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Abstract

The paper presents a robust version of a simple two-assets Merton's (1969) model where the optimal choices and the implied shadow market prices of risk for a representative robust decision maker (RDM) can be easily described.

With the exeption of the log utility case, precautionary behaviour is induced in the optimal consumption-investment rules through a substitution of investment in risky assets with both current consumption and riskless saving. For the log utility case, precautionary behaviour arises only through a substitution between risky and riskless assets.

On the financial side, the decomposition of the market price of risk in a standard consumption based component and a further price for model uncertainty risk (which is positively related to the robustness parameter) is independent of the underlying risk aversion parameter.

Keywords: Merton's Model, Knightian Uncertainty, Model Contamination, Model Misspecification, Robust Decision Making.

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1 Introduction

THIS PAPER PRESENTS a robust version of a simple two-assets Merton's (1969) model where the optimal consumption and portfolio choices as well as the implied market price of risk of a representative "robust decision maker" (RDM) (cf. Hansen, Sargent and Tallarini (1998) and Anderson, Hansen and Sargent (1999); AHS (1999) in the sequel) can be easily described.
Robustness leads generally to focusing on worst case scenarios over a restricted set of appropriately defined relevant model misspecifications. In the present formulation of robustness we model asset prices that inherently reflect a form of risk aversion to a particular kind of Knightian uncertainty (cf. Knight (1921) and Epstein and Wang (1994)). Unlike other formulations - as for instance those directly linked to the literature of risk sensitive control (cf. Whittle (1990)) - this formulation yields explicit and very easy and interpretable expressions for the relevant variables in the robust Merton's problem. In fact, the solution for the robust problem is of the same functional form as that for the classical Merton's problem.
Similarly to AHS (1999), we model robustness through a RDM determining worst case consumption and portfolio rules over a class of alternative models that are constrained in their "distance" from a reference model for asset prices. The reference model is the standard geometric Brownian motion process while the maximal admissible "distance" therefrom is measured with a continuous time version of relative entropy. This single "distance" parameter models a preference for robustness by constraining the set of model misspecifications relevant to a RDM.
The contribution of the paper consists in deriving explicit and easily understandable robust consumption and investment rules that can be compared to those of a non-robust decision maker in Merton's model. AHS (1999) develop a theoretical framework to robust decision making in continuous time that provide general characterizations of the impact of a preference for robustness on the optimal decision rules and on pricing. However, when analyzing a specific model one still has to solve the arising Bellman equations in order to fully characterize the implied optimal robust decision rules; in the Merton's model this can be done explicitly and easily.
With the exeption of the log utility case robustness affects the optimal decision rules through a substitution of investment in risky assets with both current consumption and riskless saving. For the log utility case, precautionary behaviour comes up only through a substitution between risky and riskless assets.
On the financial side, the decompositions of the market price of risk in a standard consumption based component and a further price for model un-

certainty risk (which is positively related to the robustness parameter) is independent of the underlying risk aversion parameter.
In Section 2 we present a general robust version of Merton's (1969) two assets model. Section 3 derives the implied optimal consumption and portfolio rules for isoelastic utility functions. Section 4 makes the structure of the implied market price of risk explicit while Section 5 concludes.

2 A Robust Merton's two Assets Model

There are two assets, a risk free asset with price B_t at time t and a risky asset with price P_t at time t whose dynamics are given by

$$dB_t = rB_t dt \tag{1}$$
$$dP_t = \alpha P_t dt + \sigma P_t dZ_t \quad . \tag{2}$$

The drift and volatility α and σ as well as the short rate r are assumed constant. Z is a standard Brownian motion in $\mathbb{R}$.
Let w_t be the proportion of current wealth W_t invested in the risky asset. The budget constraint for current wealth W_t is given by

$$dW_t = w_t(\alpha - r)W_t dt + (rW_t - c_t)dt + w_t W_t \sigma dZ_t \ , \tag{3}$$

where c_t is the consumption rate at time t.
Associated to the joint Markov process defined by (2) and (3) is a semigroup $(T_t)_{t\geq 0}$ of operators defined by

$$T_t\varphi(y) = E[\varphi(W_t, P_t)|(W_0, P_0) = y] \quad , \tag{4}$$

and a generator A defined by

$$A(\varphi) = \lim_{t\to 0} \frac{T_t\varphi - \varphi}{t} \quad , \tag{5}$$

for all test functions φ such that this limit exists. In the case of the classical Merton's (1969) and (1971) model the generator (A^M) is given by[1]

$$\begin{aligned} A^M(\varphi) = \alpha\frac{\partial\varphi}{\partial P} &+ (w(\alpha - r) + (rW - c))\frac{\partial\varphi}{\partial W} \\ &+\frac{1}{2}\sigma^2 P^2 \frac{\partial^2\varphi}{\partial^2 P} + \frac{1}{2}\sigma^2 w^2 W^2 \frac{\partial^2\varphi}{\partial^2 W} + \sigma^2 wWP\frac{\partial^2\varphi}{\partial W\partial P} \quad . \end{aligned} \tag{6}$$

[1]See for instance Merton (1971), Section 4.

We model a RDM by an ecomomic agent taking into account the possibility of a misspecified model (2) for asset prices. Specifically, we consider rational economic agents which are looking for decision rules that perform well not only at the reference model (2), but also over a set of relevant (local) model misspecifications of (2).
In order to define *and* measure such model misspecifications we consider absolutely continuos contaminations of (2) and (3) by introducing families $(T_t^\nu)_{t\geq 0}$ of distorted semigroups defined by

$$T_t^\nu(\varphi) = \frac{T_t(\nu\varphi)}{T_t(\nu)} \quad , \tag{7}$$

for nonnegative random variables ν such that these operators are well defined[2]. The generator $A^{M,\nu}$ associated to the "ν-contaminated" price and wealth dynamics in Merton's model is easily obtained. It is given by[3]:

$$A^{M,\nu}(\varphi) = A^M(\varphi) + A^\nu(\varphi) \tag{9}$$

where

$$\begin{aligned} A^\nu(\varphi) = \sigma^2 P^2 \frac{\partial \log\nu}{\partial P}\frac{\partial\varphi}{\partial P} + \sigma^2 w^2 W^2 \frac{\partial \log\nu}{\partial W}\frac{\partial\varphi}{\partial W} \\ +\sigma^2 PwW \frac{\partial \log\nu}{\partial P}\frac{\partial\varphi}{\partial W} + \sigma^2 PwW \frac{\partial \log\nu}{\partial W}\frac{\partial\varphi}{\partial P} \quad . \end{aligned} \tag{10}$$

The associated distorted price dynamics are

$$dP_t = \left(\alpha P_t + \left(\sigma^2 P_t^2 \frac{\partial \log\nu}{\partial P} + \sigma^2 w_t P_t W_t \frac{\partial \log\nu}{\partial W}\right)\right) dt + \sigma P_t dZ_t \tag{11}$$

while the budget constraint for distorted current wealth is:

$$\begin{aligned} dW_t = w_t \left((\alpha - r) + \left(\sigma^2 P_t \frac{\partial \log\nu}{\partial P} + \sigma^2 w_t W_t \frac{\partial \log\nu}{\partial W}\right)\right) W_t dt \\ +(rW_t - c_t)dt + w_t W_t \sigma dZ_t \quad . \end{aligned} \tag{12}$$

[2]For a more extensive discussion of these definitions we refer to AHS (1999).

[3]To proove this, remark that (6) defines the generator of the diffusions (2) and (3). Definition (5) applied to the distorted semigroup (7) then implies:

$$A^{M,\nu}(\varphi) = \frac{A^M(\nu\varphi) - \varphi A^M(\nu)}{\nu} \quad . \tag{8}$$

Using the product rule to compute $A^M(\nu\varphi)$ one then finally gets (9).

These two equations describe the dynamic budget constraint of a RDM when the conditional distributions of the given reference model are contaminated by a specific absolutely continuous change of measure ν (cf. equation (7)).
A RDM will not generally be able to determine which particular model misspecification is already affecting the assets price dynamics; he will rather have a rough perception of the set of misspecifications which are difficult to distinguish from its reference model. We assume that such a set of relevant model misspecifications can be described by a maximal continuous-time entropy radius from the given reference model. Before introducing the optimization problem of a RDM in Merton's model we define the relevant magnitudes in this respect.
Let

$$I_t(\nu) = T_t\left(\frac{\nu}{T_t(\nu)} \cdot \log\left(\frac{\nu}{T_t(\nu)}\right)\right)$$

be the relative entropy of the discrete time density of (P_t, W_t) under the contaminated model (11) and (12), relative to that implied by the reference model (2) and (3). I_t is not a metric, however it measures the discrepancy of the two densities under scrutiny by the so-called information inequality[4]. Further $I_t(\nu)$ has an important information-theoretic interpretation; it can be interpreted as the expected surprise experienced (over the time period $[0,t]$) when believing that (2) and (3) describe the model dynamics and being informed that in fact these are described by (11) and (12); cf. Renyi (1961) and (1971) for a deeper discussion of this point.
The continuous-time measure of relative entropy to be used in the sequel is[5] (see also AHS (1999)):

$$I'(\nu) := \lim_{t\to 0}\frac{I_t(\nu)}{t} = \frac{1}{\nu}A^M(\nu\log\nu) - \frac{\log\nu}{\nu}A^M(\nu) - \frac{1}{\nu}A^M(\nu) \quad . \qquad (13)$$

Using equation (8) and (9) applied to $\varphi = \log\nu$ we can write this as:

$$I'(\nu) = A^M(\log\nu) + A^\nu(\log\nu) - \frac{1}{\nu}A^M(\nu) = \frac{1}{2}A^\nu(\log\nu) \quad . \qquad (14)$$

---

[4]See for instance White (1996), Theorem 2.3, p. 9. Remark that $I_t(\nu) = 0$ if and only if the two densities to be compared are identical almost surely.

[5]To proove this formula note that:

$$\begin{aligned}\frac{I_t(\nu)}{t} &= \frac{1}{T_t(\nu)}\frac{T_t(\nu\log\nu) - T_t(\nu)\log T_t(\nu)}{t}\\ &= \frac{1}{T_t(\nu)}\left[\frac{T_t(\nu\log\nu) - \nu\log\nu}{t} - \log T_t(\nu)\frac{(T_t(\nu) - \nu)}{t} - \nu\frac{(\log T_t(\nu) - \log\nu)}{t}\right] .\end{aligned}$$

Taking limits as $t \to 0$ and using the continuity of semigroups the desired result is obtained.

We interpret this continuous time entropy measure as the marginal rate of change with which expected surprises are experienced when we are continuously informed over time about the underlying data generating mechanism.
As in AHS (1999) we model robustness as a two player (zero sum) game in which the second player (the nature say) is malevolent and chooses a worst case model ν_* from the set of model misspecifications that a RDM considers as relevant. A preference for robustness is introduced through a bound η on the maximal admissible continuous time entropy "distance" (13) between a perturbed model ν and the reference one.
Let $u(\cdot)$ be the current utility function of consumption and ϱ be a subjective discounting factor. The two player decision problem of a RDM in Merton's model is:

$$J(W) = \max_{\{c,w\}} \min_{\{\nu\}} E\left[\int_0^\infty \exp(-\varrho s)\left(u(c_s)\right) ds\right] \tag{15}$$

subject to the dynamic constraints (11) and (12) and to the "maximal continuous time entropy" restriction:

$$I'(\nu) \leq \eta \quad . \tag{16}$$

We can interpret η al the largest continuous time entropy distance for which a model misspecification is seen as relevant by the RDM. The choice of η therefore restricts the amount of (relevant) model misspecification[6]. From this perspective we can also interpret η as a parameter modelling a preference for robustness. Indeed, the larger the parameter η, the less the malevolent player is restricted in determining a worst case model ν^* over the relevant model misspecifications, the greater the incentive for robustness in determining optimal consumption and investment.
Given a preference for robustness η the minimization with respect to ν determines a worst case model ν^*. It is easy to show (see the Appendix):

$$\frac{\partial \log \nu^*}{\partial P} = -\sqrt{2\eta} \cdot \frac{J_P}{\sigma(P_t^2 J_P^2 + w_t^2 W_t^2 J_W^2 + 2w_t W_t P_t J_W J_P)^{\frac{1}{2}}} \quad , \tag{17}$$

$$\frac{\partial \log \nu^*}{\partial W} = -\sqrt{2\eta} \cdot \frac{J_W}{\sigma(P_t^2 J_P^2 + w_t^2 W_t^2 J_W^2 + 2w_t W_t P_t J_W J_P)^{\frac{1}{2}}} \quad . \tag{18}$$

In the present setting the problem can be solved by a value function depending only on current wealth. This implies:

$$\frac{\partial \log \nu^*}{\partial P} = 0 \quad , \quad \frac{\partial \log \nu^*}{\partial W} = -\frac{\sqrt{2\eta}}{\sigma w_t W_t} \quad .$$

[6] One could try to set this parameter in a way such that model misspecifications which are statistically easily detectable are outside the given maximal η-entropy "radius".

By inserting these expressions in the perturbed dynamics (11) and (12) problem (15) can be rewritten as a standard single agent Merton's problem.

3 Optimal Consumption and Portfolio Rules

The single agent problem equivalent to (15) is:

$$J(W) = \max_{\{c,w\}} E\left[\int_0^\infty \exp(-\varrho s) u(c_s) ds\right] \tag{19}$$

subject to the dynamic constraints

$$dP_t = P_t\left(\alpha - \sqrt{2\eta}\sigma\right) dt + \sigma P_t dZ_t \ , \tag{20}$$

$$\begin{aligned} dW_t = W_t w_t &\left((\alpha - r) - \sqrt{2\eta}\sigma\right) dt \\ &+ (rW_t - c_t)dt + w_t W_t \sigma dZ_t \ , \end{aligned} \tag{21}$$

The corresponding Hamilton Jacobi Bellman (HJB) equation for this problem reads

$$\varrho J(W) = \max_{c,w}\{A^{M,\nu^*} J(W) + u(c)\} \quad , \tag{22}$$

where

$$A^{M,\nu^*} = A^M + A^{\nu^*} \quad , \quad A^{\nu^*} = -\sqrt{2\eta} \cdot \sigma w W \partial_W \quad , \tag{23}$$

with $J(0) = 0$ as a boundary condition.
Assuming an isoelastic current utility of consumption

$$u(c) = \frac{c^p}{p} \quad , \quad p \in (0, 1) \quad ,$$

the HJB equation for this problem reads explicitly

$$\begin{aligned} \varrho J = \max_w &\left\{\frac{1}{2}\sigma^2 w^2 W^2 J_{WW} + \left(\alpha - r - \sqrt{2\eta}\sigma\right) w W J_W\right\} \\ &+ rW J_W + \max_c\left\{\frac{c^p}{p} - cJ_W\right\} \end{aligned} \tag{24}$$

and is of the same functional form as the HJB equation for the classical Merton's problem. It is solved by the well-known functional form

$$J(W) = K^*(\eta) W^p \ , \tag{25}$$

with an η dependent parameter $K^*(\eta)$ given by

$$\left(\frac{\varrho}{1-p} - p\left(\frac{(\alpha - r - \sqrt{2\eta}\sigma)^2}{2\sigma^2(1-p)^2} + \frac{r}{1-p}\right)\right)^{p-1} = pK^*(\eta) \quad . \tag{26}$$

The implied optimal rules are:

$$w^{RM}(\eta) = \frac{(\alpha - r - \sqrt{2\eta}\sigma)}{\sigma^2(1-p)} \ , \ c^{RM}(\eta) = (pK^*(\eta))^{\frac{1}{p-1}} W \ . \tag{27}$$

As a consequence we see[7]:

$$\frac{\partial w^{RM}}{\partial \eta} = -\frac{1}{\sigma(1-p)\sqrt{2\eta}} < 0 \quad , \quad \frac{\partial c^{RM}}{\partial \eta} = \frac{p(\alpha - r - \sqrt{2\eta}\sigma)}{\sigma(1-p)^2\sqrt{2\eta}} > 0 \quad .(28)$$

We thus conclude:

- Robustness increases optimal consumption if and only if the uncertainty adjusted market price of risk

  $$\frac{\alpha - r - \sqrt{2\eta}}{\sigma}$$

  is larger than zero[8] and lowers the optimal fraction of wealth invested in risky assets,

- When utility is logarithmic (that is for $p \to 0$), robustness only reduces the optimal fraction allocated to risky assets. Optimal consumption is independent of the financial parameters, except through the effect of changing this fraction.

4 Robust Pricing

Let (c_t^{opt}) denote the optimal consumption plans of a RDM and introduce a further risky asset that does not pay dividends, with price dynamics

$$d\tilde{P}_t = \tilde{\alpha}_t \tilde{P}_t dt + \tilde{\sigma}_t \tilde{P}_t dZ_t \quad , \tag{29}$$

where $(\tilde{\alpha}_t, \tilde{\sigma}_t)$ are some corresponding drift and diffusion processes under the given reference model.

[7]Some graphs of the implied optimal rules in dependence of p and η are presented in the Appendix for a possible choice of the parameters α, σ, ϱ and r.

[8]We discuss the market price of risk in the next section. Without loss of generality we will assume in the sequel this quantity to be non-negative.

This implies a "worst case model" market price of risk given by

$$\frac{\tilde{\alpha}_t - \sqrt{2\eta}\tilde{\sigma}_t - r}{\tilde{\sigma}_t} = -\frac{c_t^{opt} u_{cc}(c_t^{opt})}{u_c(c_t^{opt})} \cdot \sigma_{c_t^{opt}} = (1-p)\sigma_{c_t^{opt}} \quad , \tag{30}$$

where $(\sigma_{c_t^{opt}})$ is the diffusion process of optimal consumption growth.
We therefore obtain a decomposition of the market price of risk for the given reference model as:

$$\frac{\tilde{\alpha}_t - r}{\tilde{\sigma}_t} = (1-p)\sigma_{c_t^{opt}} + \sqrt{2\eta} \quad . \tag{31}$$

The first term on the right hand side of (31) corresponds to the usual consumption based motivation for the market price of risk[9]. The second term on the right hand side of (31) corresponds to an extra equilibrium reward for risk that arises because of a possible misspecification of the given reference model for asset prices. This term is positive and enhances the market price of risk resulting from the pure consumption based motives.
Using the derived robust optimal rules and the linearity in wealth of optimal consumption the dynamics of aggregate optimal consumption are

$$\begin{aligned} dc_t^{opt} = c_t^{opt} \left(r + \frac{((\alpha - r) - \sqrt{2\eta}\sigma)^2}{\sigma^2(1-p)} - (pK^*(\eta))^{\frac{1}{p-1}} \right) dt \\ + \left(\frac{(\alpha - r) - \sqrt{2\eta}\sigma}{\sigma(1-p)} \right) c_t^{opt} dZ_t \quad . \end{aligned} \tag{32}$$

A preference for robustness therefore implies:

- A lower instantaneous variance of equilibrium optimal aggregate consumption growth,
- A lower instantaneous expected growth of equilibrium optimal aggregate consumption.

The consumption based part of the implied market price of risk is:

$$\frac{\alpha - r}{\sigma} - \sqrt{2\eta} \quad . \tag{33}$$

As a consequence, robustness yields a lower consumption based shadow market price of risk in Merton's model. This part is independent of the risk

[9]Note however, that this term depends on the volatility of optimal consumption growth under the selected worth case model which will be lower than the volatility of optimal consumption growth when no model misspecification is assumed.

aversion parameter p but depends crucially on the amount of risk aversion to model misspecification (which is a function of η)[10]. The resulting market prices of risk also reflects the price of model uncertainty risk and it is higher by a factor $\sqrt{\eta}$.

5 Conclusions

We proposed a simple robust version of Merton's (1969) and (1971) model of intertemporal consumption and portfolio choice where the optimal rules and the implied market price of risk of a representative RDM can be computed explicitly.

Robustness affects the optimal rules through a substitution of risky investment with saving in riskless assets and (or) current consumption. The consumption based part of the market price of risk is lower, as a consequence of a lower volatility of consumption growth, and is enxanced by a market price for model uncertainty that is monotonically related to the robustness parameter and that is independent of the risk aversion parameter.

[10] Indeed, the substitution effect discussed in the last section produced a redistribution of optimal wealth in favour of riskless assets and current consumption. This gives a lower volatility of aggregated wealth growth. By the linearity of optimal consumption in optimal wealth this implies a lower volatility of aggregated consumption growth too, that is a lower aggregated consumption risk. Since the assumed utility function are of the constant relative risk aversion type, the consumption based part of the equilibrium shadow market price of risk implied by a concern for robustness has to be lower.

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6 Appendix

Proof of (17) and (18): We consider the minimization with respect to ν of the objective function in (15) under the entropy constraint (16). Using (14) the HJB equation corresponding to this problem reads:

$$\varrho J(W) = \min_{\nu} \left[A^{M,\nu} J(W) + u(c) + \lambda \left(\frac{1}{2} A^{\nu}(\log \nu) - \eta \right) \right] \quad , \tag{34}$$

with the compensatory slackness conditions:

$$\lambda \left(\frac{1}{2} A^{\nu}(\log \nu) - \eta \right) = 0 \quad , \quad \lambda \geq 0 \quad . \tag{35}$$

By (8) and the explicit expression (10) a differentiation with respect to $\frac{\partial \log \nu}{\partial P}$ and $\frac{\partial \log \nu}{\partial W}$ yields the optimality conditions:

$$\begin{aligned} \sigma^2 P^2 \frac{\partial \log \nu}{\partial P} + \sigma^2 P w W \frac{\partial \log \nu}{\partial W} &= -\frac{1}{\lambda}(\sigma^2 P^2 J_P + \sigma^2 P w W J_W) \quad , \\ \sigma^2 w^2 W^2 \frac{\partial \log \nu}{\partial W} + \sigma^2 P w W \frac{\partial \log \nu}{\partial P} &= -\frac{1}{\lambda}(\sigma^2 w^2 W^2 J_W + \sigma^2 P w W J_P) \quad , \end{aligned}$$

that is:

$$\frac{\partial \log \nu}{\partial P} = -\frac{1}{\lambda} \cdot J_P \quad , \quad \frac{\partial \log \nu}{\partial W} = -\frac{1}{\lambda} \cdot J_W \quad . \tag{36}$$

Finally, λ is given by the equation:

$$\begin{aligned} \eta &= \frac{1}{2} A^{\nu}(\log \nu) \\ &= \frac{1}{2} \sigma^2 P^2 \frac{\partial \log \nu}{\partial P} \frac{\partial \log \nu}{\partial P} + \sigma^2 w^2 W^2 \frac{\partial \log \nu}{\partial W} \frac{\partial \log \nu}{\partial W} \\ &\qquad + \sigma^2 P w W \frac{\partial \log \nu}{\partial P} \frac{\partial \log \nu}{\partial W} + \sigma^2 P w W \frac{\partial \log \nu}{\partial W} \frac{\partial \log \nu}{\partial P} \\ &= -\frac{1}{2\lambda^2} \left(\sigma^2 P^2 J_P^2 + \sigma^2 w^2 W^2 J_W^2 + \sigma^2 P w W J_P J_W + \sigma^2 P w W J_W J_P \right) \quad , \end{aligned}$$

using (36). This gives (17) and (18) in the paper.

Figures

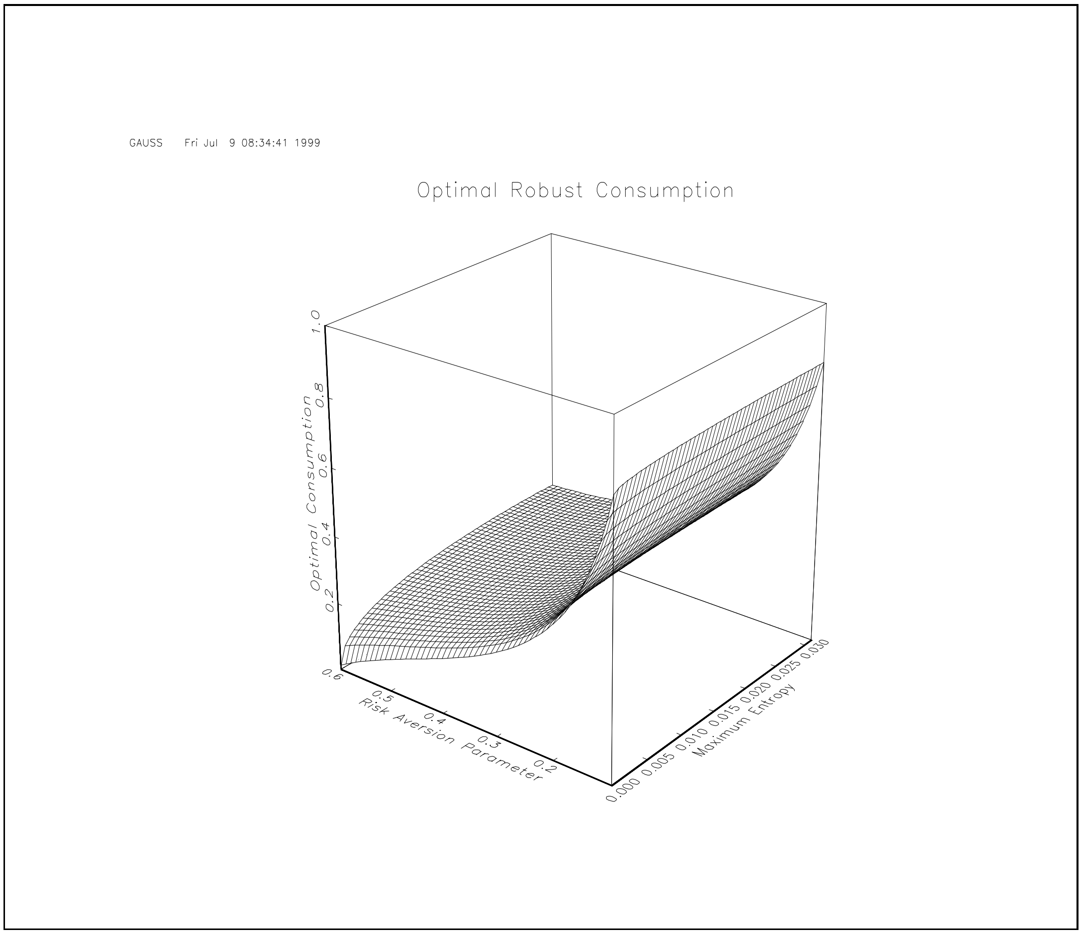

Figure 1: Optimal consumption $c^{RM}(\eta)$ (with W normalized to 1) as a function of the risk aversion parameter p (between 0.1 and 0.6) and the maximum entropy distance η (between 0 and 0.03). The other parameters were set to $\varrho = 0.08$, $r = 0.05$, $\alpha = 0.10$ and $\sigma = 0.2$.

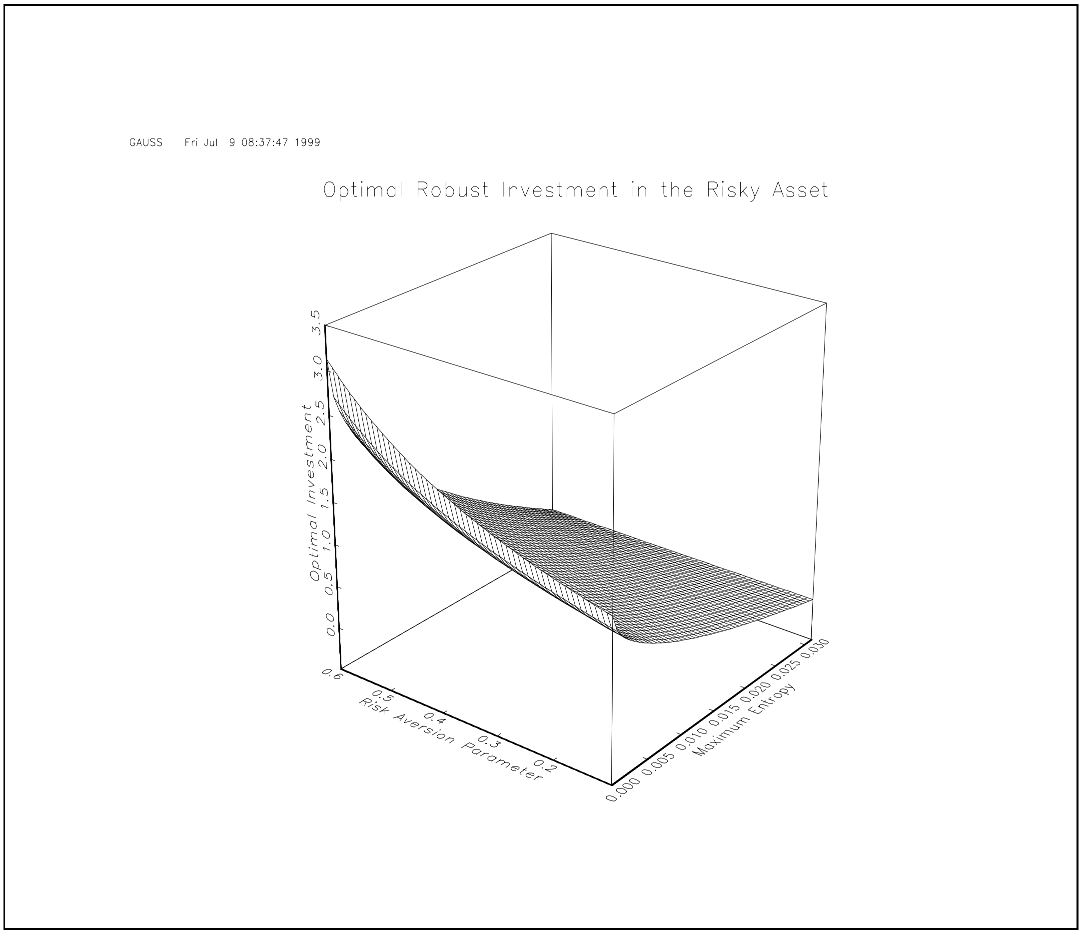

Figure 2: Optimal investment in risky assets $w^{RM}(\eta)$ as a function of the risk aversion parameter p (between 0.1 and 0.6) and the maximum entropy distance η (between 0 and 0.03). The other parameters were set to $\varrho = 0.08$, $r = 0.05$, $\alpha = 0.10$ and $\sigma = 0.2$.

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