

Optimistic Algorithms for Partial Database Replication

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Abstract

In this paper, we study the problem of partial database replication. Numerous previous works have investigated database replication, however, most of them focus on full replication. We are here interested in genuine partial replication protocols, which require replicas to permanently store only information about data items they replicate. We define two properties to characterize partial replication. The first one, *Quasi-Genuine Partial Replication*, captures the above idea; the second one, *Non-Trivial Certification*, rules out solutions that would abort transactions unnecessarily in an attempt to ensure the first property. We also present two algorithms that extend the Database State Machine [8] to partial replication and guarantee the two aforementioned properties. Our algorithms compare favorably to existing solutions both in terms of number of messages and communication steps.

1 Introduction

Database replication protocols based on group communication have recently received a lot of attention [5, 6, 8, 13]. The main reason for this stems from the fact that group communication primitives offer adequate properties, namely agreement on the messages delivered and on their order, to implement synchronous database replication. Most of the complexity involved in synchronizing database replicas is handled by the group communication layer.

Previous work on group-communication-based database replication has focused mainly on full replication. However, full replication might not always be adequate. First, sites might not have enough disk or memory resources to fully replicate the database. Second, when access locality is observed, full replication is pointless. Third, full replication provides limited scalability since every update transaction should be executed by each replica. In this paper, we extend the Database State Machine (DBSM) [8], a group-communication-based database replication technique, to partial replication. The DBSM is based on the deferred update replication model [1]. Transactions execute locally on one database site and their execution does not cause any interaction with other sites. Read-only transactions commit locally only; update transactions are atomically broadcast to all database sites at commit time for certification. The certification test ensures *one-copy serializability*: the execution of concurrent transactions on different replicas is equivalent to a serial execution on a single replica [1]. In order to execute the certification test, every database site keeps the *writesets* of committed transactions. The certification of a transaction T consists in checking that T 's *readset* does not contain any outdated value, i.e., no committed transaction T' wrote a data item x after T read x .

A straightforward way of extending the DBSM to partial replication consists in executing the same certification test as before but having database sites only process update operations for data items they replicate. But as the certification test requires storing the writesets of all committed transactions, this strategy defeats the whole purpose of partial replication since replicas may store information related to data items they do not replicate. We would like to define a property that captures the legitimacy of a partial replication protocol. Ideally, sites should be involved only in the certification of transactions that read or write data items they replicate. Such a strict property, however, would force the use of an atomic multicast protocol as the group communication primitive to propagate transactions. Since existing multicast protocols are more expensive than broadcast ones [4], this property restricts the performance of the protocol. More generally, we let sites receive and momentarily store transactions unrelated to the data items they replicate as long as this information is shortly erased. Moreover, we want to make sure each transaction is handled by a site at most once. If sites are allowed to completely forget about past transactions, this constraint cannot obviously be satisfied. We capture these two requirements with the following property:

- *Quasi-Genuine Partial Replication*: For every submitted transaction T , correct database sites that do not replicate data items read or written by T permanently store not more than the identifier of T .¹

Consider now the following modification to the DBSM, allowing it to ensure Quasi-Genuine Partial Replication. Besides atomically broadcasting transactions for certification, database sites periodically broadcast “garbage collection” messages. When a garbage collection message is delivered, a site deletes all the writesets of previously committed transactions. When a transaction is delivered for certification, if the site does not contain the writesets needed for its certification, the transaction is conservatively

¹Notice that even though transaction identifiers could theoretically be arbitrarily large, in practice, 4-byte identifiers are enough to uniquely represent 2^{32} transactions.

aborted. Since all sites deliver both transactions and garbage collection messages in the same order, they will all reach the same outcome after executing the certification test. This mechanism, however, may abort transactions that would be committed in the original DBSM. In order to rule out such solutions, we introduce the following property:

- *Non-Trivial Certification*: If there is a time after which no two conflicting transactions are submitted, then eventually transactions are not aborted by certification.

In this paper we present two algorithms for partial database replication that satisfy Quasi-Genuine Partial Replication and Non-Trivial Certification. Both algorithms make optimistic assumptions to ensure better performance. Our first algorithm is simpler and assumes *spontaneous total order*: with high probability messages sent to all servers in the cluster reach all destinations in the same order, a property usually verified in local-area networks. As a drawback, it processes a single transaction at a time. Our second algorithm is able to certify multiple transactions at a time and, as explained in Section 4, does not assume spontaneous total order.

To the best of our knowledge, [5] and [12] are the only papers addressing partial database replication using group communication primitives. In [5], every read operation is multicast to the sites replicating the data items read; write operations are multicast together with the transaction’s commit request. A final atomic commit protocol ensures transaction atomicity. In [12], the authors extend the DBSM for partial replication by adding an extra atomic commit protocol. Each replica uses as its vote for atomic commit the result of the certification test. Both of our algorithms compare favorably to those presented in [5] and [12]: they either have a lower latency or make weaker assumptions about the underlying model, i.e., they do not require perfect failure detection.

2 System Model and Definitions

We consider a system $\Pi = \{s_1, \dots, s_n\}$ of database sites. Sites communicate through message passing and do not have access to a shared memory or a global clock. We assume the crash-stop failure model. A site that never crashes is *correct*, and a site that is not correct is *faulty*. The system is asynchronous, i.e., message delays and the time necessary to execute a step can be arbitrarily large but are finite. Furthermore, the communication channels do not corrupt or duplicate messages, and are (quasi-)reliable: if a correct site p sends a message m to a correct site q , then q eventually receives m .

Throughout the paper, we assume the existence of a *Reliable Broadcast* primitive. Reliable Broadcast is defined by primitives $R\text{-bcast}(m)$ and $R\text{-deliver}(m)$, and satisfies the following properties [2]: if a correct site R-bcasts a message m , then it eventually R-delivers m (*validity*), (ii) if a correct site R-delivers a message m , then eventually all correct sites R-deliver m (*agreement*) and (iii) for every message m , every site R-delivers m at most once and only if it was previously R-bcast (*uniform integrity*). Reliable Broadcast does not ensure agreement on the message delivery order, that is, two broadcast messages might be delivered in different orders by two different sites. In local-area networks, some implementations of Reliable Broadcast can take advantage of network hardware characteristics to deliver messages in total order with high probability [9]. We call such a primitive Weak Ordering Reliable Broadcast, WOR-Broadcast.

Our algorithms also use a *consensus* abstraction. In the consensus problem, sites propose values and must reach agreement on the value decided. Consensus is defined by the primitives $propose(v)$ and $decide(v)$, and satisfies the following properties: (i) every site decides at most once (*uniform integrity*),

(ii) no two sites decide differently (*uniform agreement*), (iii) if a site decides v , then v was proposed by some site (*uniform validity*) and (iv) every correct site eventually decides (*termination*).

A database $\Gamma = \{x_1, \dots, x_n\}$ is a finite set of data items. Database sites have a partial copy of the database. For each site s_i , $Items(s_i) \subseteq \Gamma$ is defined as the set of data items replicated on s_i . A transaction is a sequence of read and write operations on data items followed by a commit or abort operation. For simplicity, we represent a transaction T as a tuple (id, rs, ws, up) , where id is the unique identifier of T , rs is the readset of T , ws is the writeset of T and up contains the updates of T . More precisely, up is a set of tuples (x, v) , where, for each data item x in ws , v is the value written to x by T . For every transaction T , $Items(T)$ is defined as the set of data items read or written by T . Two transactions T and T' are said to be conflicting, if there exists a data item $x \in Items(T) \cap Items(T') \cap (T.ws \cup T'.ws)$. We define $Site(T)$ as the site on which T is executed. Furthermore, we assume that for every data item $x \in \Gamma$, there exists a correct site s_i which replicates x , i.e., $x \in Items(s_i)$. Finally, we define $Replicas(T)$ as the set of sites which replicate at least one data item written by T , i.e., $Replicas(T) = \{s_i \mid s_i \in \Pi \wedge Items(s_i) \cap T.ws \neq \emptyset\}$.

3 The Database State Machine Approach

We now present a generalization of the Database State Machine approach. The protocol in [8] is an instance of our generalization in the fully replicated context. For the sake of simplicity, we consider a replication model where a transaction T can only be executed on a site s_i if $Items(T) \subseteq Items(s_i)$. Moreover, to simplify the presentation, we consider a client c that sends requests on behalf of a transaction T to $Site(T)$. In the following, we comment on the states in which a transaction can be in the DBSM.

- *Executing*: Read and write operations are executed locally at $Site(T)$ according to the strict two-phase locking rule (strict 2PL). When c requests to commit T , it is immediately committed and passes to the Committed state if it is a read-only transaction, event which we denote $Committed(T)_{Site(T)}$; if T is an update transaction, it is submitted for certification and passes to the Submitted state at $Site(T)$. We represent this event as $Submitted(T)_{Site(T)}$. In the fully replicated case, to submit T , sites use an atomic broadcast primitive; in a partial replication context, the algorithms of Section 4 are used.
- *Submitted*: When T enters the Submitted state, its read locks are released at $Site(T)$ and T is eventually certified. With full replication, the certification happens when T is delivered; Section 4 explains when this happens in a partially replicated scenario. Certification ensures that if a committed transaction T' executed concurrently with T , and T read a data item written by T' then T is aborted. T' is concurrent with T if it committed at $Site(T)$ after T entered the Submitted state at $Site(T)$. Therefore, T passes the certification test on site s_i if for every T' already committed at s_i the following condition holds:

$$\begin{aligned} Committed(T')_{Site(T)} \rightarrow Submitted(T)_{Site(T)} \\ \vee \\ T'.ws \cap T.rs = \emptyset, \end{aligned} \tag{1}$$

where \rightarrow is Lamport's happened before relation on events [7].

In the fully replicated DBSM, transactions are certified locally by each site upon delivery. In the partially replicated DBSM, to ensure *Quasi-Genuine Partial Replication*, sites only store the

writesets of committed transactions that wrote data items they replicate. Therefore, sites might not have enough information to decide on the outcome of all transactions. Hence, to satisfy *Non-trivial Certification*, we introduce a voting phase where each site sends the result of its certification test to the other sites. Site s_i can safely decide to commit or abort T when it has received votes from a *voting quorum* for T . Intuitively, a voting quorum VQ for T is a set of databases such that for each data item read by T , there is at least one database in VQ replicating this item. More formally, a quorum of sites is a voting quorum for T if it belongs to $VQS(T)$, defined as follows:

$$VQS(T) = \{VQ | VQ \subseteq \Pi \wedge T.rs \subseteq \bigcup_{s \in VQ} Items(s)\} \quad (2)$$

For T to commit, every site in a voting quorum for T has to vote *yes*. If a site in the quorum votes *no*, it means that T read an old value and should be aborted; committing T would make the execution non-serializable. Notice that $Site(T)$ is a voting quorum for T by itself, since for every transaction T , $Items(T) \subseteq Items(Site(T))$. If T passes the certification test at s_i , it requests the write locks for the data items it has updated. If there exists a transaction T' on s_i that holds conflicting locks with T 's write locks, the action taken depends on T' 's state on s_i and on T' 's type, read-only or update:

1. *Executing*: If T' is in execution on s_i then one of two things will happen: if T' is a read-only transaction, T waits for T' to terminate; if T' is an update transaction, it is aborted.
2. *Submitted*: This happens if T' executed on s_i , already requested commit but was not committed yet. In this case, T 's updates should be applied to the database before T' 's. How this is ensured is implementation specific.²

Once the locks are granted, T applies its updates to the database and passes to the Committed state. If T fails the certification test, it passes to the Aborted state.

- *Committed/Aborted*: These are final states.

4 Partially-replicated DBSM

In this section, we present two algorithms for the termination protocol of the DBSM in a partial replication context. These protocols ensure both one-copy serializability [1] and the following liveness property: if a correct site submits a transaction T , then either $Site(T)$ aborts T or eventually all correct sites in $Replicas(T)$ commit T . The algorithms also satisfy Quasi-Genuine Partial Replication and Non-Trivial Certification. The proof of correctness can be found in [11].

4.1 The “One-at-a-time” Algorithm

Sites execute a sequence of *steps*. In each step, sites decide on the outcome of one transaction. A step is composed of two phases, a consensus phase and a voting phase. Consensus is used to guarantee that sites agree on the commit order of transactions. In the voting phase, sites exchange the result of

²For example, a very simple solution would be for s_i to abort T' ; if T' later passes certification, its writes would be re-executed. The price paid for simplicity here is the double execution of T' 's write operations.

their certification test to ensure that the commit of a transaction T in step K induces a serializable execution.

The naive way to implement the termination protocol is to first use consensus to determine the next transaction T in the serial order and then execute the voting phase for T . We take a different approach: Based on the observation that with a high probability messages broadcast in a local-area network are received in total order [9], we overlap the consensus phase with the voting phase to save one communication step. If sites receive the transaction to be certified in the same order, they vote for the transaction before proposing it to consensus. Luckily, by the time consensus decides on a transaction T , every site will already have received the votes for T and will be able to decide on the outcome of T .

Algorithm 1 is composed of three concurrent tasks. Each line of the algorithm is executed atomically. The state transitions of transactions are specified in the right margin of lines 10, 28, and 30. Notice that the state transition happens after the corresponding line has been executed. Every transaction T is a tuple $(id, site, rs, ws, up, past, order)$. We added three fields to the definition of a transaction (c.f. Section 2), namely *site*, *past*, and *order*: *site* is the database site on which T is executed; *past* is the order of T 's submission; and *order* is T 's commit order. The algorithm also uses five global variables: K stores the *step* number; $UNDECIDED$ and $DECIDED$ are (ordered) sequences of, respectively, pending transactions and transactions for which the outcome is known; $COMMITTED$ is the set of committed transactions; and the set $VOTES$ stores the votes received, i.e., the results of the certification test. We use the operators \oplus and \ominus for the concatenation and decomposition of sequences. Let seq_1 and seq_2 be two sequences of transactions. Then, $seq_1 \oplus seq_2$ is the sequence of transactions in seq_1 followed by all the transactions in seq_2 , and $seq_1 \ominus seq_2$ is the sequence of transactions in seq_1 that are not in seq_2 . Transactions are matched using their identifiers.

To take advantage of spontaneous total order, database sites use the WOR-Broadcast primitive to submit transactions (line 10). When no consensus instance is running and $UNDECIDED$ is not empty, sites first execute the *Vote* procedure for T at the head of $UNDECIDED$ (line 17) and then propose T (line 18). In the *Vote* procedure, T is certified and the result of the certification is sent in a message of type *VOTE*.

Notice that even though $Site(T)$ is a voting quorum for T by itself ($Items(T) \subseteq Items(Site(T))$), in the algorithm, all sites replicating a data item read by T vote. This is done to tolerate the crash of $Site(T)$. If only $Site(T)$ voted, the following undesirable scenario could happen: $Site(T)$ submits T and crashes just after executing line 10. Databases *WOR-Deliver* T , propose T and decide on T . In this execution, sites would wait forever at line 24, as $Site(T)$ crashed before voting for T .

Two further remarks concern the *Vote* procedure. First, to be able to certify transactions, we need to implement the precedence relation \rightarrow between events. For two transactions T and T' , this is done by comparing the value of their *past* and *order* fields. If $T.order < T'.past$, we are sure that T committed before T' was submitted, because K is incremented after transactions commit. Second, notice that *VOTE* messages contain the *step number* K in which T was certified. This information is necessary because a transaction can be certified in different steps and the result of the certification test in steps K and K' might be different. This is precisely why sites wait for *VOTE* messages coming from step number K at line 24. Moreover, even if sites receive votes from different voting quorums, they will agree on the outcome of the transaction. Intuitively, this holds because we only take into account *voting quorums* that voted in *step* K , therefore they consider the same sequence of committed transactions. Finally, by verifying that transactions T and T' are the same at line 20, sites check if the spontaneous total order holds. If it is not the case, sites need to vote for the transaction decided by consensus.

Algorithm 1 The “One-at-a-time” algorithm - Code of database site s

```
1: Initialization
2:    $K \leftarrow 1, UNDECIDED \leftarrow \epsilon, DECIDED \leftarrow \epsilon, COMMITTED \leftarrow \emptyset, VOTES \leftarrow \emptyset$ 
3: function Certify( $T$ )
4:   return  $\forall (id, order, ws) \in COMMITTED : order < T.past \vee ws \cap T.rs = \emptyset$ 
5: procedure Vote( $T$ )
6:   if  $T.rs \cap Items(s) \neq \emptyset$  then
7:     send(VOTE,  $T.id, K, Certify(T)$ ) to all  $q$  in  $Replicas(T)$ 
8: To submit transaction  $T$  {Task 1}
9:    $T.past \leftarrow K$ 
10:  WOR-Broadcast(VOTE-REQ,  $T$ ) {Executing  $\rightarrow$  Submitted}
11: When receive(VOTE,  $T.id, K', vote$ ) from  $q$  {Task 2}
12:    $VOTES \leftarrow VOTES \cup (T.id, q, K', vote)$ 
13: When WOR-Deliver(VOTE-REQ,  $T$ )  $\wedge T.id \notin DECIDED$  {Task 3}
14:    $UNDECIDED \leftarrow UNDECIDED \oplus T$ 
15: When  $UNDECIDED \neq \epsilon$ 
16:    $T \leftarrow head(UNDECIDED)$ 
17:   Vote( $T$ )
18:   Propose( $K, T$ )
19:   wait until Decide( $K, T'$ )
20:   if  $T'.id \neq T.id$  then Vote( $T'$ )
21:    $UNDECIDED \leftarrow UNDECIDED \ominus T'$ 
22:    $DECIDED \leftarrow DECIDED \oplus T'.id$ 
23:   if  $T'.ws \cap Items(s) \neq \emptyset$  then
24:     wait until  $\exists VQ \in VQS(T') : \forall q \in VQ : (T'.id, q, K, -) \in VOTES$ 
25:     if  $\forall q \in VQ : (T'.id, q, K, yes) \in VOTES$  then
26:        $T'.order \leftarrow K$ 
27:        $COMMITTED \leftarrow COMMITTED \cup (T'.id, T'.order, T'.ws \cap Items(s))$ 
28:       commit  $T'$  {Submitted  $\rightarrow$  Committed}
29:     else
30:       if  $s = T'.site$  then abort  $T'$  {Submitted  $\rightarrow$  Aborted}
31:    $K \leftarrow K + 1$ 
32:    $VOTES \leftarrow \{(tid, q, K', v) \in VOTES \mid K' \geq K\}$ 
```

4.2 The “Many-at-a-time” Algorithm

The previous algorithm certifies transactions sequentially. Thus, if many transactions are submitted, an ever-growing chain of uncommitted transactions can be formed. Algorithm 2 solves that problem by allowing a sequence of transactions to be proposed in consensus instances and by changing the certification test accordingly.

Algorithm 2 follows the same structure and uses the same global variables as Algorithm 1. The difference lies in Task 3 and the auxiliary procedures used. In the general case, when sites notice that there is a sequence of pending transactions that have not been committed or aborted (“ $UNDECIDED \neq \epsilon$ ” at line 25), this sequence is voted for and proposed in consensus instance K (lines 26–27). In the *Vote* procedure, every pending transaction is certified considering only the previously committed transactions (lines 3–9). The results are gathered in a set and later sent to all sites that have data items updated by some transaction in the pending sequence (lines 10–12). The “ $VOTES \neq \emptyset$ ” condition at line 25 is there for garbage collection purposes: it forces the proposal of empty sequences in case there are votes for undelivered vote requests (a possible situation due to failures that would violate *Quasi-Genuine Partial Replication*).

After the K -th instance of consensus has decided on a sequence SEQ of transactions (line 28), sites

verify whether they have voted for all transactions in SEQ ; if it is not the case, they vote for the sequence SEQ (lines 29–30). Then, sites replicating data items updated by one of the transactions in SEQ sequentially certify all transactions in SEQ following their order (lines 35–45). The certification of transaction T is divided into two parts. First, T is certified considering the transactions committed in steps lower than K by taking into account the votes of a voting quorum (line 37). Second, sites certify T considering committed transactions that have been decided in the same consensus instance (line 38). This is done by gathering committed transactions in a set called $LCOMMIT$ and by verifying that there does not exist a transaction T' in this set that writes a data item read by T . If T passes both certifications and updates a data item in $Items(s)$, it is treated in exactly the same way as certified transactions in Algorithm 1 (lines 41–43).

Differently from Algorithm 1, Algorithm 2 does not rely on spontaneous total order. This is because sequences of transactions are used when voting and proposing values to a consensus instance, and the order of transactions in this sequence does not matter when it comes to voting. Recall that the vote phase in step K consists in independently certifying undecided transactions against transactions committed in previous steps (line 11 and function *Certify* at lines 3–9). This phase does not take into consideration conflicts within the sequence itself since they are solved after the consensus instance is decided. Nevertheless, votes are still optimistic in Algorithm 2 as they are sent before the consensus instance has decided on its outcome.

The optimistic assumption that allows a transaction T to be certified as soon as consensus instance K decides on a sequence containing T is that every member of at least one correct voting quorum VQ for T has voted for any sequence containing T before consensus instance K (line 26). Notice that the sequences considered by different members of VQ do not have to be the same, the only requirement is that they all contain T .

We could further relax the optimistic assumptions required at the price of having a higher number of VOTE messages. In the way both algorithms are described, sites vote for a transaction only before it is proposed to the next consensus instance (line 17 of Algorithm 1, line 26 of Algorithm 2). Consider a scenario where the vote request for a transaction T is delivered by a site s right after s has proposed transaction(s) to consensus. Site s will therefore have to wait until the instance finishes to send its vote concerning T . However, T 's vote request might have been delivered earlier by some other site and might even have been proposed to the current instance of consensus. If that is the case, and T is part of the consensus decision, the optimistic assumptions will not hold and the protocols might need an extra message step to certify T . This problem can be avoided if sites are allowed to vote while solving a consensus instance. In our example scenario, site s would vote for T even though it has already voted for its consensus proposal. Both votes would then be received by other sites and they would be used to decide on the outcome of T . This optimization relieves the need for spontaneous total order in Algorithm 1 and relaxes even more the optimistic assumption of Algorithm 2. As a secondary effect, it reduces the average latency of transaction certification since votes are sent right after the vote request is received.

5 Related Work and Final Remarks

In this section we compare our algorithms to the related work and conclude the paper. We focus here on the related works satisfying *Quasi-Genuine Partial Replication*.

In [5] the authors propose a database replication protocol based on group multicast. Every read operation on data item x is multicast to the group replicating x ; writes are multicast along with the

Algorithm 2 The “Many-at-a-time” algorithm - Code of database site s

```

1: Initialization
2:  $K \leftarrow 1, UNDECIDED \leftarrow \epsilon, DECIDED \leftarrow \epsilon, COMMITTED \leftarrow \emptyset, VOTES \leftarrow \emptyset$ 

3: function Certify( $SEQ$ )
4:  $V \leftarrow \emptyset$ 
5: for all  $T \in SEQ$  do
6:   if  $\forall (id, order, ws) \in COMMITTED : order < T.past \vee ws \cap T.rs = \emptyset$  then
7:      $V \leftarrow V \cup (T.id, yes)$ 
8:   else  $V \leftarrow V \cup (T.id, no)$ 
9:   return  $V$ 

10: procedure Vote( $SEQ$ )
11: if  $\exists T \in SEQ : T.rs \cap Items(s) \neq \emptyset$  then
12:   send (VOTE, Strip( $SEQ$ ),  $K$ , Certify( $SEQ$ )) to  $\{q \mid \exists T \in SEQ : q \in Replicas(T)\}$ 

13: function Strip( $SEQ$ )
14:  $RESULT \leftarrow \epsilon$ 
15: for all  $T \in SEQ$  in order do
16:    $RESULT \leftarrow RESULT \oplus T.id$ 
17:   return  $RESULT$ 

18: To submit transaction  $T$  {Task 1}
19:    $T.past \leftarrow K$ 
20:   R-bcast (VOTE_REQ,  $T$ ) {Executing  $\rightarrow$  Submitted}

21: When receive (VOTE,  $IDSEQ, K', V$ ) from  $q$  {Task 2}
22:    $VOTES \leftarrow VOTES \cup (IDSEQ, q, K', V)$ 

23: When R-deliver (VOTE_REQ,  $T$ )  $\wedge T.id \notin DECIDED$  {Task 3}
24:    $UNDECIDED \leftarrow UNDECIDED \oplus T$ 

25: When  $UNDECIDED \neq \epsilon \vee VOTES \neq \emptyset$ 
26:   Vote( $UNDECIDED$ )
27:   Propose( $K, UNDECIDED$ )
28:   wait until Decide( $K, SEQ$ )
29:   if  $\exists T : T \in SEQ \wedge T \notin UNDECIDED$  then
30:     Vote( $SEQ$ )
31:      $DECIDED \leftarrow DECIDED \oplus Strip(SEQ)$ 
32:      $UNDECIDED \leftarrow UNDECIDED \ominus SEQ$ 
33:     if  $\exists T \in SEQ : T.ws \cap Items(s) \neq \emptyset$  then
34:        $LCOMMIT \leftarrow \emptyset$ 
35:       for all  $T \in SEQ$  in order do
36:         wait until
37:            $\exists VQ \in VQS(T) : \forall q \in VQ : \exists (SEQ_q, q, K, V_q) \in VOTES : T \in SEQ_q$ 
38:            $\wedge (\exists T' \in LCOMMIT : T'.ws \cap T.rs \neq \emptyset)$  then
39:              $LCOMMIT \leftarrow LCOMMIT \cup \{T\}$ 
40:           if  $T.ws \cap Items(s) \neq \emptyset$  then
41:              $T.order \leftarrow K$ 
42:              $COMMITTED \leftarrow COMMITTED \cup (T.id, T.order, T.ws \cap Items(s))$ 
43:             commit  $T$  {Submitted  $\rightarrow$  Committed}
44:           else
45:             if  $s = T.site$  then abort  $T$  {Submitted  $\rightarrow$  Aborted}
46:            $K \leftarrow K + 1$ 
47:            $VOTES \leftarrow \{(tid, q, K', v) \in VOTES \mid K' \geq K\}$ 

```

commit request. The delivered operations are executed on the replicas using strict two-phase locking and results are sent back to the client. A final atomic commit protocol ensures transaction atomicity. In the atomic commit protocol, every group replicating a data item read or written by a transaction T sends its vote to a *coordinator* group, which collects the votes and sends the result back to all participating groups. The protocol ensures *Quasi-Genuine Partial Replication* because a transaction operation on data item x is only multicast to the group replicating x and the atomic commit protocol is executed among groups replicating data item read or written by the transaction. In [12] the authors extend the DBSM to partial replication. They use an optimistic atomic broadcast primitive and a variation of atomic commit, called resilient atomic commit. In contrast to atomic commit, resilient atomic commit may decide to commit a transaction even though some participants crash. When a transaction T is optimistically delivered, replicas certify T and execute a resilient atomic commit protocol using the result of the certification test as their vote. If the optimistic order of T corresponds to the final order, the protocol ends; otherwise when the final order is known, T is certified again and a second resilient atomic commit protocol is executed. The protocol ensures *Quasi-Genuine Partial Replication*, since only sites replicating data item written by T keep T in their committed transaction sequence.

We now compare the cost of the protocols in [5, 12] with the two algorithms presented in this paper. We compare the number of communication steps and the number of messages exchanged during the execution of a transaction T . To simplify the analysis, we assume that all messages have a delay of δ . We consider two cases, one where the algorithms' respective optimistic assumptions hold and one where they do not (c.f. Section 4). In both cases, we consider the best achievable latency and the minimum number of messages exchanged, when neither failures nor failure suspicions occur, the most frequent case in practical settings. We first present in Figure 1 the cost of known algorithms used by the protocols compared in this section. Variable k is the total number of participants in the protocol.

Problem	steps	unicast msgs.	broadcast msgs.
Non-Uniform R. Broadcast (RBCast) [2]	1	$k(k - 1) + 1$	k
Uniform Consensus (Consensus) [10]	2	$2k(k - 1)$	$2k$
Non-Blocking A. Commit (NBAC) [3] ³	2	$2k(k - 1)$	$2k$
Uniform A. Broadcast (ABcast) [2]	3	$3k(k - 1) + 1$	$3k$
Uniform A. Multicast (AMcast) [4]	4	$4k(k - 1) + 1$	$4k$

Figure 1: Cost of different agreement problems

Figure 2 presents the cost of the different algorithms. To compute the cost of the execution of T , we consider that T consists of a read and a write operation on the same data item x . For all the protocols, we consider that d database sites replicate data item x and that n is the total number of database sites in the system.

In [5], one multicast is used to read x , d messages are sent containing the result of the read, one multicast is used to send the write along with the commit request and a final atomic commit protocol among d participants is executed. Notice that none of the optimistic assumptions assumed by the algorithms in this paper influence the cost of this protocol. In [12], the transaction is atomically broadcast and one communication step later it is optimistically delivered. A resilient atomic commit protocol is then executed among the d database sites. Resilient atomic commit is implemented in one communication step, in which all participants exchange their votes. To guarantee agreement on the outcome of a transaction, the implementation requires perfect failure detection, an assumption that we do not need in this paper. In the best-case scenario, i.e., spontaneous total order holds, the number of

³This cost corresponds to the case where all participants spontaneously start the protocol. This assumption makes sense here because in [5] participants deliver a transaction's commit request before starting the atomic commit protocol.

Algorithm	steps	unicast msgs.	broadcast msgs.
[5]	11	$10d^2 - 9d + 2$	$11d$
[12]	3	$3n(n-1) + d(d-1) + 1$	$3n + d$
Algorithms 1 & 2	3	$3n(n-1) + d(d-1) + 1$	$3n + d$

(a) Optimistic assumption holds

Algorithm	steps	unicast msgs.	broadcast msgs.
[5]	11	$10d^2 - 9d + 2$	$11d$
[12]	4	$3n(n-1) + 2d(d-1) + 1$	$3n + 2d$
Algorithms 1 & 2	4	$3n(n-1) + 2d(d-1) + 1$	$3n + 2d$

(b) Optimistic assumption does not hold

Figure 2: Comparison of the database replication protocols

communication steps is equal to $\max(2, \text{steps}(ABcast))$. If the optimistic order is not the final order of the transaction, another resilient atomic commit protocol is needed and therefore the number of communication steps becomes $\text{steps}(ABcast) + 1$.

For Algorithms 1 and 2, the cost is computed as follows. In the best-case scenario, the number of communication steps is equal to $\text{steps}(RBCast) + \max(\text{steps}(Consensus), \text{steps}(vote\ phase))$, where *vote phase* corresponds to d broadcast messages. If the algorithms' respective optimistic assumptions do not hold, after deciding on T in consensus, another *vote phase* has to take place and therefore the number of communication steps becomes $\text{steps}(RBCast) + \text{steps}(Consensus) + \text{steps}(vote\ phase)$. For simplicity, we assume that in this second *vote phase*, all participants vote, generating an extra $d(d-1)$ messages.

Considering latency, Algorithms 1, 2, and [12] give the best results. However, to achieve such latency, [12] uses perfect failure detection. In terms of the number of messages generated, [5] is cheaper than the DBSM-based solutions if d is much smaller than x . Nonetheless, this protocol has a serious drawback: its number of communication steps highly depends on the number of read operations, as every read operation adds 5 message steps (4 for the multicast and 1 to send back the result). As a final remark, notice that in this analysis we consider the execution of only one transaction. The cost of the protocols might however change if we considered multiple transactions. In this scenario, the following observations can be made. First, even though Algorithms 1 and 2 have equal costs in Figure 2, the overhead might be higher for Algorithm 1 when multiple transactions are submitted. This stems from the fact that in Algorithm 2, the cost of running consensus might be shared among a set of transactions, therefore reducing the number of generated messages. Second, in [5, 12], each transaction requires a separate instance of atomic commit to decide on its outcome. In Algorithm 2, however, at most two voting phases are needed to decide on the outcome of the sequence of transactions decided in the same consensus instance. Therefore, the longer this sequence, the cheaper Algorithm 2 will be compared to [5, 12].

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A Appendix

We here only give the proof of Algorithm 2, as Algorithm 1 is a special case of Algorithm 2.

A.1 The proof of Algorithm 2

Lemma 1 *No two consensus instances k, k' ($k \neq k'$) decide on the same transaction T .*

Proof: Suppose, by way of contradiction, that two instances k and k' decide on the same transaction T . Without loss of generality, suppose that $k' > k$. Consequently, by the validity property of consensus, there exists a database site s_i that proposes T for instance k' . If s_i proposes T in instance $k' > k$, s_i decided in instance k . By the uniform agreement of consensus, s_i decided on T in instance k . Therefore, $T.id$ is added to $DECIDED_i$ at line 31 and T is removed from $UNDECIDED_i$

at line 32 when $K_i = k$. Since no transaction is removed from $DECIDED_i$, T cannot be inserted in $UNDECIDED_i$ afterwards at line 24. Therefore, T cannot be in $UNDECIDED_i$ at line 27 when $K_i > k$ and therefore s_i does not propose T in consensus instance k' , a contradiction. \square

Lemma 2 *For any transaction T , any database site s_i , and any K_i , between two invocations of $Certify(-)$, namely $Certify(-)_1$ and $Certify(-)_2$, such that K_i is the same at the time $Certify(-)_1$ and $Certify(-)_2$ are called, $Certify(-)_1$ reads the same $COMMITTED$ set as $Certify(-)_2$.*

Proof: The function $Certify$ is called indirectly either (a) from line 26 or (b) from line 30. The set $COMMITTED$ is modified at line 42. Therefore, $Certify(-)_2$ reads the same $COMMITTED$ set as $Certify(-)_1$ when K_i is the same at the time $Certify(-)_1$ and $Certify(-)_2$ are called. \square

Lemma 3 *For any transaction T and any database site s_i , in every step K_i , any two invocations of $Certify(-)$, namely $Certify(SEQ_1)$ and $Certify(SEQ_2)$, return the same value for all transactions T such that $T \in SEQ_1 \cap SEQ_2$.*

Proof: By Lemma 2, the values of $COMMITTED$ read in $Certify(SEQ_1)$ and $Certify(SEQ_2)$ are the same. By the fact that $T.past$ and $T.rs$ are constant, the returned values of function $Certify(SEQ_1)$ and $Certify(SEQ_2)$ are the same for all $T \in SEQ_1 \cap SEQ_2$. \square

Definition 1 *From Lemma 2, we can define $COMMITTED_i^k$ as the value of variable $COMMITTED$ read at line 7 on s_i when $K_i = k$. From Lemma 3, we can define $Certify(T, k)_i$ as the returned value of the function $Certify(SEQ)$ for T ($T \in SEQ$), called on s_i when $K_i = k$. If there exists no invocation of function $Certify(SEQ)$ on s_i when $K_i = k$ such that $T \in SEQ$, we say that $Certify(T, k)_i$ is undefined, and we write $Certify(T, k)_i = \perp$.*

Definition 2 *We define $vote(VQ, T, k)$, voting quorum VQ 's vote for transaction T ($VQ \in VQS(T)$), considering the $VOTE$ messages of all $q \in VQ$ cast when $K_q = k$ as follows:*

- $vote(VQ, T, k) = yes \iff \forall s_i \in VQ : Certify(T, k)_i = yes$
- $vote(VQ, T, k) = no \iff \forall s_i \in VQ : Certify(T, k)_i \neq \perp \wedge \exists s_j \in VQ : Certify(T, k)_j = no$
- $vote(VQ, T, k) = \perp \iff \exists s_i \in VQ : Certify(T, k)_i = \perp$

Lemma 4 *For all k and every transaction T , there does not exist $VQ_1, VQ_2 \in VQS(T)$ such that $vote(VQ_1, T, k) = yes$ and $vote(VQ_2, T, k) = no$.*

Proof: The proof is by induction on k . (Base step - $k = 1$) When the algorithm initializes, the set $COMMITTED$ is empty. Therefore, for all s_i such that $Certify(T, 1)_i \neq \perp$, we have that $Certify(T, 1)_i = yes$. Therefore, for all $VQ \in VQS(T)$ such that $vote(VQ, T, 1) \neq \perp$, $vote(VQ, T, 1) = yes$. (Induction step) Suppose Lemma 4 holds for all m such that $1 \leq m < k$, we show that Lemma 4 holds for k . Suppose, by way of contradiction, that there exist a transaction T and two voting quorums $VQ_1, VQ_2 \in VQS(T)$, such that $vote(VQ_1, T, k) = yes$ and $vote(VQ_2, T, k) = no$. Therefore, for every database s_i in VQ_1 , $Certify(T, k)_i = yes$ and there exists a database site s_j in VQ_2 such that

$Certify(T, k)_j = no$. Therefore, there exists a transaction $T' \in COMMITTED_j^k$ such that $T'.ws \cap T.rs \neq \emptyset$ and $T'.order \geq T.past$. Let k' be the consensus instance in which T' was decided ($k' < k$). By the uniform agreement property of consensus, all databases decide on T' in instance k' . Since VQ_1 is a voting quorum for T and because $T'.ws \cap T.rs \neq \emptyset$, there exists a database site s_l in VQ_1 such that $Items(s_l) \cap T'.ws \neq \emptyset$. By the induction hypothesis, for all $VQ \in VQS(T') : vote(VQ, T', k') = yes$, therefore $T' \in COMMITTED_l^{k'+1}$. Thus, $Certify(T, k)_j = Certify(T, k)_l = no$, a contradiction. \square

We now show that Algorithm 2 ensures 1-SR. A replicated data history H is 1-SR if it is equivalent to a one-copy serial history $1H$ [1]. Notice that in these histories we only consider operations of committed transactions. Our proof is composed of two steps. We first show that every history H_i of database s_i is serializable. Second we show how to construct a serial one-copy history $1H$ that is equivalent to the replicated data history H .

Lemma 5 *Every history H_i of database site s_i is serializable.*

Proof: In the following, we use the multi-version serialization graph formalism introduced in [1]. A multi-version serialization graph of a history H_i $MVSG(H_i, \ll)$ is a directed graph, in which the nodes represent committed transactions and \ll defines the version order on data items. An edge $T_a \rightarrow T_b \in MVSG(H_i, \ll)$ can be of three types:

1. Read-from: T_b reads data item x from T_a .
2. Version-order type I: T_a and T_b write x such that $x_a \ll x_b$.
3. Version-order type II: T_a reads x_c from T_c and T_b writes x_b such that $x_c \ll x_b$.

To show that history H_i is serializable, we prove that $MVSG(H_i, \ll)$ is acyclic. To prove that $MVSG(H_i, \ll)$ is acyclic, we show that for every node T_a, T_b : $T_a \rightarrow T_b \in MVSG(H_i, \ll) \Rightarrow C(T_a) \prec_i C(T_b)$.⁴

1) *Read-from edge*: Because databases use the strict 2PL locking policy, T_b cannot read uncommitted data and therefore $C(T_a) \prec_i C(T_b)$.

2) *Version-order edge type I*: Since the commit order induces the version order, we have that $C(T_a) \prec_i C(T_b)$.

3) *Version-order edge type II*: Suppose, by way of contradiction, that $C(T_b) \prec_i C(T_a)$. When T_b requests the write locks to apply its updates at s_i , T_a can either be in the Executing state or in the Submitted state:

- *Executing state*: According to rule 1 on how locks for T_b are granted, if T_a is a read-only transaction, T_b waits for T_a to commit, contradicting the fact that $C(T_b) \prec_i C(T_a)$. If T_a is an update, T_a is aborted, a contradiction to the fact that T_a commits in H_i .
- *Submitted state*: There are two cases to consider, either (a) T_a and T_b are decided in the same consensus instance k or (b) T_b is decided in instance k and T_a is decided in instance k' ($k < k'$). In case (a), because $C(T_b) \prec_i C(T_a)$, T_b is in variable $LCOMMIT$ at line 38 when sites try to

⁴ \prec_i defines the commit order at site s_i

commit T_a . Because T_b writes x and T_a reads x , $T_b.ws \cap T_a.rs \neq \emptyset$ and therefore no site commits T_a after consensus instance k . By Lemma 1, only instance k decides on T_a and therefore T_a never commits, a contradiction.

In case (b), because T_a is in the Submitted state, T_a was submitted using the algorithms' submit primitive and therefore $T_a.past \leq T_b.order$. Because T_a commits, there exists a voting quorum VQ_1 for T_a such that $vote(VQ_1, T_a, T_a.order) = yes$. By Lemma 4, for all voting quorums $VQ \in VQS(T_a)$ such that $vote(VQ, T_a, T_a.order) \neq \perp$, $vote(VQ, T_a, T_a.order) = yes$. We now prove that there exists a voting quorum $VQ_2 \in VQS(T_a)$ such that $vote(VQ_2, T_a, T_a.order) = no$, contradicting Lemma 4. Because for all transactions T , there exists a correct voting quorum for T , there exists a correct voting quorum VQ_2 for T_a . By the definition of a voting quorum, there exists a correct site $s_j \in VQ_2$ such that $x \in Items(s_j)$ and therefore s_j commits T_b . Consequently, we have that $T_b \in COMMITTED_j^{T_b.order+1}$. Therefore, $certify(T_a, T_a.order)_j = no$, because $T_b.ws \cap T_a.rs \neq \emptyset$ and $T_a.past \leq T_b.order$. Thus, $vote(VQ_2, T_a, T_a.order) = no$. \square

Proposition 1 (Safety) *There exists a serial one-copy history $1H$ that is equivalent to H .*

Proof: From Lemma 5 and because databases use strict two-phase locking, for each s_i , there is a serial execution $1H_i$ equivalent to H_i , where no operation of different transactions are interleaved and operations of transactions follow the order of the commits. By the uniform agreement property of consensus, databases agree on the sequence of update transactions. By the uniform integrity of Reliable Broadcast, for any transaction T and any site s_i , T can only be R-Delivered once on s_i and thus T can only be added once to $UNDECIDED_i$. Therefore, no database site proposes a sequence SEQ to consensus such that a transaction T appears more than once in SEQ . Hence, by Lemma 1, for all transactions T , T can only be committed once on s_i . We thus construct $1H$ in the following way:

1. A read operation $r_i[x]$ of transaction T_i in H is mapped to the same operation $r_i[x]$ of T_i in $1H$. Write operations $w_i[x_A], w_i[x_B], \dots, w_i[x_N]$ of transaction T_i in H is mapped to a single write operation $w_i[x]$ of transaction T_i in $1H$.
2. The commit order of the updates in $1H$ follow the order defined by the consensus instances.
3. A read-only transaction T_q that commits just after the commit of an update transactions T_u on $site(T_q)$ in H , appears after T_u and before any update transaction $T_{u'}$ in $1H$.
4. In $1H$, for any two transactions T_i and T_j , their respective operations do not interleave.

We now show that $1H$ is view equivalent to H . For $1H$ to be view equivalent to H , two conditions have to be fulfilled [1]:

1. H and $1H$ have the same *read-x-from* relationships on data items: $\forall T_i, T_j \ T_j \text{ read-x-from } T_i \text{ in } H \iff T_j \text{ read-x-from } T_i \text{ in } 1H$.
2. For each final write $w_i[x]$ in $1H$, $w_i[x_A]$ is also a final write in H for some copy x_A of x .

H is view equivalent to $1H$:

1. (\Rightarrow) Let T_i, T_j be two transactions such that T_j *read-x-from* T_i in H . We prove that T_j *read-x-from* T_i in $1H$. There are two cases to consider, either (a) T_j is an update transaction or (b) T_j is a read-only transaction. In case (a), either (a-i) T_i and T_j are decided in the same consensus instance k or (a-ii) T_i and T_j are decided in consensus instance k and k' .

(a-i) By the fact that databases use the strict 2PL locking policy, T_j cannot read uncommitted data, and therefore $Site(T_j)$ cannot submit T_j before T_i commits. Hence T_i and T_j cannot be decided in the same consensus instance k .

(a-ii) In H , by Lemma 5 and the fact that databases use strict two-phase locking, there exists no transaction T_a that updates data item x that commits between instance k and k' , otherwise T_j would not *read-x-from* T_i in H . Therefore, by construction step 2 of $1H$, there exists no such transaction T_a in $1H$ and therefore by step 4 of $1H$, T_j *read-x-from* T_i in $1H$. In case (b), by using construction step 3 and 4 of $1H$, we conclude that T_j *read-x-from* T_i in $1H$.

(\Leftarrow) Let T_i, T_j be two transactions such that T_j *read-x-from* T_i in $1H$. We prove that T_j *read-x-from* T_i in H . If T_j *read-x-from* T_i in $1H$, by construction step 4, (*) there exists no transaction T_a that commits between T_i and T_j and updates data item x in $1H$. There are two cases to consider, either (a) T_j is an update transaction or (b) T_j is a read-only transaction. In case (a), either (a-i) T_i and T_j are decided in the same consensus instance k or (a-ii) T_i and T_j are decided in consensus instance k and k' .

(a-i) For the same reason as in (\Rightarrow), this case is impossible.

(a-ii) By (*) and construction step 2 of $1H$, there exists no consensus instance k'' , $k < k'' < k'$, such that T_a is decided in consensus instance k' . Therefore, by Lemma 5 and by the fact that databases use strict two-phase locking, T_j *read-x-from* T_i in H . In case (b), by using construction step 3, by Lemma 5 and by the fact that databases use strict two-phase locking, we conclude that T_j *read-x-from* T_i in H .

2. Clear from the fact that both histories contain the same update transactions and the fact that update transactions in $1H$ follow the order of update transactions in H . □

Lemma 6 *For any correct database site s_i , s_i does not wait forever at line 36.*

Proof: If s_i waits at line 36, s_i executed $Propose(K, -)$. By the termination property of consensus, all correct sites decide in instance K . Let SEQ be the decision of consensus instance K . Because there exists a correct voting quorum VQ for every transaction T and because channels are quasi-reliable, after all sites in VQ executed line 29, all correct sites eventually receive enough VOTE message for all $T' \in SEQ$ to constitute a voting quorum. Therefore, s_i eventually stop waiting at line 36. □

Lemma 7 *For any transaction T , if a correct database site s_i R-Delivers(VOTE_REQ, T), then there exists a k such that consensus instance k decides on T .*

Proof: Suppose, by way of contradiction, that there exists a transaction T such that a correct database site s_i R-Delivers(VOTE_REQ, T) and no consensus instance decides on T . If s_i R-Delivers(VOTE_REQ, T), by the agreement property of Reliable Broadcast and because s_i is correct, every correct database s_j eventually R-Delivers(VOTE_REQ, T). Since no consensus instance k decides on T , it is always true that $T.id \notin DECIDED_j$ and therefore s_j adds T to $UNDECIDED_j$ at line 24. By the termination

property of consensus, by Lemma 6, and because for all s_j , T is in $UNDECIDED_j$ forever, all s_j start infinitely many consensus instances at line 27. Let t_1 be the time after which all the faulty databases have crashed and let t_2 be the time at which the last correct site adds T to $UNDECIDED$. After t_1 and t_2 , all values proposed to the consensus instances contain T , therefore by the validity and termination property of consensus, there exists a consensus instance k that decides on SEQ such that $T \in SEQ$, a contradiction. \square

Proposition 2 (Liveness) *For any transaction T and any correct database site s_i , if s_i submit(T) then eventually all correct database sites $s_j \in Replicas(T)$ either commit T or s_i aborts T .*

Proof: Since s_i is correct, s_i R-Broadcasts(VOTE_REQ, T) and by the validity property of Reliable Broadcast, s_i eventually R-Delivers(VOTE_REQ, T). By Lemma 7, there exists a k such that consensus instance k decides on T . Therefore, by Lemma 6, all correct database sites $s_j \in Replicas(T)$ either commit T or s_i aborts T . \square

Proposition 3 (Quasi-Genuine Partial Replication) *For every submitted transaction T , correct database sites that do not replicate data items read or written by T permanently store not more than the identifier of T .*

Proof: From the algorithm, information other than the identifier of T is stored (a) in the $UNDECIDED$ sequence, (b) in the $COMMITTED$ set, (c) in the consensus instances, and (d) in the $VOTES$ set. Notice that in [2], the implementation of Reliable Broadcast only keeps the identifiers of messages.

(a) First, notice that because task 3 executes the last two *when blocks*, these *when blocks* are executed atomically. A correct site s_i can only add a transaction T to $UNDECIDED_i$ at line 24 if s_i R-Delivers(VOTE_REQ, T). By the integrity property of Reliable Broadcast, s_i can only R-Deliver(VOTE_REQ, T) once. If s_i adds T to $UNDECIDED_i$ at line 24, $T.id \notin DECIDED_i$. If $T.id \notin DECIDED_i$, s_i has not executed line 31 yet. By Lemma 7, eventually all correct database sites decide on T and s_i removes T from $UNDECIDED_i$ at line 32.

(b) A correct site s_i adds T to $COMMITTED_i$ only if $T'.ws \cap Items(s_i) \neq \emptyset$.

(c) We here only give an example of a consensus algorithm that satisfies this property. Consider the consensus algorithm given in [10]. It can be used in our termination protocol because it assumes the same system model, namely sites are crash-stop and channels are quasi-reliable. In this algorithm, after a correct site *delivers* and processes the *decide* message (by the termination property of consensus, this eventually happens), it stops taking part in the algorithm, and therefore all the variables of that consensus instance can be safely *garbage collected*.

(d) If for some correct site s_i $VOTES_i \neq \emptyset$, by the condition of line 25, s_i proposes a sequence of transactions to consensus. By the termination property of consensus, s_i eventually decides. By Lemma 6, s_i does not wait forever at line 36 and s_i eventually deletes all votes from the $VOTES$ set that concern the current and previous consensus instances. \square

Proposition 4 (Non-Trivial Certification) *If there is a time after which no two conflicting transactions are submitted, then eventually transactions are not aborted by certification.*

Proof: Let t_1 be the time after which no two conflicting transactions are submitted. Let $t_2 > t_1$ be the time after which the last transaction T submitted before t_1 commits. Let $t_3 > t_2$ be the time after which all transactions T' submitted after t_1 are such that $T.order < T'.past$. Because after t_1 , no two conflicting transactions are submitted, after t_3 we have that:

1. All call of function *Certify* return *yes* and the condition at line 37-38 always evaluates to true,
2. No transaction is aborted by rules 1 and 2 of Section 3 on how locks are granted.

Thus, after t_3 , no transaction is aborted by certification. □