the finding that (13) represents the instance in which the least unfavorable results to the efficiency hypothesis were found. Estimation is performed by standard two-stages least squares (TSLS) algorithms.

As in CP, for given maturity  $t + \tau$ , past implied volatility  $IV(t - k, \tau + k)$ ,  $k \geq 1$ , is a natural instrument: for instance the first five  $k$ -th order autocorrelations are all in excess of 0.5 for short-term ( $\tau \in [8, 25]$ ) and medium-term contracts ( $\tau \in [26, 50]$ ); moreover, they are all highly significant in statistical terms. Hence it is plausible that  $IV(t - k, \tau + k)$  be highly correlated with the true but unobserved implied volatility; furthermore, being separated by at least  $k$  calendar days, it is likely that  $IV(t - k, \tau + k)$  be unrelated to the measurement error associated with implied volatility  $k$  days later. As in CP (p. 138) we also employ past historical volatility  $\sigma_{MA}^*$  as an additional instrument. In the following we set  $k = 3$  and investigate the TSLS versions of (9)-(10) for closing, near-ATM, short- and medium-term option prices only.<sup>35</sup> The first-stage estimates are:

$$\begin{aligned} (\text{short-term}) \quad IV(t, \tau) &= 0.088 + 0.416 IV(t - 3, \tau + 3) + 0.297 \sigma_{MA}^*(t - 33, t - 3) + \varepsilon(t, \tau) \\ (\text{medium-term}) \quad IV(t, \tau) &= 0.048 + 0.645 IV(t - 3, \tau + 3) + 0.188 \sigma_{MA}^*(t - 33, t - 3) + \varepsilon(t, \tau). \end{aligned}$$

The second-stage estimates are then obtained from (9) when  $IV(t, \tau)$  is replaced by the residuals derived from the first-stage regressions (standard errors are appropriately adjusted):

$$\begin{aligned} (\text{short-term}) \quad \sigma^*(t, \tau) &= -0.138 + 1.326 IV(t, \tau) + u(t, \tau) \quad (R^2 = 0.371) \\ (\text{medium-term}) \quad \sigma^*(t, \tau) &= 0.092 + 0.398 IV(t, \tau) + u(t, \tau) \quad (R^2 = 0.026). \end{aligned}$$

We obtain evidence that measurement error issues are unlikely to drive our results in Section 5.4: the null that  $\hat{\beta} = 1$  may be rejected on both maturity class; in fact, for medium term options we cannot even reject the hypothesis that  $\hat{\beta}$  is zero, and for short-term contracts there evidence that  $\hat{\beta} > 1$  (the standard error of  $\hat{\beta}$  is 0.137). For short-term ATM options, also the null of  $\hat{\alpha} = 0$  fails to be rejected in tests of standard size.<sup>36</sup>

CP (pp. 144-147) also stress that sampling procedures that allow options maturities to overlap induce upward biases in the explanatory power of past historical volatility in regressions like (10). Therefore they build a time series of non-overlapping ATM implied volatilities for contracts one month to expiration and find that S&P 100 index options may actually reflect unbiased estimates of future, realized volatility. One additional concern about our results is therefore related to the use of overlapping data. As a final robustness check, we distill a non-overlapping sample of near ATM, closing option prices with maturity of 15 days from our 10 month high-frequency data set.

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<sup>35</sup>We also estimate regressions with  $k = 2$  and 4 without noticing relevant changes in the estimated coefficients.  $k = 1$  is probably not a good instrument as with daily observations, it is possible that some correlation may exist between  $IV(t - 1, \tau + 1)$  and measurement errors. We do not present results for very-short term and long term contracts as these sub-samples fail to span in a uniform manner the period April 1999 - January 2000.

<sup>36</sup>The second-stage estimates for (10) also reject the null of informational efficiency.

Clearly, the resulting sample of observations is necessarily short, 14 non-overlapping observations only. Estimation of (9)-(10) on this non-overlapping data set gives:

$$\begin{aligned}\sigma^*(t, \tau) &= 0.015 + 0.634 IV(t, \tau) + u(t, \tau) \quad (R^2 = 0.2808) \\ &\quad (0.9047) \quad (0.2212) \\ \sigma^*(t, \tau) &= 0.253 - 1.858 IV(t, \tau) + 2.065 \sigma_{MA}^*(t - 30, t) + u(t, \tau) \quad (R^2 = 0.5283). \\ &\quad (0.2650) \quad (0.3545) \quad (0.2209)\end{aligned}$$

None of the coefficients is significant. However unbiasedness and efficiency can be rejected as the null of  $\hat{\beta} = 0$  is supported by both regressions. It would be of course interesting to replicate CP's tests on longer series of non-overlapping MIBO data. On the other hand, these results shows that overlapping sampling methods are unlikely to be responsible for the finding of informational inefficiency of the MIBO.<sup>37</sup>

## 6. Conclusion

This paper has analyzed the pricing and informational efficiency of the Italian market for options on the most important stock index, the MIB30. We find several indications inconsistent with previous findings in the literature (see Cavallo and Mammola (2000)) that have led to the conclusion that the Italian MIBO30 is quite an efficient options market favoring risk-sharing activities and unbiased aggregation and dissemination of information. On the opposite, we report that a striking percentage of the data consists of option prices violating some basic no-arbitrage condition. This percentage declines but never becomes negligible when we relax the no-arbitrage restrictions to accommodate for the presence of bid/ask spreads and other frictions. The result holds generally for all levels of moneyness and time-to-expiry. We also tentatively map the presence of niches of resilient arbitrage opportunities to a few microstructural features of the Italian derivatives market, such as the mismatch between the calendar cycle of futures (FIB30) and options markets, and the fact that maximum bid/ask spreads are set in absolute values and therefore can be particularly wide for low-price contracts (DOTM). Finally, using a variety of econometric tools and approaches for the treatment of our high-frequency panel, we investigate the informational efficiency of the MIBO and conclude that option prices are indeed very poor aggregators and predictors of future volatility of MIB30 returns.

There are many possible extensions of this work. First, we feel that a better comprehension of the effects of market microstructure would be highly desirable. To this end, it would be interesting to consider making frictions and costs endogenous, as the outcome of the maximizing behavior of market makers. Clearly this would also imply the need for a more detailed data set including the characteristics of the trader, etc. (practically, using information from market makers' books). Second, although Section 5 insists on checking the robustness of the results on informational efficiency to the existence of basic arbitrage violations, we believe that our understanding of the relationship between these two basic components of market efficiency should be improved. *Third, given our*

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<sup>37</sup>CP also argue that a possible cause of difference relative to results in CF is their use of pre-October 1987 crash data. There is evidence of substantial heterogeneity in the way index options were priced before and after the crash. However, in our (in fact, rather short) sample we fail to find any evidence of outliers in MIB30 returns or interest rates, of unusual volatility in stock or bond returns, and of significant regulatory changes.

conclusions on the efficiency of the MIBO market, we need to stress that our results are strictly sample-specific, i.e. we cannot rule out that the market might have evolved in a much more efficient allocative and operative mechanism in the three years following the end of our sample. The reform of the IDEM rules that has taken place in the following years points in this direction.<sup>38</sup> Only more data and repeated tests will allow financial economists to reach firm conclusions.

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<sup>38</sup>Currently a much wider range of strike prices have to be quoted: four above and four below the strike price next to the index.

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**Table I**  
**Summary Statistics.**

Summary statistics of the financial prices (options, the MIB30 index, and the interest rate) used in the paper. All the values are expressed in MIB30 index points. MIB30 index returns are continuously compounded and annualized.

	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std Dev.</b>
Call prices	1	5,260	1,003.99	855.41
Put prices	1	4,300	882.25	667.97
All contracts - price	1	5,260	942.55	768.97
Strike price	31,000	44,000	37,500	3,968.63
Residual Life	1	109	26.07	16.93
Black-Scholes implied volatility	0.0393	1.5474	0.2548	0.0775
ATM – BS implied volatility	0.0515	0.7755	0.2437	0.0477
MIB30 index	31,518	43,476	35,821	2,923.63
MIB30 index returns (%)	-107.15	68.22	0.141	0.178
Risk-free Rate (LIBOR)	2.48	3.54	2.99	0.3605

**Table II**  
**Summary Statistics – Percentage Composition of the Data Set By Moneyness and Time to Maturity.**

Moneyness is measured by  $z = S/K$  for a call option and  $z = K/S$  for a put option. In the case of a call, moneyness classes correspond to:  $z < 0.95$  (DOTM),  $0.95 \leq z < 0.98$  (OTM),  $0.98 \leq z \leq 1.02$  (ATM),  $1.02 < z \leq 1.05$  (ITM) and  $1.05 < z$  (DITM). The classes for time to maturity ( $\tau$ ) are defined as follows:  $\tau \leq 7$  (Very Short),  $7 < \tau \leq 25$  (Short),  $25 < \tau \leq 50$  (Medium),  $50 < \tau \leq 90$  (Long) and  $\tau > 90$  (Very Long).

	<b>Very Short</b>	<b>Short</b>	<b>Medium</b>	<b>Long</b>	<b>Very Long</b>	<b>Total</b>
DOTM	1,29	5,13	5,69	0,69	0,01	12,81
OTM	2,55	9,97	9,02	2,25	0,13	23,92
ATM	4,36	15,76	13,75	3,39	0,22	37,48
ITM	2,14	8,12	5,97	1,29	0,10	17,62
DITM	1,06	3,62	3,03	0,39	0,07	8,16
Total	11,40	42,60	37,46	8,01	0,52	100

**Table III**
**Arbitrage Opportunities with Bid/Ask Spreads: Percentage Incidence and Average Profits.**

The table reports the percentage ratio (on the total sample size), and the average profit deriving from violations of the no-arbitrage conditions listed in the first column of the table, and under the structure of transaction costs assumed in Section 4. Each column refers to a different half-size of the bid/ask spreads for options and spot index markets. These scenario simulations impose the restriction  $\alpha = \beta$ . In the last line (“Overall”) we report the percentage of contracts violating at least one of the preceding conditions except the maturity spread and the average of the maximum arbitrage profits.

	Half-size of the bid-ask spread ( $\alpha$ )										
	0	1	2	3	4	5	6	7	8	9	10
L. Bound	%	3.12	0.35	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	349.26	919.90	600.02	698.60	109.95	0.00	0.00	0.00	0.00	0.00
Strike Mon.	%	1.77	1.68	1.63	1.58	1.55	1.50	1.47	1.42	1.35	1.30
	$\pi$	231.47	227.18	217.99	209.96	199.74	192.28	181.33	173.24	167.89	160.11
Rev. Mon.	%	2.98	1.77	1.19	0.82	0.54	0.39	0.27	0.21	0.17	0.14
	$\pi$	114.63	141.53	160.35	179.99	220.91	260.73	325.25	370.04	422.17	478.98
Butterfly	%	14.48	8.80	5.42	3.53	2.25	1.53	1.05	0.77	0.61	0.52
	$\pi$	61.11	65.44	72.07	77.34	88.77	101.20	120.71	143.90	162.49	173.92
Box– Short	%	23.08	15.41	9.80	6.51	4.59	3.39	2.63	2.13	1.73	1.42
	$\pi$	150.47	177.47	231.22	301.65	383.40	476.71	574.07	669.89	786.42	923.65
Box – Long	%	21.66	13.77	8.16	5.04	3.25	2.09	1.36	0.95	0.69	0.50
	$\pi$	89.95	91.28	102.49	115.75	130.31	153.10	188.25	223.72	263.75	322.10
P/C – Short	%	24.64	0.34	0.25	0.20	0.16	0.09	0.02	0.00	0.00	0.00
	$\pi$	127.17	1905.29	1696.55	1287.55	823.86	499.51	65.27	0.00	0.00	0.00
P/C – Long	%	15.26	0.13	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	80.31	677.53	323.24	186.50	0.00	0.00	0.00	0.00	0.00	0.00
Mat. Mon.	%	0.04	0.03	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00
	$\pi$	130.37	142.63	151.60	194.03	167.19	133.19	143.63	85.07	43.37	0.00
Mat. – Short	%	0.29	0.16	0.10	0.05	0.04	0.03	0.02	0.02	0.01	0.00
	$\pi$	93.60	102.45	99.86	126.28	132.16	120.45	106.22	70.91	82.03	94.94
Mat. – Long	%	17.69	17.43	17.12	16.72	16.09	15.32	14.52	13.41	12.21	10.97
	$\pi$	394.81	364.04	334.30	306.15	282.26	261.26	241.09	226.93	215.64	207.50
Overall	%	51.74	33.35	22.51	15.34	10.84	7.97	6.08	4.91	4.07	3.45
	$\pi$	165.23	153.88	170.55	202.56	243.91	292.82	349.02	399.55	452.24	505.27
											560.80

**Table IV**  
**Arbitrage Opportunities with Bid/Ask Spreads as a Function of Moneyness: Percentage Incidence and Average Profits.**

This table is entirely analogous to table III save that costs are modeled as a function of moneyness. In particular, the value of  $\alpha$  reported in each columns refers to the spread for ATM options. If the option is OTM or ITM the spread is increased by 25% with respect to ATM; if the option is dOTM or dITM then the spread is increased by 50% with respect to ATM.

		Half-size of the bid-ask spread ( $\alpha$ )										
		0	1	2	3	4	5	6	7	8	9	10
Effective $\alpha$		0	1.21	2.42	3.63	4.83	6.04	7.25	8.46	9.67	10.88	12.09
L. Bound	%	3.12	0.35	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	349.26	918.29	597.58	692.64	100.95	0.00	0.00	0.00	0.00	0.00	0.00
Strike Mon.	%	1.77	1.67	1.59	1.54	1.49	1.43	1.39	1.31	1.22	1.17	1.08
	$\pi$	231.47	225.64	217.46	207.80	196.90	186.82	175.85	168.53	163.45	153.62	149.69
Rev. Mon.	%	2.98	1.77	1.19	0.82	0.54	0.39	0.27	0.21	0.17	0.14	0.12
	$\pi$	114.63	141.53	160.35	179.99	220.91	260.73	325.25	370.04	422.17	478.98	519.22
Butterfly	%	14.48	7.91	4.45	2.60	1.57	0.97	0.70	0.53	0.43	0.38	0.33
	$\pi$	61.11	65.88	72.91	82.10	98.51	124.57	147.47	170.45	189.32	194.62	202.53
Box– Short	%	23.08	14.14	8.24	5.15	3.47	2.52	1.94	1.47	1.11	0.88	0.74
	$\pi$	150.47	184.01	254.48	349.08	463.60	588.21	714.89	897.99	1142.74	1390.72	1619.26
Box – Long	%	21.66	12.43	6.65	3.77	2.17	1.31	0.84	0.58	0.44	0.33	0.29
	$\pi$	89.95	91.35	104.85	121.83	149.70	187.26	238.08	289.42	330.02	386.24	404.02
P/C – Short	%	24.64	0.34	0.25	0.20	0.16	0.09	0.01	0.00	0.00	0.00	0.00
	$\pi$	127.17	1921.58	1682.24	1272.51	796.47	440.38	73.06	0.00	0.00	0.00	0.00
P/C – Long	%	15.26	0.13	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	80.31	671.72	315.89	173.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mat. Mon.	%	0.04	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	130.37	128.47	139.31	138.39	143.63	55.78	0.00	0.00	0.00	0.00	0.00
Mat. – Short	%	0.29	0.15	0.06	0.03	0.02	0.02	0.01	0.00	0.00	0.00	0.00
	$\pi$	93.60	92.41	109.31	147.91	113.80	66.42	45.90	21.38	0.00	0.00	0.00
Mat. – Long	%	17.69	17.28	16.75	15.88	15.05	14.08	12.95	11.82	10.57	9.34	8.40
	$\pi$	394.81	359.64	327.27	302.76	278.70	258.67	243.29	229.77	221.20	216.00	207.39
Overall	%	51.74	31.08	19.30	12.32	8.26	5.92	4.60	3.67	3.00	2.57	2.26
	$\pi$	165.23	156.66	181.38	225.44	285.60	353.48	416.27	485.75	560.77	624.90	681.69

**Table V**  
**Arbitrage Opportunities with Bid/Ask Spreads as a Function of the Option Delta:**  
**Percentage Incidence and Average Profits.**

This table is entirely analogous to table III save that the parameter  $\alpha$  corresponding to the bid/ask spread is no longer assumed to be constant but rather an increasing function of the absolute value of the delta of the option. The sample of option contracts is divided into four classes of possible values of  $|\delta|$ :  $|\delta| \leq 25\%$ ,  $25\% < |\delta| \leq 50\%$ ,  $50\% < |\delta| \leq 75\%$  and  $75\% < |\delta|$ . The value of the spread applied to each class is respectively:  $\alpha$ ,  $1.25\alpha$ ,  $1.5\alpha$  and  $1.75\alpha$ , where the value assigned to  $\alpha$  appears in the first row. In the second line of the table it is computed the average level of the spread across the whole sample for given value of  $\alpha$ .

		Half-size of the bid-ask spread ( $\alpha$ )										
		0	1	2	3	4	5	6	7	8	9	10
Effective $\alpha$		0	1.34	2.68	4.02	5.36	6.70	8.04	9.38	10.72	12.06	13.40
L. Bound	%	3.12	0.35	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	349.26	918.20	597.58	692.64	100.95	0.00	0.00	0.00	0.00	0.00	0.00
Strike Mon.	%	1.77	1.66	1.60	1.55	1.49	1.43	1.35	1.26	1.18	1.12	1.03
	$\pi$	231.47	223.71	211.06	197.10	185.33	173.76	163.24	155.59	147.21	137.02	130.79
Rev. Mon.	%	2.98	1.42	0.77	0.43	0.26	0.18	0.14	0.11	0.09	0.07	0.06
	$\pi$	114.63	148.72	186.60	248.71	330.80	408.27	470.68	539.87	610.05	743.77	765.48
Butterfly	%	14.48	7.19	3.85	2.14	1.31	0.88	0.67	0.54	0.47	0.43	0.38
	$\pi$	61.11	66.24	72.80	84.43	100.88	120.70	137.00	150.93	153.59	151.23	151.70
Box– Short	%	23.08	12.48	6.67	4.04	2.74	1.98	1.48	1.09	0.82	0.67	0.58
	$\pi$	150.47	199.08	296.75	419.75	557.89	710.55	897.08	1156.52	1494.92	1781.43	2008.46
Box – Long	%	21.66	10.73	5.18	2.67	1.43	0.83	0.54	0.39	0.30	0.24	0.18
	$\pi$	89.95	95.73	114.34	139.94	182.92	241.45	306.43	359.62	406.05	446.44	533.68
P/C – Short	%	24.64	0.34	0.25	0.20	0.16	0.09	0.01	0.00	0.00	0.00	0.00
	$\pi$	127.17	1920.93	1711.42	1278.84	807.45	463.56	69.37	0.00	0.00	0.00	0.00
P/C – Long	%	15.26	0.13	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	80.31	673.98	308.55	161.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mat. Mon.	%	0.04	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\pi$	130.37	122.64	211.74	143.12	111.52	32.37	0.00	0.00	0.00	0.00	0.00
Mat. – Short	%	0.29	0.14	0.05	0.03	0.02	0.02	0.00	0.00	0.00	0.00	0.00
	$\pi$	93.60	90.18	134.88	130.77	97.19	49.26	52.18	0.00	0.00	0.00	0.00
Mat. – Long	%	17.69	17.25	16.54	15.60	14.42	12.86	11.20	9.58	7.92	6.76	5.60
	$\pi$	394.81	350.35	311.25	277.49	249.14	229.36	214.87	204.84	202.87	195.06	194.08
Overall	%	51.74	27.92	15.83	9.62	6.45	4.73	3.71	3.00	2.50	2.18	1.94
	$\pi$	165.23	164.55	202.34	262.14	333.51	406.51	476.40	551.99	626.72	684.65	740.33

**Table VI**  
**Arbitrage opportunities with bid/ask Spreads. Sample composition.**

The table reports the distribution of arbitrage opportunities across moneyness and maturity. Each cell Number of arbitrage opportunities for each condition as distributed over maturity and moneyness classes (such classes are defined as in Table I). The b/a spread is fixed at 0 and 5%.

		Very Short		Short		Medium		Long		Total		
		$\alpha=0$	$\alpha=5$									
DOM	Butterfly	13.47	10.92	7.69	2.30	8.96	1.80	10.02	0.57	8.96	2.85	
	Box spread	Short	31.84	10.00	24.86	3.45	18.60	4.00	8.13	0.00	21.87	4.16
		Long	30.41	7.45	22.84	2.20	14.18	1.50	9.64	0.19	19.04	2.31
	Mat Spread	Short	0.82	0.00	0.69	0.00	0.00	0.00	0.00	0.00	0.36	0.00
		Long	43.78	20.31	24.83	6.64	1.06	0.12	0.00	0.00	14.84	4.76
OTM	General		52.76	32.65	36.74	8.20	30.93	7.81	19.47	1.70	34.83	10.13
	Butterfly		5.03	3.39	3.69	0.16	13.99	1.05	14.83	0.30	8.76	0.85
	Box spread	Short	38.24	8.06	31.44	3.90	22.98	1.86	16.18	0.35	27.56	3.25
		Long	37.94	8.32	30.02	2.37	20.90	1.39	14.35	0.65	25.97	2.48
	Mat Spread	Short	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
ATM	Overall		49.13	48.31	32.68	31.40	0.99	0.86	0.00	0.00	19.47	18.79
	Butterfly		2.94	0.63	8.70	0.38	22.78	1.63	25.77	1.97	14.80	1.02
	Box spread	Short	30.69	7.87	33.66	4.78	22.02	1.85	16.72	1.63	27.45	3.77
		Long	30.03	6.82	31.64	2.34	20.85	1.58	18.13	0.52	26.22	2.41
	Mat Spread	Short	0.06	0.00	0.02	0.00	0.01	0.00	0.00	0.00	0.02	0.00
ITM	Overall		44.20	44.17	36.62	36.49	1.15	1.05	0.00	0.00	21.05	20.95
	Butterfly		53.60	15.53	54.46	8.53	54.96	5.98	53.87	4.19	54.49	8.00
	Box spread	Short	27.33	3.56	21.58	2.26	23.23	2.26	21.55	1.13	22.83	2.33
		Long	14.37	4.73	18.99	3.71	12.87	1.78	10.59	1.51	15.69	3.01
	Mat Spread	Short	10.57	1.35	18.51	1.58	11.37	0.92	11.91	1.42	14.61	1.32
DTIM	Overall		0.37	0.06	0.37	0.10	0.18	0.00	0.00	0.00	0.28	0.05
	Butterfly		27.21	23.89	23.84	19.78	0.83	0.35	0.00	0.00	14.57	12.14
	Box spread	Long	78.99	12.84	65.02	8.88	55.60	6.50	45.84	3.50	62.01	8.13
		Short	5.96	1.12	8.99	1.78	7.83	1.90	4.86	0.57	7.94	1.67
	Mat Spread	Short	6.21	0.50	8.81	0.91	7.70	1.08	1.43	0.00	7.65	0.87
Total	Overall		8.07	0.25	2.57	0.43	0.30	0.00	0.00	0.00	2.30	0.22
	Butterfly		14.16	4.84	14.24	5.07	0.74	0.09	0.00	0.00	8.42	2.91
	Box spread	Long	79.75	5.09	71.91	6.52	63.39	8.83	37.14	1.43	67.81	6.91
		Short	27.17	6.94	27.19	3.96	19.11	2.17	14.18	1.07	23.08	3.39
	Mat Spread	Long	25.99	5.62	25.76	2.06	17.26	1.38	14.45	0.65	21.66	2.09
	Overall		0.96	0.03	0.38	0.06	0.06	0.00	0.00	0.00	0.29	0.03
	Butterfly		39.29	34.97	29.94	25.85	1.01	0.67	0.00	0.00	17.69	15.32
	Box spread	Long	60.41	17.06	53.31	8.00	49.15	6.28	43.45	2.94	51.74	7.97

**Table VII**  
**Informational Content of MIBO30 Implied Volatility – Forecasting and Encompassing**  
**Regression Tests for Classes of Moneyness and Time-to-Maturity (GMM).**  
**Original Data (Purged of Violations of the Lower Bound Condition)**

The table reports the GMM estimates of the coefficients  $\alpha$ ,  $\beta$ , and  $\gamma$  in the regression tests of forecast rationality:

$$\sigma^*(t, \tau) = \alpha + \beta IV(z_t, t, \tau) + u(z_t, t, \tau) \text{ and}$$

$$\sigma^*(t, \tau) = \alpha + \beta IV(z_t, t, \tau) + \gamma x_t + u(z_t, t, \tau)$$

where  $\sigma^*(t, \tau)$  is the annualized standard deviation of MIB30 (infra-daily) log- returns, and  $u(z_t, t, \tau)$  is a white-noise residual.  $x_t$  corresponds to either the rolling window standard deviation of (infra-daily) MIB30 returns over the 30 days preceding  $t$ , or the lagged value if  $IV$ . \* indicates that a coefficient is significant at 5%, while \*\* means significant at 1%. In the case of encompassing regressions we only report the estimate of  $\gamma$  for the two definitions of  $x_t$ . When the IVs represent rational forecasts of future volatility,  $\alpha = 0$  and  $\beta = 1$ . In encompassing regressions,  $\alpha = \gamma = 0$  and  $\beta = 1$ .

		Very short	Short	Medium	Long	All maturities
DOTM	$\hat{\alpha}^{GMM}$	0.1868	0.1050	0.1246	0.1192	0.1395
	$\hat{\beta}^{GMM}$	-0.0143	0.2561*	0.1231	0.2562**	0.0907
	R <sup>2</sup>	0.0060	0.1320	0.0781	0.1003	0.0773
	Numb. Obs.	(688)	(2.567)	(2.685)	(367)	(6.307)
	$\hat{\gamma}^{GMM} x_t = RW_{vol}$	-1.0621**	-0.1839*	0.2043**	-1.1331**	0.0653
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	-0.0101	0.1755*	0.0891	0.1663**	0.0383
OTM	$\hat{\alpha}^{GMM}$	0.1571	0.1322	0.1693	0.1224	0.1467
	$\hat{\beta}^{GMM}$	0.0183	0.1217	-0.0414	0.2206*	0.0590
	R <sup>2</sup>	0.0038	0.0711	0.0064	0.0561	0.0137
	Numb. Obs.	(2.006)	(6.362)	(5.322)	(1.335)	(15.025)
	$\hat{\gamma}^{GMM} x_t = RW_{vol}$	-0.3080**	-0.1861	-0.2519*	-1.0194**	-0.1985
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0132	0.0594*	-0.0242	0.1354	0.0410
ATM	$\hat{\alpha}^{GMM}$	0.1005	0.1292	0.2014	0.0971	0.1408
	$\hat{\beta}^{GMM}$	0.2230*	0.1357	-0.1638	0.2992**	0.0846
	R <sup>2</sup>	0.1516	0.0467	0.0848	0.0660	0.0104
	Numb. Obs.	(5.210)	(10.399)	(10.419)	(2.590)	(28.618)
	$\hat{\gamma}^{GMM} x_t = RW_{vol}$	-0.1307	-0.1433	-0.2845*	-1.1863**	-0.2796*
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.1362	0.0894	-0.1072	0.1953	0.0529
ITM	$\hat{\alpha}^{GMM}$	0.1332	0.1541	0.2048	0.1116	0.1740
	$\hat{\beta}^{GMM}$	0.0698	0.0317	-0.1643*	0.2312**	-0.0505
	R <sup>2</sup>	0.0589	0.0143	0.1349	0.0470	0.0066
	Numb. Obs.	(2.183)	(5.801)	(5.741)	(1.447)	(15.172)
	$\hat{\gamma}^{GMM} x_t = RW_{vol}$	0.2034**	0.0199	-0.3555**	-1.2281**	-0.1569
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0495	0.0140	-0.1097	0.1605*	-0.0167
DITM	$\hat{\alpha}^{GMM}$	0.1457	0.1722	0.1890	0.1336	0.1796
	$\hat{\beta}^{GMM}$	0.0155	0.0165	-0.0742	0.0544*	-0.0340
	R <sup>2</sup>	0.0088	0.0068	0.0977	0.0200	0.0138
	Numb. Obs.	(1.041)	(3.119)	(3.689)	(558)	(8.407)
	$\hat{\gamma}^{GMM} x_t = RW_{vol}$	0.3547**	-0.3296*	-0.2239**	-1.3585**	-0.1870*
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0119	0.0037	-0.0379	0.0282	-0.0154
All levels of moneyness	$\hat{\alpha}^{GMM}$	0.1423	0.1062	0.1882	0.1438	0.1508
	$\hat{\beta}^{GMM}$	0.0435	0.2394	-0.1113	0.1000	0.0351
	R <sup>2</sup>	0.0264	0.0771	0.0298	0.0136	0.0055
	Numb. Obs.	(11.128)	(28.248)	(27.856)	(6.297)	(73.529)
	$\hat{\gamma}^{GMM} x_t = RW_{vol}$	-0.0481	-0.1382	-0.2874	-1.1610**	-0.1747
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0348	0.1617	-0.0737	-0.0720	0.0236

**Table VIII**  
**Informational Content of MIBO30 Implied Volatility – Forecasting and Encompassing**  
**Regression Tests for Classes of Moneyness and Time-to-Maturity (GMM).**  
**Arbitrage-Free Data.**

The table reports the GMM estimates of the coefficients  $\alpha$ ,  $\beta$ , and  $\gamma$  in the regression tests of forecast rationality:

$$\sigma^*(t, \tau) = \alpha + \beta IV(z_t, t, \tau) + u(z_t, t, \tau) \text{ and}$$

$$\sigma^*(t, \tau) = \alpha + \beta IV(z_t, t, \tau) + \gamma x_t + u(z_t, t, \tau)$$

where  $\sigma^*(t, \tau)$  is the annualized standard deviation of MIB30 (infra-daily) log- returns, and  $u(z_t, t, \tau)$  is a white-noise residual.  $x_t$  corresponds to either the rolling window standard deviation of (infra-daily) MIB30 returns over the 30 days preceding  $t$ , or the lagged value if  $IV$ . \* indicates that a coefficient is significant at 5%, while \*\* means significant at 1%. In the case of encompassing regressions we only report the estimate of  $\gamma$  for the two definitions of  $x_t$ . When the IVs represent rational forecasts of future volatility,  $\alpha = 0$  and  $\beta = 1$ . In encompassing regressions,  $\alpha = \gamma = 0$  and  $\beta = 1$ . ♦ indicates that a slope coefficient is not significantly different from 1 at 5%.

		Very short	Short	Medium	Long	All maturities
DOTM	$\hat{\alpha}^{GMM}$	0.1737	0.0709	0.1182	0.1221	0.1326
	$\hat{\beta}^{GMM}$	0.0016	0.4079**	0.1522	0.2485**	0.1249
	R <sup>2</sup>	0.0001	0.2967	0.0948	0.0860	0.1067
	Numb. Obs.	(463)	(2.507)	(2.550)	(363)	(5.883)
	$\hat{\gamma}^{GMM} x_t = RW vol$	-0.8217**	0.2860**	0.2051**	-1.1238**	0.3685**
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0008	0.2510*	0.1052	0.1531	0.0632
OTM	$\hat{\alpha}^{GMM}$	0.1458	0.0461	0.1761	0.1856	0.1275
	$\hat{\beta}^{GMM}$	0.0474	0.5168	-0.0515	-0.0269	0.1580
	R <sup>2</sup>	0.0175	0.3774	0.0054	0.0003	0.0600
	Numb. Obs.	(1.580)	(6.182)	(5.012)	(1.274)	(14.048)
	$\hat{\gamma}^{GMM} x_t = RW vol$	-0.0015	0.2344*	-0.2107*	-1.2297**	0.1569
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0359	0.2870*	-0.0344	-0.0204	0.1071
ATM	$\hat{\alpha}^{GMM}$	0.0799	0.0346	0.1962	0.3269	0.1093
	$\hat{\beta}^{GMM}$	0.3076**	0.5587*	-0.1026	-0.6441**	0.2567
	R <sup>2</sup>	0.2721	0.2971	0.0108	0.0938	0.0686
	Numb. Obs.	(3.642)	(10.114)	(9.760)	(2.572)	(26.088)
	$\hat{\gamma}^{GMM} x_t = RW vol$	0.1620*	0.1912	-0.1293	-1.6943**	0.0370
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.1784	0.2899	-0.0635	-0.4245*	0.1552
ITM	$\hat{\alpha}^{GMM}$	0.1288	0.0261	0.2098	0.3565	0.1452
	$\hat{\beta}^{GMM}$	0.0922	0.5966**♦	-0.1058	-0.7103**	0.1342
	R <sup>2</sup>	0.0778	0.2745	0.0096	0.1555	0.0232
	Numb. Obs.	(1.746)	(5.505)	(5.526)	(1.385)	(14.162)
	$\hat{\gamma}^{GMM} x_t = RW vol$	0.3840**	0.3213*	0.0292	-1.8175**	0.2741
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0646	0.3386	-0.0701	-0.4664**	0.1247
DITM	$\hat{\alpha}^{GMM}$	0.1510	0.0639	0.1648	0.3605	0.2002
	$\hat{\beta}^{GMM}$	0.0163	0.4859	0.1580	-0.6795**	-0.0042
	R <sup>2</sup>	0.0090	0.2762	0.0225	0.3107	0.0001
	Numb. Obs.	(892)	(2.872)	(3.471)	(546)	(7.781)
	$\hat{\gamma}^{GMM} x_t = RW vol$	0.4820**	0.3881*	0.1979	-3.1775**	0.4971
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0124	0.3156	0.1003	-0.3712**	0.0485
All levels of moneyness	$\hat{\alpha}^{GMM}$	0.1399	0.0388	0.1413	0.2542	0.1374
	$\hat{\beta}^{GMM}$	0.0598	0.5497	0.1539	-0.3101	0.1455
	R <sup>2</sup>	0.0476	0.3431	0.0227	0.0404	0.0482
	Numb. Obs.	(8.323)	(27.180)	(26.319)	(6.140)	(67.962)
	$\hat{\gamma}^{GMM} x_t = RW vol$	0.0459	0.2146	-0.1039	-1.5882**	0.2308
	$\hat{\gamma}^{GMM} x_t = IV_{t-1}$	0.0480	0.3275	0.0975	-0.2160	0.1057

**Table IX**

**Informational Content of MIBO30 Implied Volatility – Regression Tests for Classes of Moneyness and Time-to-Maturity (FGLS)**

The table reports the FGLS estimates of the coefficients  $\alpha$  and  $\beta$  in the regression tests of forecast rationality:

$$\sigma^*(t, \tau) = \alpha + \beta IV(z_t, t, \tau) + u(z_t, t, \tau).$$

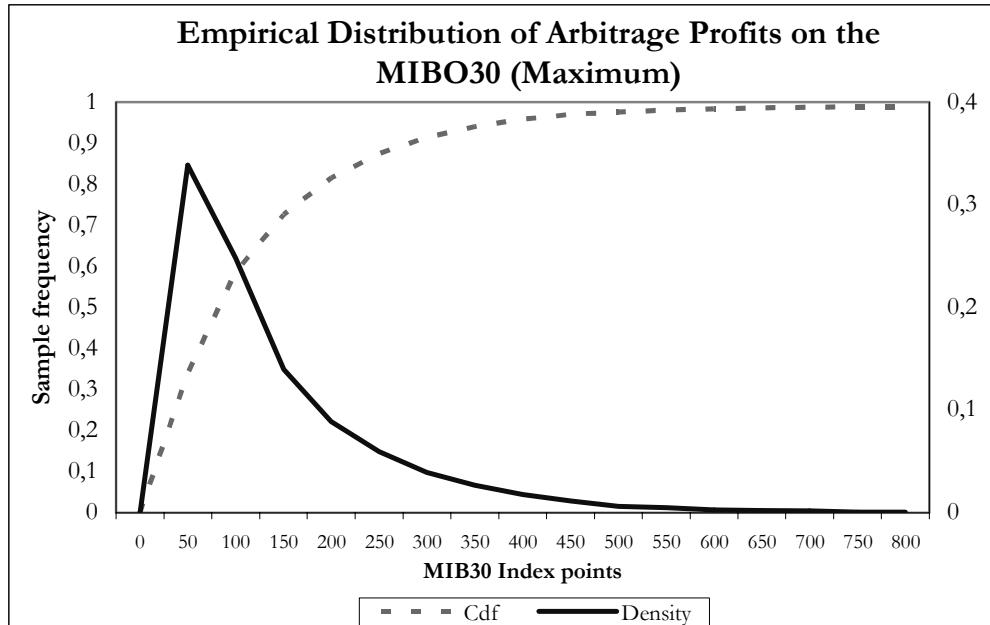
where  $\sigma^*(t, \tau)$  is the annualized standard deviation of MIB30 (infra-daily) log- returns. and  $u(z_t, t, \tau)$  is a white-noise residual. \* indicates that a coefficient is significant at 5%. while \*\* means significant at 1%. The two panels report results for the two balanced panels built by reduction of the original data sets (lower-bound violations and arbitrage violations-free) in Section 5. When the IVs represent rational forecasts of future volatility.  $\alpha = 0$  and  $\beta = 1$ .

		Panel A — panel derived from lower bound violation-free data (21.589 obs.)			
		Very short	Short	Medium	Long
DOTM	$\hat{\alpha}^{FGLS}$	0.2000**	0.1610**	0.1538**	0.1720**
	$\hat{\beta}^{FGLS}$	-0.0060	0.0024	-0.0040**	-0.0002
	R <sup>2</sup>	0.0137	0.0007	0.0245	0.0000
	Numb. Obs.	(210)	(976)	(720)	(147)
OTM	$\hat{\alpha}^{FGLS}$	0.3800*	0.1556**	0.2680*	0.7700**
	$\hat{\beta}^{FGLS}$	-0.0039*	-0.0000	-0.0005	0.0008
	R <sup>2</sup>	0.0095	0.0000	0.0005	0.0056
	Numb. Obs.	(594)	(1.768)	(1.747)	(445)
ATM	$\hat{\alpha}^{FGLS}$	0.1607**	0.1667**	0.1900**	0.2000**
	$\hat{\beta}^{FGLS}$	-0.0020	0.0100**	-0.0012*	-0.0026*
	R <sup>2</sup>	0.0001	0.0059	0.0022	0.0133
	Numb. Obs.	(794)	(1.999)	(2.052)	(491)
ITM	$\hat{\alpha}^{FGLS}$	0.1593**	0.2070**	0.1860**	0.1840**
	$\hat{\beta}^{FGLS}$	-0.0043	-0.0026*	0.0002	0.0015
	R <sup>2</sup>	0.0045	0.0030	0.0002	0.0001
	Numb. Obs.	(660)	(1.682)	(1.742)	(472)
DITM	$\hat{\alpha}^{FGLS}$	0.1778**	0.2190**	0.9540**	0.1930**
	$\hat{\beta}^{FGLS}$	-0.0002	-0.0055**	-0.0019**	-0.0035
	R <sup>2</sup>	0.0001	0.0139	0.0120	0.0044
	Numb. Obs.	(458)	(865)	(1.109)	(186)
		Panel B — panel derived from arbitrage-free data (21.165obs.)			
		Very short	Short	Medium	Long
DOTM	$\hat{\alpha}^{FGLS}$	0.1969**	0.1610**	0.1500**	0.1552**
	$\hat{\beta}^{FGLS}$	-0.0086*	0.0076*	-0.0003	-0.0005
	R <sup>2</sup>	0.0297	0.0041	0.0001	0.0001
	Numb. Obs.	(201)*	(976)	(720)	(147)
OTM	$\hat{\alpha}^{FGLS}$	0.2330	0.1584**	0.2777*	0.7720**
	$\hat{\beta}^{FGLS}$	-0.0010	0.0114*	-0.0000	0.0007
	R <sup>2</sup>	0.0005	0.0056	0.0000	0.0043
	Numb. Obs.	(594)	(1.722)	(1.726)	(445)
ATM	$\hat{\alpha}^{FGLS}$	0.1600**	0.1679**	0.3360**	0.1828**
	$\hat{\beta}^{FGLS}$	-0.0034**	-0.0004	-0.0012	-0.0018
	R <sup>2</sup>	0.0006	0.0001	0.0022	0.0007
	Numb. Obs.	(792)	(1.950)	(2.017)	(573)
ITM	$\hat{\alpha}^{FGLS}$	0.1647**	0.1711**	0.1917**	0.1843**
	$\hat{\beta}^{FGLS}$	-0.0078**	0.0029	0.0008	0.0015
	R <sup>2</sup>	0.0181	0.0008	0.0013	0.0002
	Numb. Obs.	(580)	(1.628)	(1.722)	(467)
DITM	$\hat{\alpha}^{FGLS}$	0.2138**	0.1820**	0.3710**	0.1920**
	$\hat{\beta}^{FGLS}$	-0.0027*	-0.0073	0.0006	-0.0037
	R <sup>2</sup>	0.0148	0.0092	0.0021	0.0040
	Numb. Obs.	(405)	(824)	(1.071)	(186)

**Figure 1**

### Empirical Distribution of Arbitrage Profits on the MIBO30 Market

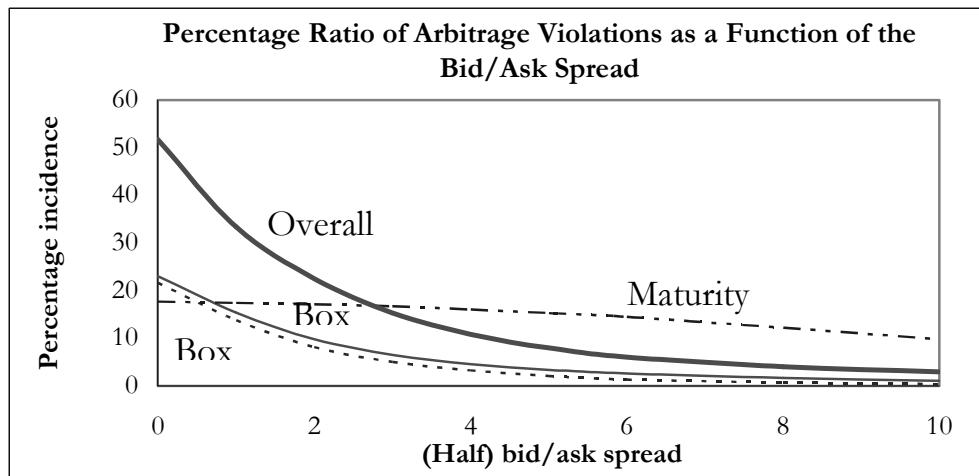
The graph reports the empirical density and distribution function of the maximum arbitrage profits across the different conditions. Arbitrage profits are expressed in MIB30 index points.



**Figure 2**

### Percentage Incidence of Arbitrage Violations As a Function of Alternative Levels of the Bid/Ask Spread.

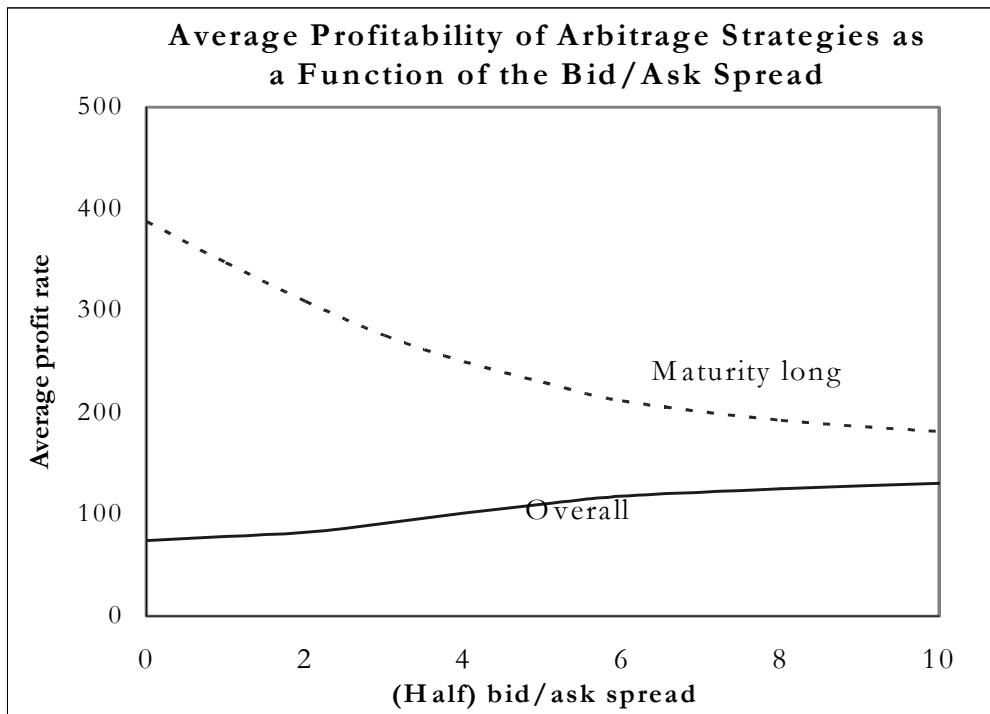
The graphs plot the changes in the percentage of the data displaying violations of the basic no-arbitrage conditions derived in Section 4 as a function of the (half-) size of the bid/ask spreads  $\alpha$  and  $\beta$  characterizing the MIBO30 (options) and the MIB30 index (the underlying) markets, respectively. These scenario simulations impose the restriction  $\alpha = \beta$ .



**Figure 3**

**Average Arbitrage Profit Rates as a Function of Alternative Levels  
of the Bid/Ask Spread.**

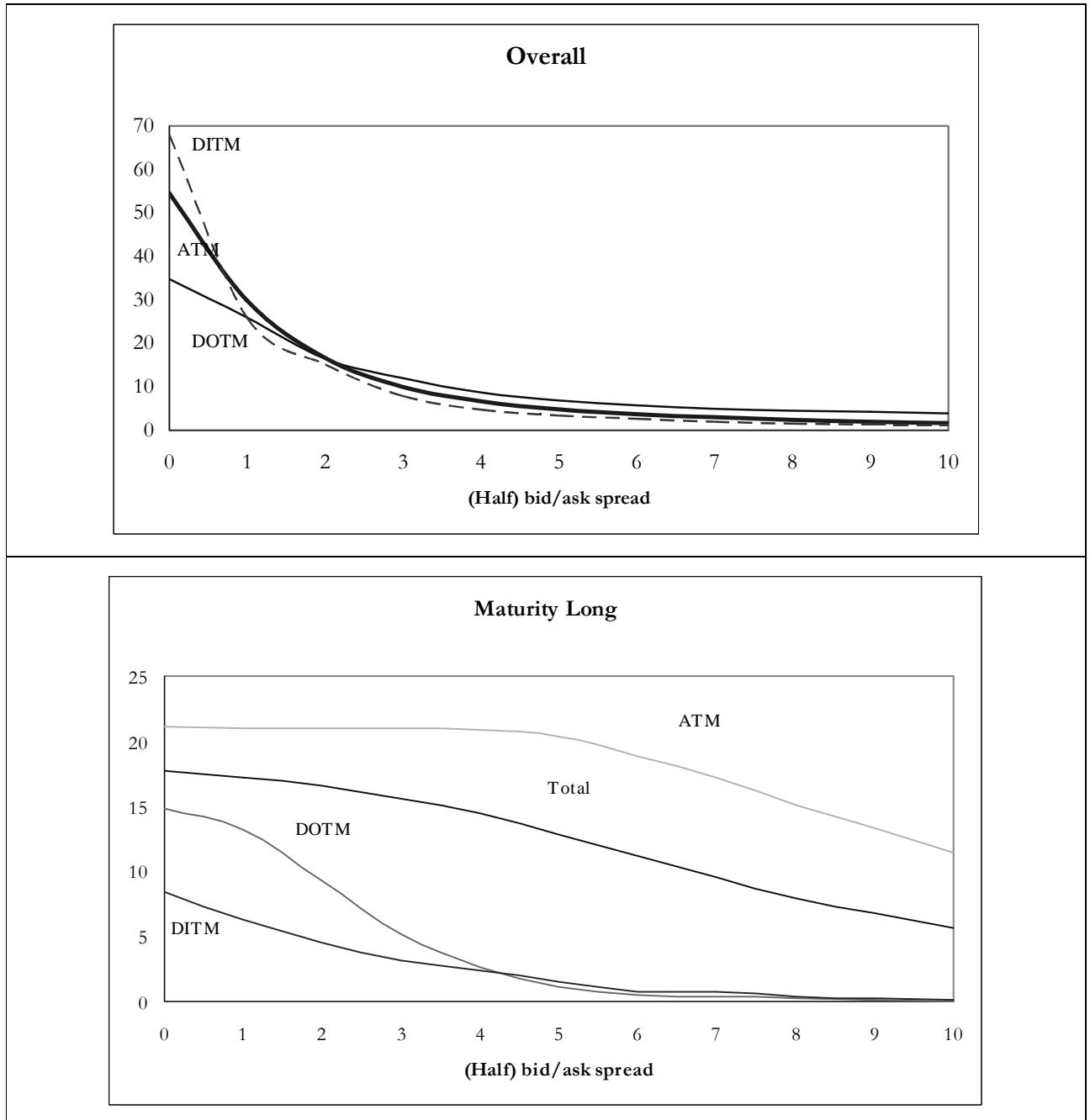
The graph plots the changes in the average profit rates obtained by exploiting the presence of violations of the basic no-arbitrage conditions derived in Section 4 as a function of the (half-) size of the bid/ask spreads  $\alpha$  and  $\beta$  characterizing the MIBO30 (options) and the MIB30 index (the underlying) markets, respectively. These scenario simulations impose the restriction  $\alpha = \beta$ .



**Figure 4**

**Percentage Incidence of Arbitrage Violations As a Function of Alternative Levels  
of the Bid/Ask Spread and Moneyness Classes**

The graphs plot the ratio of arbitrage opportunities arising from any of the conditions listed in Section 3 (“Overall”) or just the Maturity spread long side as a function of the (half-) size of the bid/ask spread,  $\alpha$ , for different moneyness classes; the line labeled “Total” refers to the whole sample. For the Overall condition the lines of remaining moneyness classes remain within those relative to DOTM and DITM. In particular ATM almost coincides with the line corresponding to the whole sample (represented in Figure 2).



**Figure 5**

**Daily Incidence of the Percentage Incidence of Arbitrage Violations on the MIBO30 Market and the value of the MIB30 index**

The graph plots the percentage of arbitrage violations of any type detected and the value of the MIB30 index for each day in the between April 1999 and January. The upper line describes the value reached by the MIB30 index as a ratio of the maximum over the sample period. The intermediate line corresponds to daily ratio of arbitrage violations when  $\alpha=0$ . i.e. in the case of no frictions; the lower line corresponds to the case  $\alpha=5\%$ . The straight horizontal lines represent the corresponding mean sample values.

