Giovanni Barone-Adesi

Does Volatility Pay?

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Decanato della Facoltà di Scienze economiche
Via Ospedale, 13 CH-6900 Lugano. Tel. 091 912 46 09 / 08. Fax 091 912 46 29

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Facoltà di Economia USI

Via Ospedale 13,Lugano 6900 CH

baroneg@lu.unisi.ch

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Abstract

An investor with quadratic utility invests amounts changing with his perceptions of risk and expected return in a market with changing risk. Optimal investment policies are derived under several hypotheses for expected returns. These policies are combined in a Bayesian framework to yield a policy that performs better than the 'buy and hold' policy in our tests, except in the case of the FTSE index.

Does Volatility Pay?

Introduction. A major focus of the literature in financial economics is the predictability of excess stock returns. Variables such as interest rates and dividend yields appear to predict to some degree the variation of expected returns over time.

Ferson and Harvey (1991) and Evans (1994) ascribe most of this variation to variations in risk premia. Other authors, including Campbell (1987),Glosten, Jagannathan and Runkle (1993) and Whitelaw (1994) try to explain changing market risk premiums with changing volatility. They link market volatility to explanatory variables, such as interest rates.However the link between changing volatility and expected return is elusive.

To cast more light on market risk premia He, Kan, Ng and Zhang (1996) model conditional covariances as well as reward-to-volatility variables. They find that reward-to-covariances variables change with economic conditions. Their results are supported by the findings of Li (1998), based on aggregate returns on the stock and bond markets. Changing reward-to-risk variables seem to dominate changing risk as explanatory variables because they capture a larger fraction of predictable excess returns.

If changing risk is not a good explanatory variable for changing expected return an obvious question is whether it pays an investor interested only in mean and variance to be in the market at times of high risk. To answer this question we consider the problem of an investor with constant proportional risk aversion that invests in a market with changing risk. The optimal investment policies we derive change with risk perceptions through time.

It is well known that high risk periods can be predicted by time-series models. Black (1976), Bollerslev (1986) and Engle and Ng (1993) present models of changing volatility based on available information. Our methodology is closely related to Bollerslev and Engle and Ng because we focus on modelling risk. Rather than attempting to link changes in expected returns to risk we simply measure the impact of changing risk on financial performance. We find that the link between risk and expected return is weak, but there is some evidence that higher expected returns accompany higher risk.

To exploit changing reward-to-risk ratios we consider the optimal investment policies under two competing hypotheses. That leads us to try to improve performance by investing more in the risky asset when its expected volatility is low and shifting to the riskless asset when expected volatility is high. Three stock indices from the major international markets are tested in a variety of ways to assess the robustness of our findings.

Models. Volatilities are modeled as an AGARCH process of the first order:

$$\sigma_t^2 = \beta\,\sigma_{t-1}^2 + \alpha(\varepsilon_{t-1}+\gamma)^2 + \psi \qquad (1)$$

Therefore the volatility tomorrow is a function of the volatility today plus the square of the daily residual return, ε, added to a constant, γ, that introduces an asymmetric response of volatility to residual returns. Estimates of the constant ψ are not significant and are neglected henceforth.

The AGARCH model assumes that the distribution of ε is normal:

$$\varepsilon_t \sim N(0,\sigma_t) \qquad (2)$$

This is generally not true in financial markets, resulting into a loss of efficiency in the model estimation procedures. In keeping with the common practice in the literature the daily expected return is ignored in the computation of ε, that is set to be therefore equal to the daily return, r.

In our tests we use the volatility forecast from equation (1) to rank or partition index returns over time. Our purpose is to test whether expected index returns change with changing volatility. Our model for this relationship is:

$$E(r_t) = c + d\sigma_t \qquad (3)$$

If the chosen index proxies the market portfolio, market equilibrium considerations suggest that c be equal to the risk-free rate. A first testable hypothesis is therefore:

$$H_0: \qquad c = r_F \qquad (4)$$

The reward for taking risk can be expressed by the Sharpe ratio, SR_t:

$$SR_t = E(r_t - r_F)/\sigma_t \qquad (5)$$

Substituting equations 3 and 4 into equation 5 it follows that under the Sharpe-Lintner hypothesis the Sharpe ratio is constant:

$$SR_t = d \qquad (6)$$

An investor interested in mean and variance only would solve the following optimization problem:

$$\max_{x_t}\left(x_t E(r_t - r_F) - p x_t^2 \sigma_t^2 \right) \qquad (7)$$

where x_t is the instantaneous leverage (the percentage of wealth invested in the risky asset) and p is a risk aversion parameter, assumed to be constant.

Substituting equations (5) and (6) into equation (7) we find that the optimal leverage ratio for our investor is given by:

$$x^*_t=d/2p\sigma_t \qquad (8)$$

To evaluate performance, we will set $d/2p=\sigma$ initially, where σ is the unconditional volatility. That implies no loss of generality, because other values would multiply x^*_t by a constant, with no other effect on its evolution through time.

If equation 6 is verified excess returns for taking risk are proportional to the risks being taken, that is the reward for taking risk is constant. In this case the optimal policy, described in equation (8), is to have a leverage ratio inversely related to the current volatility.

An alternative policy stems from the special case of equation(5) in which the excess return is constant, that is

$$H_1: \qquad E(r_t-r_F)=c' \qquad (9)$$

The hypothesis H_1 is consistent with investors ignoring short term fluctuations in volatility, possibly because transaction costs and misspecifications in the model make dynamic policies ineffective.

If the expected return is constant, the reward for taking risk is inversely related to volatility:

$$SR_t = c'/\sigma_t \qquad (10)$$

An investor interested in mean and variance could then attempt to improve her performance by shifting from risky into riskless assets when the forecast volatility is high. The optimal policy then finds the instantaneous optimal leverage ratio, x_t , that maximizes expected utility. Under a quadratic utility function the investor's problem is now:

$$\max_{x_t}(x_t c'-p x_t^2\sigma_t^2) \qquad (11)$$

. A convenient value of p is $c'/2\sigma^2$. The same value of p would be obtained from the market clearing condition applied to the unconditional moments of market return, c and σ^2 . However our constant risk aversion policies are not sufficient to achieve market clearing. Therefore $c'/2\sigma^2$ is just a convenient choice.With this value of p the optimal policy invests in the index with a leverage ratio:

$$x_t=\sigma^2/\sigma_t^2 \qquad (12)$$

Naturally, if equation 9 is true and also:

$$c' > 0 \qquad (13)$$

it follows that to reduce market exposure when forecast volatility is high implies for our portfolios a sacrifice of expected return to reduce risk. It remains to be checked whether the risk reduction compensates for the loss of expected return.

Note that both of our optimal policies depart from the buy-and-hold policy. In our framework, an investor with constant proportional risk aversion would find buy-and-hold to be optimal only if excess expected returns increase proportionally with the variance of return.

Data Analysis

To evaluate the proposed models we collected data on three of the main stock indices, the Dow Jones Industrials, the FTSE 100 and the Nikkei from January 1993 to January 1999. To ensure comparability all of our tests begin October 21 1993.

The behavior of the returns of the three indices is shown in Figures 1 to 3. There is some evidence of clustering of large returns, especially in the later period for the Dow Jones. This clustering is pointing to the presence of GARCH effects.

The statistics of the three indices are summarized in Table 1. The three indices have average annualized returns ranging from 18% to –6%. Their average annualized volatilities range from 14% to 20. Histograms of the three indices in Figures 4 to 6 reveal that their unconditional distributions are leptokurtic.

Table 2 shows the autocorrelations of the squared daily returns of the three indices. If the return distributions were stationary these autocorrelations would be zero.
The estimates in Table 2 suggest that GARCH effects play an important role in the dynamics of our indices. These effects were modeled following the BFGS methodology. GARCH parameter estimates for the three indices are summarized in Table 3. Plugging these estimates into equation 1 leads to the ranges of volatility shown in Figures 7 to 9. Volatility for the Dow and the FTSE increases to about 25% annualized through time, while the Nikkei volatility ranges up to 35% throughout our test period.

The daily series of returns and estimated volatilities allow us to test for the existence of linkages between expected return and volatility. To explore these linkages we plot return against volatility in Figures 10 to 12. The dispersion of returns is higher for the higher volatility estimates on the right of these figures, confirming the ability of our volatility estimates to explain the changing variability of index returns.

An important question to be answered is whether investors are compensated for taking higher risk. To answer this question we regress returns on GARCH volatilities . All the slope estimates in Table 4 are positive, but their t-statistics show that there is no reliable linkage between volatility and expected returns. We cannot reject the hypothesis of constant expected returns introduced in equation 7. Therefore there is some evidence that the ratios of risk to expected reward for our indices change through time. It may be possible to exploit these changes to improve portfolio performance.

A breakdown of average returns at different volatility levels is provided in Table 5. The erratic relationship between volatility and return is confirmed, with low volatility being associated with the highest average returns for the Dow Jones and the lowest returns for the FTSE. The highest average returns for our three indices are found in the first, the second and the fourth volatility quartiles respectively.

Trading strategies. The weak evidence of compensation for volatility risk found in the previous section suggests that policies different from buy and hold may produce superior performances. In fact under our hypothesis H_o the optimal policy invests in the index with investment weights inversely related to the forecast standard deviation. That contrasts with the optimal policy under the hypothesis H_1, to use investment weights proportional to forecast variance.

The results of the trading strategies implied by our hypotheses H_0 and H_1 are summarized in Tables 6 and 7 respectively. Estimated volatilities and returns for our strategies are generally lower than the corresponding indices values, only the H_1 strategy return for the Dow and the average volatility in Japan under H_1 depart from this trend.

Results for the Dow suggest that it is possible to exploit information on market volatility to predict returns. However the evidence for the FTSE and the Nikkei does not support that. Results for the FTSE and the Nikkei may be influenced by their abnormally low returns associated with low GARCH volatilities within our sample. These returns are levered under our optimal investment policies, receiving a higher weight than returns associated with higher GARCH volatilities.

Our two hypotheses, H_0 and H_1, are not nested. Both of them introduce one constraint on the parameters of equation (3). Their likelihood ratio can be interpreted as odds ratios in a Bayesian setting. We use the normal model and diffuse prior distributions to compute the odds ratios:

$$H_0 / H_1 = (\sigma_1 / \sigma_0)^n \tag{14}$$

where σ_1 and σ_0 refer to the standard deviations of residuals of the regression in equation (3) under the two hypotheses and n is the number of observations used in the regression. The odds ratios for the three indices, in Table 8, show that H_o is slightly more likely for the Dow and the Ftse, while H_1 is more likely for the Nikkei.

Because of the modest values of the estimated odds ratios, we consider also the combined optimal policy. This policy combines the two optimal policies according to their odds ratios. It is important to recognize that it is necessary to scale the two policies before combining them. In fact earlier we found to be convenient to set p to be equal to $d/2\sigma$ to test H_o and to be equal to $c'/2\sigma^2$ under H_1, where d=0. The two expressions for p are not consistent and cannot be used simultaneously. To remedy that it is necessary to use an arbitrary common value of p under the two hypotheses. The combined optimal policy can then be written as:

$$x^*_t=(yd/\sigma_t+(1-y)c'/\sigma^2_t)/2p \tag{15}$$

where y/(1-y) represents the odds ratio of the two models. Because the optimal leverage is linear in the risk aversion parameter p, choosing an arbitrary value for it does not affect the evolution of our optimal policy except through a scale factor.

It is easy to verify that setting y = 0.5 corresponds to the optimal policy derived from the unconstrained form of equation (3). Therefore equation (15) uses the odds ratio to introduce a bias in favor of either of our hypotheses to improve out-of-sample performance of our combined policy. A graphic representation of the hypotheses on which our policies are based is shown in Fig. 13.

The results for the policy described in equation (15) are also reported in Table 8. This table is constructed setting p=1/2. Statistics corresponding to more realistic values of p , such as 3 or 5, can be obtained by setting p to the new value of the risk aversion parameter. It appears that our combined policy outperforms the Dow and the Nikkei, but not the FTSE. The result from the Sharpe ratios is confirmed by the regressions of excess policy return on excess index return:

$$E(r_t-r_F)=\beta E(r_{Mt}-r_F)+\omega \tag{16}$$

In equation (16) the constant ω measures the performance of the optimal policy relative to the index. The values of ω for the Dow and the Nikkei are positive, but the FTSE has a marginally negative value, corresponding to a yearly underperformance of 20 basis points for our combined policy.

In summary it appears that it was possible to outperform the Dow and the Nikkei during our testing period, but not the FTSE. The different result is due to the very low average returns of the FTSE at low volatility levels, that are levered up by our optimal policies. Our policies were developed to maximize performance in the mean-variance space. They do not necessarily produce better performance within any given period.

Our combined policy relies on estimates of parameters, such as the excess market return, that are likely to fluctuate substantially over time. To have a first assessment of the performance of our policy out-of –sample we divided our test period in two subperiods. The first subperiod was used to estimate our policy parameters, the second to measure performance. Results, in Table 9, mirror the pattern of Table 8, with the combined policy outperforming the index for the Dow and the Nikkei, but not the FTSE. Therefore our policy does not appear to be unduly sensitive to its estimation period.

Summary statistics for the returns of the three indices and our combined policy in Table 10 show that our policy reduces substantially the kurtosis of the distribution of daily returns for two indices, mitigating the risk due to the fat tails of security returns. Minimum, maximum returns and ranges are comparable only by taking the differences in variances into account, because our combined policy was computed for a very low value of the investor’s risk aversion parameter . The results for skewness are more mixed, but overall there is no evidence that improvements in mean-variance performance have come at the expense of other dimensions of risk.

Conclusion

We have shown that changing risk implies that investors with quadratic utility function will not find buy and hold policies to be optimal. Only a risk premium changing proportionally to variance through time would lead investors to follow buy and hold policies.

The optimal policy for our investor depends on parameters, such as the excess market return, that are difficult to estimate precisely. To overcome that we have introduced a Bayesian policy that weights alternatives according to their likelihoods. The preliminary tests in this paper suggest that this policy is fairly robust and it leads to better performance for two of our three tested indices. Further research will be necessary to investigate more thoroughly the performance of our investment policies through time as well as their implementation in practice, possibly using index futures to reduce transaction costs.

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Table1

Statistics of the Stock Indices

	Dow Jones	FTSE	Nikkei
Average daily return	0.000715	0.000528	-0.000173
Standard deviation	0.008623	0.008588	0.012882
t-statistics	3.281769	2.435186	-0.530508

Table2

First Order Autocorrelations of Squared Returns

	Dow Jones	FTSE	Nikkei
Autocorrelation	0.255898	0.164598	0.140074
t-statistics	10.15507	6.540946	5.570133

Table3

AGARCH Parameters

	Dow Jones	FTSE	Nikkei
α	0.073213 (4.14)	0.047208 (6.39)	0.060762 (4.63)
β	0.899317 (31.98)	0.922799 (83.24)	0.918815 (54.54)
γ	-0.004379 (-3.23)	-0.007026 (-6.48)	-0.007062 (-6.04)

t-statistics in parentheses

Table4

Regressions of returns on AGARCH volatilities

Dow Jones	coefficient	st. error	t-statistics	p-value
Intercept	-0.0002086	0.000898	-0.23226	0.816368
slope	0.00768842	0.007079	1.086112	0.277614
FTSE				
Intercept	-0.000030	0.000893	-0.03388	0.972976
slope	0.004709	0.006829	0.689644	0.490531
Nikkei				
Intercept	-0.002323	1.310769	-1.77223	0.076571
slope	0.101321	6.411859	1.58022	0.11428

Table 5

Volatility quartiles	Dow Jones Av. Volatility	Average Return	FTSE Av. Volatility	Average Return	Nikkei Av. Volatility	Average Return
First	0.087325	0.000963	0.094296	0.000214	0.136281	-0.000661
Second	0.108313	0.000298	0.110259	0.000687	0.176220	-0.000885
Third	0.128731	0.000725	0.129049	0.000415	0.211324	-0.000452
Fourth	0.170270	0.000947	0.175870	0.000683	0.273094	0.000633

Table 6

Optimal policy under H_0

	Dow	FTSE	Nikkei
Av. excess ret.	0.000559	0.000243	-0.000353
st.dev. of return	0.008417	0.008414	0.013615
av. index ex. ret	0.000558	0.000273	-0.000189
std. of index return	0.009139	0.009034	0.014529
Sharpe r. policy	0.066466	0.028982	-0.025984
Sharpe r. index	0.061056	0.030304	-0.013070

Table 7

Optimal Policy under H_1

Dow	FTSE		Nikkei
Av. excess ret.	0.000624	0.000222	-0.000521
std. of return	0.009044	0.009031	0.014751
av.index ex. ret	0.000558	0.000273	-0.000189
std. of index ret.	0.009139	0.009034	0.014529
Sharpe r. policy	0.069081	0.024626	-0.035367
Sharpe r. index	0.061056	0.030304	-0.013070

Table 8

Combined Policy

	Dow	FTSE	Nikkei
odds ratio	5:3	11:9	6:7
Av. Ex. Ret	0.002603	0.000514	0.000355
St. deviation	0.037658	0.020302	0.010059
t-statistic	2.494541	0.913752	1.273632
Sharpe R.	0.069132	0.025323	0.035297
Index S.R.	0.061056	0.030304	-0.013070
ω	0.000598	-0.000008	0.000241

Table 9

Combined Policy:robustness

	Dow	FTSE	Nikkei
Av. Ex.Ret	0.001838	0.000603	0.000282
St. deviation	0.034478	0.020169	0.010034
t-statistic	1.361418	1.079897	1.015235
Sharpe R.	0.053317	0.029928	0.028135
Index S.R.	0.052434	0.039147	-0.033606

Table10

Index excess return				Strategy excess return			
	Dow	*FTSE*	*Nikkei*		*Dow*	*FTSE*	*Nikkei*
Mean	0.000553	0.000235	-0.00019	Mean	0.002581	0.000458	0.000355
Standard Error	0.000253	0.00025	0.000403	Standard Error	0.001044	0.000563	0.000279
Median	0.000763	0.000478	-8.1E-05	Median	0.003092	0.001258	3.86E-05
Standard Deviation	0.009147	0.009013	0.01453	Standard Deviation	0.037699	0.020317	0.01006
Sample Variance	8.37E-05	8.12E-05	0.000211	Sample Variance	0.001421	0.000413	0.000101
Kurtosis	7.16029	2.105816	3.051328	Kurtosis	2.334275	0.827886	4.313889
Skewness	-0.61325	-0.06188	0.222877	Skewness	-0.43572	-0.18637	-0.1488
Range	0.121644	0.080349	0.148013	Range	0.372302	0.166914	0.141269
Minimum	-0.07204	-0.03619	-0.06844	Minimum	-0.23247	-0.09097	-0.07479
Maximum	0.049606	0.044159	0.079575	Maximum	0.139827	0.075943	0.066475
Sum	0.720422	0.30605	-0.24746	Sum	3.363629	0.597333	0.462679
Count	1303	1303	1303	Count	1303	1303	1303

Figure 1: Dow Jones Return

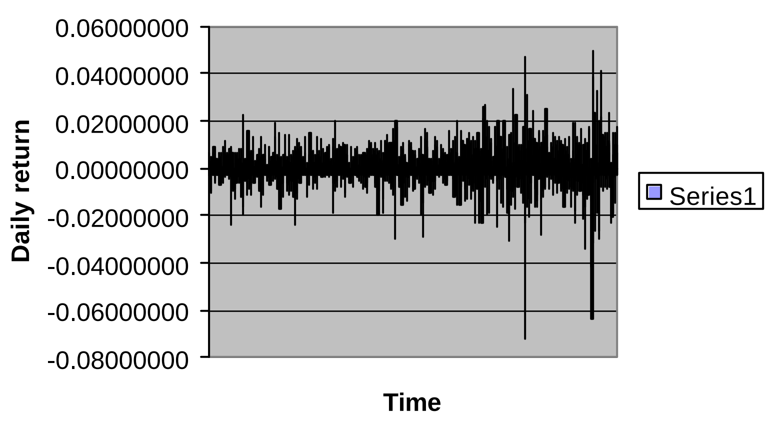


Figure 2: FTSE Return

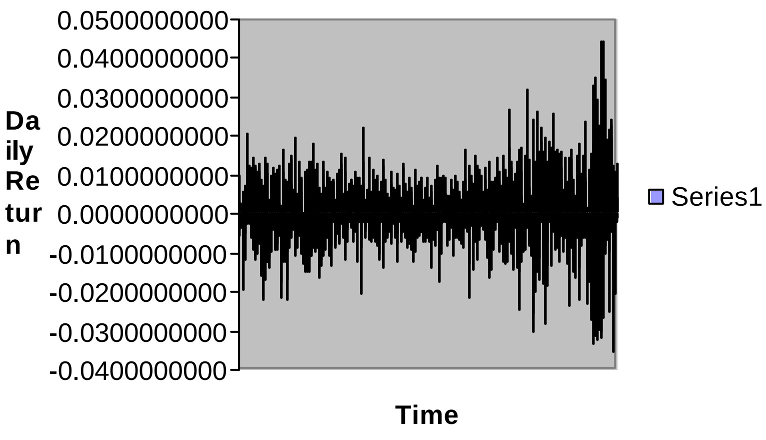


Figure 3: Nikkei Return

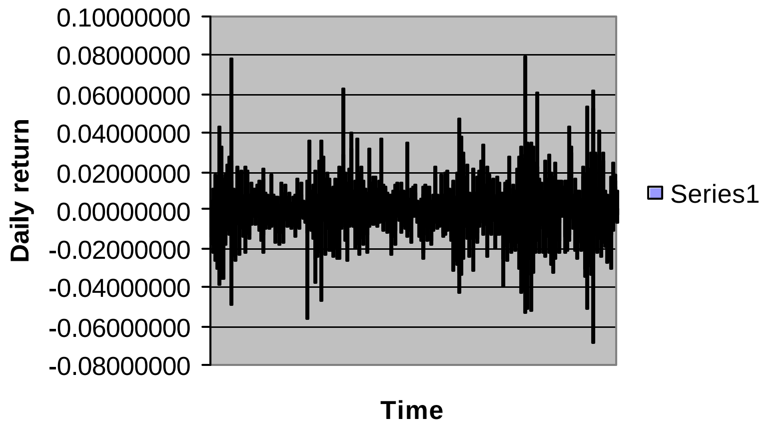


Figure 4:Histogram of Dow Jones Return

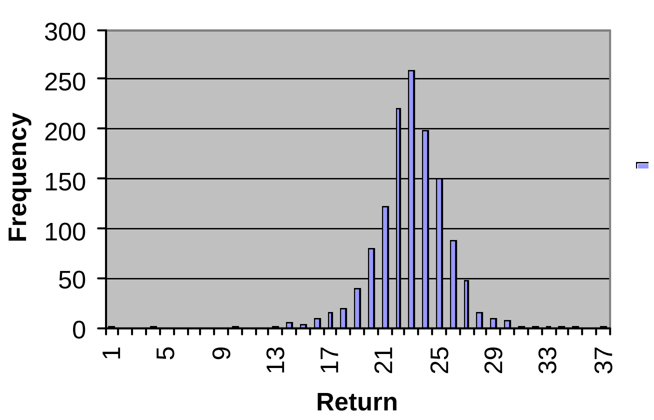


Figure 5: Histogram of FTSE return

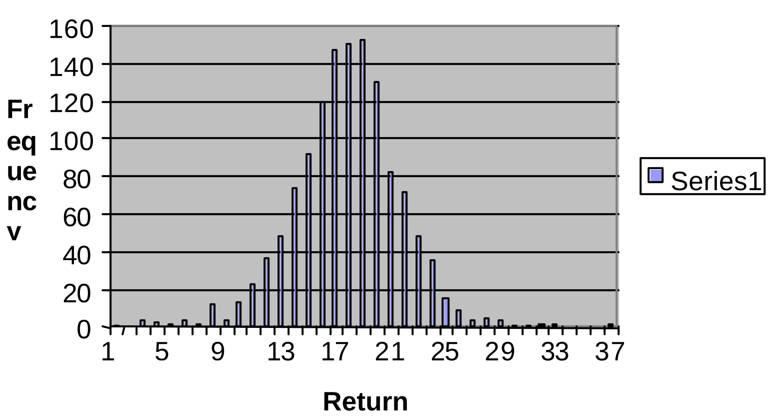
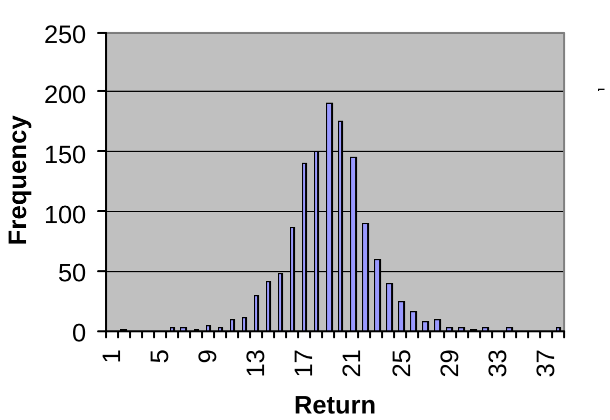


Figure 6:Histogram of Nikkei Return



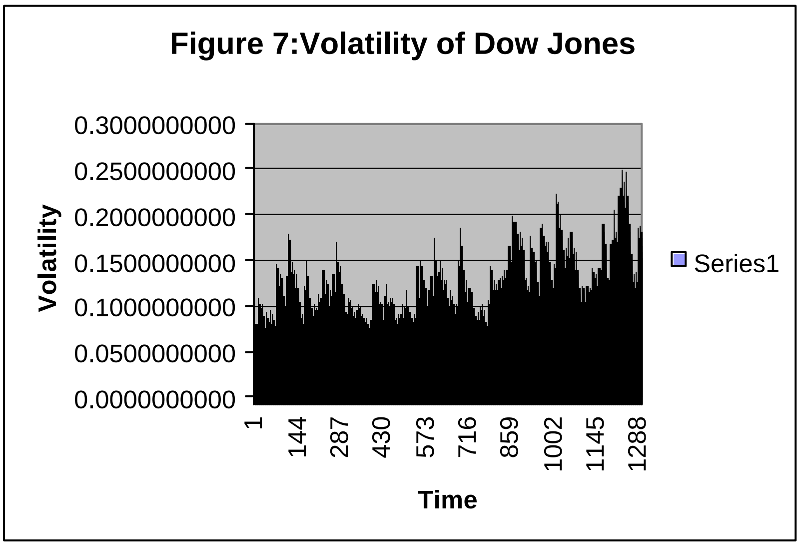


Figure 7:Volatility of Dow Jones

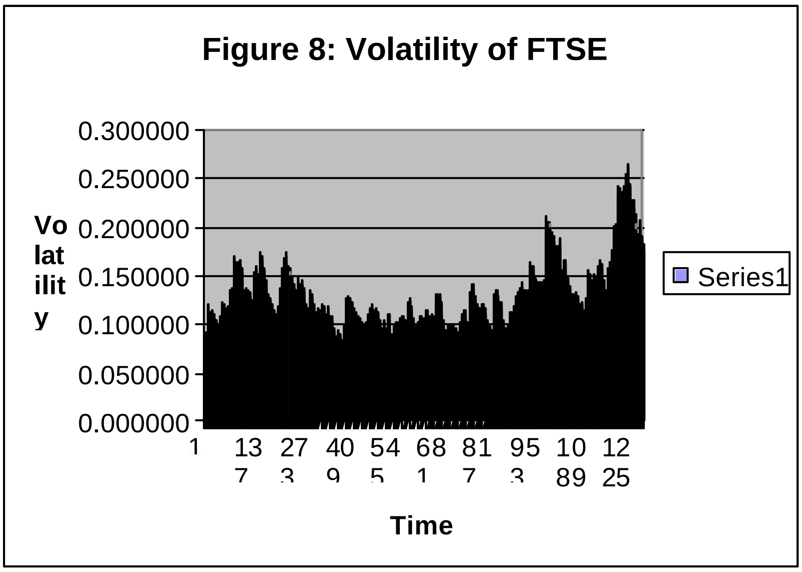


Figure 8: Volatility of FTSE

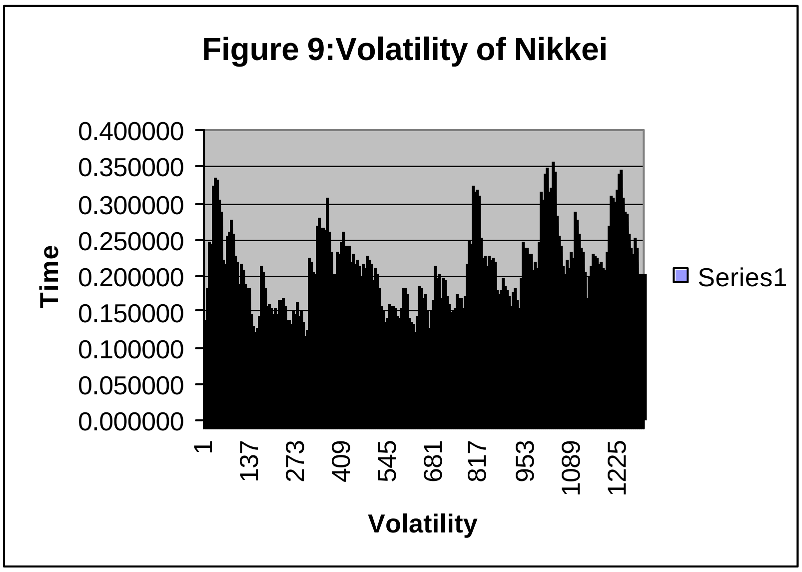


Figure 9:Volatility of Nikkei

Figure 10: Dow Volatility and Return

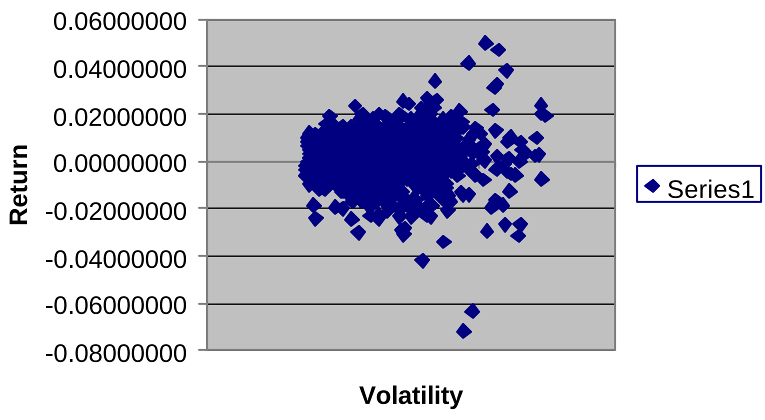


Figure 11: FTSE Volatility and Return

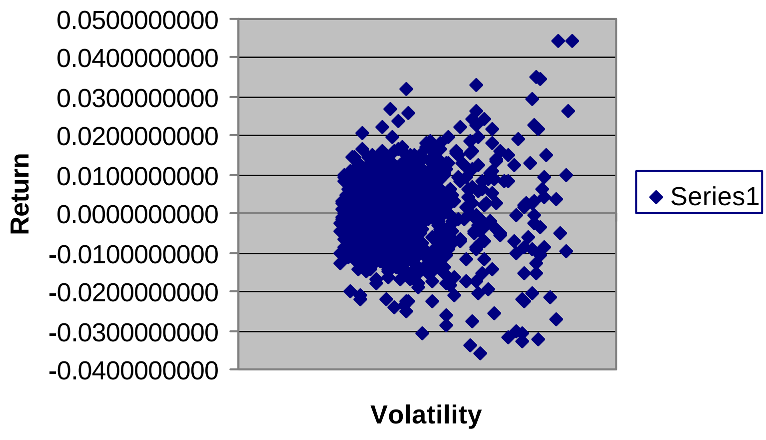


Figure 12:Nikkei Volatility and Return

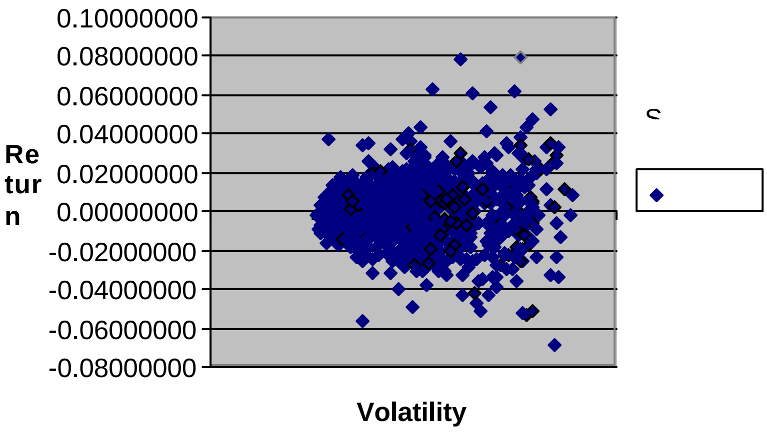
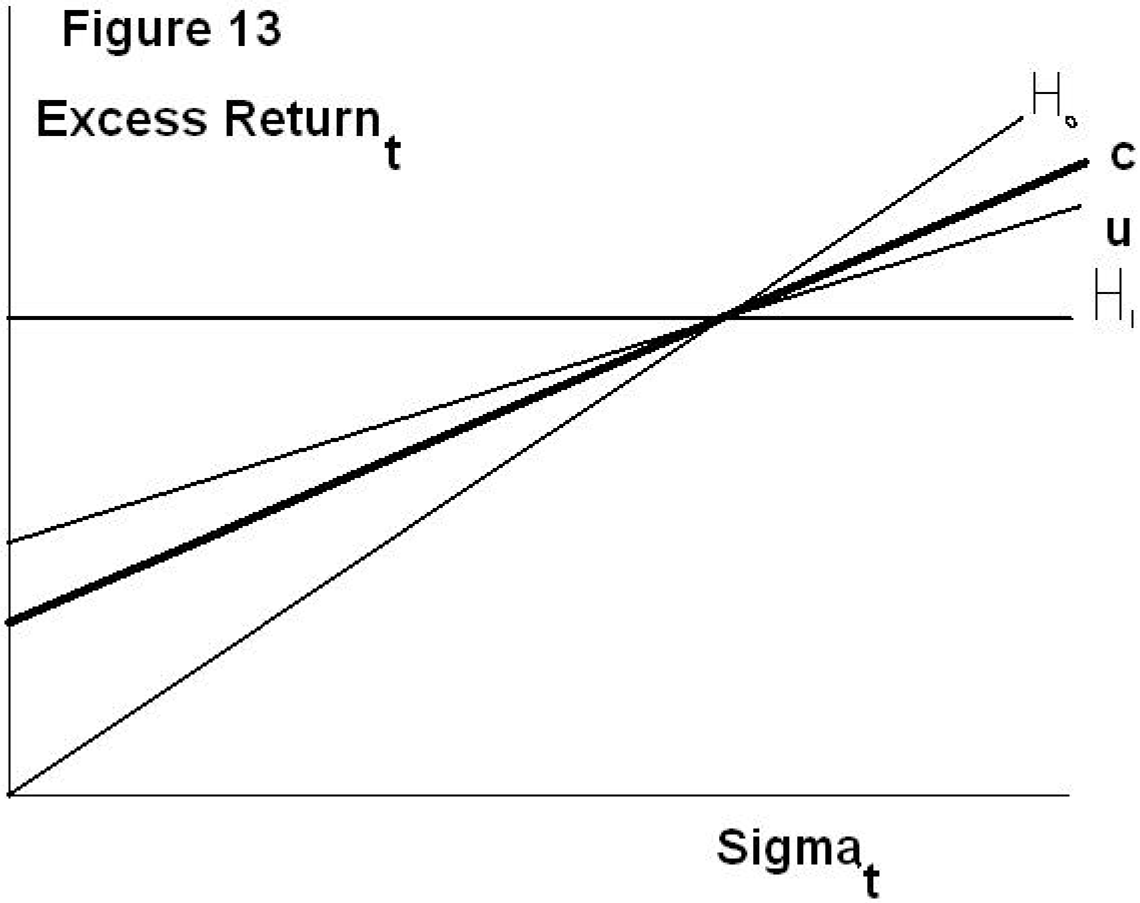


Figure 13



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